Quantum Scattering Project: Using the Lippmann–Schwinger equation for scattering off a non-local potential

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1 Introduction

In high energy physics and the study of quantum mechanical phenomena, particle properties and composition, scattering is an important event to study and understand. This is because the scattering amplitude is something that can be directly measure. It was due to the Rutherford gold foil experiment that an early model of the atom was formed, this model was built on the fact that alpha particles would scatter via Coulombic forces. When shot through a piece of gold foil there was many angles that could only be justified by the idea of a nucleus and the particle participating in an elastic scattering event. Clearly scattering is integral to our understanding of the universe.

Rather than dealing with the wave equation itself, instead the reaction amplitude is solved using the Lippmann–Schwinger equation. The Lippmann–Schwinger equation is an integral form of the Schrodinger equation and constructs a reaction amplitude matrix, the R matrix, on which the diagonal represents an observable in momentum space.

$$R(k',k) = V(k',k) + \frac{2}{\pi}\rho \int_0^\infty dp \frac{(p^2 V(k',p) R(p,k)}{(k_O^2 - p^2)/2\mu}$$

This program employs a method of breaking down the Lippmann–Schwinger integral equation into a system of linear equations to be solved using a matrix.

$$R = V_i + \frac{2}{\pi} \sum_{j=1}^{N} \frac{k_j^2 V_i j R_j w_j}{(k_o^2 - k_j^2)/2\mu} - \frac{2}{\pi} V_i o R_o \sum_{m=1}^{N} \frac{w_m}{(k_o^2 - k_m^2)/2\mu}$$

Where V is velocity, k_o is observable momentum, k_j is quadrature points, w is weight and R is the R matrix. The weights and denominators are turned into a single vector D abd the equation takes on the matrix form:

$$R - DVR = [1 - DV]R = V$$
$$[1\text{-}DV] = F$$

Finally you can solve for the R matrix using matrix inversion

$$[R] = [F]^{-}1[V]$$

2 Error Analysis

For the error analysis we used a known analytical solution. The fitting was very good in some regions but poor in other. The region of good fitting it from 0 to pi, where we are seeing resonant behavior, specifically at pi/2 where sin =1. After be pass region around pi the the computation begins to fit poorly. There was three regions that I decided to look at. Each one corresponds to a peak or saddle point in the analytical solution. The region of best fitting occurs where there is resonance of the system. I chose to fit this as the program is highly intensive computationally and this region shows the power law dependence best for the number points in the system. The error of Gaussian quadrature is expected to follow a power law in respect to N. The error scales as e = xlogN where x is the power. In this case we found that the power of the error is 1/2, meaning that the error scales as the square root. So it was indeed shown to follow a power law with respect to the number of points