Nonlinear emergent behaviour in a spatial tropical forest model

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Arxiv Preprint: 2207.14118

OUTLINE

1. Stochastic cellular automaton of forest and fire

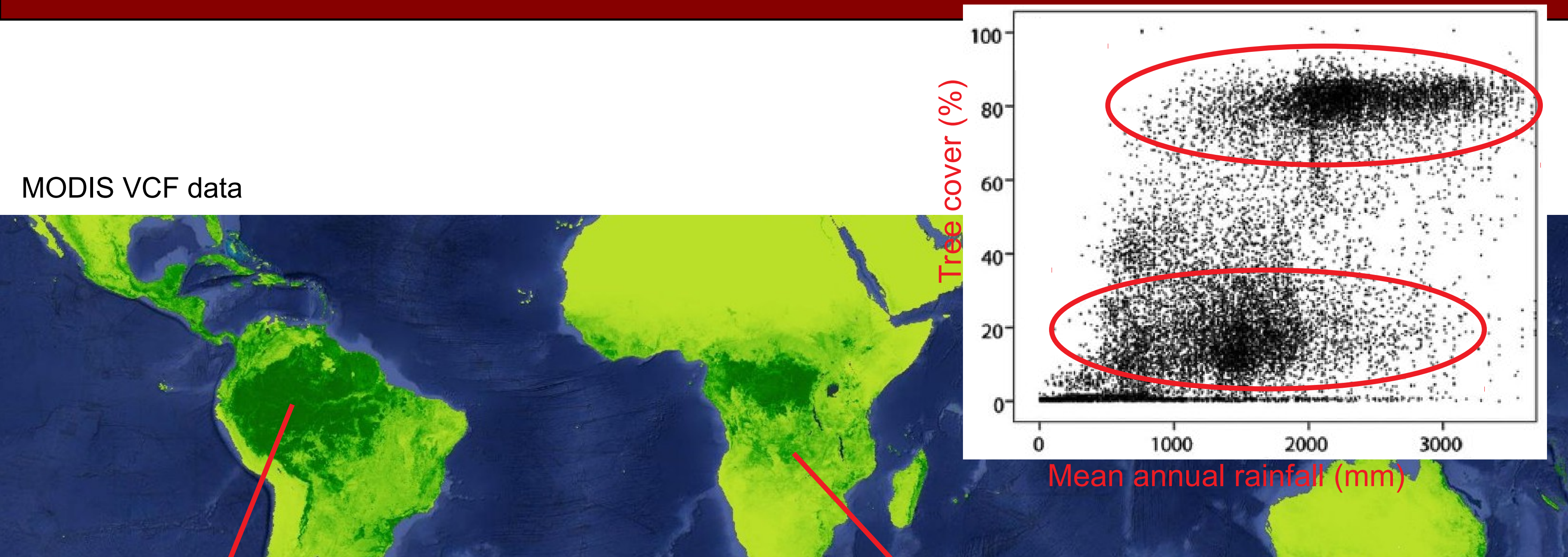
2. Slow and fast processes

3. Analysis: steady states & dynamics

   => emergent structure-dynamics relations

Tropical tree cover bimodality

MODIS VCF data



Staver et al., 2011
Hirota et al., 2011

Stochastic cellular automaton:

fire and forest dynamics

CA FOREST AND FIRE

Square Lattice (each cell ~ 30m x 30m) $N \times N \quad (N = 100)$

4 Species: **T**ree, **G**rass, **B**urning, **A**sh

Empirical facts:

- fire ignites and spreads in grassland
- trees block fires but get damaged
- **fast** fire spread (hours-days)
- **slow** tree spread (years-decades)

Staver et al., 2011; 2012
Dufresne et al., 2018

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Dufresne et al., 2018
Wuyts & Sieber, 2022

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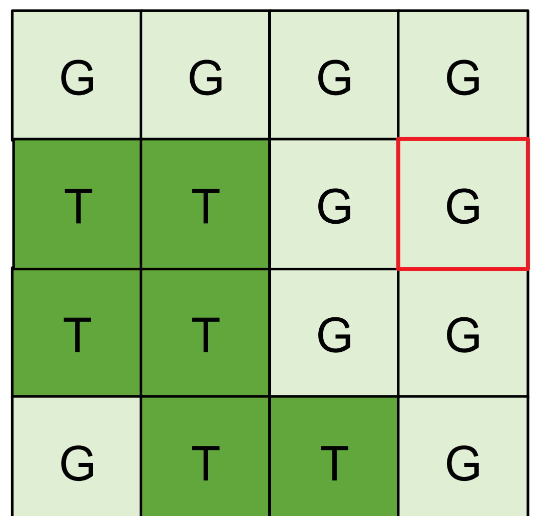

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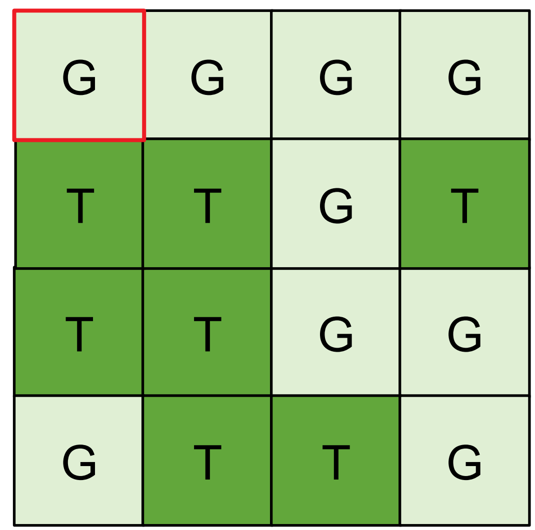

