

Granular local labor market multipliers. \*

## **JOB MARKET PAPER**

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### **Abstract**

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# 1 Introduction

Attracting new employers through tax breaks or subsidies is a common practice of local governments in both developed (Bartik, 2019) and developing countries (Duranton and Venables, 2021). Given the recent trend of firms growing by expanding to new regions Hsieh and Rossi-Hansberg (2023), the policymakers’ targets are mostly large, multi-location employers. When deciding on such policies, it is important to consider not only the jobs brought by the new employer but also the spillover effects, such as job losses or gains among incumbent firms. Moreover, a new source of labor demand might affect not only the number but also the quality of jobs at incumbent firms. Specifically, increased competition for workers may pressure other employers to raise wages.

Previous research has not provided an explicit answer on the gross effect of a new source of labor demand on the local labor market. The literature on local multipliers (Greenstone et al., 2010; Moretti, 2010; Bartik and Sotheland, 2019), which has focused mostly on the tradable sector, generally finds positive employment and wage-bill spillovers at the local economy level. Conversely, research on the effects of multi-location retailers has found ambiguous effects of large retailer entry (Basker, 2002; Neumark et al., 2008; Wiltshire, 2021) or no spillover effects from rapid changes in large retailers’ wage policies (Derenoncourt and Weil, 2024). The aim of this study is to provide a new, unifying specification that covers different sectors and types of employers. Unlike previous studies that relied on aggregate or selective employment/wage time series, I open the ‘black box’ of local employers’ responses to large external labor demand shocks using administrative Brazilian employer-employee data.

In this paper, I examine the spillovers from a common yet understudied source of labor demand shocks in local labor markets—multi-location (herein referred to as national) employers’ expansions. Specifically, I ask how local firms react to intense hiring activity and wage increases of national, typically well-paying, employers. Do national employers bid workers away from other firms or attract potential hires from other firms, causing local employers to increase wages? Are these effects stronger in certain types of firms (e.g., those

with closer ties to the national firm)? Do lower productivity firms, in particular, lose out when the national firm expands? Answering these questions allows us to understand how local employers adjust to new sources of labor demand, and therefore, estimate the gross effects of many place-based policies.

Analyzing the spillovers from national employer expansions, as with most labor demand shocks, requires handling endogeneity issues. For example, a national employer’s wage increase might simply be a reaction to a local productivity shock, leading to an overestimation of spillover size. To establish the conditions for causal identification of spillovers, I follow the structural approach of the outside option studies (Beaudry et al., 2012, 2018; Caldwell and Danieli, 2021). Specifically, I develop and derive the empirical specification from a simple search model of the labor market with continuous wage bargaining and convex vacancy costs, as in Lise and Robin (2017). The model provides testable implications for the size of positive wage spillovers and negative employment spillovers from national employers’ wage and vacancy shocks. Particularly, I show that local employers’ wage and employment changes can be decomposed into their own productivity changes, changes in local labor market productivity, and effects of national employers’ idiosyncratic wage and vacancy changes. Moreover, it is impossible to distinguish national employers’ shocks from local labor market productivity changes based solely on national employers’ wage changes in one location.

Therefore, the theoretical setting suggests that examining national employers’ entries—a seemingly natural way of investigating their expansions’ effects—would be biased unless entry is motivated purely by the national employer’s idiosyncratic labor demand changes. However, entries are difficult to model and instrument, as many regional characteristics might influence firms’ location choices, especially if the research is not limited to one specific company. Moreover, their impact might be concentrated in a relatively small area. Therefore, I propose a new empirical strategy that exploits the multi-location firms’ expansions, which are decided on the national level and focused on the locations where those firms are already present, providing a likely exogenous shock.

Specifically, to address potential confounding from unobserved changes in local labor demand, I have developed a portable quasi-experimental strategy based on recent research on national wage-setting (Hazell et al., 2021). Using rich Brazilian administrative employer-employee data, I track the policies of national employers across various locations and construct a measure of their idiosyncratic labor demand shocks. This measure is derived from relative changes in wages and employment of national employers in Brazil’s major cities. Such defined shocks allow me to develop a novel shift-share instrument to measure the exposure to idiosyncratic employment and wage shifts of national employers in labor markets beyond the largest urban centers.

Consistent with (Hazell et al., 2021) and (Schubert et al., 2021), my findings reveal that a 1% wage increase by national employers in major city regions (relative to other large-city employers) is accompanied by approximately a 0.5% wage increase in smaller labor markets within the same occupation. Similarly, a 1% relative increase in employment by national employers is matched by a 0.6% increase in employment in less populous locations. The strength of these co-movements confirms that the shift-share instrument effectively captures the idiosyncratic labor demand shocks of national employers.

Next, I proceed to the main research design, a matched event study for national employers’ expansions, defined as large increases in the constructed shift-share instrument. The analysis reveals that such expansions trigger an outflow of incumbent workers from local employers who cannot replace incumbent workers with new hires, leading to about a 2% employment decrease. Local employers subsequently raise wages for both remaining and newly hired employees. I observe positive, though slowly increasing, wage spillovers at the job and worker level, which rise to about 2.5% on the job level and about 1.7% on the worker level. Importantly, incumbent workers who remain with their employers also experience wage increases that converge to the same level as for all workers. This suggests that local firms adapt to more intense labor market competition not by selecting workers but by changing their wage policy.

