# Lab Assignment 4

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ECSE 403

## 3.1 Controllability and the implication on state feedback

1. Write the definition of controllability of a dynamical system and explain its importance.

*A dynamical ISO system {u,x,Φ,y,n) is controllable from an initial state x0 at a time t0 in positive time, if theres exists a later time tx > t0 and an input u, then x(tx, t0, x0, u) = x , ie. given an initial state and input, the trajectory of the state from an initial time to a later time terminates at x. This means that given some initial state and an appropriate input function, the initial state can be steered towards any other state over a time interval, which defines the system as controllable.*

*The importance of controllability of a dynamical system lies in the system's ability to possess any dynamical behaviour that may be desired for the system. If a system is controllable, its poles can be placed at any desired location in the complex plane, which consequently allows for the any desired dynamical behavior to take place, such as SHM, exponential decay of desired rates, damped harmonic motion, and also allows us to place poles of a system in a way that makes the system stable (or unstable).*

2. Check the rank controllability condition for the pair (A1;B1) in Lab 2 and (A2;B2) in Lab2. Find the number of uncontrolled state variables.[5 marks]

```
Jm = 0.01; %kgm^2 , inertia of the rotor and shaft
b = 0.001; %Nmsec , viscous friction coefficient
Ke = 0.02; %Vsec  , back emf constant
Kt = 0.02; %Nm/A  , motor torque constant
Ra = 10;   %Ω     , armature resitance
La = 0.5;

A1 = [-b/Jm Kt/Jm ; -Ke/La -Ra/La];
B1 = [0 ; 1/La];

A2 = [0 1 0 ; 0 -b/Jm Kt/Jm ; 0 -Ke/La -Ra/La]
B2 = [0 ; 0 ; 1/La]

Ctl_1 = ctrb(A1, B1);
Ctl_2 = ctrb(A2, B2);
```

```
A2 = 3×3
         0    1.0000         0
         0   -0.1000    2.0000
```

```
rank_ctl_1 = rank(Ctl_1)

rank_ctl_2 = rank(Ctl_2)
```

```
B2 = 3×1
     0
     0
     2
```

```
rank_ctl_2 = 3

rank_ctl_1 = 2
```

*In both cases, the controllability matrices are full rank, so there are no uncontrolled state variables.*

3. Using MATLAB and the state feedback gain for which the closed loop system of (A1;B1) has two poles on [2;3], repeat this question for poles [i;1 +i], and[2i;1+i].

```
p1 = [-2 -3];
K1 = place(A1, B1, p1)


p2 = [-1+i ,-1-i];
K2 = place(A1, B1, p2)


p3 = [-2-i ,-1+i];
K3 = place(A1, B1, p3)
```

```
K1 = 1×2
     1.3575   -7.5500

K2 = 1×2
     0.4325   -9.0500
Warning: A complex gain matrix is requirec
conjugate pairs.
K3 = 1×2 complex
   0.6575 - 0.2500i  -8.5500 - 0.0000i
```

4. Consider system (A1;B1). Can state feedback places the poles on two complex non-conjugate positions? Can state feedback place the poles on arbitrary real number and conjugate complex numbers? why? [5 marks]

*The complex conjugate root theorem states that the the non-real roots of a poynomial (with all real coefficients) appear on the complex plane as complex conjugate poles. In the case of system A1,B1, unless the state is controlled by non-conjugate complex gain, then the placed poles will appear either as real valued poles or complex conjugate ones. Moreover, since transfer functions are rational functions of complex variables, the coefficients of the denominator must be real, and by extent the poles must be real and complex conjugates.*

*By the argument above, state feedback can only place poles on arbitrary real numbers and complex conjugate numbers, and not complex non-conjugate numbers.*

5. Calculate eigenvalues of A1-B1K for all three pair of poles in Q3. What is the relation ofthese eigenvalues with the place of the poles? [5 marks]

```
eig1 = eig(A1-B1*K1)
eig2 = eig(A1-B1*K2)
eig3 = eig(A1-B1*K3)
```

```
eig1 = 2×1
    -2.0000
    -3.0000
eig2 = 2×1 complex
   -1.0000 + 1.0000i
   -1.0000 - 1.0000i
eig3 = 2×1 complex
   -1.0000 + 1.0000i
   -2.0000 - 1.0000i
```

