Lab Assignment 5

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ECSE 403

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Observability and Observer Design

1. Write the definition of observability of a dynamical system and explain its importance.

An ISO system {u,x,ø, y,n} is observable on an interval of tine for any initial conditions or inputs, there is a function which takes in input and output parameters of the system and gives a state value or estimate of the state. The function should give the initial condition of the system if the input and output parameters are taken over the time interval.

2 Check the rank observability condition for the pair (A1; C1) in Lab 2 and (A2; C2) in Lab 2.Find the number of unobserved state variables.[5 marks]

```
Jm = 0.01; %kgm^2 , inertia of the rotor and shaft
b = 0.001; %Nmsec , viscous friction coefficient
Ke = 0.02; %Vsec  , back emf constant
Kt = 0.02; %Nm/A  , motor torque constant
Ra = 10;   %Ω      , armature resitance
La = 0.5;

A1 = [-b/Jm Kt/Jm ; -Ke/La -Ra/La];
B1 = [0 ; 1/La];
C1 = [1 0];

A2 = [0 1 0 ; 0 -b/Jm Kt/Jm ; 0 -Ke/La -Ra/La]
B2 = [0 ; 0 ; 1/La]
C2 = [1 0 0 ]0.0

obs_1 = obsv(A1, C1);
obs_2 = obsv(A2, C2);
rank_obs_1 = rank(obs_1)
                            rank_obs_2 = rank(obs_2)
```

```
A2 = 3x3
       0    1.0000         0
       0   -0.1000    2.0000
       0   -0.0400  -20.0000
B2 = 3x1
     0
     0
     2
C2 = 1x3
     1     0     0
rank_obs_1 = 2
rank_obs_2 = 3
```

No unobserved state variables since both observability matrices are of full ranks.

3. Consider the state space system with A3=A2 and C3= [0 ;1 ;0]. Check the rank observability condition for this system. Find the number of unobserved state variables. [5 marks]

```
A3 = A2;
C3 = [0,1,0];


obs_3 = obsv(A3, C3);
rank_obs_3 = rank(obs_3)
```

```
rank_obs_3 = 2
```

Since the rank of the observability matrix is 2, there is one uncontrolled variable.

4. Given the above explanations, should the poles of the error state equation be much larger than dominant poles of the system? why? [5 marks]

The poles of the error state equation should be much larger than the dominant poles of the system because the state estimate should converge towards the system's state. This is done quickest if the poles are greater in magnitude and negative, as the exponential will decay towards the state value at a rapid rate.

5. Consider the (A2; C2) system. Using the place command, and the observer gain which places the poles of the error state equation on [50;20 + 20j;2020j]. Hint: You can use the place command as L = place (A2'; C2'; [p1; p2; p3]), to place [p1, p2, p3] poles, and the observer gain is L'

```
p2 = [-50, -20 + 20j, -20 - 20j];
L2 = place(A2',C2',p2);
```

```
obsGain = L2'
```

```
obsGain = 3×1
10^3 ×
    0.0699
    1.3929
    5.9980
```

6. The observer system (A, B, C, D) can be seen as a dynamical system with two inputs u and y – yˆ, the state of the system as system state estimate xˆ, and the output as output estimate yˆ = C2xˆ. Suppose we want to design an observer system for the system (A2,B2,C2,D2). Based on Eq.1 we have A = A2 and C=C2. Find B and D. [5marks]

```
B_hat = [B2 obsGain]

D_hat = [0 0; 0 0 ; 0 0]
```

```
B_hat = 3×2
10^3 ×
         0    0.0699
         0    1.3929
    0.0020    5.9980
D_hat = 3×2
     0     0
     0     0
     0     0
```

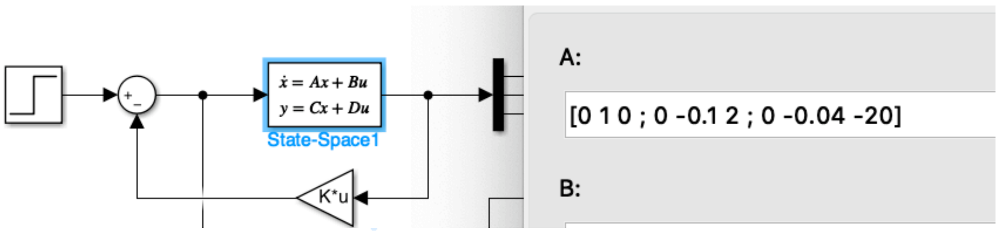
7. Implement sys = (A2, B2, I(3), D2) and obs = (A, B, I(3), D) on Simulink(I(3) is the identity matrix of dimension 3. The Matlab command is eye(3)). With this implementation we have access to the state of system(x) and state of observer(xˆ) which is system state estimate. Using the demux block find the output of the system y = C2x and the output estimate yˆ = C2xˆ. Using signals yˆ,y implement state observer of Q5 and Q6 (get the observer gain from Q5 and get the observer equations from Q6). Compare the step response of each state with its estimate. [15 marks]

