Lab Assignment8

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ECSE 403

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1. Consider the state space representation of the inverted pendulum system in Question 6 of Lab 7. The first approach is to design the state feedback such that two dominant poles control the behavior of the system. To do that assume $\zeta = 0.4$ and $\omega_n = 2.4$ and find place of the dominant poles using:

$$P_{D_1,D_2} = -\zeta.\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$$

Choose the non-dominant poles 10 times of the dominant poles and find the state feedback gains which moves the poles of the system to the desired poles. [10 marks]

```
km = 0.0077; %N.m amp
ke = 0.0077;%N.m amp
mp = 0.106;  %kg
mc = 0.526 ; %kg
lp = 0.168 ; %m
Ra = 2.6   ; %ohms
rg = 0.0064 ;%m
kg = 3.7    ;%kg
g  = 9.8    ; %m/s2

x = -(kg*kg*km*ke)/(mc*rg*rg*Ra);


A = [0 1 0 0 ; 0 x -mp*g/mc 0 ; 0 0 0 1 ; 0  -x/lp g/lp 0]
B = [0 ; (kg*km)/(mc*rg*Ra) ; 0 ; -(kg*km)/(rg*Ra*mc*lp)]
C = [1 0 0 0 ; 0 0 1 0]

damp_coef = 0.4;
wn = 2.4;
Pd1 = -damp_coef*wn + 1i*wn*sqrt(1-damp_coef^2)
Pd2 = -damp_coef*wn - 1i*wn*sqrt(1-damp_coef^2)
Pd3 = 10*Pd1
Pd4 = 10*Pd2

%Pd3 = 10*real(Pd1)
%Pd4 = 10*real(Pd2)

poles = [Pd1 Pd2 Pd3 Pd4];
K = place(A,B,poles)
```

```
A = 4×4
         0    1.0000         0         0
         0  -14.4899   -1.9749         0
         0         0         0    1.0000
         0   86.2495   58.3333         0

B = 4×1
         0
    3.2550
         0
  -19.3751

C = 2×4
     1     0     0     0
     0     0     1     0
```

```
Pd1 = -0.9600 + 2.1996i
Pd2 = -0.9600 - 2.1996i
Pd3 = -9.6000 + 21.9964i
Pd4 = -9.6000 - 21.9964i

K = 1×4
  -21.8832  -12.4754  -38.6159   -2.4381
```

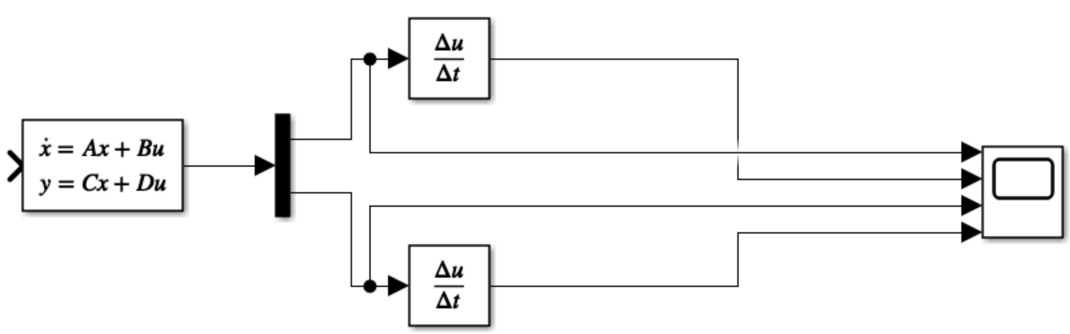
2. Follow the steps in Q1 but this time, place the non-dominant poles 100 times of the dominant poles. By looking at the state feedback matrix explain the drawback in having large non-dominant poles with large absolute values. [5 marks]

```
Pd3 = 100*Pd1
Pd4 = 100*Pd2
poles = [Pd1 Pd2 Pd3 Pd4];
K = place(A,B,poles)
```

```
Pd3 = -9.6000e+01 + 2.1996e+02i
Pd4 = -9.6000e+01 - 2.1996e+02i
K = 1×4
10^3 ×
   -2.1883   -0.7412   -3.3629   -0.1338
```

Increasing the magnitude of the non dominant poles increases the values of the feedback gain matrix. The drawback of this is that the trajectories of the states will remain bounded and won't be able to reach other state values.

3. Implement the state space model of the pendulum in the Simulink(choose matrices A,B,C,D in the Simulink model similar to lab 7 Question 6). Using the relation between outputs and states of the system, construct all the states of the system from outputs of the system. By setting the input to 0, observe the trajectory of all states of the system to $\theta_0 = 0.05rad$ as initial condition of the system. Is the open loop system stable? Does non-zero initial condition in θ_0 affects responses of other states of the system? Why? [10 marks]



A:

[0 1 0 0 ; 0 -14.4899 -1.9749 0 ; 0 0 0 1 ; 0 86.2495 58.3333 0]

B:

[0 ; 3.255 ; 0 ; -19.3751]

C:

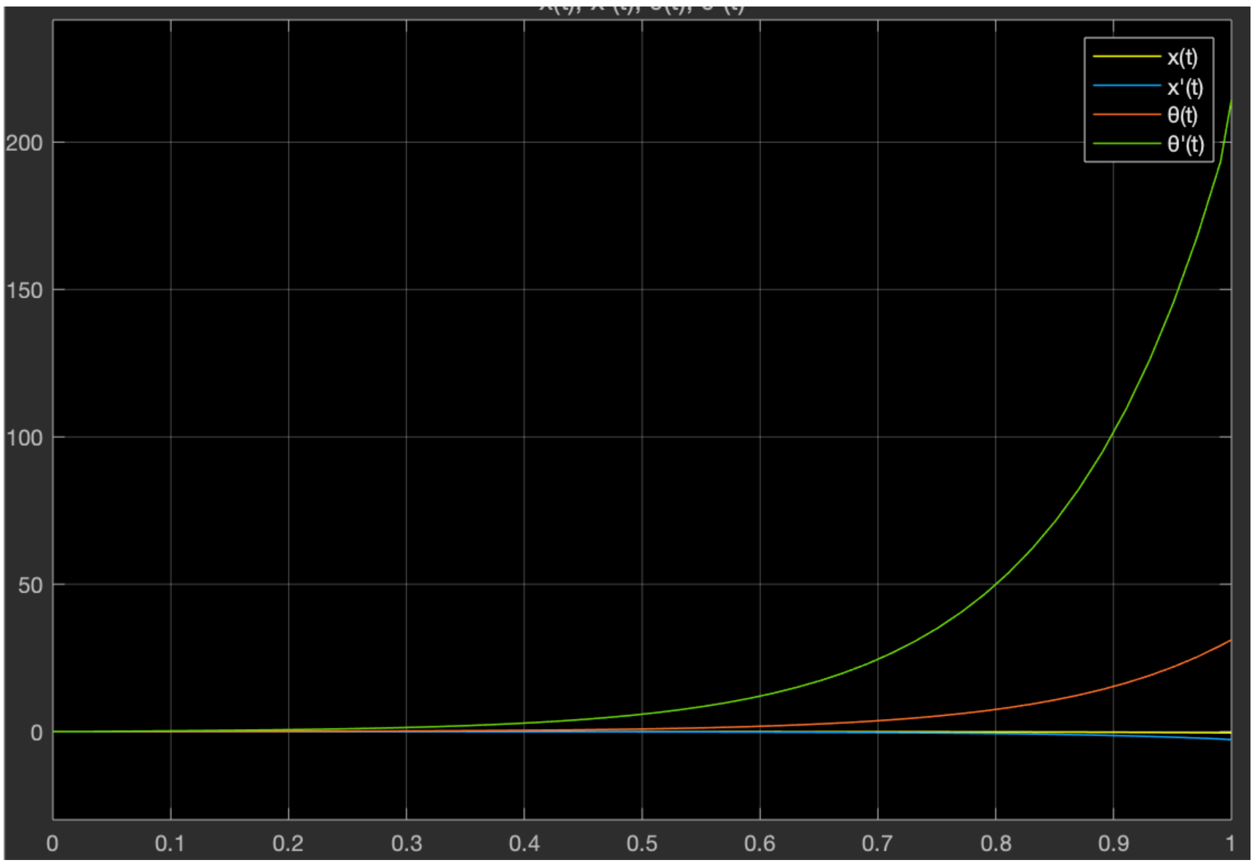
[1 0 0 0 ; 0 0 1 0]

D:

[0;0]