*These eigenvalues are the same as the placed poles for each case.*

6. Using MATLAB and the state feedback gain for which the closed loop system of (A2,B2) has three poles on [-1+ i , -1 - i , -10]. [5 marks]

```
p4 = [-1 + i, -1 - i , -10];
K4 = place(A2,B2,p4)
eig4 = eig(A2 - B2*K4)
```

```
K4 = 1×3
     5.0000    5.1825   -4.0500
eig4 = 3×1 complex
  -10.0000 + 0.0000i
   -1.0000 + 1.0000i
   -1.0000 - 1.0000i
```

7. Consider the state space system with A3=A2 and B3= [1; 0; 0]. Check the rank controllability condition for the system. Find the number of uncontrolled state variables.

```
A3 = A2;
B3 = [1 ; 0 ; 0];
ctl_3 = ctrb(A3,B3)
rank_ctl_3 = rank(ctl_3)
```

```
ctl_3 = 3×3
     1     0     0
     0     0     0
     0     0     0
rank_ctl_3 = 1
```

*Since the matrix is of dimension 3, but rank 1, there are 2 uncontrolled state variables.*

8. Using MATLAB and the state feedback gain for which the closed loop system of (A3;B3) has poles on [4;2;3]. Does such a gain exist? Why?

*No gain can do this since the system is uncontrollable and the new poles of the system cannot be placed far away fom the current poles of the system.*

```
p5 = [-4,-2,-3];
%K5 = place(A3, B3, p5)
eig_A3  = eig(A3)
```

```
eig_A3 = 3×1
         0
   -0.1040
  -19.9960
```

9. Is it true that no set of desired poles for the closed loop system of (A3;B3) exists that can be achieved via state feedback? To answer this question you can try to and state feedback for the poles [4;0.1040;19.9960]. Why state feedback gain exists for these poles? [5 marks]

State feedback exists for the uncontrollable system A3, B3 as the latters poles without state feedback are {0, -0.104, -19.996}. Since the controllabiltiy matrix of this system is rank 1, there is 1 controllable state variable, whch allows the last pole to be placed at a new desired location. If we try to place the poles of the system such that we are changing the location of 2 or 3 poles, or if we change the matrix A3 matrix such that it is rank 0, then we cannot control any state variables.

So, no, there is a set of desired poles which spans 1D space, as we can only control 1 state variable.

```
p6 = [-4,  -0.104, -19.9960]
K6 = place(A3,B3,p6)
```

```
p6 = 1×3
   -4.0000   -0.1040  -19.9960
K6 = 1×3
10^15 ×
    0.0000   -0.0100   -4.9083
```

10. By experimenting different state feedback gains K, on the eigenvalues of A3-B3K, and find out which eigenvalue can be moved by state feedback.

```
p7 = [0, -4, -19.9960]
K7 = place(A3,B3,p7);
eig7 = eig(A3 - B3*K7)


p8 = [0, -0.1040, -20]
K8 = place(A3,B3,p8);
eig8 = eig(A3 - B3*K8)
```

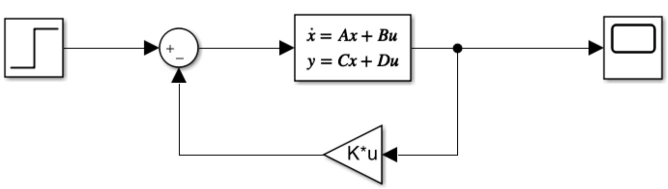
By experimentation, only the eigenvalue of 0 can changed to move the poles of the system. If either of the other two eigenvalues are changed while keeping 0 and the second eigenvalue constant, then state feedback is unable to steer the state to that position, unless the altered

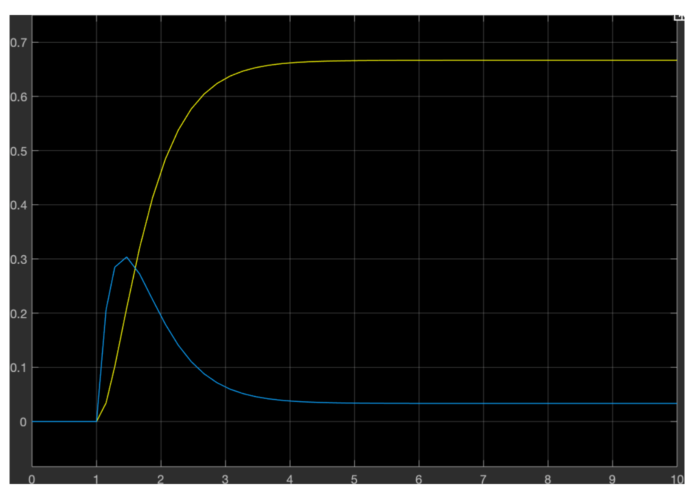
```
p7 = 1×3
         0   -4.0000  -19.9960
Warning: Achieved pole locations differ by more than 10 percent from specified locations.
eig7 = 3×1
   -2.5990
   -0.1040
  -19.9960
p8 = 1×3
         0   -0.1040  -20.0000
eig8 = 3×1
    0.0198
   -0.1040
  -19.9960
```

eigenvalue is sufficiently close to its previous value.