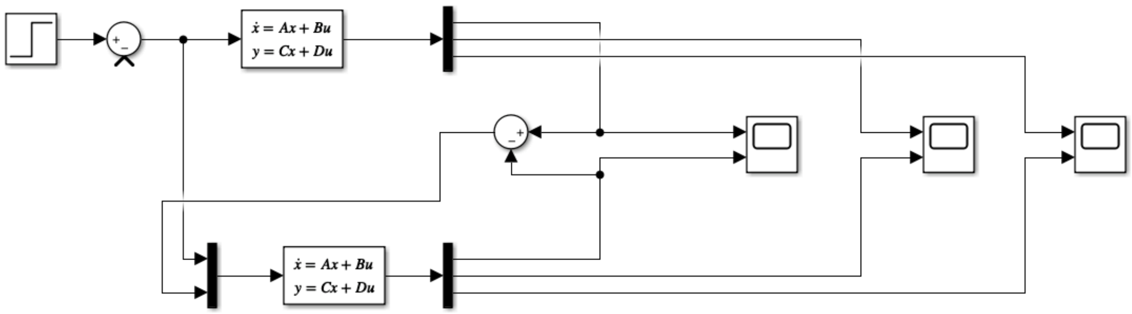
Hint: You can use mux block in the Simulink to compare x(i)(ith element of x) with xˆ(i) and then passing the signal to the scope.

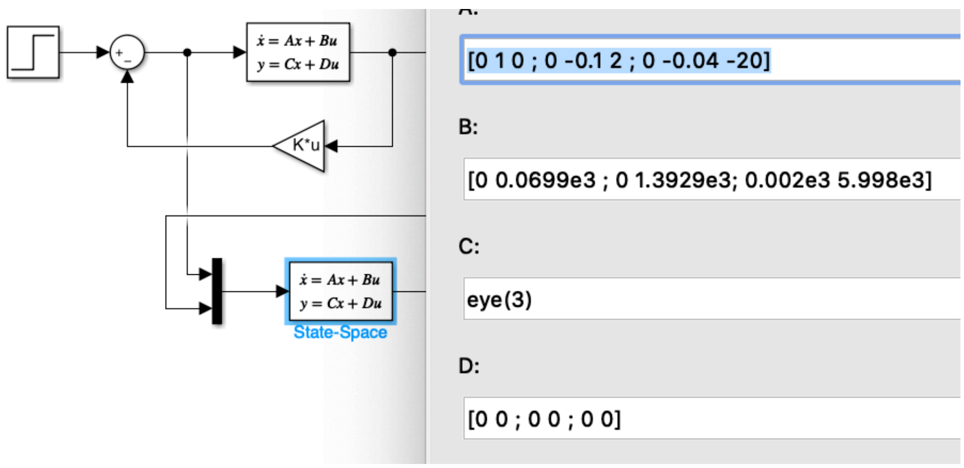
```
p3 = [-1-i, -1+i, -10];

K = place(A2,B2,p3)
```

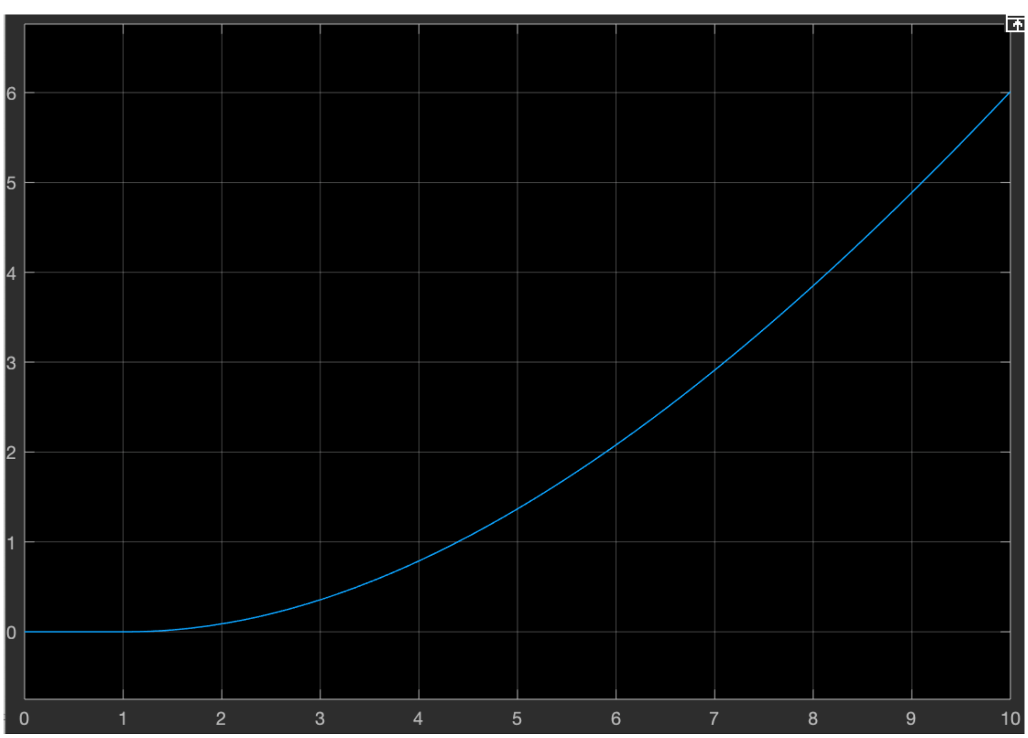
```
K = 1×3
    5.0000    5.1825   -4.0500
```


ẋ = Ax + Bu
y = Cx + Du
State-Space1
K*u
A:
[0 1 0 ; 0 -0.1 2 ; 0 -0.04 -20]
B:

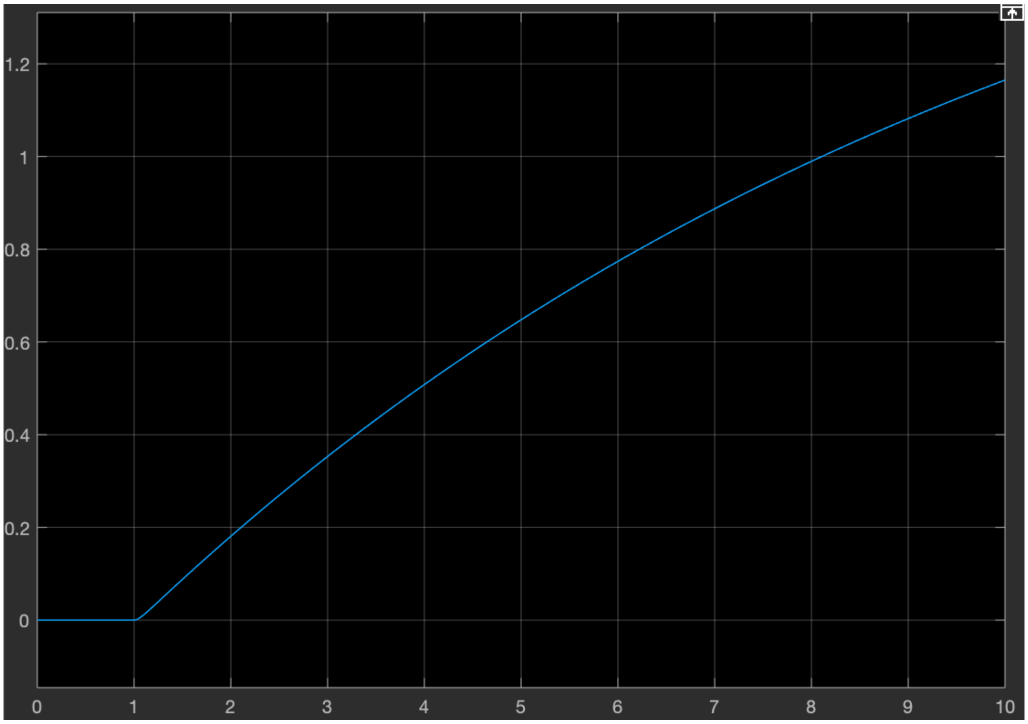

ẋ = Ax + Bu
y = Cx + Du
ẋ = Ax + Bu
y = Cx + Du


ẋ = Ax + Bu
y = Cx + Du
K*u
ẋ = Ax + Bu
y = Cx + Du
State-Space
[0 1 0 ; 0 -0.1 2 ; 0 -0.04 -20]
B:
[0 0.0699e3 ; 0 1.3929e3; 0.002e3 5.998e3]
C:
eye(3)
D:
[0 0 ; 0 0 ; 0 0]