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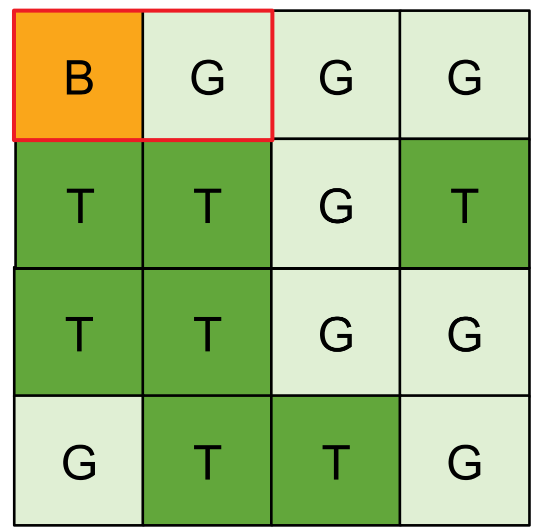

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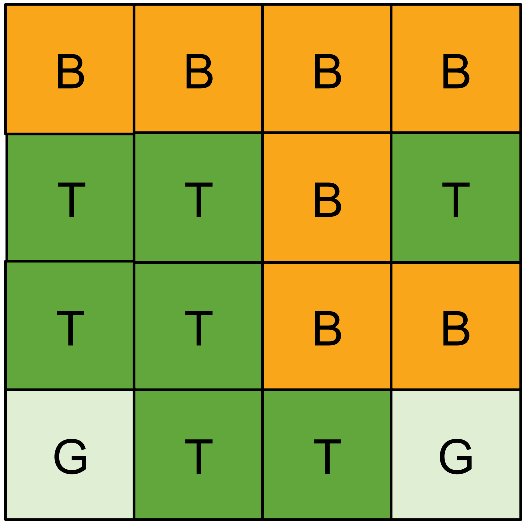

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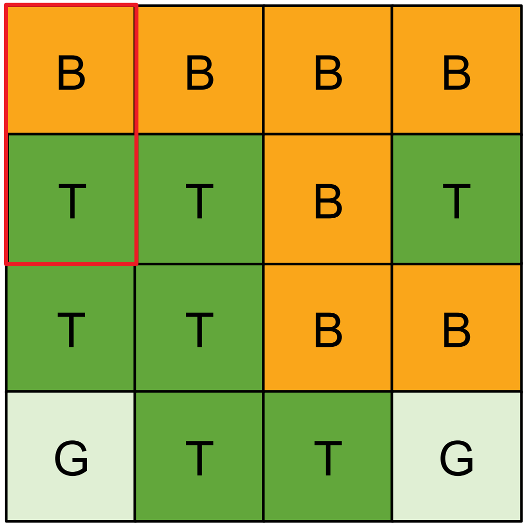

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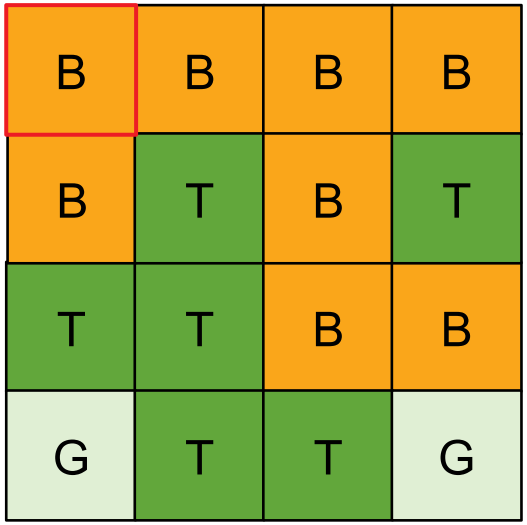

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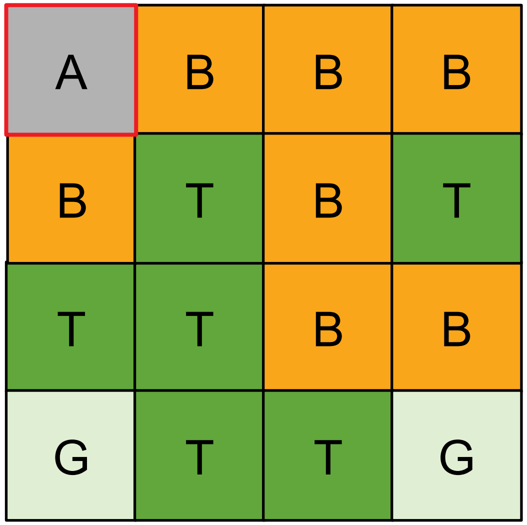

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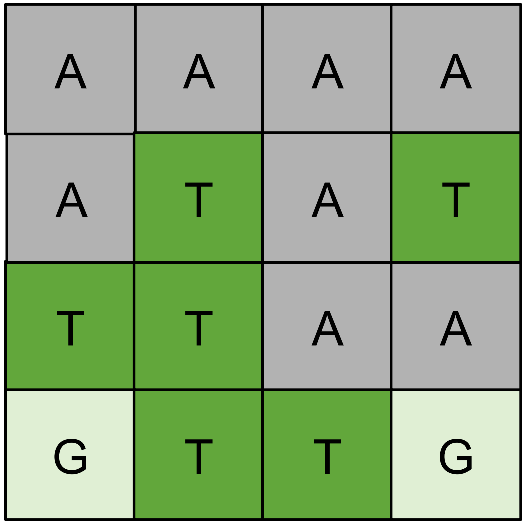

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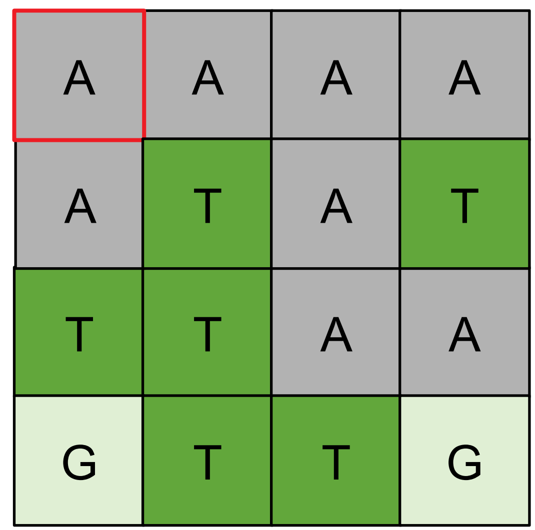