The theoretical analysis also provides guidance on who, or what type of workers, would gain from national employers' expansions, highlighting the relationship between the probability of joining the national employer and the workers' wage increase. The event study results confirm this connection. Workers of local employers in the baseline period closer (in the same municipality) to the expanding national employer receive about 50% higher wage increases than workers in other municipalities but within the same commuting zone. Moreover, using the [Caldwell and Harmon \(2019\)](#) measure of workers' connections to national employers, I found that workers with past co-workers now working for national employers are 3 times more likely to join the national employer and experience 2 times higher wage increases than other workers.

Besides estimating the size of the spillovers, I also explore *how* national employers' expansions influence the local labor market. I show that in the model, their impact can be decomposed into two channels: (1) higher labor market tightness (more vacancies make workers more valuable) and (2) better outside options (higher quality jobs allow workers to bargain for higher wages). I later empirically estimate the size of the effects of each channel (HERE WILL BE DESCRIPTION OF THE RESULTS).

The last set of results measures the general spillovers of national employers' policies, not only restricted to the expansion periods. Using a standard instrumental variables design, I estimate spillover effects that are similar in direction to those observed in the event study but are quantitatively weaker.

My work contributes to the literature that estimates cross-employer spillovers. Seminal papers by [Beaudry et al. \(2012\)](#), [Caldwell and Danieli \(2021\)](#), and [Gathmann et al. \(2020\)](#) relied on sectoral variation (or sector and job characteristics variation), while I use individual employers' variation. In this dimension, my paper is similar to the recent work of [Bassier \(2021\)](#) and [Green et al. \(2022\)](#). Unlike these papers, I do not base my outside option measure on changes in union contracts but on the expansions of large employers. Lastly, this paper is related to studies of outside option shocks on the individual worker level ([Caldwell and](#)

Harmon, 2019; Lachowska et al., 2022; Urena et al., 2021), which aim to identify workers' renegotiation. I view my paper as complementary; whereas these studies focus on individual worker wage renegotiation, I document the outside-option effects of the total local labor market shocks.

I estimate a different margin of adjustment than the related paper by Derenoncourt and Weil (2024). While they study large employer wage changes that lead to decreased separations and lower hiring, I estimate the effects of a more standard labor demand shock with large employers increasing wages but even more increasing their employment level and hiring intensity. Moreover, I focus more on labor market exposure to different national employers than on specific firms' policies.

My work is also related to the local multipliers literature (Bartik, 1991; Moretti, 2010; Bartik and Sotherland, 2019; Bartik, 2019, 2021). Unlike these papers, I focus on a single labor market defined by occupation and commuting zone, rather than on the whole local economy (defined as state, commuting zone, or county). My approach allows for precise identification of wage spillovers, which were typically ignored by this literature. Conversely, I do not account for possible product market spillovers, which likely extend beyond the occupation-defined labor market (Dhyne et al., 2022). I leave this research direction for future work.

Lastly, in constructing large firm expansion shocks, I build on research that discusses the multi-location employers' decisions. Particularly, the related work of Hazell et al. (2021) defines the concept of national employers: firms that plan their wages on the country level, making their wage decisions likely exogenous to the labor market conditions in a particular location. Similar to Hazell et al. (2021), I found that wages and employment of multi-location firms are strongly correlated across locations. Therefore, wage and employment shocks observed in large cities (with the most flexible labor market) are likely to represent the firms' idiosyncratic labor demand shocks. A similar approach was also used by Schubert et al. (2021).

## 2 Theoretical model

This section develops a simple model of a search and continuous bargaining labor market. The model combines the wage-setting mechanism from [Beaudry et al. \(2012\)](#) with employment [Lise and Robin \(2017\)](#) model of convex vacancies costs. The aim of this section is threefold. First, I derive the estimating equation for the wage and employment spillovers resulting from changes in national employers' wages and vacancies. This helps clarify the identification issues and distinguish between the effects of a labor market productivity shock and the spillovers from national employer expansions.

Secondly, the model allows me to study *how* national employer expansions impact local employers' policies. For wage spillovers, I show that two channels can be identified: the tightness channel (where more vacancies lead to higher market tightness) and the outside option channel (where better outside options enhance workers' negotiating positions). I further demonstrate that local labor market tightness is affected first by local employers wage increases and, secondly, by changes national employers vacancies. The decomposition of spillovers not only provides insight into the key labor market mechanisms but also allows for the development of policy guidelines. Should local policymakers focus on attracting more employers (thus increasing market tightness), or should they prioritize attracting a smaller number of high-paying jobs (or those providing other valuable amenities)? The model provides a micro-founded framework to estimate local market responses to each of these channels and to answer such questions.

Finally, the model provides testable predictions on *who* gains from the expansion. Specifically, if wages are set through a bargaining process, the workers with the highest probability of joining national employers are those who should gain the most through this process.

Firstly, I present the basic version of the model with a set number of employers and no match effects. In subsection [2.3](#) I describe the structure of the steady state. Subsection [2.4](#) derives the effects of the national employers wage and vacancies changes, while subsection [2.5](#) describes the identification issues for two main research designs. In the section [2.7](#).