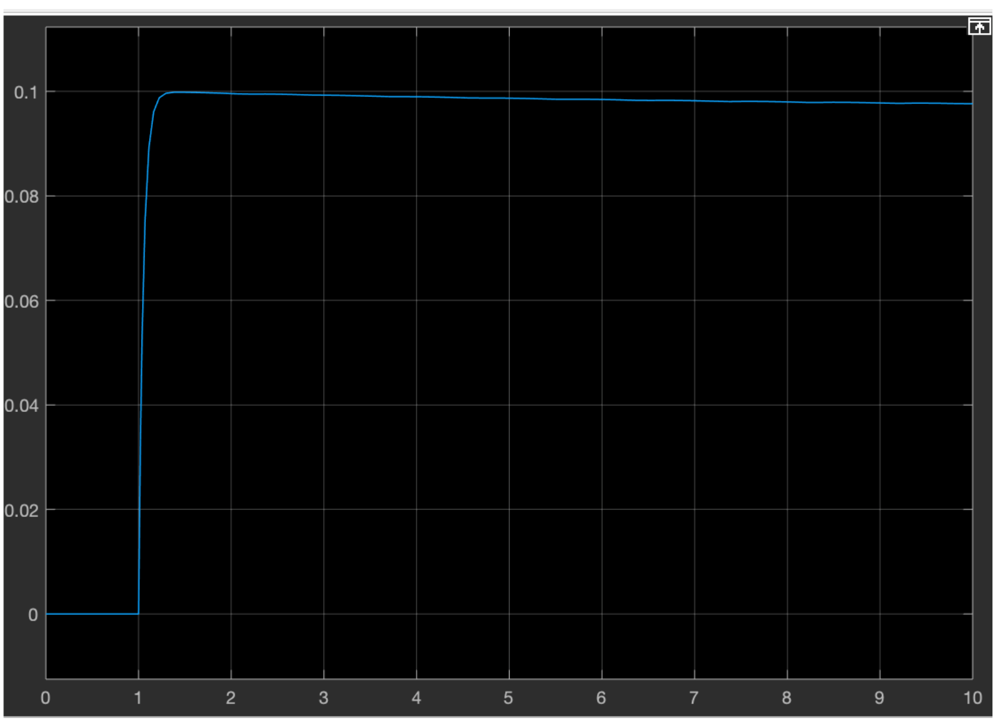
x1 and $\hat{x}1$


0
1
2
3
4
5
6
7
8
9
10

x2 and x̂2



x3 and x̂3



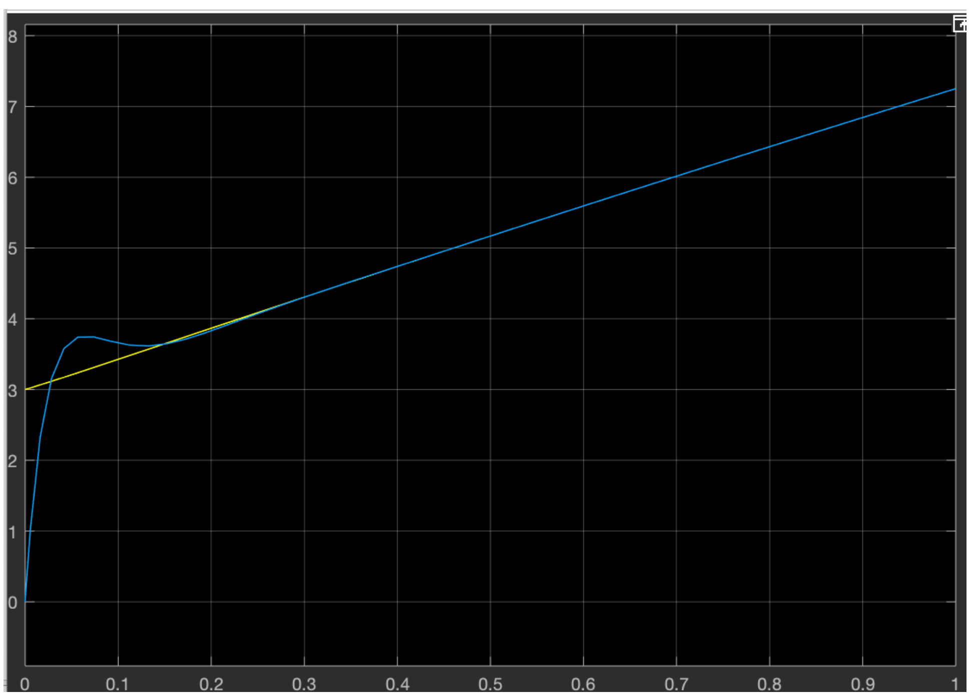
Since both the system and the system's observer are initialized at the same initial conditions, there is no difference between the states over their trajectories, so there is no error.

8. By setting initial condition of the system (sys) to [3; 4; 5] and initial condition of the observer to 0, compare the step responses of each state with its estimate. Explain intuitively the role of different initial conditions in system and observer

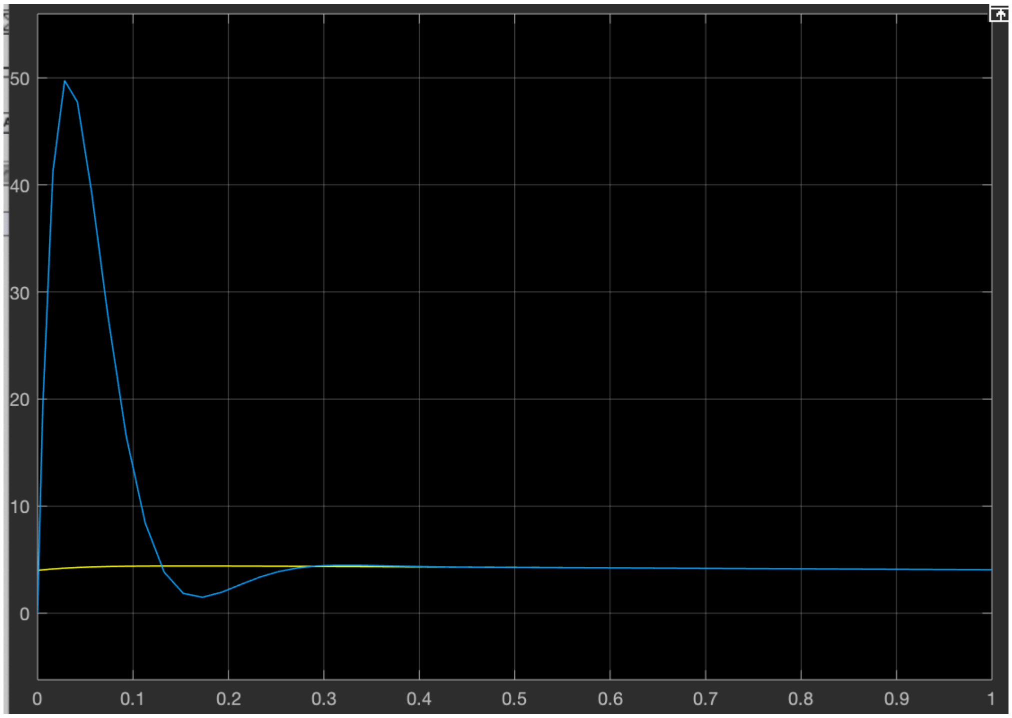
Initial conditions:

[3;4;5]