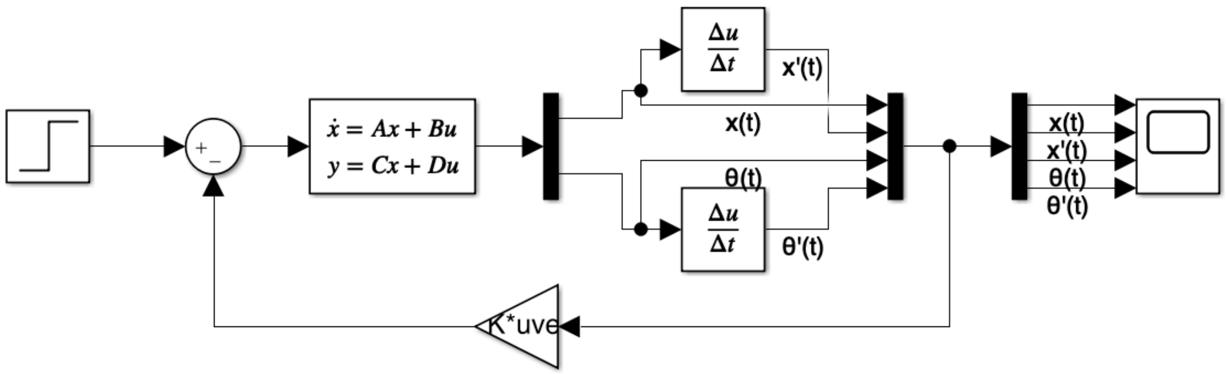
Initial conditions:

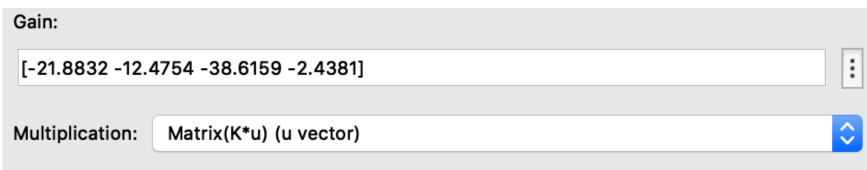
[0,0,0.05,0]

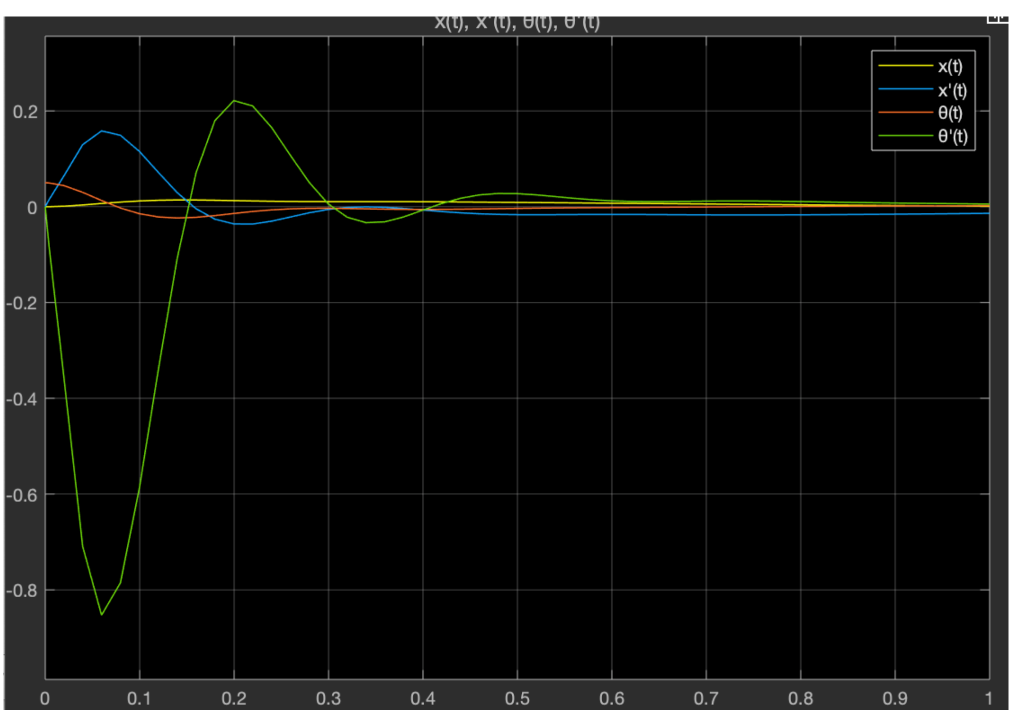


As there is no state feedback, the angle of the inverted pendulum does not come back to an equilibrium state of 0, so the system is unstable as seen by the trajectories of theta and theta' above. The non-zero initial condition of θ(0) affects the other state variables. This is obvious at first because the A matrix is non diagonal, so non theta states also depend on theta. Naturally, an initial angle on the pendulum will affect future values of angular velocity. The non zero theta initial condition also minimally affects the displacement and velocity because of conservation of momentum.

4. Implement the state feedback using the gains in Question 1. Observe the states of the system witl



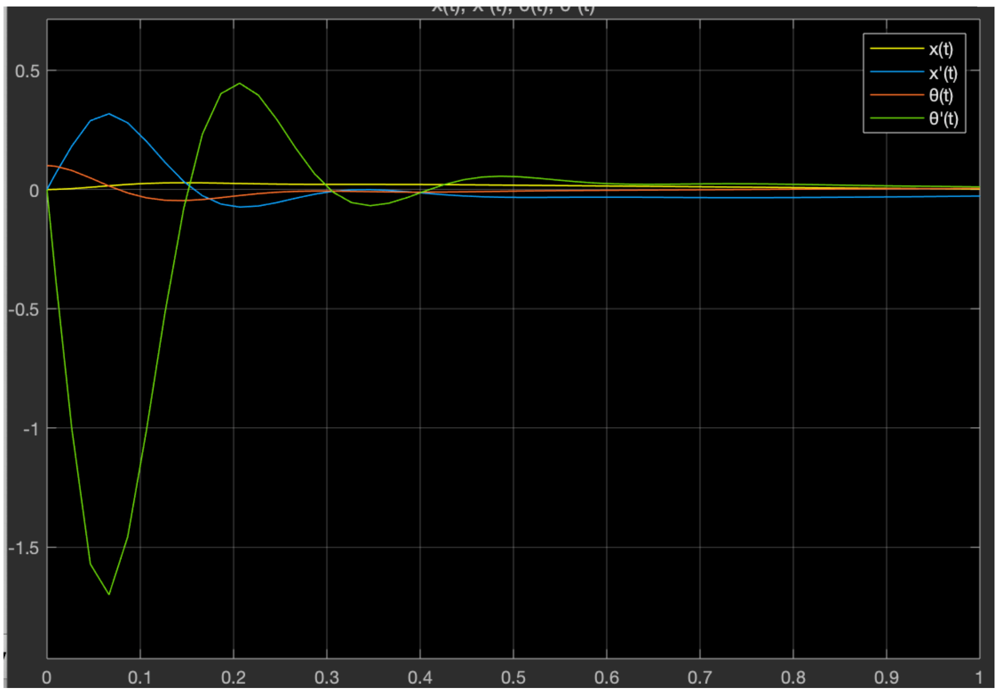




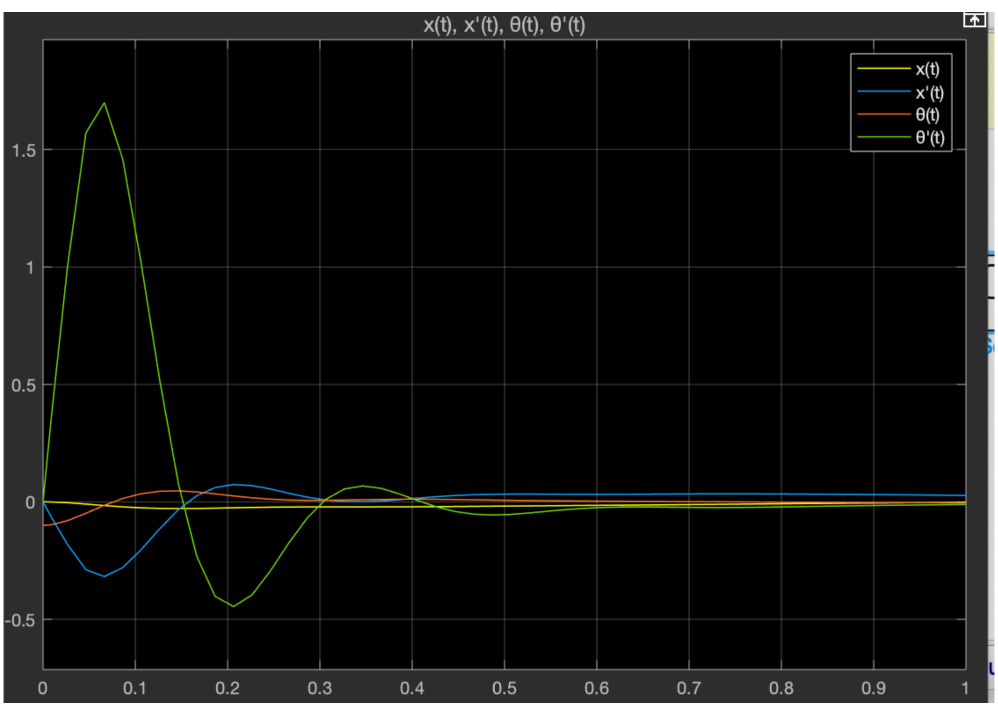
The system is stable due to the feedback loop.