I extend the model to introduce the employers' entry choice and selection through match effects.

## 2.1 Model Setup

In the labor market for occupation  $o$  and commuting zone  $m$  in period  $t$ , there are  $n - 1$  local employers (denoted by subscript  $j$ ) and one national employer<sup>1</sup> (denoted by  $n$ ). Measure  $L_{o,m,t}$  of workers is either employed and receive the wage or is unemployed and receive unemployment benefit  $b_{o,m,t}$ . Matches get destroyed with exogenous probability  $\delta$ . Time is discrete, and both employers and workers discount the future with the rate  $\rho$ .

Local employers vary by productivity  $\epsilon_{j,o,m,t}$ . When employer  $j$  vacancy is filled, the match produces  $y_{o,t} + \epsilon_{j,o,m,t}$ , where  $y_{o,t}$  is the average output level in the occupation  $o$ . Local firms inherit from the previous period  $(1 - \delta)l_{o,m,j,t-1}$  matches and post  $V_{j,o,m,t}$  new vacancies. The national employer posts wage  $w_{n,o,m,t}$ , inherits  $(1 - \delta)l_{n,o,m,t-1}$  matches and post vacancies  $V_{n,o,m,t}$ . Producing the vacancies generates the cost given by the convex function  $c(V)$ .

The unemployed workers are matched with vacancies via the matching technology with constant returns to scale, defined by the function  $M(u_{o,m,t}L_{o,m}, \sum_{k=1}^n V_{k,o,m,t})$ , where  $u_{o,m,t}$  is the unemployment rate. Under the standard assumption, the probability of matching is defined by the market tightness  $\theta_{o,m,t} = \frac{\sum_{j=1}^n V_{j,o,m,t}}{u_{o,m,t}L_{o,m}}$ : the probability that unemployed worker finds jobs  $p(\theta)_{o,m,t} = \theta_{o,m,t}^\nu$ , while the probability that vacancy is filled  $q(\theta)_{o,m,t} = \theta_{o,m,t}^{\nu-1}$ . Conditional on getting matched, the probability that workers is matched with concrete local employer  $j$  or national employer  $n$  depends on their vacancy share  $\gamma_{j,o,m,t} = \frac{V_{j,o,m,t}}{\sum_{k=1}^n V_{k,o,m,t}}$ .

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<sup>1</sup>I define just one national employer for simplicity; this analysis can be generalized to  $K_n \geq 1$  national employers.



## 2.2 Local employers vacancies and wage determination

Here, I describe the value functions. I assume  $n - 1$  is large, so employers take  $\theta_{o,m,t}$  as given.

The value of a match  $Y_{j,o,m,t}^f$  for the local employer is:

$$Y_{j,o,m,t}^f = y_{o,t} + \epsilon_{j,o,m,t} - w_{j,o,m,t} + \rho(1 - \delta)Y_{j,o,m,t+1}^f \quad (1)$$

Local employers set vacancies  $V_{j,o,m}$  to maximize profits by solving:

$$\max_{V_{j,o,m}} Y_{j,o,m,t}^f ((1 - \delta)l_{j,o,m,t-1} + q(\theta_{o,m,t})V_{j,o,m,t}) - c(V_{j,o,m,t}) \quad (2)$$

For a worker employed by local employer  $j$ , the value of a match  $W_{j,o,m,t}$  depends on the wage received  $w_{j,o,m,t}$  and the value of unemployment  $U_{o,m,t+1}$ :

$$W_{j,o,m,t} = w_{j,o,m,t} + \rho(\delta U_{o,m,t+1} + (1 - \delta)W_{j,o,m,t+1}) \quad (3)$$

Finally, the value of unemployment depends on the benefit  $b_{o,m}$ , the probability of being matched with an employer  $p(\theta_{o,m,t+1})$ , and the expected value of the job provided by an employer  $\mathbf{E}W_{k,o,m,t}$ :

$$U_{o,m,t} = b_{o,m} + \rho(p(\theta_{o,m,t})\mathbf{E}W + (1 - p(\theta_{o,m,t}))U'_{o,m,t}) \quad (4)$$

$$\mathbf{E}W_{k,o,m,t} = \sum_{j=1}^{n-1} \gamma_{j,o,m,t} W_{j,o,m,t} + \gamma_{n,o,m,t} W_{n,o,m,t} \quad (5)$$

Local employers' wages are set via a continuous bargaining process after matching takes place. The bargaining process splits the surplus between the firm's value of a filled vacancy (over producing nothing) and the worker's value of being employed by  $j$  over being unemployed:

$$\kappa Y_{j,o,m,t}^f = (1 - \kappa) (W_{j,o,m,t} - U_{o,m,t}) \quad (6)$$

## 2.3 Steady state equations

The steady-state equilibrium is represented by a system of equations for each local firm's wages and employment, taking the national employers' wages and vacancy postings as given. To simplify notation, I drop the time subscript for steady-state equations.