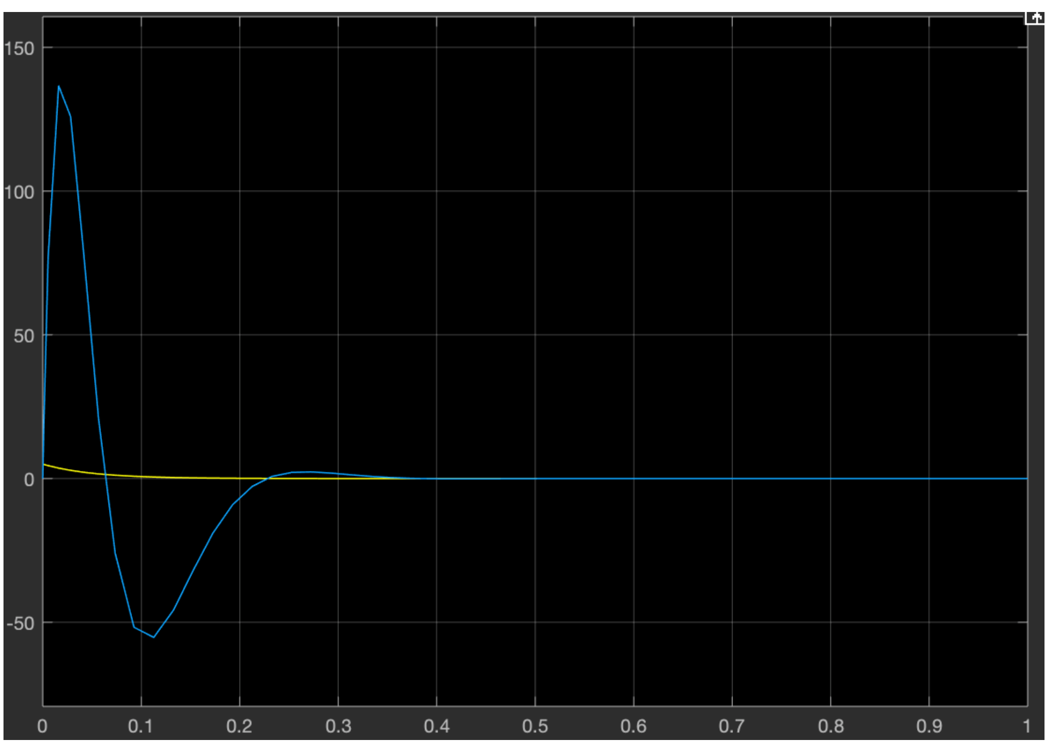
x1 and x̂1



x2 and x̂2



x3 and x̂3

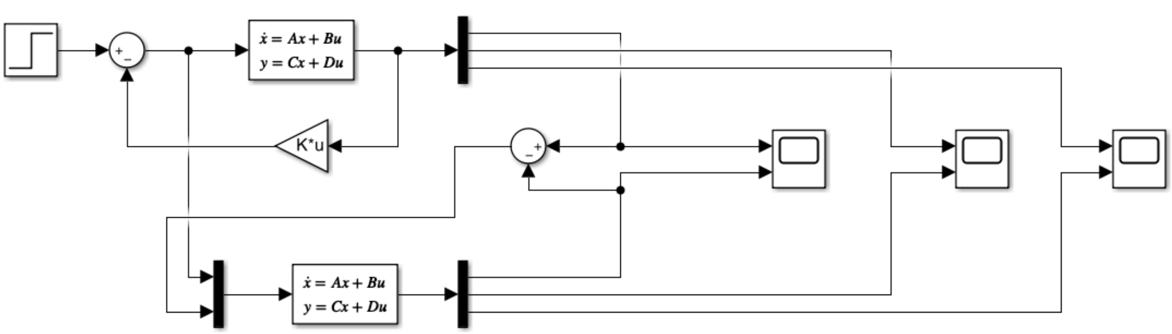


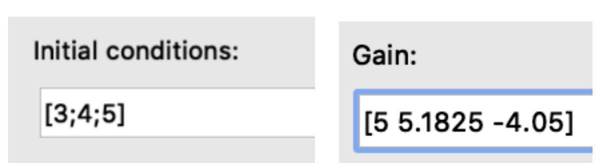
Because the system is initialized with initial conditions, whereas the observer is initialized to zero initial conditions, there is a difference or error between the system and its observer, as the observer is not following the output of the system. The observer compensates its output according to the output of the system, so the two eventually have the same trajectory.

9. Using the commands and implementation in Lab4 find state feedback gains which places the closed loop poles on [−1 − i, −1 + i, −10], and implement the state feedback on Simulink. Compare the step response of each state with its estimate. [15 marks]

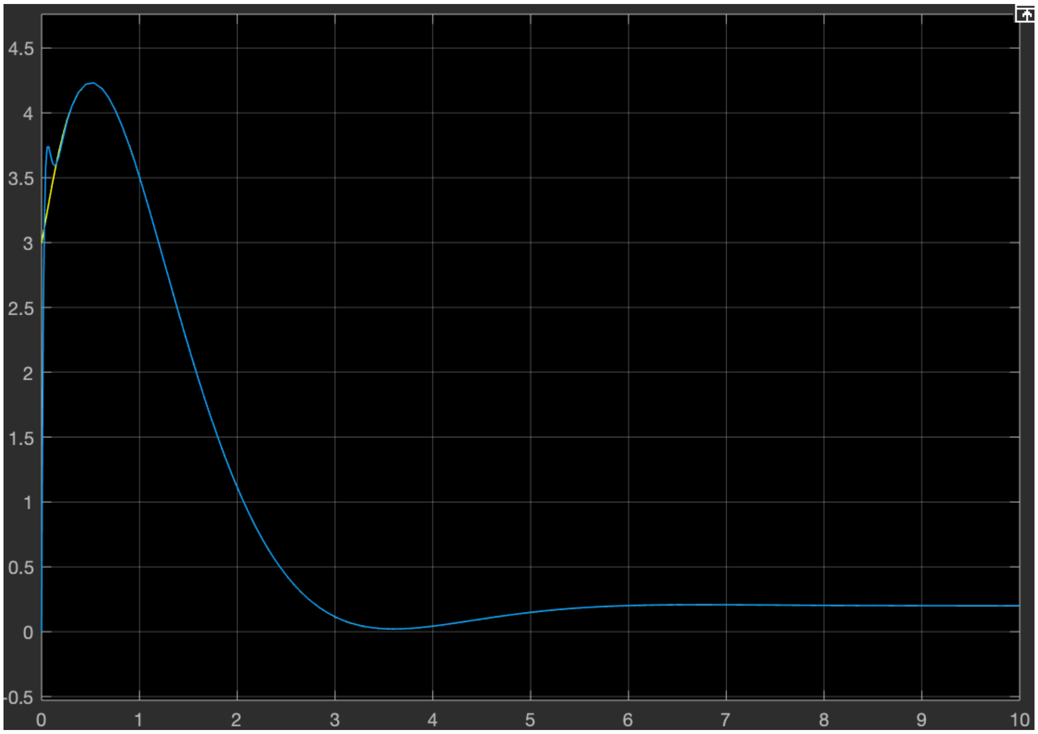
```
p4 = [-1 + i, -1 - i , -10];
K4 = place(A2,B2,p4)
```

```
K4 = 1x3
     5.0000    5.1825   -4.0500
```