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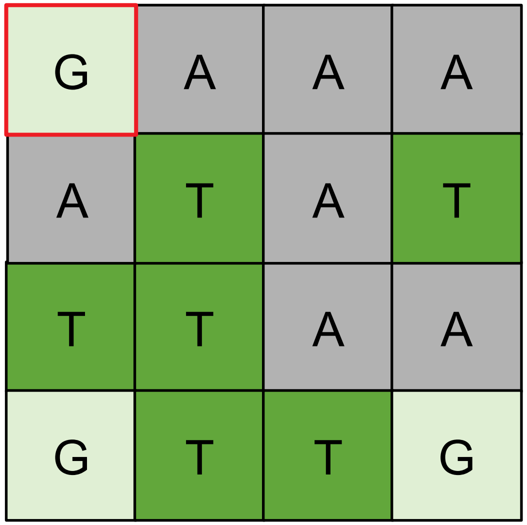

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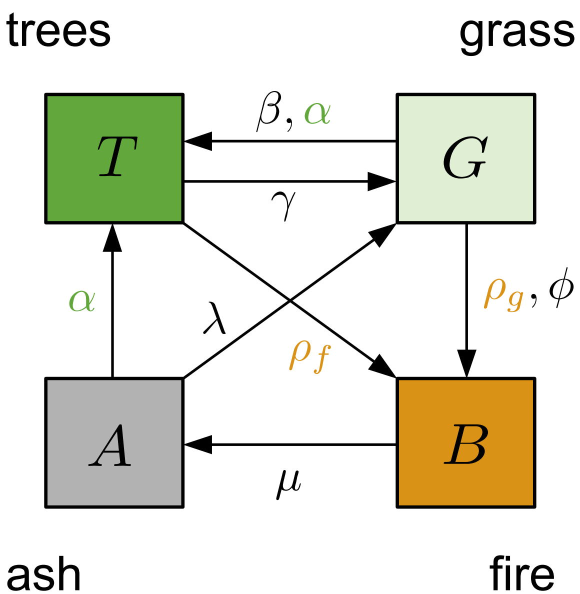


$$\mu, \rho_G > \rho_T \gg \lambda \gg 1 \gg \alpha, \beta, \gamma$$

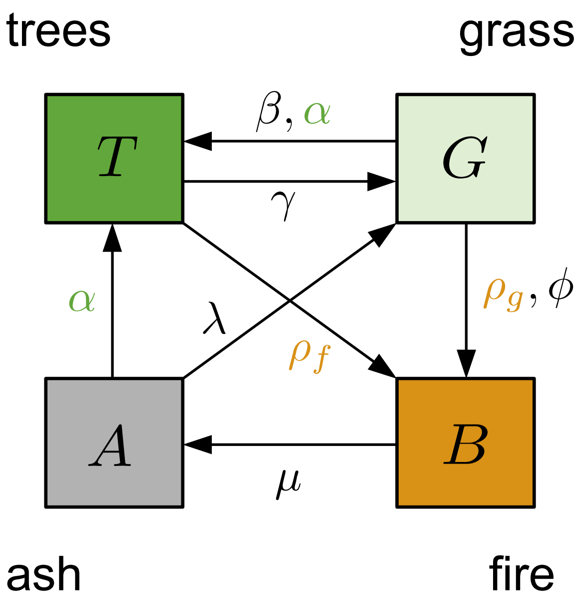
fast fire spread (<hours)

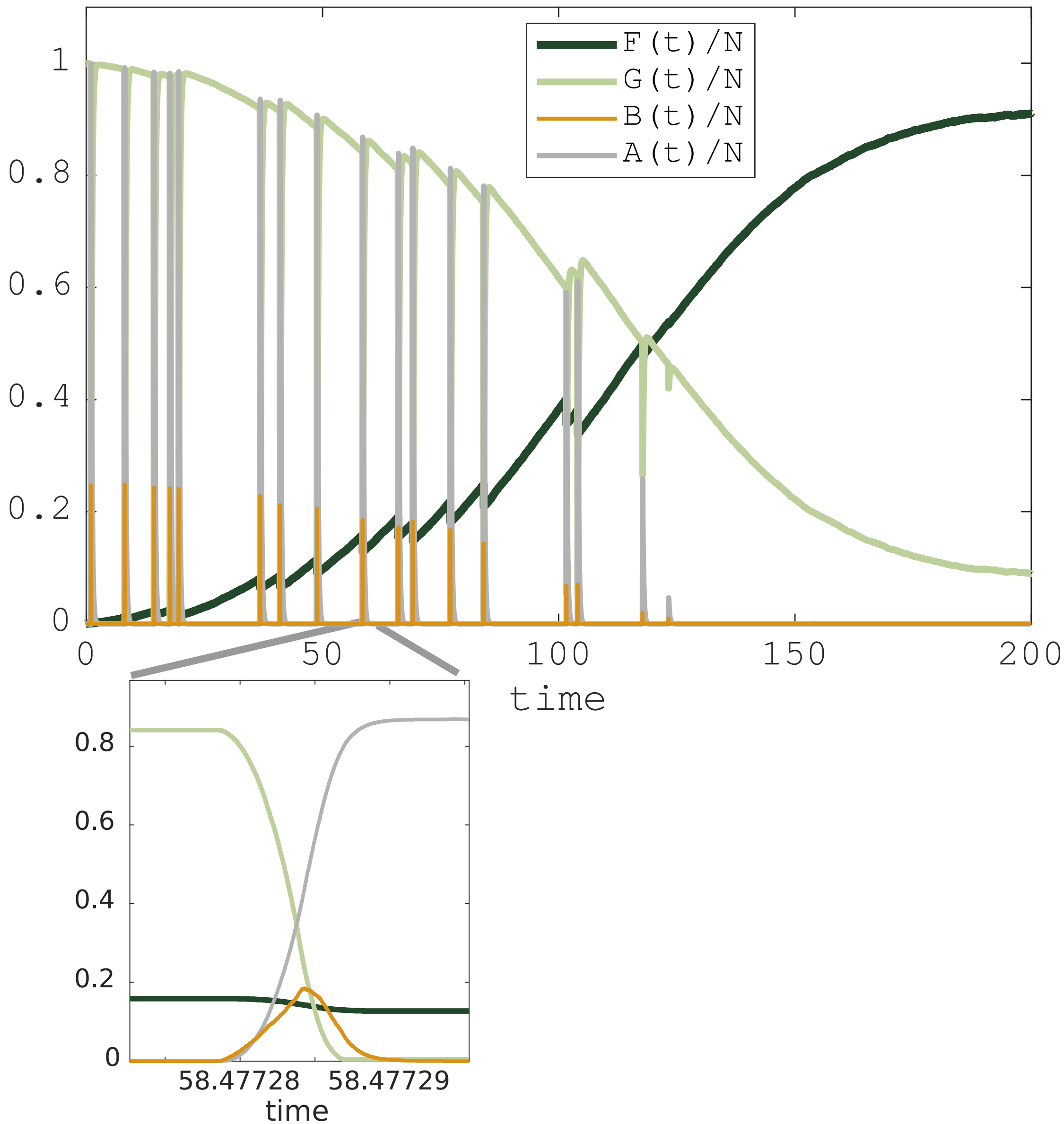
medium grass regrowth (months)