5. Suppose we consider the linearized model valid as long as the angle of the pendulum satisfies $|\theta| \leq 0.1rad$. Find the range of initial conditions for θ_0 for which controller can stabilize the pendulum while the linearized model is considered valid.(exact range is not required, only write a few positive and negative cases) [10 marks]

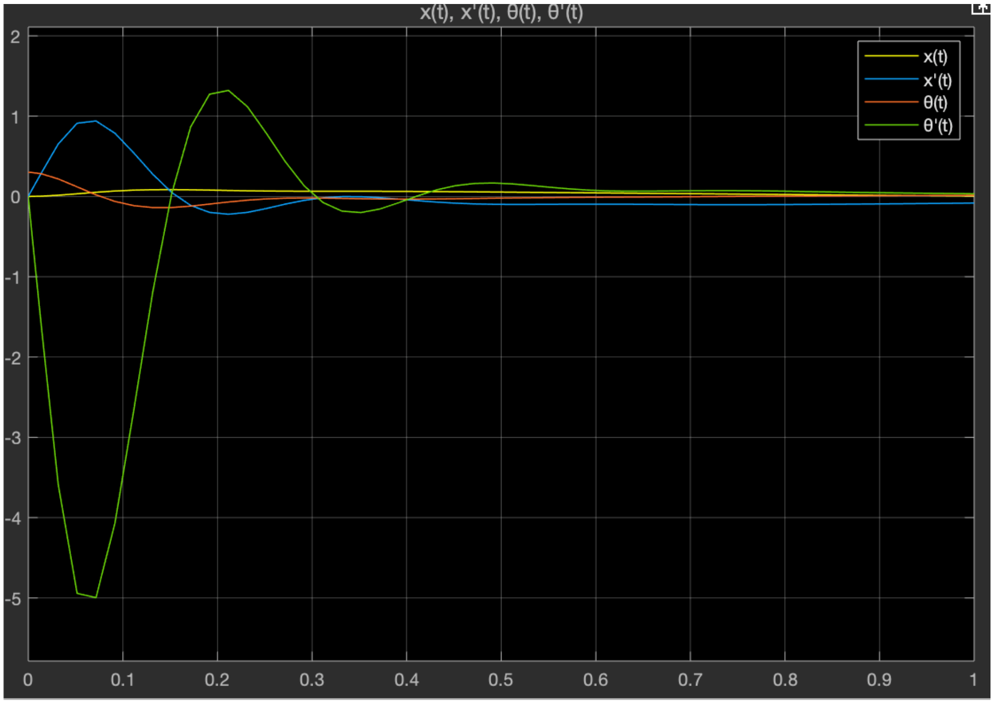
θ = 0.1



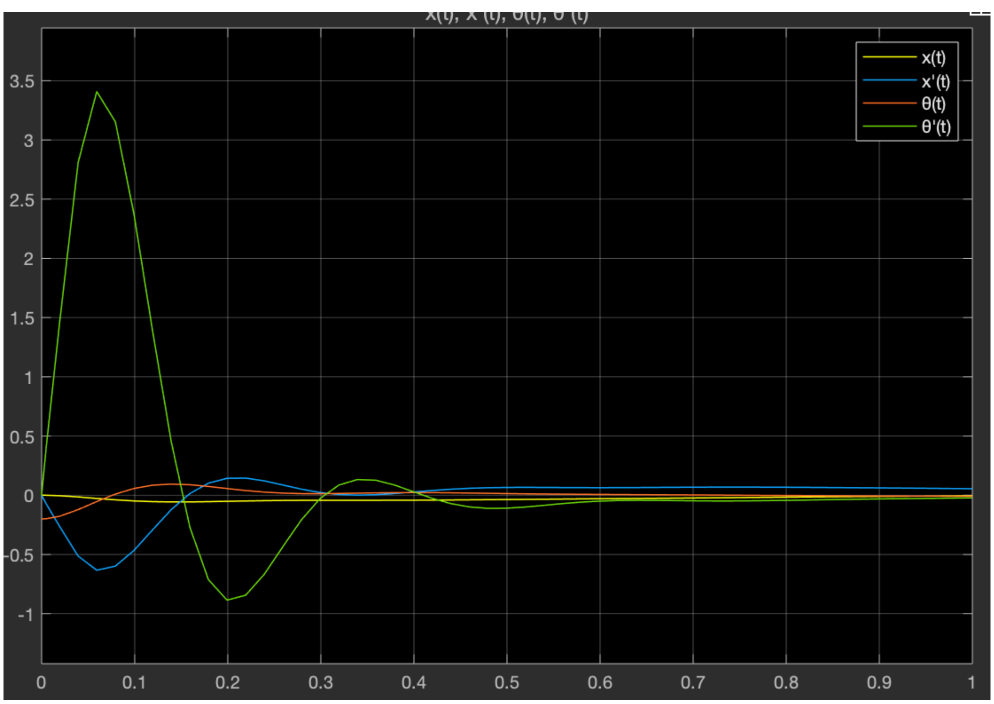
θ = - 0.1



θ = 0.3



θ = -0.2

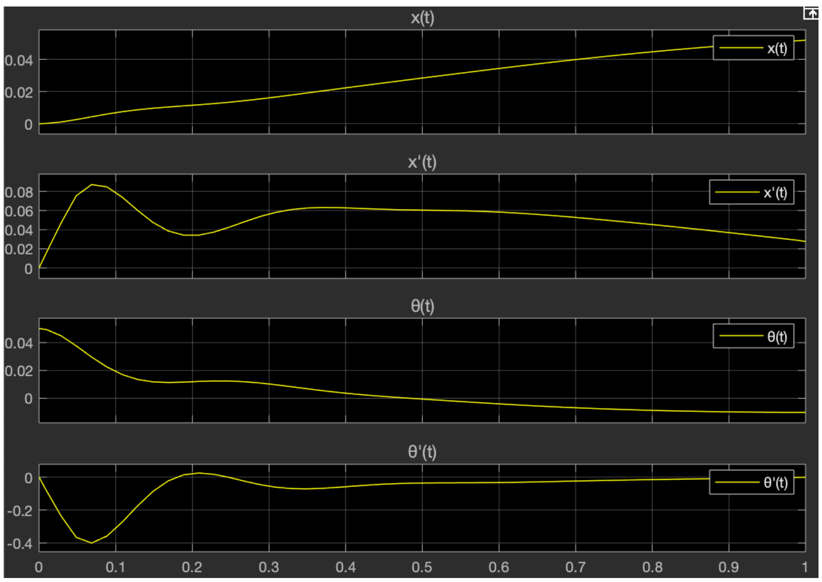


The range of theta IC for which the linearized model is considered valid is the small angle limit $|\theta| \leq 0.1$rad.

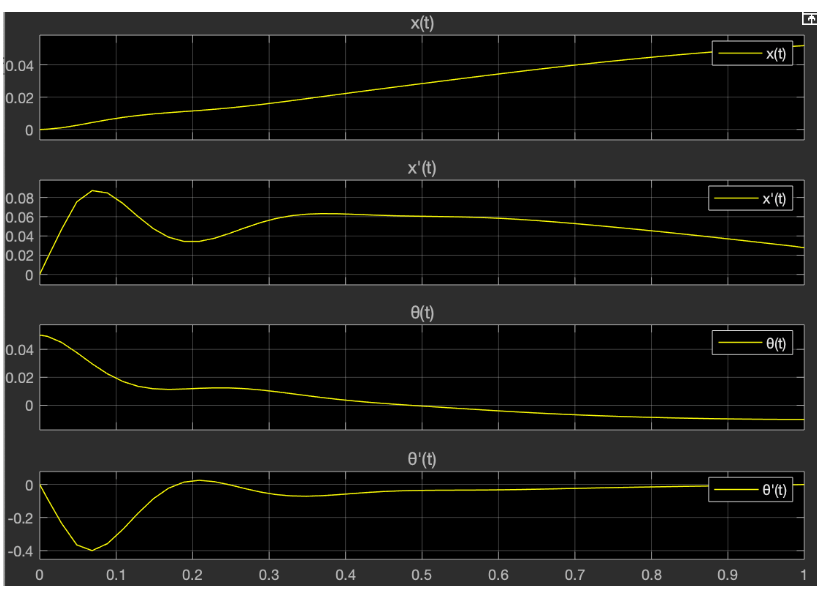
6. Suppose we consider the linearized model valid as long as the angle of the pendulum satisfies $|\theta| \leq 0.1rad$. Suppose a square-wave signal is applied to the system as the input. By experimenting different amplitudes and frequencies, find a range for these two parameters for which controller can stabilize the pendulum while the linearized model is considered valid.(exact range is not required, only write a few positive and negative cases)[10 marks]

For all cases below. θ = 0.05.