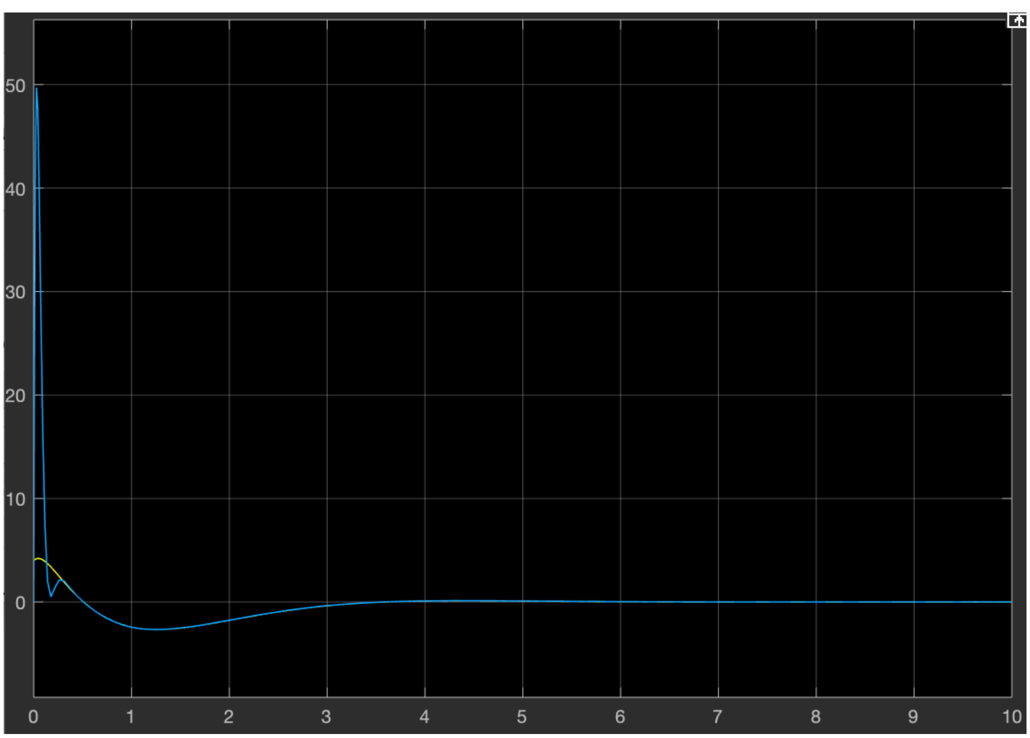




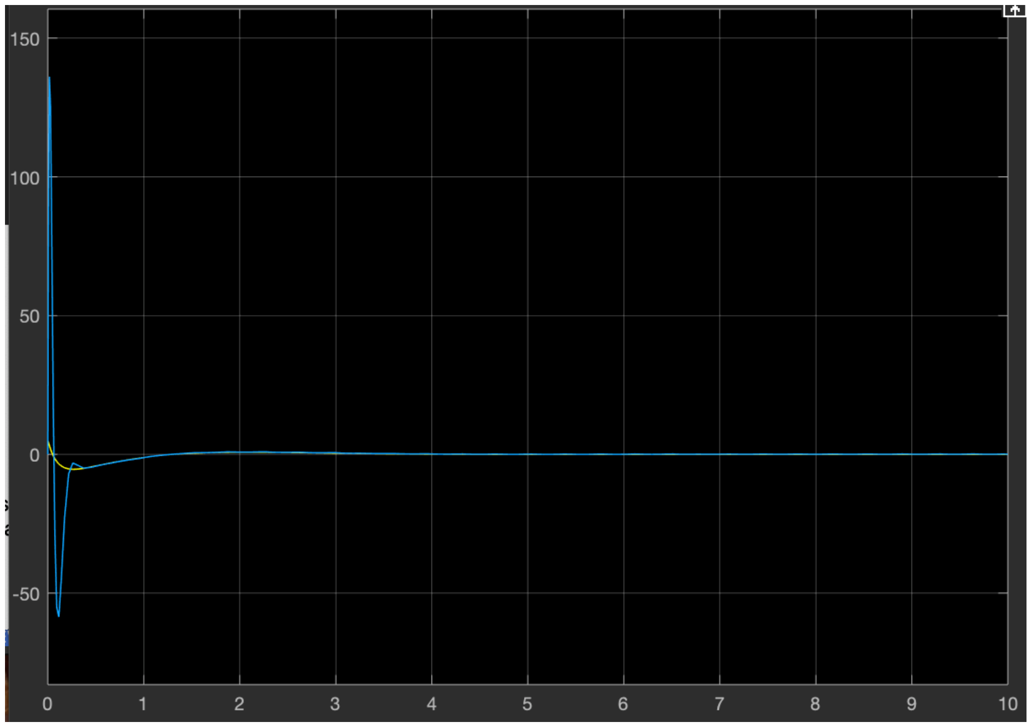
x1 and x̂1



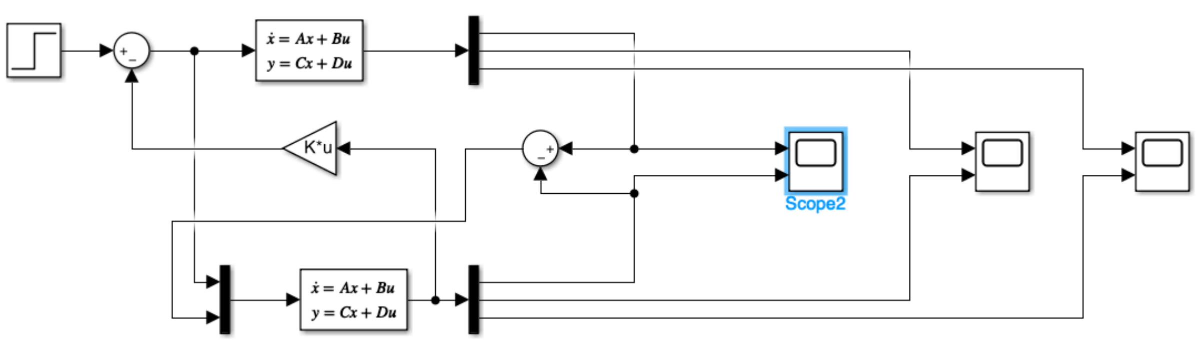
x2 and x̂2



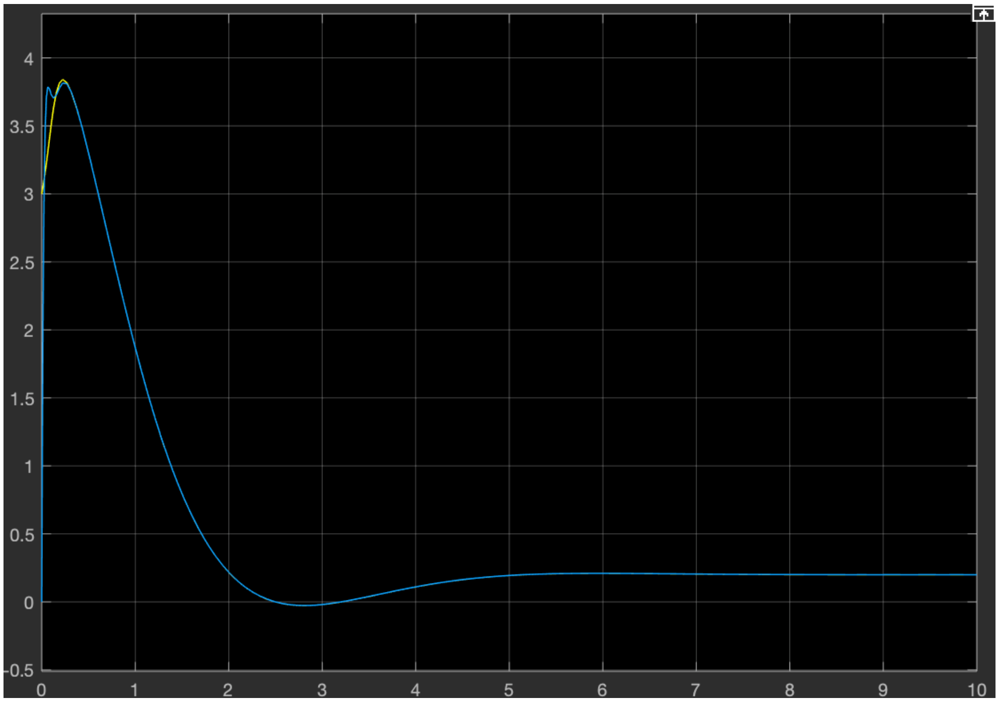
x3 and x̂3



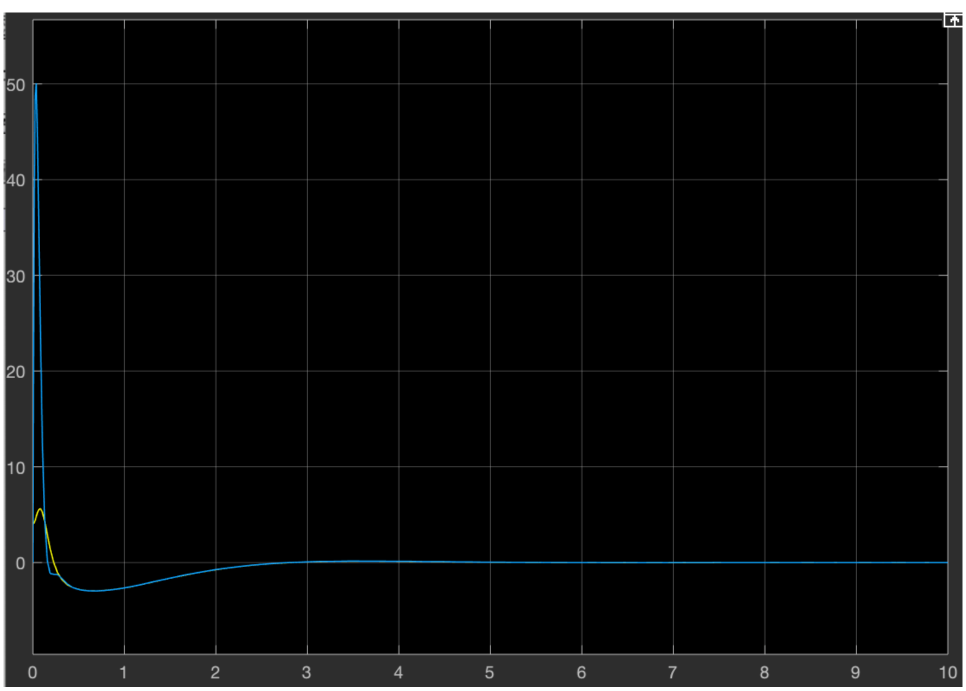
10. In the last question, you implemented the state feedback