slow tree dynamics (decades)

Dufresne et al., 2018
Wuyts & Sieber, 2022

CA FOREST AND FIRE





video

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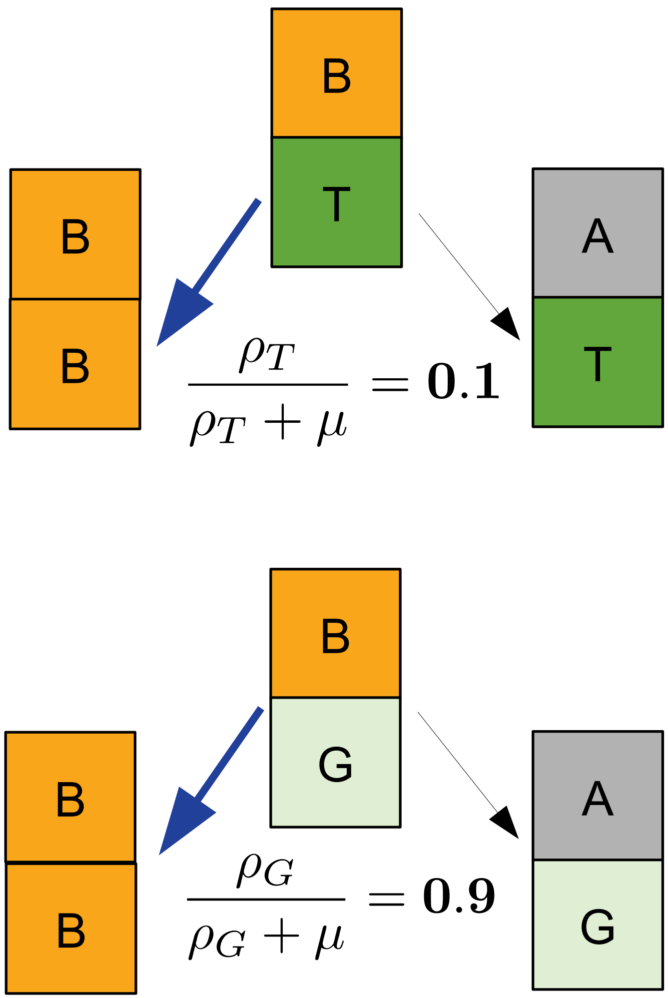
Analysis of the forest & fire automaton