For simplicity, I assume that the vacancy creation cost function is given by  $c(V) = cV_{j,o,m}^2$  and  $L_{o,m} = 1$ . Moreover, I define  $\epsilon_{j,o,m}$  as the sum of labor-market productivity  $\epsilon_{o,m}$  and idiosyncratic employer productivity  $\varepsilon_{j,o,m}$ . Combining equations 1–6, I obtain a system of equations<sup>2</sup>:

$$w_{j,o,m} = A_1 y_o + A_2 (\theta_{o,m}) b_{o,m} + A_3 (\theta_{o,m}) \left( \gamma_{n,o,m} w_{n,o,m} + \sum \gamma_{j,o,m} w_{j,o,m} \right) \quad (7)$$

$$+ A_1 (\epsilon_{o,m} + \varepsilon_{j,o,m}) \quad (8)$$

$$\delta l_{j,o,m} = V_{j,o,m} = \tilde{\rho}_1 \frac{q(\theta_{o,m})}{c} (y_o + \epsilon_{o,m} + \varepsilon_{j,o,m} - w_{j,o,m}) \quad (9)$$

$$\theta_{o,m} = \frac{V_{n,o,m} + \sum V_{j,o,m}}{u_{j,o,m}} \quad (10)$$

$$u_{j,o,m} = \frac{\delta}{\delta + p(\theta_{o,m})} \quad (11)$$

$$\gamma_{j,o,m} = \frac{V_{j,o,m}}{V_{n,o,m} + \sum V_{j,o,m}} \quad (12)$$

Where  $V_{o,m} = \sum V_{j,o,m} V_{n,o,m} + \sum V_{j,o,m}$ .

Therefore, the local employers wage is define by productivity  $y_o + \epsilon_{o,m} + \varepsilon_{j,o,m}$ , tightness  $\theta_{o,m}$  and outside option  $(\gamma_{n,o,m} w_{n,o,m} + \sum \gamma_{j,o,m} w_{j,o,m})$ , while the local employer's employment is defined by its own wage and productivity. Hence the potential impact of national employers is through alternating 1) outside option 2) tightness.

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<sup>2</sup>Where:  $\tilde{\rho}_1 = \frac{1}{1-\rho(1-\delta)}$ ,  $\tilde{\rho}_2(\theta_{o,m}) = \frac{1}{1-\rho(1-p(\theta_{o,m}))}$ ,  $A_1 = \kappa \tilde{\rho}_1$ ,  $A_2 = (1 - \kappa)(1 - \rho \tilde{\rho}_1) \frac{\tilde{\rho}_2(\theta_{o,m})}{1-\rho^2 \delta p(\theta_{o,m}) \tilde{\rho}_1 \tilde{\rho}_2(\theta_{o,m})}$  and  $A_3 = \rho p(\theta_{o,m}) \tilde{\rho}_1 A_2$

## 2.4 Effects of national employers' shock

This section describes how national employers' wage and vacancy-level changes affect local employers' steady-state wage and employment policies. In the ideal research design, the first steady state is the control (counterfactual) state of the market if the treatment (national employers policy change) won't happen. I derive the first-order equations for the general case. In section 2.5, I discuss its identification.

I define the  $\Delta x$  operator as the difference in  $x$  values between two steady states. For example, let  $x$  be at time  $t_1$  in steady state 1 and at time  $t_2$  in steady state 2. Then,  $\Delta x = x_{t_2} - x_{t_1}$ .

### 2.4.1 National employers wage and vacancies changes

I assume that national employers determine wages through an unknown process, which depends on both the local productivity shock,  $\epsilon_{o,m,t}$ , and their idiosyncratic wage and vacancies level,  $\Omega_{n,o,m,t}^w, \Omega_{n,o,m,t}^V$ , which are positively correlated. Therefore, I can decompose the national employers' wage and vacancies changes:

$$\Delta w_{n,o,m} = \Delta \Omega_{n,o,m}^w + \psi^w \epsilon_{o,m} \quad (13)$$

$$\frac{\Delta V_{n,o,m}}{V_{n,o,m}} = \Delta \Omega_{n,o,m}^E + \psi^E \epsilon_{o,m} \quad (14)$$

$$(15)$$

Consequently, the national employer wage and vacancy changes are a sum of exogenous idiosyncratic shocks and local labor market shock.

### 2.4.2 Linearization across steady states

Non-linear system 7 -12 shows full response of local employers to  $\Delta w_{n,o,m}$ ,  $\Delta V_{n,o,m}$ . To capture the first-order effects and receive an estimable equations, I take linear approximation 7 -12, for  $\Delta w_{n,o,m}$ ,  $\Delta V_{n,o,m}$  and local labor market shocks  $\Delta \epsilon_{o,m}$  and idiosyncratic employer

shock  $\Delta\varepsilon_{j,o,m}$ .

