Nonlinear Problems in applications (MTH311) — Coursework 1

This is the first of two courseworks which combine for 100% of the credits for this unit. The maximum number of marks for this coursework is 35, worth 35% of the credits. This coursework consists of **one** question. Parameter `q` (see Question) is **personalised**.

Instructions and rules

- Deadline: 31 March 2011.
- Material to be handed in:
  - (**Hardcopy**) Printed document containing graphs and solutions with necessary headings, annotations and comments (no essay!),
  - (**Hardcopy**) printout of program codes with comments.
  - (**Victory**) Upload a script `cw1.m` and all functions (also `m`-files) necessary to run the script to the Victory assignment `CW1`. When I run the script `cw1.m` it should recover all graphs and numerical outputs of your document.

  The working scripts and functions account for 80% of the credits. The hardcopy (20%) is only there to show the output, the code, and give additional comments.
- Credit for the coding part of each question:

  100% code performs all computations correctly, is well structured and commented;
  ≥**80%** code performs all computations correctly but has problems with its structure[1];
  ≥**60%** code performs most of the computations correctly;
  ≥**40%** code does not perform computations correctly but could be made to work with minor corrections;
  ≥**20%** the intentions behind the code are discernible with some effort.
- This is **individual** coursework. If you are unable to implement some of the functions or scripts you may use a copy of these functions' or scripts' `m`-files from other students **if you declare at the top of your coursework document** which `m`-files are borrowed. You will then get zero credit only for these functions. If you do not declare 'borrowed code' but use it to get your results then you are plagiarising.

  Working code that you wrote yourself will always get more credit than a neat plot obtained by calling other students' functions.
- For questions, clarifications and further help contact:

  Jan Sieber (`jan.sieber@port.ac.uk`, office LG.146).

---

[1] examples of bad structure:
- hard-coded 'magic' numbers spread throughout the code,
- functions that should be general but only work for this example,
- one part of the code is a repetition of another part.

Question 1: A predator-prey model with harvesting

The following set of nonlinear differential equations is a simple model for interaction between two species, x and y, where y is the predator and x is the prey (for example, seal and cod). At the same time the prey species x is also harvested (for example, by fishing).

$$\begin{aligned} \dot{x} &= ax(1-x) - xy - p\left[1-\exp(-qx)\right], && x \geq 0 \\ \dot{y} &= -y + bxy\exp(-y/c), && y \geq 0. \end{aligned} \tag{1}$$

The values of x and y express the number of existing animals but are scaled such that x and y are both of order 1 (say, existing cod is measured as $x \times 10^3$ tonnes, seal in thousands). The system has 5 parameters, of which

$$a = 5, \qquad b = 5, \qquad c = 5$$

are constant. The parameter a measures how rapidly the prey multiplies, b measures how many predators a certain abundance of prey supports, c measures the saturation of this support. The parameters p and q determine the harvesting strategy (p is the maximum amount of x harvested, pq determines the harvesting rate near $x = 0$. You obtain your personal parameter q by calling

```
q=GeneratePar(XXXYYY);
```

where `XXXYYY` is your six-digit student ID. Parameter p will be varied during your study. See Hints below for information where to get the (encrypted) Matlab function file `GeneratePar.p`.

(a) **Equilibria** Find all equilibria of the system (1) for parameter $p > 0$. Only equilibria, for which both components are non-negative, are of interest. Plot the curves of equilibria in the (p, x)-plane. [10 marks]

(b) **Stability** Indicate the type of each equilibrium along the curves you obtained in part a. For example, use a dot for equilibria that are stable (sinks), a cross for equilibria that have one unstable eigenvalue (saddles), a square for equilibria that have two unstable eigenvalues (sources). You may use colors instead of different symbols. [10 marks]

(c) **Bifurcations** Calculate the following special equilibria and values of p to 4 significant digits:

i. **(Branch point 1)** The value of p at which complete extinction ($x = y = 0$) becomes stable. [3 marks]

ii. **(Fold)** The maximal value of p for which an equilibrium with positive x and $y = 0$ exists (and the corresponding value of x). [4 marks]

iii. **(Branch point 2)** The minimal value of p for which no equilibrium with positive y exists (and the corresponding value of x). [4 marks]

iv. **(Hopf)** The value of p for which the equilibrium with positive x and y loses its stability (and the corresponding value of x and y). [4 marks]

Total for Question 1: 35 marks

Question 1 continued

Hints:

- You may assume that there is only one banch of equilibria of (1) with non-zero y.
- In the folder `Coursework 1` on Victory is the file `GeneratePar.p`, which you should download into your Matlab working folder before you can execute `GeneratePar`.
- The file `cw1.m` is a starting point, which you can use for your script.
- On Victory in the folder `Useful Functions` you will find Matlab functions that are potentially useful, and that you can call as part of your own scripts and functions after downloading them. Beware that they are written by me and only provided for your convenience. This means that they may not give meaningful error messages if you call them with inconsistent arguments. Report difficulties to me.