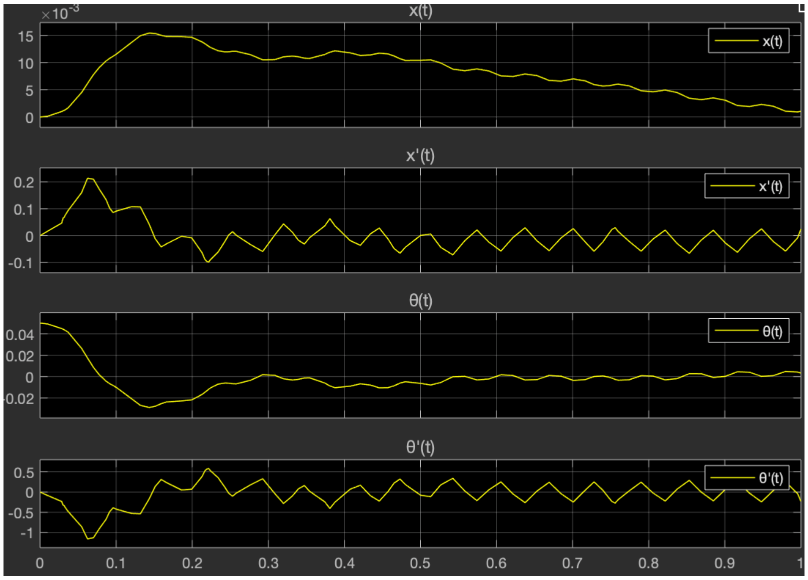
Case 1: Amplitude = 1, Frequency = 0.1



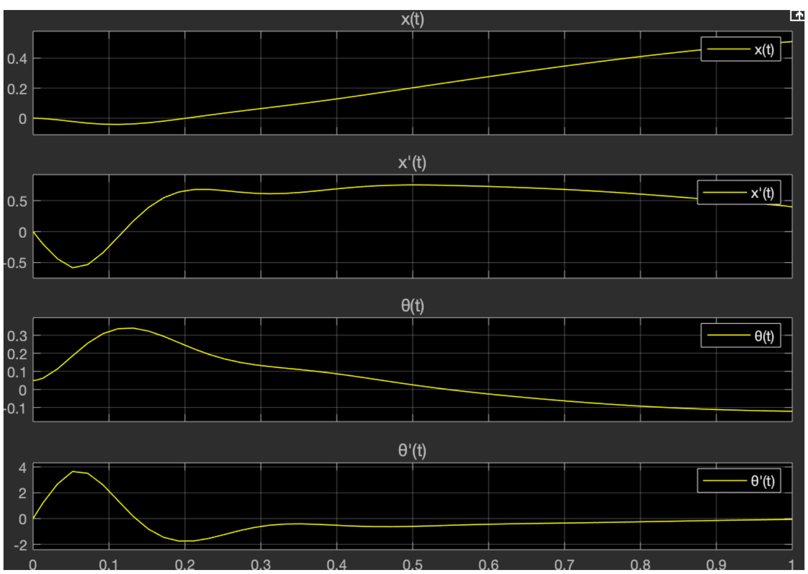
Cause 2: Amplitude = 1, Frequency = 1



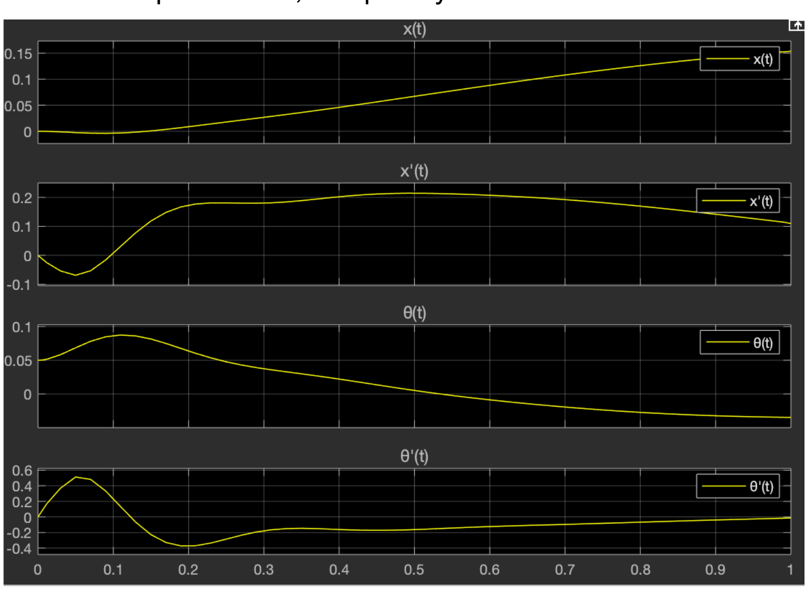
Case 3: Amplitude = 1, Frequency = 100



Case 4: Amplitude = 10, Frequency = 1



Case 5: Amplitude = 3, Frequency = 1

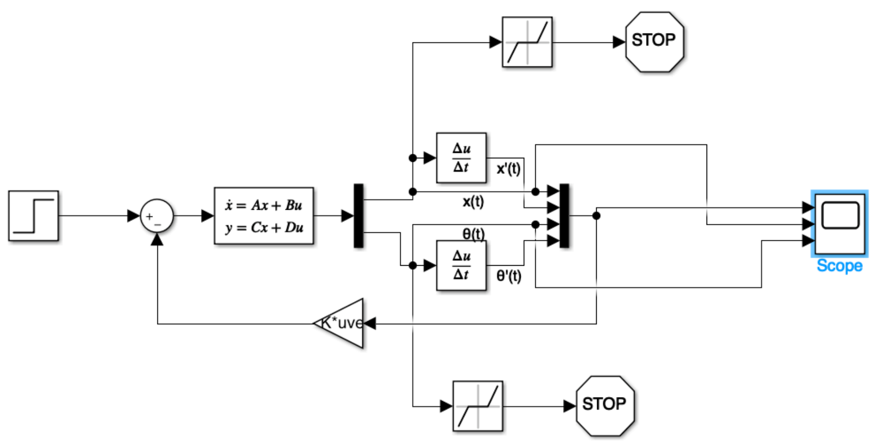


Varying the frequency appears to have no effect on the stabilization of the pendulum using the linearized model.

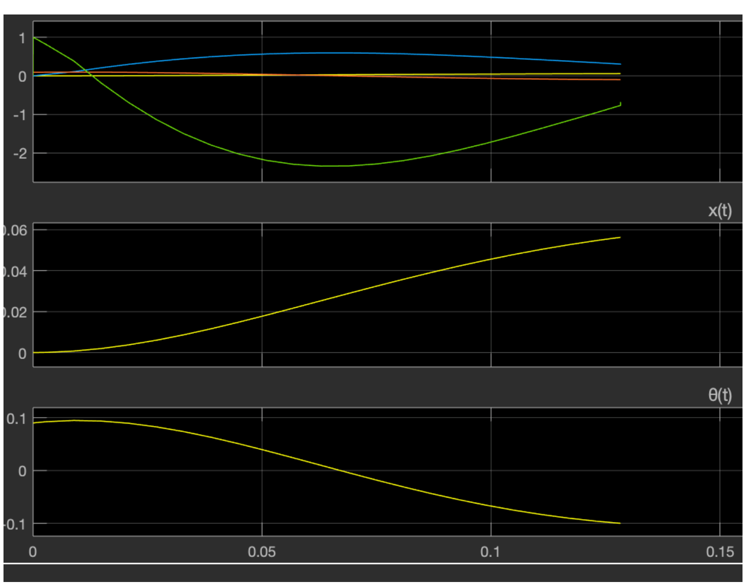
Varying the amplitude does however have an effect. When the amplitude is greater than 3, the the pendulum swings beyond 0.1 rad, making the linearized model invalid for stabilizing the pendulum.

7. Using the *Stop Simulation* block from *Sink* library and *Dead Zone* from *Discontinuities* library, design a mechanism which stops the simulation when the system is out of valid linear area. Suppose the cart position has maximum range of $1m$. Design similar safety procedure to avoid exiting maximum range.(ignore momentum of the cart for the second part)[10 marks]