steady states,
dynamics
and
structure-dynamics relations

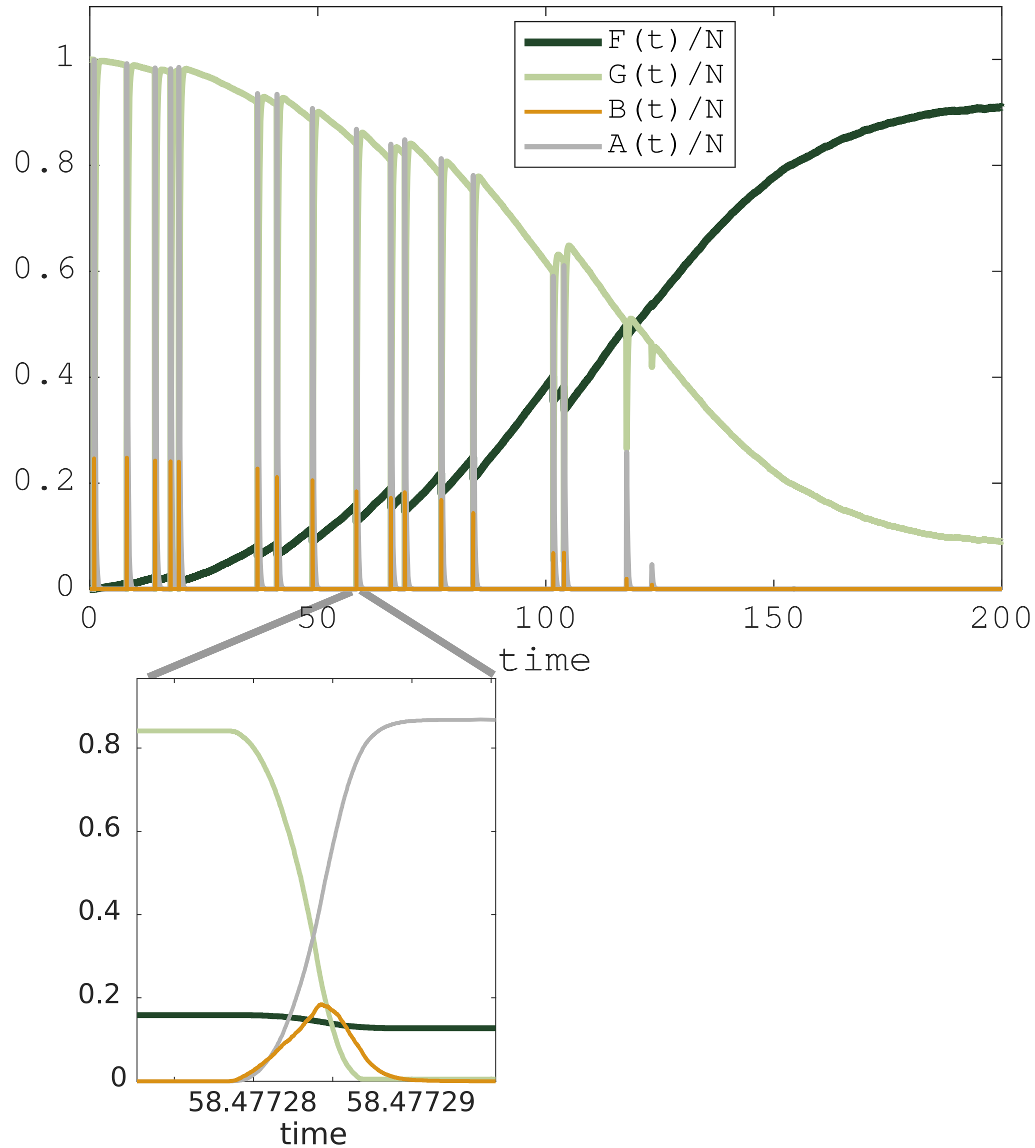
CA FOREST AND FIRE

Fire spreading probability



$$\frac{\rho_T}{\rho_T + \mu} = \mathbf{0.1}$$

$$\frac{\rho_G}{\rho_G + \mu} = \mathbf{0.9}$$



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MACROSCOPIC QUANTITIES

Steady states and dynamics

=> first define macroscopic quantities:

- frequency of T or G cells: **[T], [G] (FOREST/GRASS AREA)**
- frequency of TG pairs: **[TG] (FOREST PERIMETER)**

e.g.:

G	G	G	G
G	T	G	G
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normalised by N:

$[T] = 3/16$

$[G] = 13/16$

$[TG] = 8/16$

NOTE: mean field assumes

$[TG] = 4\,[T][G]$

but:

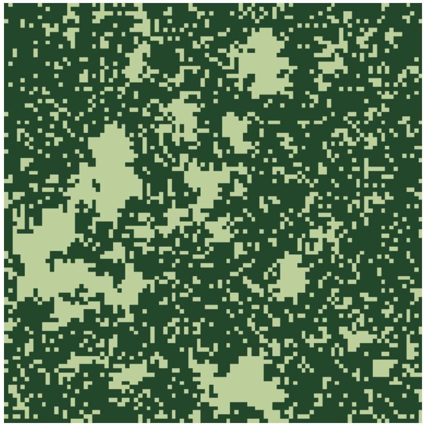
$[TG] = 0.5 < 4\,[T][G] = 0.61$

SLOW-FAST DYNAMICS

Fast dynamics

- For each grass patch, fires ignite and spread through entire patch to bounding forest:

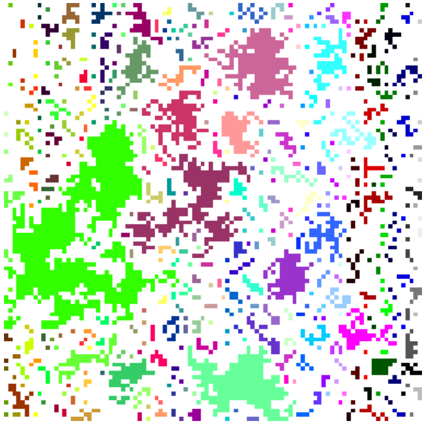
  forest exposure to fire damage: $\phi N \sum_{j=1}^{n_c} [G]_j [TG]_j$



Slow dynamics

- fire-induced loss:

$$\Delta_T^{\text{loss}} := \sum_{j=1}^{n_c} \underbrace{\phi N [G]_j}_{\text{\# fires per time in cluster } j} \times \underbrace{\frac{\rho_T}{\rho_T + \mu}[TG]_j}_{\text{loss per fire in cluster } j}$$



- forest spread and spontaneous conversion:

$$\Delta_T^{\text{gain}} := \beta[G] - \gamma[T] + \alpha[TG]$$

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BALANCE ON SLOW TIMESCALE

$$\frac{\mathrm{d}[T]}{\mathrm{d}t} = \beta[G] - \gamma[T] + \alpha[\underline{TG}] - \phi N q[G]\langle[\underline{TG}]\rangle_{\mathrm{cg}}$$

Overall
rate of change

forest perimeter
for spread

"**grassland-weighted**
forest perimeter"
for loss

undetermined

$$\Downarrow \quad \langle[TG]\rangle_{\mathrm{cg}} := \sum_{j=1}^{n_c} \frac{[G]_j}{[G]}[TG]_j$$