gain –Kx. Using same gains (K) and the state of the observer (xˆ), implement a state feedback gain on the estimate of the state –Kxˆ. Compare the step response of each state with its estimate.



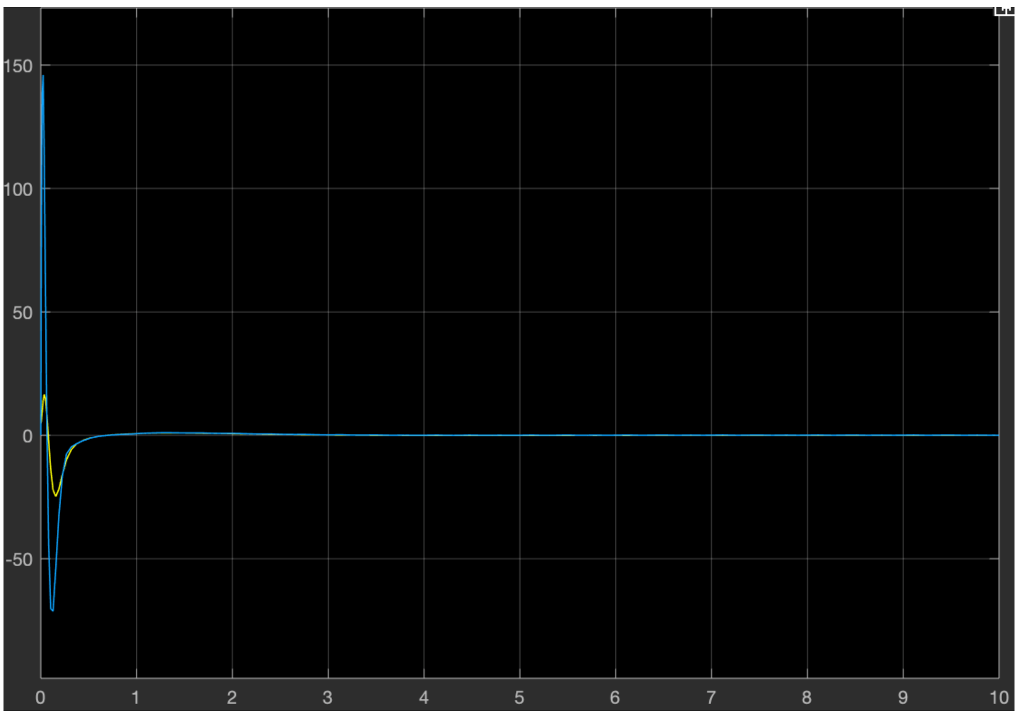
x1 and x̂1



x2 and x̂2

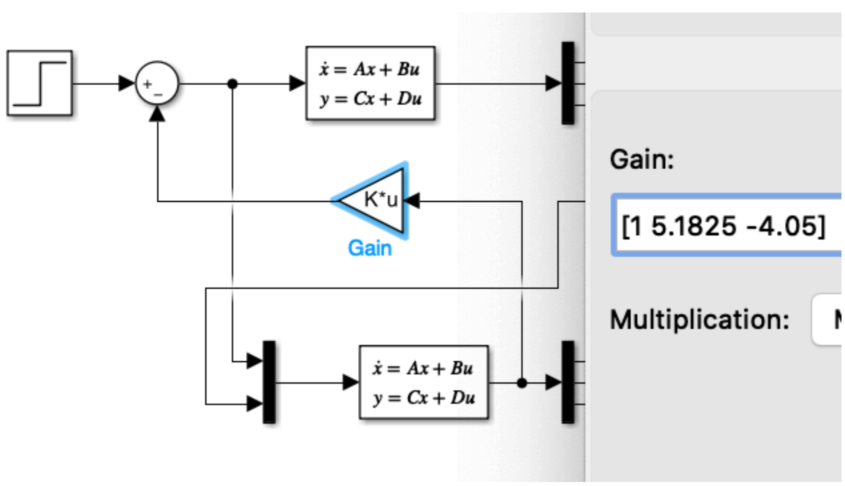


x3 and x̂3



11. Is the steady state of the output signal to the unit step is equal to 1? Why? Find the reference gain which fixes the steady state of the output. [15 marks]

The steady state of x1 is 0.2 as the feedback for x1 is 5. For unity gain, the K1 value should be set to 1.



$x1$ and $\hat{x}1$