Control Lab Assignment 2
Instructor: Prof. Caines
ECSE 403, Fall 2020
Due October 3^{rd} 2020

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1 Objective

The main goal of this assignment is to review some concepts from the linear control course(ECSE 307), and to become familiar with some useful tools in MATLAB which helps us in designing and implementing controllers. Following concepts are covered in this assignment: response of a second order system, root-locus diagram, state space representation, pole-zero plots, and stability of uncontolled dynamics.

2 Your duty

Your duty is to answer all questions which have been asked throughout this assignment and submit all your answers in addition to MATLAB codes in MyCourses website. Please submit your work *both* in the PDF format and MATLAB executable notebook files.

3 Model Description

The final goal of this lab is to model and control an inverted pendulum system. In that system the input is applied to a DC motor connected to a cart-pole. The first step to model the whole system is to model the DC motor. We saw the (approximate) model of the DC motor in lab 1 as:

$$J_m\ddot{\theta} + (b + \frac{K_t K_e}{R_a})\dot{\theta} = \frac{K_t}{R_a} v_a$$

where θ is the shaft angle (in radians) of the motor and v_a is the applied voltage.
System's parameters are as following:

- $J_m = 0.01kgm^2$ be the inertia of the rotor and the shaft.
- $b = 0.001Nmsec$ be the viscous friction coefficient
- $K_e = 0.02Vsec$ be the back emf constant

- $K_t = 0.02 Nm/A$ be the motor torque constant.
- $R_a = 10\Omega$ be the armature resistance

Note that using SI units $K_e = K_t$.

4 Questions

4.1 Proportional Controller

1. Consider the transfer function $\frac{\theta(s)}{v_a(s)}$ in lab assignment 1 as open loop system. Suppose we are using a proportional gain with a unity feedback loop. Using the standard form of second order systems:
$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$
find the parameters ω_n, ζ of the *closed loop system* as a function of proportional controller(gain) K. [5 marks]

2. For the closed loop system in Question 1, plot the the roots of the closed loop system for proportional gains $K \in [0.01, 0.1, 1, 10, 100, 10^3]$ in *one plot* to see the movement of poles as a function of proportional gain K.[5 marks]
Hint: You can use command pzplot(sys) to plot the poles and zeros on the complex plane.

3. Explain what root locus diagram describes and plot the root locus diagram for $\frac{\theta(s)}{v_a(s)}$ as open loop function.[5 marks]
Hint: You can use command rlocus(sys) to plot the root locus diagram. Notice that this command receives the *open loop system* as *input.*

4. Using the time-domain step response of second order systems find a value of proportional gain K such that closed loop step response has approximately 20% overshoot. Verify your proportional controller using step response of closed loop system.[10 marks]
Hint: You can use the formulas required in Question 4 and 5 from any reference.

5. Ignoring the over shoot constraint in Question 4, find the proportional gain K such that closed loop step response has peak time of approximately $4s$. Verify your proportional controller using step response of closed loop system.[10 marks]

6. Suppose a controller with transfer function $\frac{s+0.9}{s+0.75}$ is added to the system. Is there a proportional gain for this system which makes it unstable?[5 marks]
Hint: You can find the gain by visual inspection of root locus diagram(theoretical derivation is not required.)

4.2 state space representation

7. The electormechanical equations of the DC motor can be described as following:

$$J_m\ddot{\theta} + b\dot{\theta} = K_t i$$
$$L_a\dot{i} + R_a i = v_a - K_e\dot{\theta},$$

where i and $L_a = 0.5H$ are the current and inductance of the armature. The transfer function in Lab 1, is an approximation of the model which is described here. We can find the exact model of a DC motor by state space representation. Describe the state space representation of the DC motor in the standard from of:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$
$$D = 0$$

with two different setups:

   (a) Variables $(\dot{\theta}, i_a)$ are assumed to be states of the system and variable $\dot{\theta}$ is assumed to be the output.[5 marks]
   (Please denote matrices corresponding to state space representation by A_1, B_1, C_1.)

   (b) Variables $(\theta, \dot{\theta}, i_a)$ are assumed to be states of the system and variable θ is assumed to be the output.[5 marks]
   (Please denote matrices corresponding to state space representation by A_2, B_2, C_2.)

8. Consider the state space model (A_1, B_1, C_1). Find the transfer function(denote it by $H_1(s)$) and plot the step response.[5 marks]
   Consider transfer function of $\dot{\theta}(s)/v_a(s)$ in Lab 1(denote it by $G_1(s)$). Are transfer functions $H_1(S)$ and $G_1(s)$ exactly the same? Compare the step responses of $H_1(s)$ and $G_1(s)$, are they exactly the same?[5 marks]
   Hint : You can use the command [num,den] = ss2tf(A,B,C,D) to find the numerator and denumerator of the transfer function corresponding to state space representation of (A, B, C, D).

9. Graph the pole-zero plot for both $H_1(s)$ and $G_1(s)$. Based on these Graph explain why the approximation in Lab 1 was a "good" approximation. [5 marks]

10. Consider the state space model (A_2, B_2, C_2). Find the transfer function(denote it by $H_2(s)$) and plot the step response.[5 marks]
    Consider transfer function of $\theta(s)/v_a(s)$ in Lab 1(denote it by $G_2(s)$). Are transfer functions $H_2(S)$ and $G_2(s)$ exactly the same? Compare the step responses of $H_2(s)$ and $G_2(s)$, are they exactly the same? [5 marks]

11. Graph the pole-zero plot for both $H_2(s)$ and $G_2(s)$. Based on these Graph explain why the approximation in Lab 1 was a "good" approximation. [5 marks]

12. Consider the uncontrolled dynamics(or zero input dynamics):

$$\dot{x} = A_1 x(t)$$

Find the eigen-values and eigen-vectors of A_1. Find the response of the uncontrolled system to the eigen-values as initial conditions. Is there an initial condition which results in non exponentially stable response?[5 marks]
Hint: Command [V1,D1] = eig(A1) returns eigen-values and eigen-vectors in matrix format.
Hint: Command initial(sys,x0) plots the response of the system to the initial condition x_0.
Hint: Any exponentially stable response goes to 0 as time goes to infinity.

13. Plot the system's response to initial conditions $[1; 0]$ and $[0; 1]$. Explain intuitively what is the physical interpretation of these initial conditions. Explain physical interpretation of the final value of the system response to each of these initial conditions. (for example if system starts with an initial speed and zero initial current, intuitively why this speed approaches 0?) [10 marks]

14. Consider uncontrolled dynamics

$$\dot{x} = A_2 x(t)$$

Find the eigen-values and eigen-vectors of A_2. Find the response of the uncontrolled system to the eigen-values as initial conditions. Is there an initial condition which results in non exponentially stable response?[5 marks]