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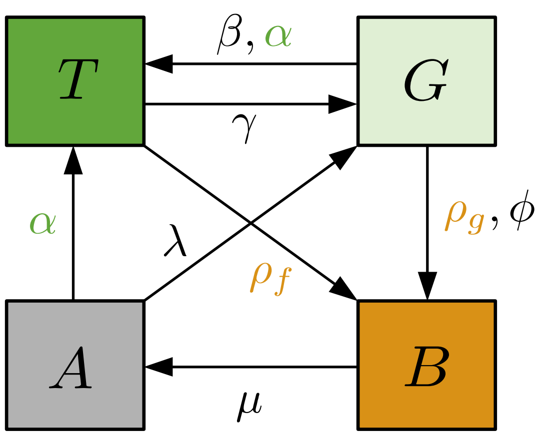
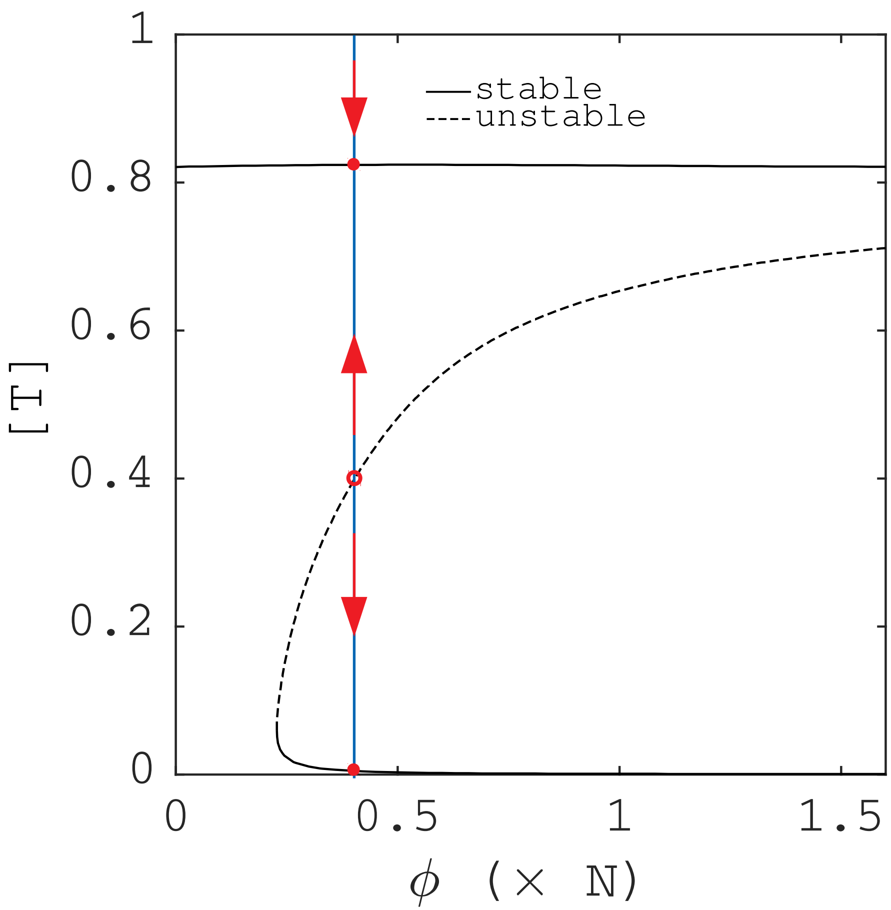
STEADY STATES VIA FEEDBACK CONTROL





conventional simulation

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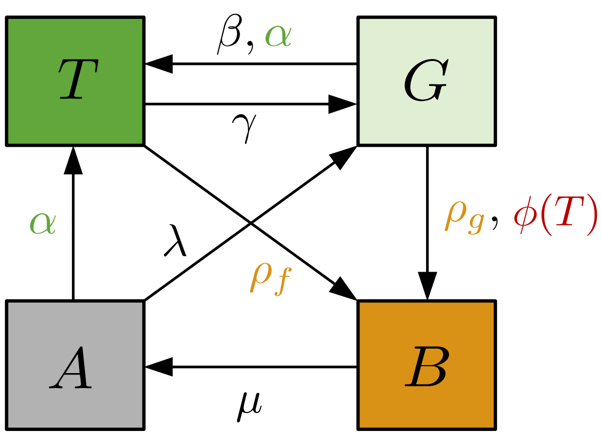
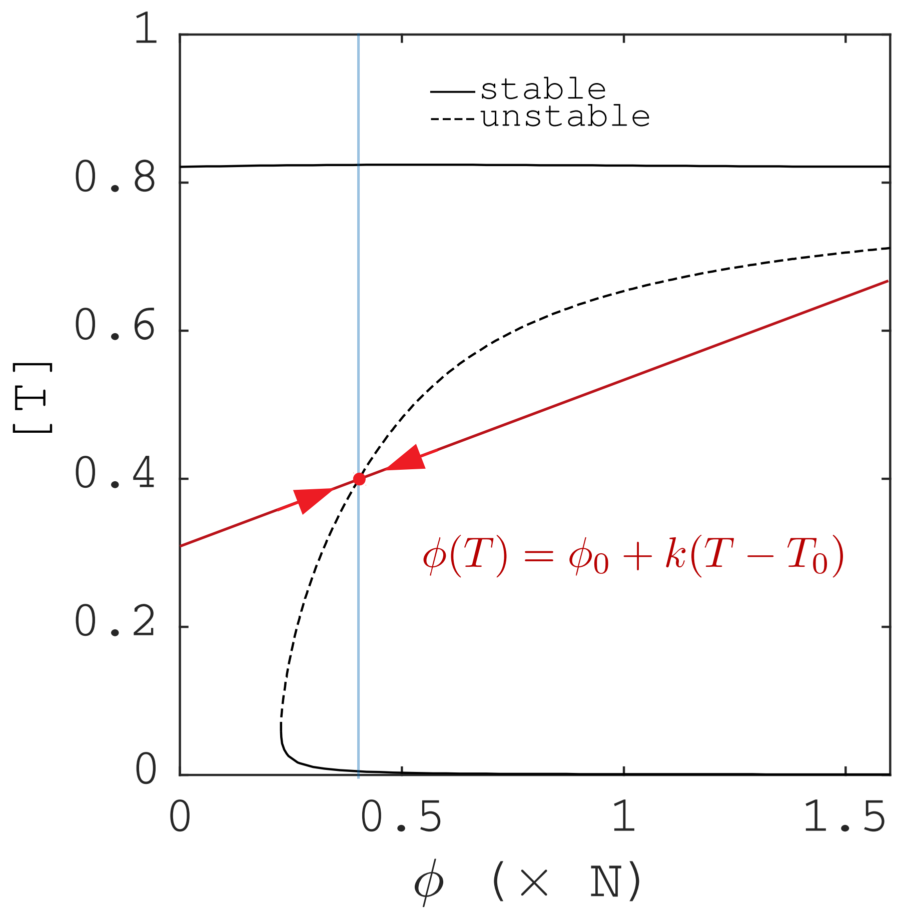
see also MT1