In the appendix, I show that a first-order approximation for local employers wage and employment changes, is defined by the following system:

$$\Delta w_{j,o,m} = a_1 \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + a_2 \gamma_{n,o,m} \Delta w_{n,o,m} + a_3 \Delta \epsilon_{o,m} + a_4 \Delta \varepsilon_{j,o,m} \quad (16)$$

$$\Delta l_{j,o,m} = b_1 \Delta \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + b_2 \gamma_{n,o,m} \Delta w_{n,o,m} + b_3 \Delta \epsilon_{o,m} + b_4 \Delta \varepsilon_{j,o,m} \quad (17)$$

## 2.5 Spillover identification

Based on the equations 16, 17, the simple OLS cannot identify  $a_1, a_2, b_1, b_2$ . Consider the specification when econometrician observes only  $\Delta w_{n,o,m}$  and  $\Delta V_{n,o,m}$ . In such a case, the estimating equations are:

$$\Delta w_{j,o,m} = \zeta_1^1 \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \zeta_2^1 \gamma_{n,o,m} \Delta w_{n,o,m} + u_{1,j,o,m} \quad (18)$$

$$\Delta l_{j,o,m} = \zeta_1^2 \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \zeta_2^2 \gamma_{n,o,m} \Delta w_{n,o,m} + u_{2,j,o,m} \quad (19)$$

Then the estimating Equations 18 and 19 represents:

$$\begin{aligned} \Delta w_{j,o,m} &= \underbrace{a_1}_{\zeta_1^1} (\gamma_{n,o,m} w_{n,o,m}) \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \underbrace{a_1}_{\zeta_2^1} \gamma_{n,o,m} \Delta w_{n,o,m} + \underbrace{a_3 \Delta \epsilon_{o,m} + a_4 \Delta \nu_{j,o,m}}_{u_{1,j,o,m}} \\ \Delta l_{j,o,m} &= \underbrace{b_1}_{\zeta_1^2} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \underbrace{a_1}_{\zeta_2^2} \gamma_{n,o,m} \Delta w_{n,o,m} + \underbrace{b_3 \Delta \epsilon_{o,m} + b_4 \Delta \nu_{j,o,m}}_{u_{2,j,o,m}} \end{aligned}$$

The defined equations do not satisfy the exogeneity constraints, as  $\mathbf{E} \zeta_i^k \Delta \epsilon_{o,m} \propto \text{Var}(\Delta \epsilon_{o,m}) \neq 0$  for  $i, k = 1, 2$ . To address this issue, I propose a novel shift-share measure, based on the results elaborated in Section ?? and related to the recent literature on national wage-setters Hazell et al. (2021). This strategy primarily isolates the exogenous variation in  $\Delta \Omega_{n,o,m}^w$  and  $\Delta \Omega_{n,o,m}^E$ , allowing for the identification of key parameters.

In this section, I describe the estimating equation in two cases relevant to this study's research designs. In the first case, I consider when the exogenous event of a national firm "expansion" is identified. In the second case, I consider when the exogenous variation of  $\Delta\Omega_{n,o,m}^E$  and  $\Delta\Omega_{n,o,m}^W$  is known, allowing for the isolation of different channels through which national employers' wage and vacancy changes affect local employers' policies.

For both designs, the starting point is to rewrite 16 and 17, isolating  $\Delta\Omega_{n,o,m}^E$  and  $\Delta\Omega_{n,o,m}^W$  from  $\psi^w\epsilon_{o,m}$  and  $\psi^E\epsilon_{o,m}$ .

$$\Delta w_{j,o,m} = a_1(\gamma_{n,o,m}w_{n,o,m})\Delta\Omega_{n,o,m}^E + a_2\gamma_{n,o,m}\Delta\Omega_{n,o,m}^W \quad (20)$$

$$+ a'_3\Delta\epsilon_{o,m} + a_4\Delta\epsilon_{j,o,m}$$

$$\Delta l_{j,o,m} = (b_1 + b_2a_1\gamma_{n,o,m}w_{n,o,m})\Delta\Omega_{n,o,m}^E + b_2a_2\gamma_{n,o,m}\Delta\Omega_{n,o,m}^W \quad (21)$$

$$+ b'_3\Delta\epsilon_{o,m} + b_4\Delta\epsilon_{j,o,m}$$

### 2.5.1 National Employers Expansion Event

Firstly, consider the case when the national firm's idiosyncratic expansion event is identified. That is, the Bernoulli variable  $\mathbf{1}_{EXP}$  is such that  $\mathbf{E}[\mathbf{1}_{EXP}\epsilon_{o,m}] = 0$  and  $\forall_j \mathbf{E}[\mathbf{1}_{EXP}\epsilon_{j,o,m}] = 0$ , while  $\mathbf{E}[\gamma_{n,o,m}\Delta\Omega_{n,o,m}^W|\mathbf{1}_{EXP}] = \gamma_{n,o,m}\Delta\bar{\Omega}^W$  and  $\mathbf{E}[\gamma_{n,o,m}\Delta\Omega_{n,o,m}^E|\mathbf{1}_{EXP}] = \gamma_{n,o,m}\Delta\bar{\Omega}^E$ .