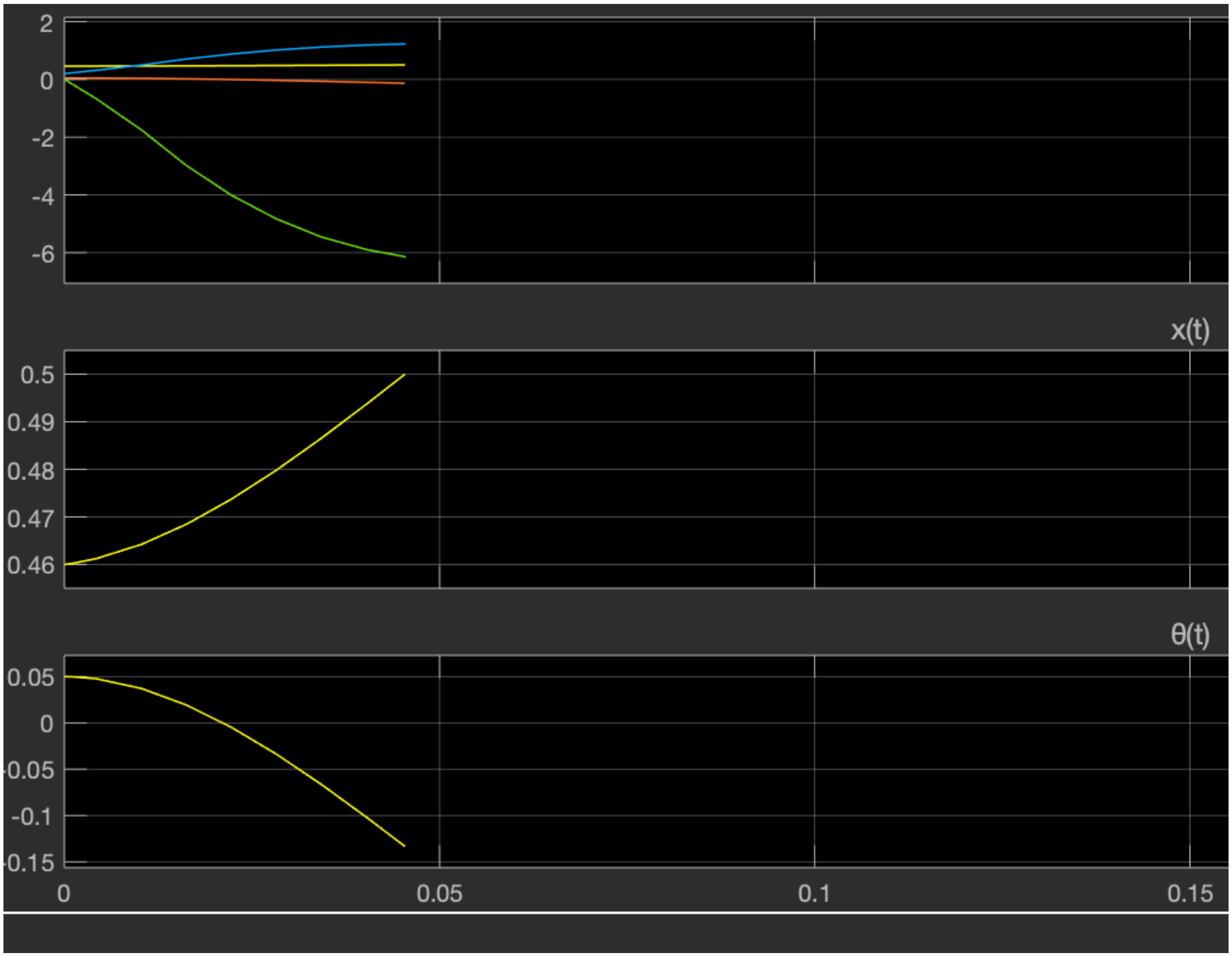
Dead zone for |θ| > 0.1 and for |x| > 0.5



θ(0) = 0.09, θ'(0) = 0.09, u = 1



$\theta(0) = 0.05$; $x(0) = 0.45$, $v(0) = 0.15$, $u = 1$



As seen in the first set of trajectories above, once theta reaches 0.1, the trajectory is discontinued. Likewise, when the displacement reaches 0.5, the trajectories are discontinued.

8. Define stabilization of the inverted pendulum problem as an LQR problem. Explain which element of Q corresponds to the cost of deviation of pendulum from $0°$. Explain intuitively the consequence of $R = 0$.[10 marks]

From solution of lab 7:

$$\frac{d}{dt}\begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{k_e k_m k_g^2}{r_g^2 m_c R_a} & -\dfrac{m_p g}{m_c} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{k_e k_m k_g^2}{r_g^2 m_c l_p R_a} & \dfrac{g}{l_p} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \dot{x}_c \\ \theta_p \\ \dot{\theta}_p \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{k_g k_m}{m_c r_g R_a} \\ 0 \\ \dfrac{-k_g k_m}{m_c l_p r_g R_a} \end{bmatrix} v_a$$

We define stabilization with optimal control u minimizing $J(x_0, u)$

$$LQR(LSS)\colon\ J(x0,\ u)\ = min\ u\ J(x0, u)$$

$$LQR(LSS)\colon\ J(x0,\ u)\ =\ min\ u\ (\frac{1}{2}\int_0^{\tau} (xT(t)\ Q\ (x(t))\ +\ uT(t)Ru(t))dt\ +\ \frac{1}{2}xT(\tau)Q\tau x(\tau)$$

The third element in the third row (Q33) corresponds to the cost of deviation from theta = 0. If we set R = 0, the value of Q will grow fast in order to get x(t) to converge.

9. Using *lqr* command in Matlab, find the optimal state feedback gains for the linear quadratic regulator problem for

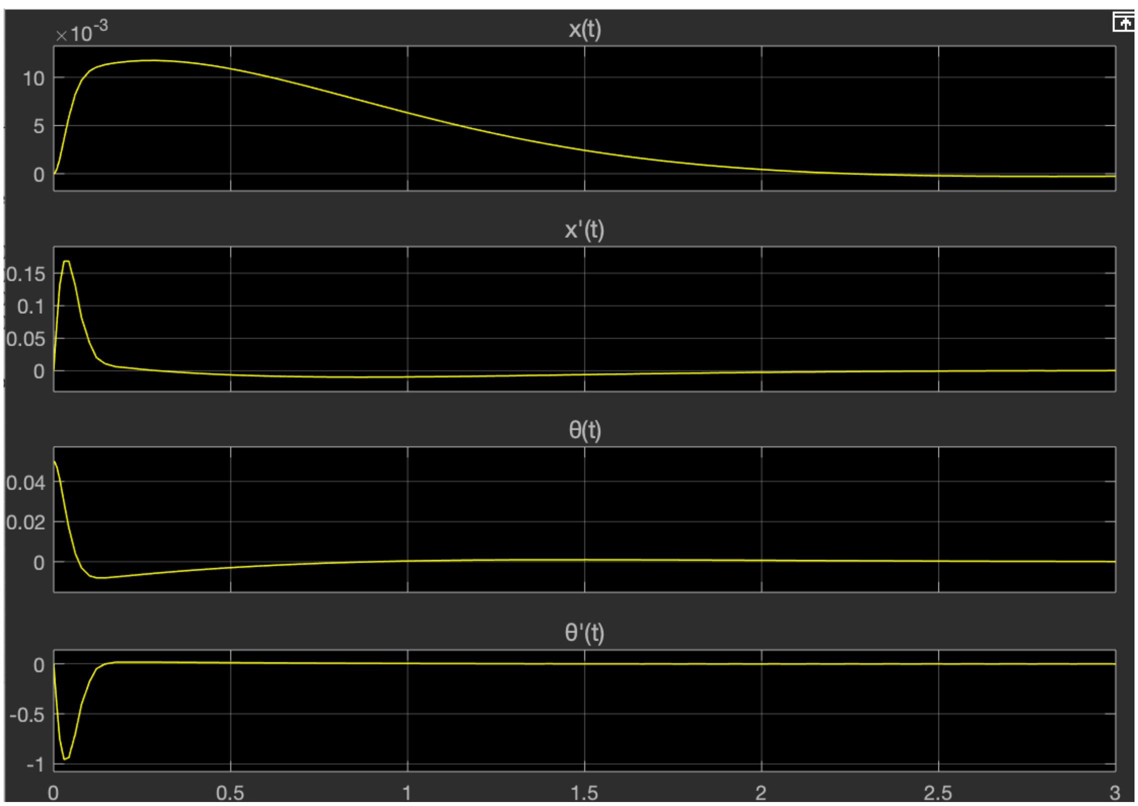
$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 5000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = 1$$

and implement it in the Simulink. Observe zero input response of the system and initial condition $\theta_0 = 0.05$. [10 marks]

```
Q1 = [1000  0   0   0;
        0   0   0   0;
        0   0  5000 0;
        0   0   0   0;];

R1 = 1;
K = lqr(A, B, Q1, R1)
```

```
K = 1×4
   -31.6228  -31.0047  -87.7942   -7.4732
```