STEADY STATES VIA FEEDBACK CONTROL



video (T0 = 40%)



simulation with feedback control

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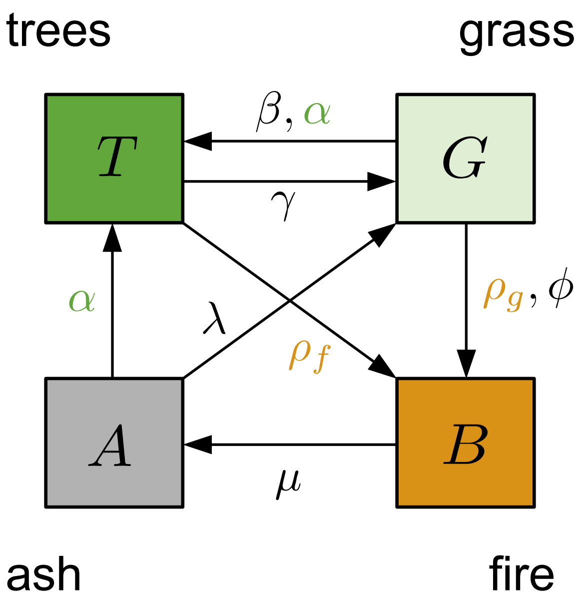
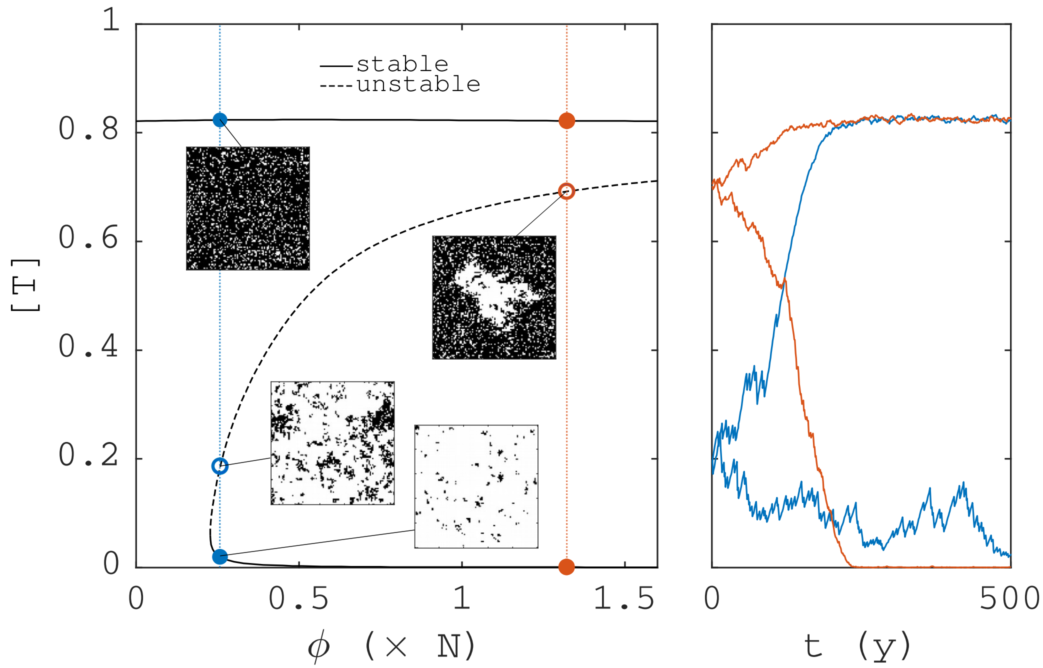
STEADY STATES & BISTABILITY





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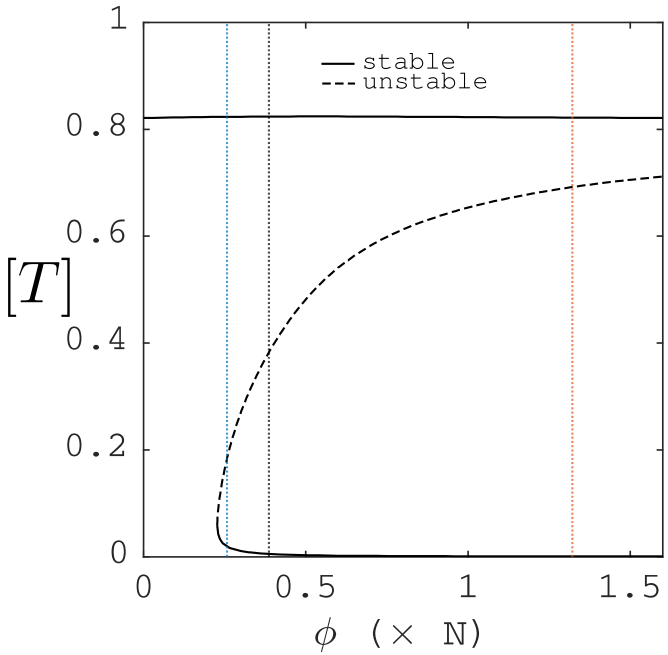
CLOSURE



$$\frac{\mathrm{d}[T]}{\mathrm{d}t} = \beta[G] - \gamma[T] + \alpha[TG] - \phi N q[G]\langle[TG]\rangle_{\mathrm{cg}}$$