Therefore, the regression equation for estimation is:

$$\Delta w_{j,o,m} = \zeta_1^{EXP}\mathbf{1}_{EXP} + u_{1,j,o,m}^{EXP}, \quad (22)$$

$$\Delta l_{j,o,m} = \zeta_2^{EXP}\mathbf{1}_{EXP} + u_{2,j,o,m}^{EXP}. \quad (23)$$

From 20 and 21, I can identify the following<sup>3</sup>:

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<sup>3</sup>In the empirical specification I consider the logs variables. It is not that important here: I will discuss it slightly more in the research design for event study. It is more important for continuous design and I discuss it carefully in the next subsection.

$$\mathbf{E}[\zeta_1^{EXP}] = \mathbf{E}[a_1\gamma_{n,o,m}\Delta\bar{\Omega}^E + a_2\gamma_{n,o,m}\Delta\bar{\Omega}^W], \quad (24)$$

$$\mathbf{E}[\zeta_2^{EXP}] = \mathbf{E}[b_1\gamma_{n,o,m}\Delta\bar{\Omega}^W + b_2\gamma_{n,o,m}\Delta\bar{\Omega}^E]. \quad (25)$$

Where  $\mathbf{E}[a_1\gamma_{n,o,m}\Delta\bar{\Omega}^E + a_2\gamma_{n,o,m}\Delta\bar{\Omega}^W]$  is LATE for wages for each pair of firms from control and treated market. (I will expand this part).

Consequently, the event study analysis allows us to identify the joint effect of idiosyncratic changes in  $\gamma_{n,o,m}\Delta\Omega_{n,o,m}^W$  and  $\gamma_{n,o,m}\Delta\Omega_{n,o,m}^E$  but does not enable identification of the separate effects of each channel—specifically, how much the total effect depends on changes in workers’ outside options versus its influence on tightness (or national vacancies).

Nevertheless, the event study design allows for a more comprehensive test of inference and identification assumptions, such as the independence of the expansion event from local labor market conditions. Hence, it is used as the main research design.

### 2.5.2 Wage Channels Decomposition

Lastly, I consider the case where the variation in  $\gamma_{n,o,m}\Delta\Omega_{n,o,m}^W$  and  $\gamma_{n,o,m}\Delta\Omega_{n,o,m}^E$  can be identified, which provides identification for their coefficients in 20 and 21.

However, 20 and 21 are not informative about how national employers affect local employers’ wage policies. As described in Section 2.3, national employers impact local wage policies by altering workers’ outside options (i.e., changing the wage available in an alternative job) and increasing market tightness (i.e., increasing the number of active vacancies). Therefore, I transform the original system to obtain the equation 26 which enables to identify the strength of each channel. In the appendix, Section A, I demonstrate that this equation is equivalent to 20. Equation 26 is in log-linear form, with redefined  $\Delta\gamma_{n,o,m}\tilde{\Omega}_{n,o,m}^W, \gamma_{n,o,m}\Delta\tilde{\Omega}_{n,o,m}^E$ , which denotes respectively the idiosyncratic change in national employer log-wage and log-employment:

$$\begin{aligned}
\Delta \ln w_{j,o,m} = & \underbrace{\tilde{a}_1 \frac{\bar{w}_{o,m}}{w_{j,o,m}} \gamma_{n,o,m} \Delta \tilde{\Omega}_{n,o,m}^E}_{\text{Tightness channel}} + \underbrace{\tilde{a}_2 \frac{w_{n,o,m}}{w_{j,o,m}} \gamma_{n,o,m} \left( \Delta \tilde{\Omega}_{n,o,m}^E + \Delta \tilde{\Omega}_{n,o,m}^W \right)}_{\text{Outside option channel}} \\
& + \tilde{a}_3 \Delta \epsilon_{o,m} + \tilde{a}_4 \Delta \varepsilon_{j,o,m}
\end{aligned} \tag{26}$$

The first component: "tightness" channel show the effect on the  $\Delta \ln w_{j,o,m}$ , effect of increasing number of jobs, but not changing the average wage. The second part pins down the effect of outside option: effect of the higher expected wage conditional of unemployed being matched with vacancy.

In Section 4, I will decompose the estimates into these two channels. This will help us understand the relative importance of each channel, highlighting the different effects of the quality (wage changes) and quantity (vacancy increases) components of the labor demand shock.

Again I need to expand it, but here LATE is again the integral over distribution of labor markets. I dont think I can assume the same national employers share as in Beaudry et al. (2012)

## 2.6 Outside Option Heterogeneity

In the previous section, I assumed that all workers have the same probability of joining the national employer, which is proportional to its employment share  $\gamma_{n,o,m}$ . However, some workers are more likely to get hired by national employer. For example, Le Barbanchon et al. (2020) showed the importance of distance to the workplace for workers' search intensity. Alternatively, Caldwell and Harmon (2019) demonstrated larger wage increases for workers connected to expanding firms through their past coworkers.

A simple way to analyze such heterogeneity in the model is to assume that, for some group of workers  $g$ , the probability of joining the national employer (given being matched),

$\gamma_{n,o,m}^g$ , is greater than the national employer's employment share  $\gamma_{n,o,m}$ . In this case:

$$\begin{aligned} \Delta \ln w_{j,o,m}^g = & \tilde{a}_1 \frac{\bar{w}_{o,m}^g}{w_{j,o,m}^g} \gamma_{n,o,m}^g \Delta \tilde{\Omega}_{n,o,m}^E + \tilde{a}_2 \frac{w_{n,o,m}}{w_{j,o,m}} \gamma_{n,o,m}^g \left( \Delta \tilde{\Omega}_{n,o,m}^E + \Delta \tilde{\Omega}_{n,o,m}^W \right) \\ & + \tilde{a}_3 \Delta \epsilon_{o,m} + \tilde{a}_4 \Delta \epsilon_{j,o,m} \end{aligned} \quad (27)$$

Therefore, the testable model prediction is that the group more likely to be employed by the national employer will receive higher wage increases and will be less likely to be employed by local employers.