10. Only considering diagonal Q matrices, try following combinations of Q matrices and repeat steps of Question 9:

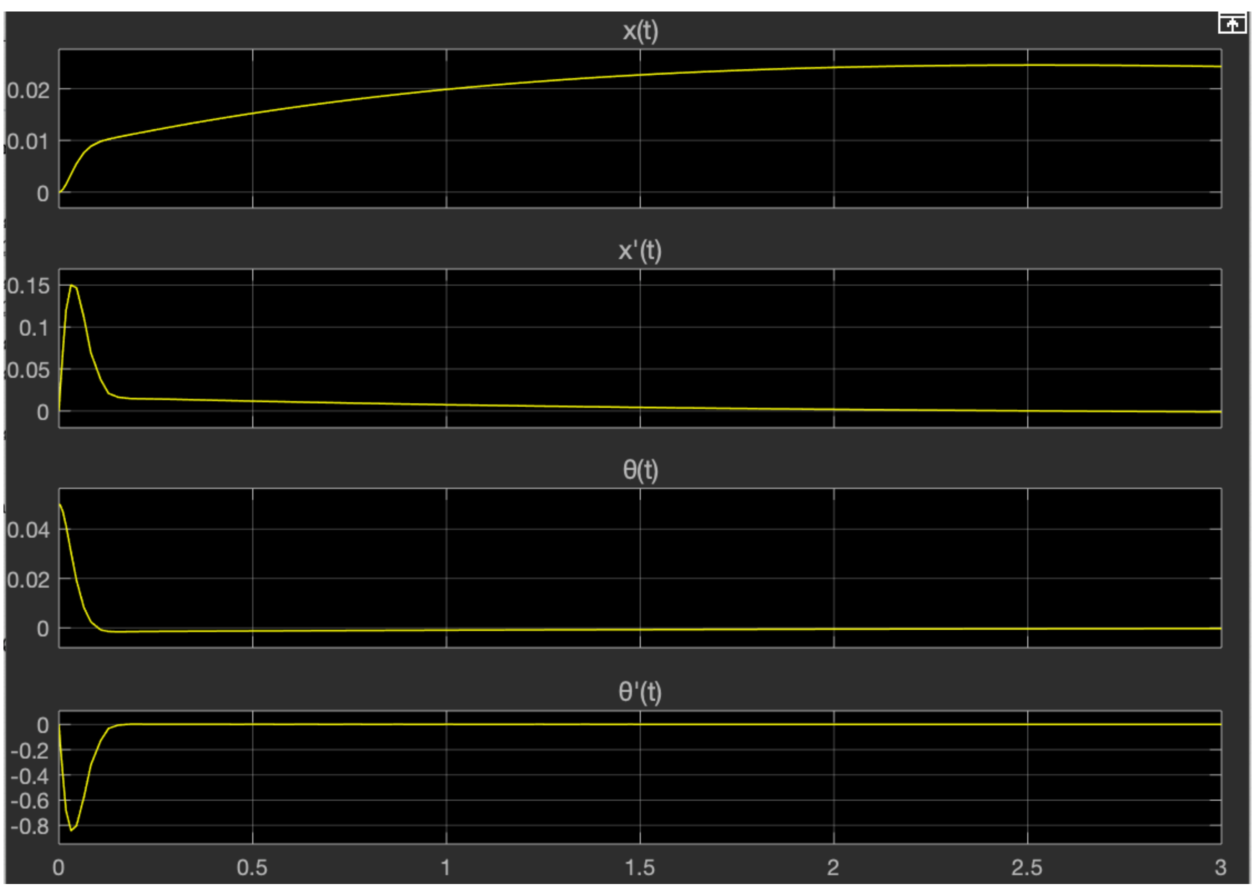
$$Q_{11} = 1, Q_{22} = 0, Q_{33} = 5000, Q_{44} = 0$$
$$Q_{11} = 100, Q_{22} = 0, Q_{33} = 5000, Q_{44} = 0$$
$$Q_{11} = 1, Q_{22} = 1, Q_{33} = 5000, Q_{44} = 0$$
$$Q_{11} = 1, Q_{22} = 100, Q_{33} = 5000, Q_{44} = 0$$
$$Q_{11} = 1, Q_{22} = 0, Q_{33} = 5000, Q_{44} = 1$$
$$Q_{11} = 1, Q_{22} = 0, Q_{33} = 5000, Q_{44} = 100$$
$$Q_{11} = 1, Q_{22} = 0, Q_{33} = 5, Q_{44} = 0$$
$$Q_{11} = 1, Q_{22} = 0, Q_{33} = 5000, Q_{44} = 0$$

Explain intuitively the effect of each of the diagonal elements of the Q by comparing consecutive pairs. [15 marks]

```
Q =     [1    0    0    0;
         0    0    0    0;
         0    0   5000 0;
         0    0    0    0;];

R = 1;
K = lqr(A, B, Q , R)
```

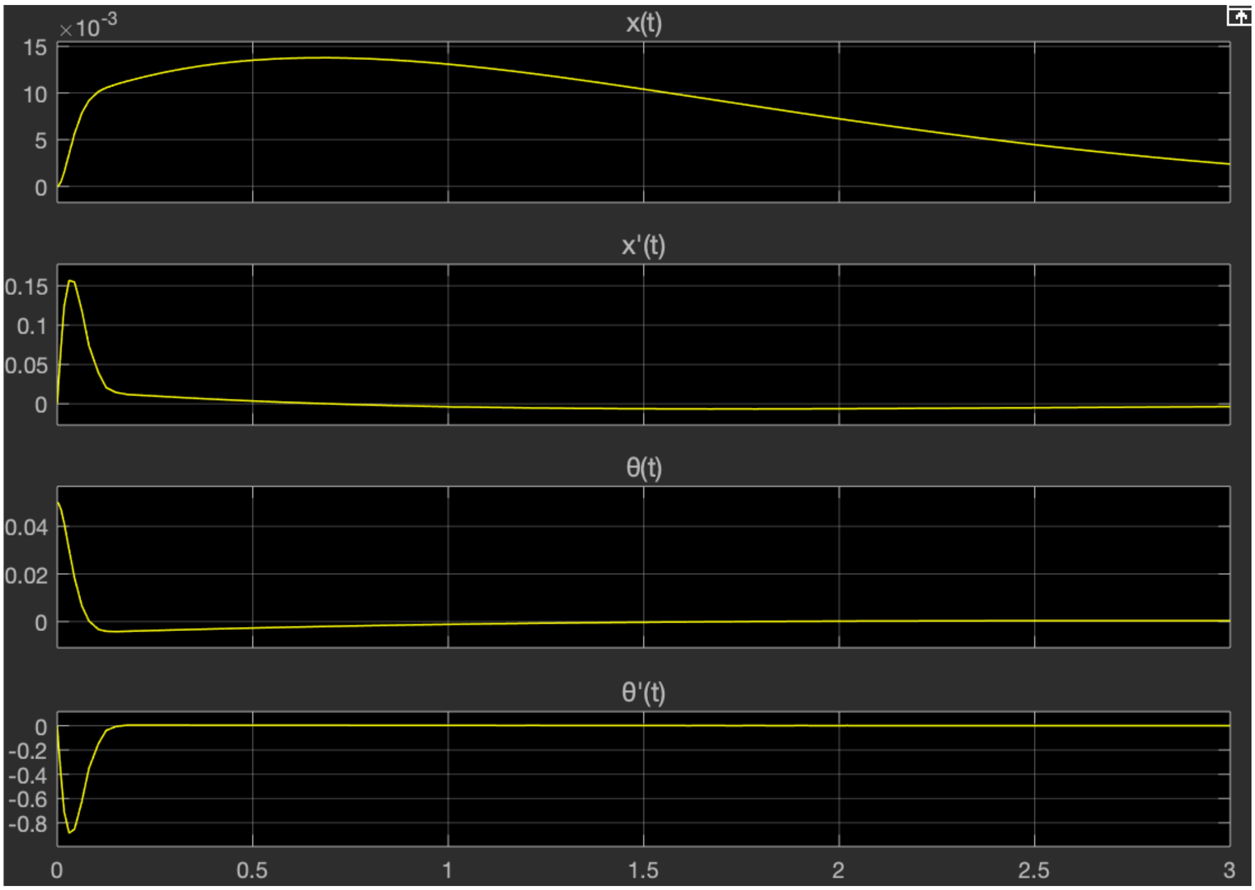
```
K = 1×4
     -1.0000   -10.6597   -76.2693    -3.9438
```



```
Q =   [100   0   0    0;
         0   0   0    0;
         0   0   5000 0;
         0   0   0    0;];

R = 1;
K = lqr(A, B, Q , R)
```

