Toolbox usage & 2d continuation

toolboxes: ep ~ equilibria & their bifurcations
coll ~ boundary value problems
po ~ periodic orbits & their bifurcations (on top of coll)

input for ep, po: f for $x' = f(x,p)$ or $x' = f(t,x,p)$
for coll: f & b, $x' = f(x,p)$ (opt t), $0 = b(t_0, T, x(t_0), x(t_0+T), p)$

Constructors

toolbox does coco_add_func internally in 'constructors"

ode_x2y, x = 'isol' -> arg is initial guess
x = 'ep', ... -> arg is saved solution from previous run (label)

Symbolic generation

for bifurcation tracking: derivatives of r.h.s. needed
for coll & po: vectorization recommended
=> use symcoco to generate r.h.s. & derivatives sco_symfuncs
saved in file, wrapper fcn = sco_gen(fname)
f = fcn(''), dfx = fcn('x'), dfxp = fcn({'x','p'})
where names 'x','p' were set in sco_symfuncs

Demo for (modified)

Problem for initial ep run:

```
>> tmp=prob_fcn_info(prob)
tmp =
  3×5 table
                  type           fidx          uidx          midx            data

   ep          {'zero'     }  {3×1 double}  {9×1 double}  {0×1 double}  {1×1 coco_func_dat
   ep.pars     {'inactive'}   {6×1 double}  {6×1 double}  {6×1 double}  {0×0 double
   ep.test     {'regular' }   {0×1 double}  {9×1 double}  {3×1 double}  {1×1 coco_func_dat
```

Hopf bifurcation tracking in 2 parameters
Where is Hopf frequency? ep.test.BTP ← tests for Takens-Bogdanov points

```
>> tmp=prob_fcn_info(prob)
tmp =
  6×5 table
                    type              fidx             uidx             midx              data
                ______________    _____________    _____________    _____________    ________________

1   ep          {'zero'      }    {3×1 double}    { 9×1 double}    {0×1 double}    {1×1 coco_func_d
2   ep.pars     {'inactive'}      {6×1 double}    { 6×1 double}    {6×1 double}    {0×0 double
3   ep.var      {'zero'      }    {6×1 double}    {21×1 double}    {0×1 double}    {1×1 coco_func_d
4   hb_glue     {'zero'      }    {3×1 double}    { 6×1 double}    {0×1 double}    {1×1 struct
5   ep.HB       {'zero'      }    {5×1 double}    { 7×1 double}    {0×1 double}    {1×1 coco_func_d
6   ep.test     {'regular' }      {0×1 double}    {[       22]}    {[        7]}    {0×0 double
```

1) $0 = f(u)$ $\quad u = (x, p)$

2) $u_p \to 'p'$

3) $0 = \partial_x f(x,p) v_1 - w_1$

$0 = \partial_x f(x,p) v_2 - w_2$

4) $v_2 - w_1 = 0$

5) $k \cdot v_1 + w_2 = 0$

$v_1^T v - 1 = 0$

$v_{ref}^T v = 0$

6) $k \to$ 'BTP'

$\sim$ frequency ω

Periodic Orbits

(provide initial guess, branch off from Hopf, ...)

ode_isol2po $\qquad$ ode_HB2po

MXCL : monitors error estimate exceeding tolerance

coll_settings() $\leftarrow$ check for options

Plotting

thm = coco_plot_theme() then modify

Torus bifurcation - detect resonances

2 eigenvalues of monodromy matrix on unit circle in $\mathbb{C}$ (complex pair)

$\alpha = (\arg \mu)/2\pi$ (μ is e.v.) is rotation number

if r is rational $\alpha = p/q$ $\to$ resonance tongue

branches of from there with periodic orbits of period $q \cdot T$

(T original period)

α is not recorded in bifurcation diagram

find out that po.TR stores a_idx, b_idx in its data

and that $\mu = a + ib$

add equation

$$0 = b \cdot \cos(2\pi\alpha) - a \cdot \sin(2\pi\alpha) \to a = \cos(2\pi\alpha),\ b = \sin(2\pi\alpha)$$

and assign $\alpha \to$ 'rotation'

Create RE events when $\alpha = p/q$ with small q $(q \leq 13)$

2D continuation of periodic orbits of autonomous ODEs

add active continuation parameters that don't depend on mesh or phase

I used $m_{kj} = \left[\int_0^1 b^1_{kj}(t)^T x(t)\,dt\right]^2 + \left[\int_0^1 b^2_{kj}(t)^T x(t)\,dt\right]^2$

- where $b^1_{kj}(t) = c_{kj}\cos(2\pi k t),\ b^2_{kj} = c_{kj}\sin(2\pi k t)$

  c vector selecting component & scaling

added

$$m_j = \sqrt{\sum_{kj} m_{kj}}$$

$$(x(t_0), x(t_1), \ldots, x(t_0+T))$$

p

$\Phi(u) = 0$ ← 'zero'

$\Psi(u) = \mu$ some Ψ 'regular'

some Ψ ~~'inactive'~~ active

$\Phi(u) = u_2 + u_3 u_1 - u_1^3$

$\Psi(u) = \begin{bmatrix} x \\ a \\ b \end{bmatrix}$ ←

μ

$u_1 = x$

$u_2 = a$

$u_3 = b$

$\Phi : \mathbb{R}^5 \to \mathbb{R}^3$

$\Phi(u) = 0$

$\Psi_2(u) = c$

$(x, a, b) \in \mathbb{R}^3$

$\Phi(u_1, u_2, u_3, u_4) = 0$ ← Φ_1, Φ_2

$\Psi(u) = \begin{bmatrix} x \\ y \\ z \\ p \end{bmatrix}$ ←

$\Phi : \mathbb{R}^5 \to \mathbb{R}^2$ ← 2 $\Phi(u) = f(x, p)$ $x \in \mathbb{R}^2, p \in \mathbb{R}^3$

$\Psi(u) = u$ $\begin{bmatrix} x \\ y \\ a \\ b \\ c \end{bmatrix}$ ← 5 $u \in \mathbb{R}^5,\ \mu \in \mathbb{R}^5$

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$(x, y, a, b) \in \mathbb{R}^4$

a, c