## 2.7 Extension

I don't know if I will have time for this part, at least in a short time.

## 3 Identification

Here I will add revised identification section.

## 4 Model estimation

Here I will add the results of estimation of equation 26.

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## A Linear approximation around steady states

To compute the first order approximation of difference between two steady states. In the first national employers post  $V_{n,o,m}$  vacancies and offered wage  $w_{n,o,m}$ ,  $\epsilon_{o,m} = 0$ , while each  $j$  has  $\varepsilon_{j,o,m}$ . In the second, they post  $V_{n,o,m} + \Delta V_{n,o,m}$  vacancies for the wage  $w_{n,o,m} + \Delta w_{n,o,m}$  and there were drawn  $\Delta \epsilon_{o,m}$ ,  $\Delta \varepsilon_{j,o,m}$ . For convenience, I denote  $V_{o,m} = V_{n,o,m} + \sum V_{j,o,m}$  and average wage in the market  $\bar{w}_{o,m} = \gamma_{n,o,m} w_{n,o,m} + \sum \gamma_{j,o,m} w_{j,o,m}$ .

Idiosyncratic local employers shocks are zero-sum. Therefore, it is convenient to focus on the  $\Delta \bar{w}_{j,o,m}$ ,  $\Delta \bar{l}_{j,o,m}$  which an average wage/employment change. Moreover,  $\Delta w_{j,o,m} = \Delta \bar{w}_{j,o,m} + A_1 \Delta \varepsilon_{j,o,m}$  and  $\Delta V_{j,o,m} = \Delta \bar{V}_{j,o,m} + A_1 \Delta \varepsilon_{j,o,m}$ .

Equations the first order approximation of differences between two steady states, from 7 -12

$$\Delta \bar{w}_{j,o,m} = \frac{1}{1 - \gamma_{n,o,m}} (A'_2(\theta) b + A'_3(\theta) \bar{w}_{o,m}) \Delta \theta_{o,m} \quad (28)$$

$$\begin{aligned} & + \frac{A_3(\theta)}{1 - \gamma_{n,o,m}} (\theta) [\Delta \gamma_{n,o,m} w_{n,o,m} + \gamma_{n,o,m} \Delta w_{n,o,m}] + \frac{A_1}{1 - \gamma_{n,o,m}} \Delta \epsilon_{o,m} \\ \Delta \bar{V}_{j,o,m} = & - \frac{q(\theta_{o,m})}{c} \Delta w_{j,o,m} + \frac{q'(\theta_{o,m})}{c} \Delta \theta_{o,m} + \frac{q(\theta_{o,m})}{c} \Delta \epsilon_{o,m} \end{aligned} \quad (29)$$

$$\Delta \gamma_{j,o,m} = \frac{1}{V_{o,m}} \Delta V_{j,o,m} (1 - \gamma_{j,o,m}) + (1 - \gamma_{j,o,m}) \frac{n-1}{V_{o,m}} \Delta \bar{V}_{j,o,m} \quad (30)$$

$$\Delta \theta_{o,m} = \frac{V_{o,m}}{\delta} p'(\theta_{o,m}) \Delta \theta_{o,m} + \frac{\delta + p(\theta_{o,m})}{\delta} \left[ \Delta V_{n,o,m} + \sum_j \Delta V_{j,o,m} \right] \quad (31)$$

Transforming equations:

$$\begin{aligned}
\Delta\theta &= \frac{\delta + p(\theta)}{p'(\theta)} \left[ \frac{\Delta V_n}{V_{o,m}} + (n-1) \frac{\Delta V_j}{V_{o,m}} \right] \\
\Delta\gamma_{n,o,m} &= (1 - \gamma_{n,o,m}) \frac{\Delta V_{n,o,m}}{V_{o,m}} + \gamma_{n,o,m} (n-1) \frac{\Delta V_j}{V_{o,m}} \\
\frac{\Delta V_j}{V_{o,m}} &= -\tilde{\rho} \frac{q(\theta_{o,m})}{c\theta_{o,m}} (\Delta w_{j,o,m} - \Delta\epsilon_{o,m}) + \tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta)} \left[ \frac{\Delta V_n}{V_{o,m}} + (n-1) \frac{\Delta V_j}{V_{o,m}} \right] \\
\Leftrightarrow \frac{\Delta V_j}{V_{o,m}} &= \frac{\tilde{\rho}}{c\theta} \left( \frac{-q(\theta_{o,m})(\Delta w_{j,o,m} - \Delta\epsilon_{o,m}) + \frac{q'(\theta)\delta}{p'(\theta)} \frac{\Delta V_n}{V_{o,m}}}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta)}} \right)
\end{aligned}$$