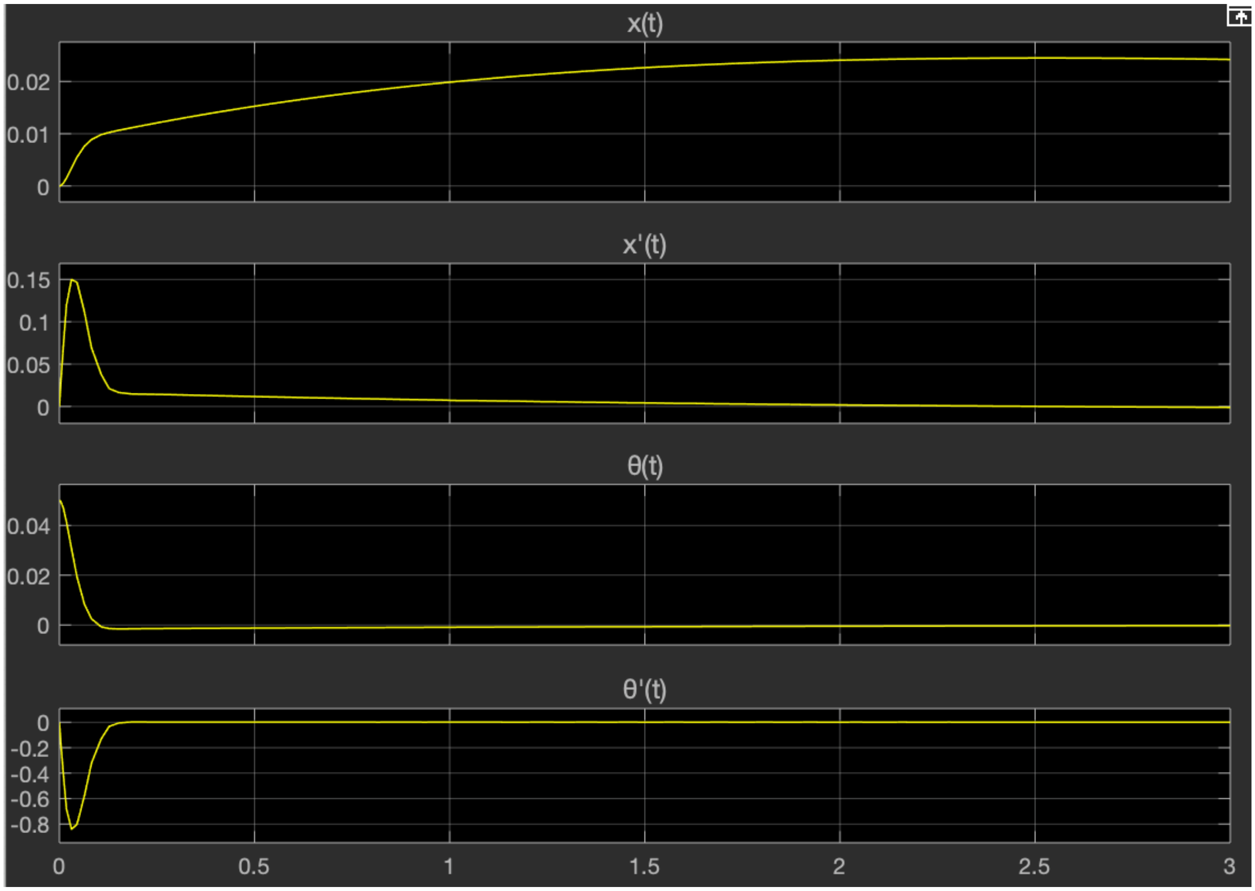
```
K = 1×4
    -10.0000   -19.2060   -80.4307    -5.4263
```



```
Q =     [1    0    0    0;
         0    1    0    0;
         0    0   5000 0;
         0    0    0    0;];

R = 1;
K = lqr(A, B, Q , R)
```

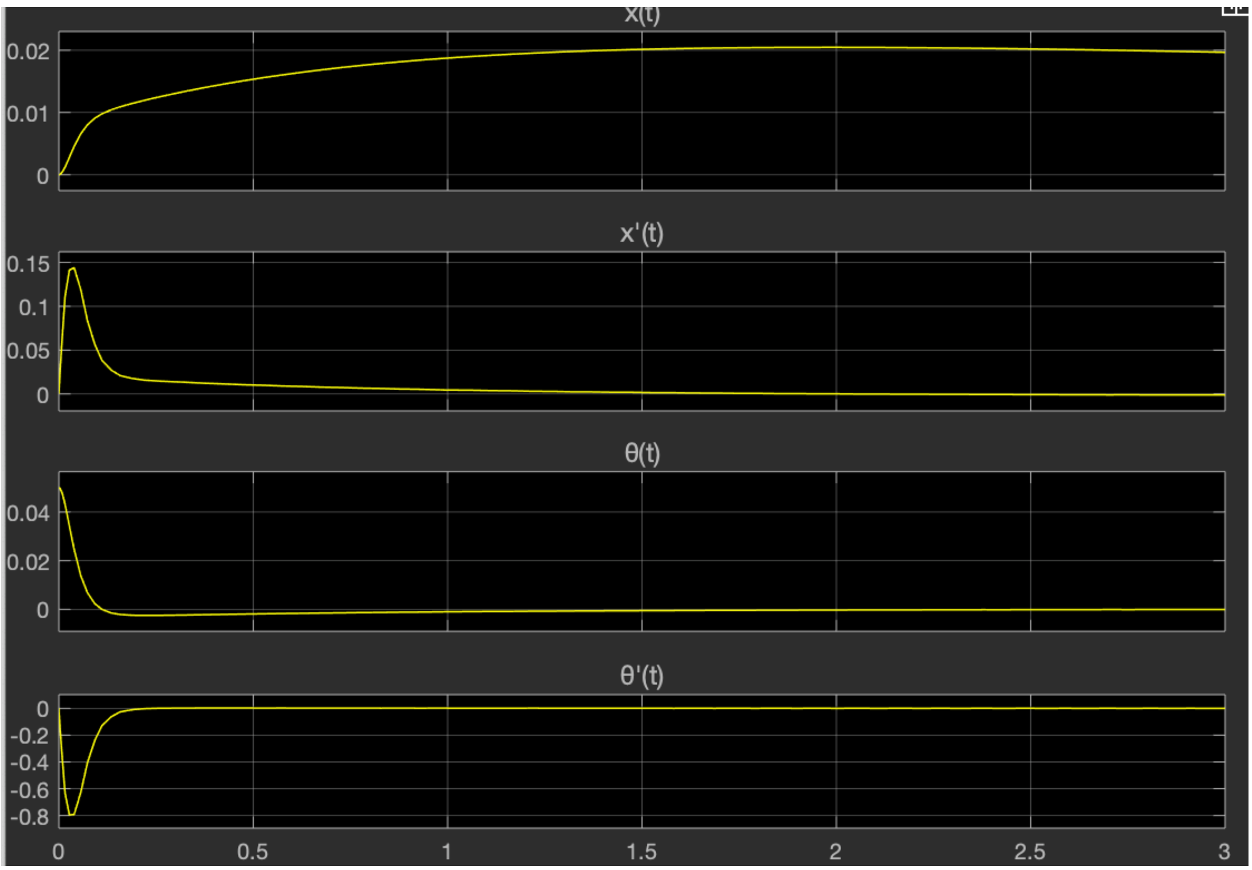
```
K = 1×4
     -1.0000   -10.7406   -76.3162    -3.9631
```



```
Q =     [1   0    0    0;
         0   100  0    0;
         0   0    5000 0;
         0   0    0    0;];

R = 1;
K = lqr(A, B, Q , R)
```

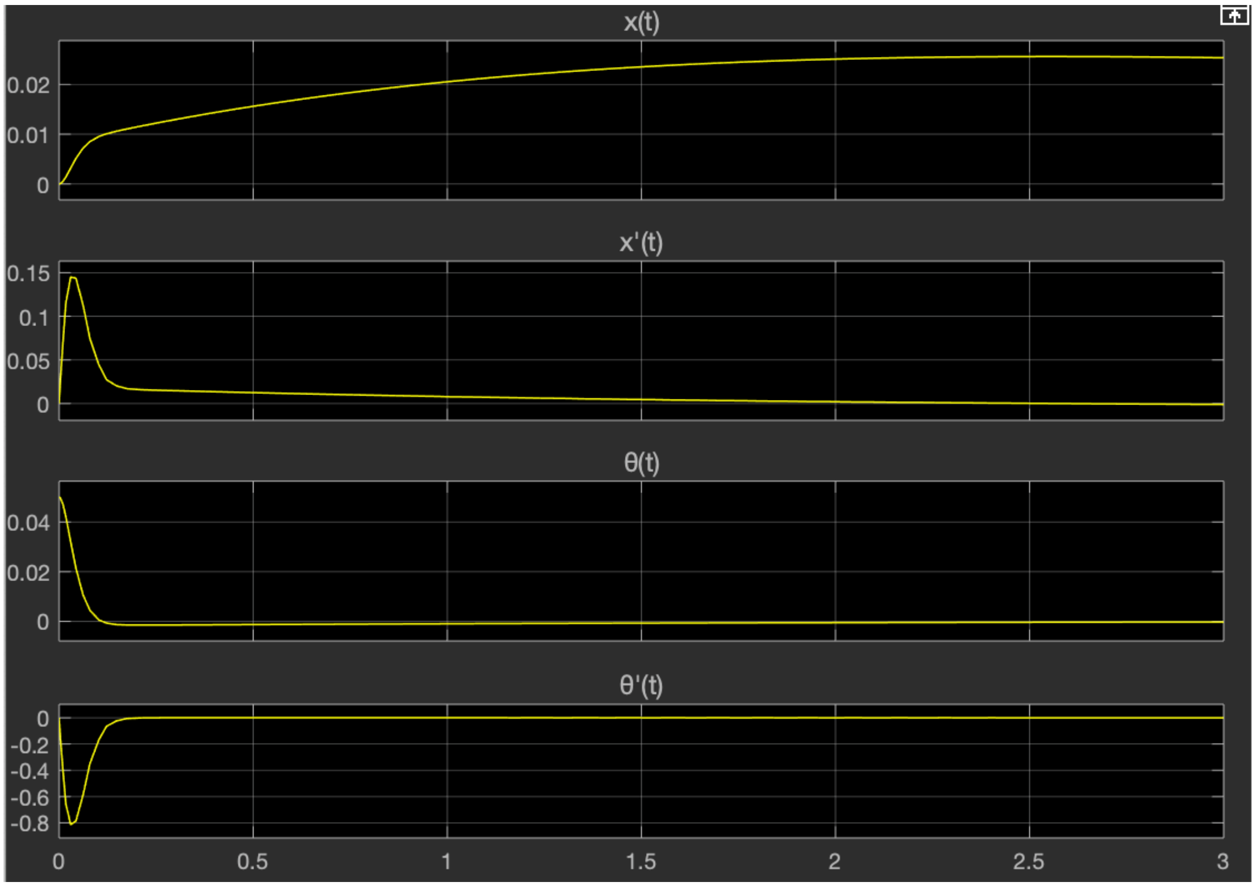
```
K = 1×4
   -1.0000  -16.2660  -80.3450   -5.3992
```



```
Q =     [1   0   0   0;
         0   0   0   0;
         0   0   5000 0;
         0   0   0   1;];

R = 1;
K = lqr(A, B, Q , R)
```