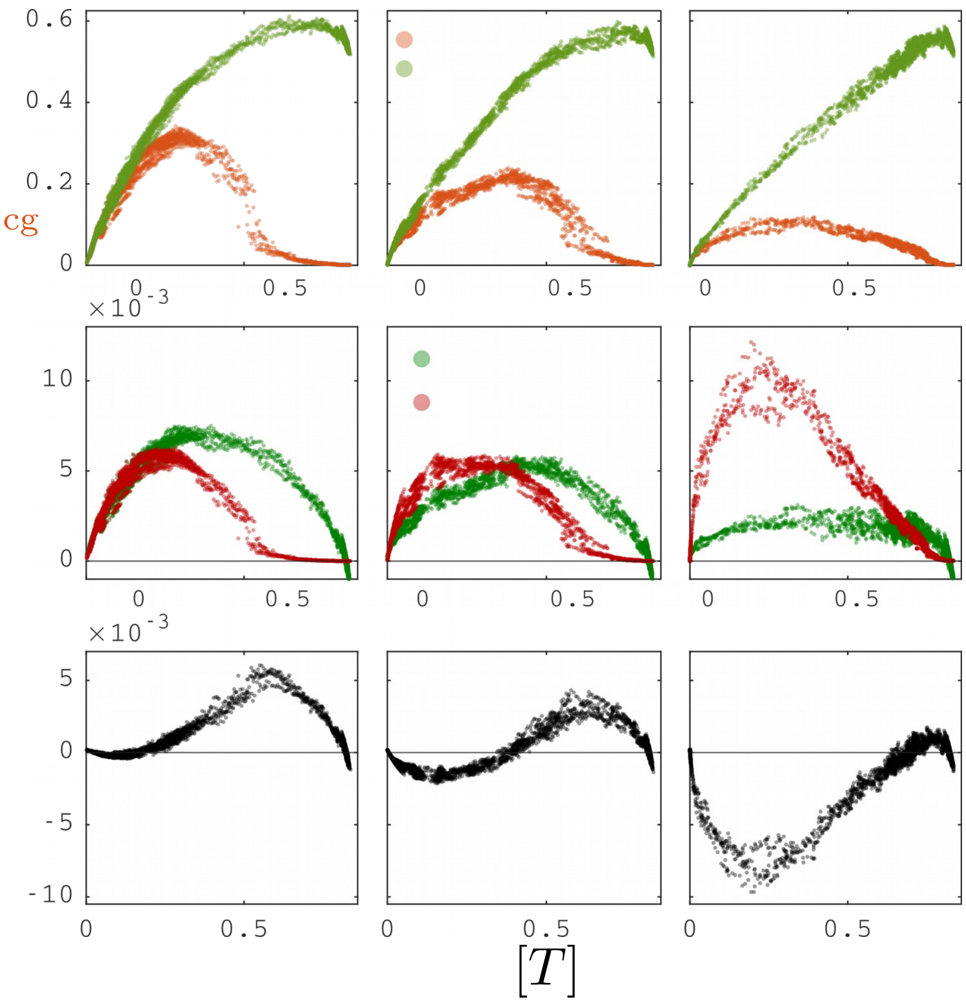
$[TG]$

$\langle[TG]\rangle_{\mathrm{cg}}$

gain

loss

R.H.S. of emerging scalar ODE $\frac{\mathrm{d}[T]}{\mathrm{d}t}$



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STRUCTURE-DYNAMICS RELATIONS

$$\frac{\mathrm{d}[T]}{\mathrm{d}t} = \beta[G] - \gamma[T] + \alpha[TG]^* - \phi N q[G]\langle[TG]\rangle^*_{\mathrm{cg}}$$

linear terms
(independent of spatial structure)

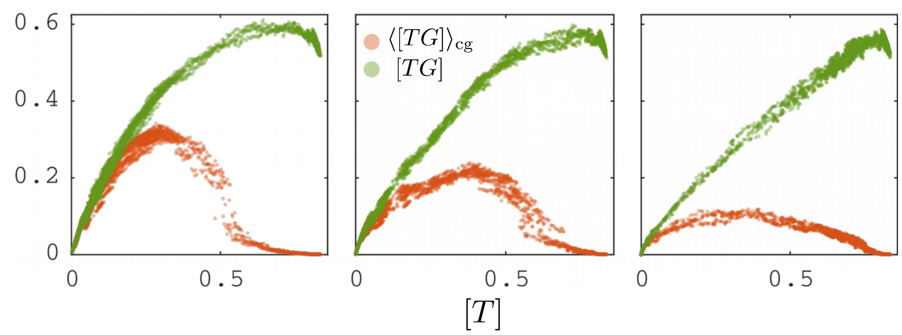
nonlinear terms
(emerge from interactions at forest perimeter)

$[TG]^*([T])$: **perimeter-area** relation

$\langle[TG]\rangle^*_{\mathrm{cg}}([T])$: **weighted perimeter-area** relation

Can be calculated from spatial data

=> test where fire feedbacks are strong enough to cause bistability



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CONCLUSIONS

- Hypothesis of tropical tree cover **bistability** relies on **bimodality** in observations, but alternative explanations exist
  => **more specific indicators** are required
- Most models are **mean-field models**
  assume that vegetation patches are **well mixed**
  => neglect spatial nature of fire and forest spread
  => unable to provide spatial predictions
- **Spreading processes occur near the forest perimeter**
  => structure-dynamics relations emerge
  => test where fire-vegetation feedbacks cause bistability
- Explore **saddle landscapes**
- Explore **Ash blocking fire** spread for frequent fires