Thus:

$$\begin{aligned}
\Delta\theta &= \left[ \frac{\delta + p(\theta_{o,m})}{p'(\theta_{o,m})} + \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{(n-1) \frac{q'(\theta_{o,m})\delta}{p'(\theta_{o,m})}}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right] \frac{\Delta V_n}{V_{o,m}} \\
&\quad + \left[ \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{-(n-1)q(\theta_{o,m})}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right] (\Delta w_{j,o,m} - \Delta\epsilon_{o,m}) \\
\Delta\gamma_{n,o,m} &= \left[ (1 - \gamma_{n,o,m}) + \gamma_{n,o,m} \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{(n-1) \frac{q'(\theta_{o,m})\delta}{p'(\theta_{o,m})}}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right] \frac{\Delta V_n}{V_{o,m}} \\
&\quad + \gamma_{n,o,m} \left[ \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{-(n-1)q(\theta_{o,m})}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right] (\Delta w_{j,o,m} - \Delta\epsilon_{o,m})
\end{aligned}$$

I can denote the following constants<sup>4</sup>:

$$\begin{aligned}
C_1 &= \frac{\delta + p(\theta_{o,m})}{p'(\theta_{o,m})} + \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{(n-1) \frac{q'(\theta_{o,m})\delta}{p'(\theta_{o,m})}}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} > 0 \\
C_2 &= \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{-(n-1)q(\theta_{o,m})}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} < 0 \\
C_3 &= \gamma_{n,o,m} \left[ \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{-(n-1)q(\theta_{o,m})}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right] < 0
\end{aligned}$$

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<sup>4</sup>As  $q'(\theta)$  is negative

Since  $\gamma_{n,o,m} \frac{\tilde{\rho}}{c\theta_{o,m}} \frac{(n-1) \frac{q'(\theta_{o,m})\delta}{p'(\theta_{o,m})}}{1-(n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \frac{\Delta V_n}{V_{o,m}}$  is quadratic in  $\gamma_{n,o,m}$ , I skip this term. Therefore, the wage change can be written as:

$$\Delta \bar{w}_{j,o,m} = \alpha_1 \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \alpha_2 (w_{n,o,m}(1 - \gamma_{n,o,m}) \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \gamma_{n,o,m} \Delta w_{n,o,m}) + \alpha_3 \epsilon_{o,m} \quad (32)$$

$$\Delta \bar{V}_{n,o,m} = (\beta_1 + \beta_2 \alpha_1) \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \beta_2 (w_{n,o,m}(1 - \gamma_{n,o,m}) \gamma_{n,o,m} \frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \gamma_{n,o,m} \Delta w_{n,o,m}) + \beta_3 \epsilon_{o,m} \quad (33)$$

Where:

$$\begin{aligned} \alpha_1 &= \frac{(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_1}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3} \\ \alpha_2 &= \frac{1}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3} \\ \beta_1 &= \frac{\tilde{\rho}}{c\theta_{o,m}} \left( \frac{\frac{q'(\theta_{o,m})\delta}{p'(\theta_{o,m})}}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right) \\ \beta_2 &= \frac{\tilde{\rho}}{c\theta_{o,m}} \left( -\frac{q(\theta_{o,m})}{1 - (n-1)\tilde{\rho} \frac{q'(\theta_{o,m})}{c\theta_{o,m}} \frac{\delta}{p'(\theta_{o,m})}} \right) \end{aligned}$$

Finally, adding  $\varepsilon_{j,o,m}$ , and using a steady state relationship between vacancies and total employment, I receive equations 16–17.

## A.1 Channel decomposition

This section derives the equation 26. For tractability, I assume  $b_{o,m} = bw_{o,m}$ . Dividing 32 by  $w_{j,o,m}$ , I can decompose the  $\frac{\alpha_1}{w_{j,o,m}}$  and taking the log approximation:

$$\underbrace{\frac{(A'_2(\theta_{o,m})b + A'_3(\theta_{o,m}))C_1}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3}}_{\text{market level}} \times \underbrace{\frac{w_{o,m}}{w_{j,o,m}}}_{\text{job to average ratio}} \gamma_{n,o,m} \Delta \ln V_{n,o,m}$$

Similarly  $\frac{\alpha_2}{w_{j,o,m}}(w_{n,o,m}(1 - \gamma_{n,o,m})\gamma_{n,o,m}\frac{\Delta V_{n,o,m}}{V_{n,o,m}} + \gamma_{n,o,m}\Delta w_{n,o,m})$  and taking the log approximation:

$$\begin{aligned} & \underbrace{\frac{1 - \gamma_{n,o,m}}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3}}_{\text{market level}} \\ & \times \underbrace{\frac{w_{n,o,m}}{w_{j,o,m}}}_{\text{job to national emp. ratio}} \times \underbrace{\gamma_{n,o,m}(\Delta \ln V_{n,o,m} + \Delta \ln w_{n,o,m})}_{\text{national employers change}} \end{aligned}$$

Therefore:

$$\begin{aligned} \tilde{a}_1 &= \frac{(A'_2(\theta_{o,m})b + A'_3(\theta_{o,m}))C_1}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3} \\ \tilde{a}_2 &= \frac{1 - \gamma_{n,o,m}}{1 - \frac{1}{1-\gamma_{n,o,m}}(A'_2(\theta_{o,m})b_{o,m} + A'_3(\theta_{o,m})\bar{w}_{o,m})C_2 + \frac{1}{1-\gamma_{n,o,m}}C_3} \end{aligned}$$

Then, using the steps same as in 2.5 for  $\tilde{\Omega}$ , I obtain 26.