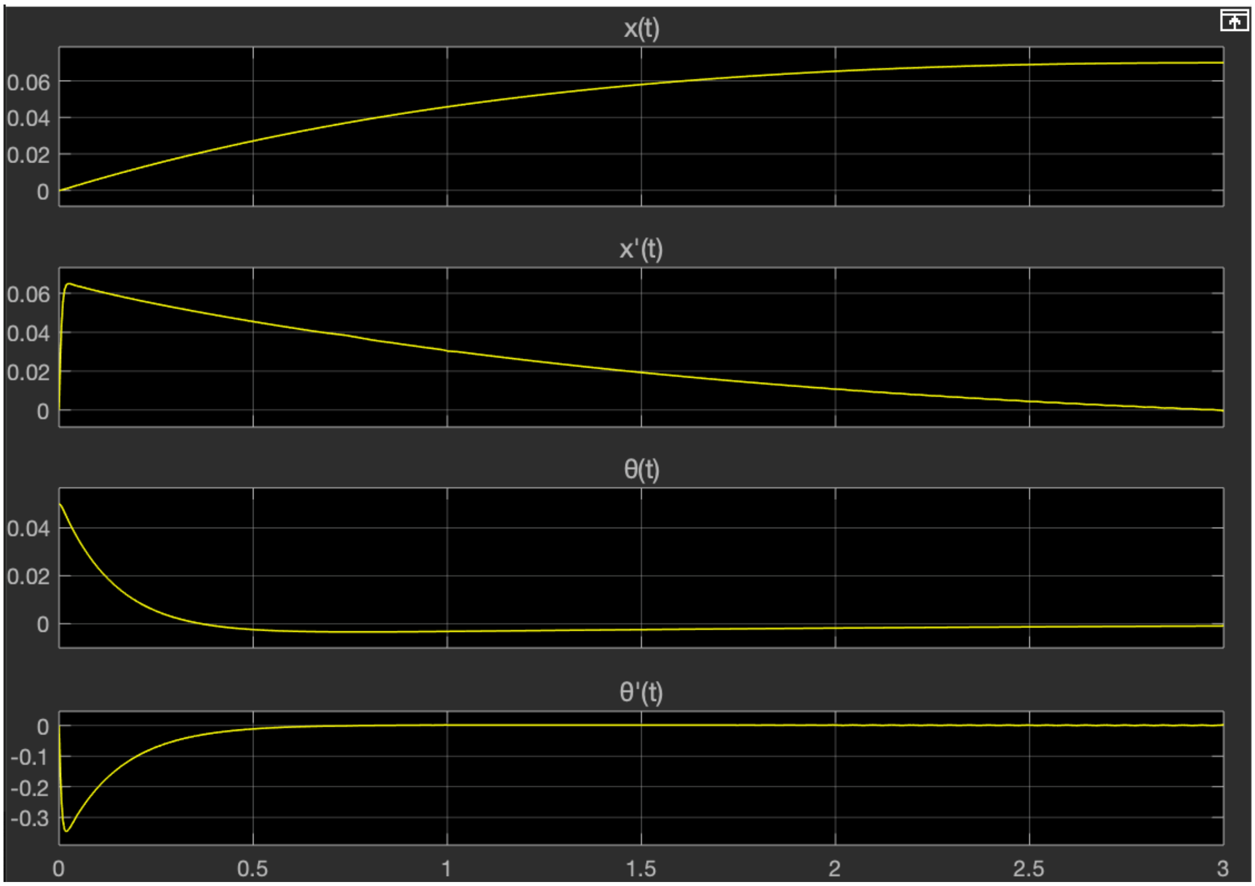
```
K = 1×4
     -1.0000   -10.6620   -76.3826   -4.1136
```



```
Q =     [1    0    0    0;
         0    0    0    0;
         0    0   5000 0;
         0    0    0   100;];

R = 1;
K = lqr(A, B, Q , R)
```

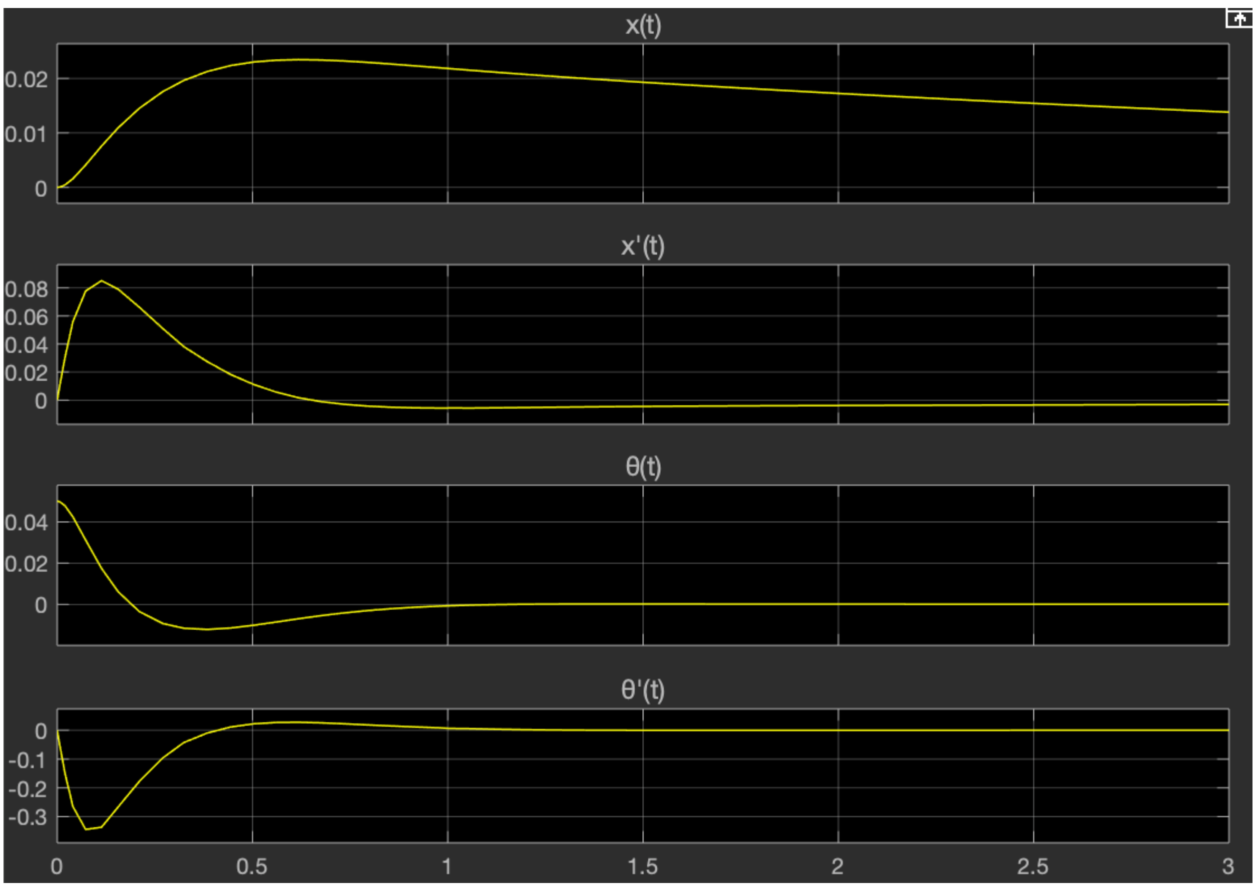
```
K = 1×4
     -1.0000   -10.7608   -81.2231   -11.4967
```



```
Q =     [1   0   0   0;
          0   0   0   0;
          0   0   5   0;
          0   0   0   0;];

R = 1;
K = lqr(A, B, Q , R)
```

```
K = 1×4
     -1.0000   -9.2760  -16.5442   -2.3104
```



If we increase Q11, then x will converge to the origin quicker, as Q11 corresponds to the weighting of x in the cost function.

If we increase Q22, the x will convene to the origin more slowly, as Q22 corresponds to the weighting of x’ in the cost function.

If we increase Q33, the angle theta will converge to the origin quicker, as Q33 corresponds to the weighting of theta in the cost function.

If we increase Q44, the angle theta will converge to the origin more slowly as Q44 corresponds to the weighting of theta’ in the cost function.