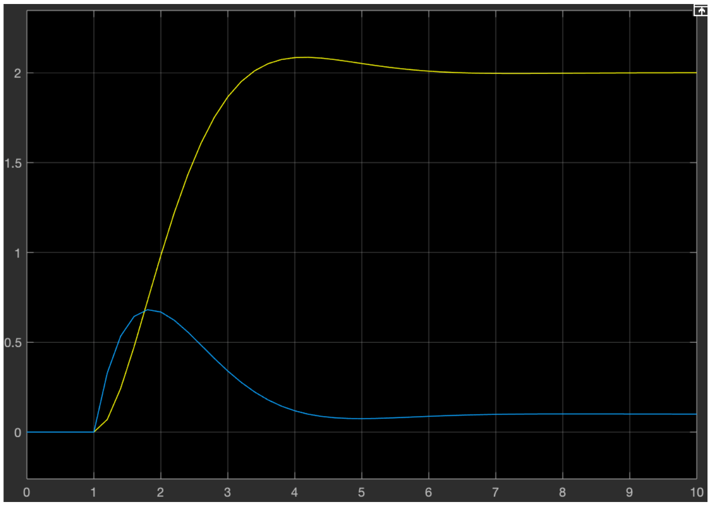
11. Implement state feedbacks of Q3 and Q6 in Simulink and observe the step response.[15 marks] Hint: In order to access to the states of the state space model you have to choose a different C matrix to observe all the states. $C_1$ and $C_2$ matrices in Lab 2 where returning single output signals and not the entire states. Hint: By choosing corresponding C matrix, the output of the state space model in the Simulink is a vector signal.
Hint: You can use gain block in the Simulink to multiply matrices to vector signals.

*Using appropriate given values for A1 (Q3) , C = 2x2 identity matrix, pole placement at [-2,-3] (using MATLAB results for K), and system below, The state trajectory is plotted:*



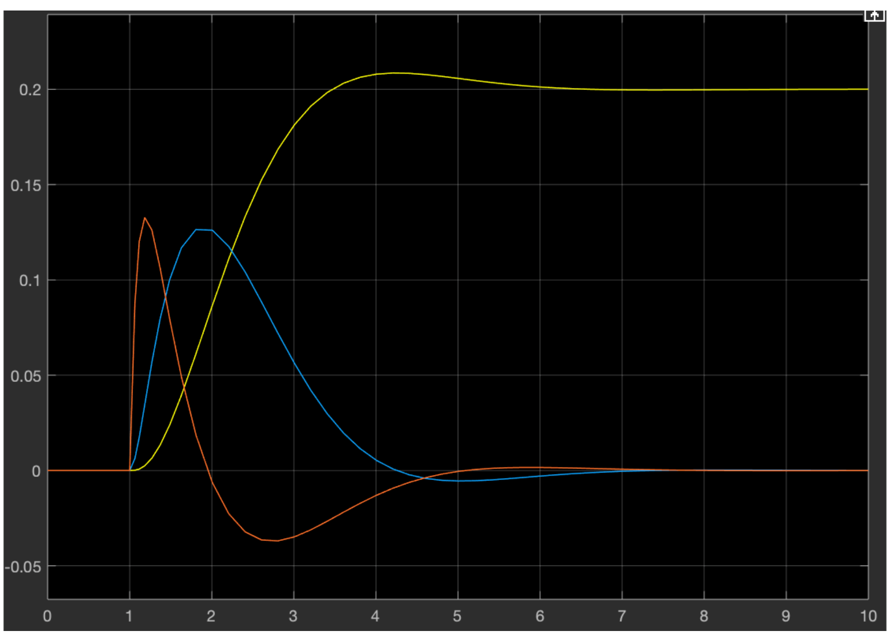



*Similarly for appropriate A1 and pole placement at* $[-1 - i, -1 + i]$




*Similarly for appropriate A1 and pole placement at* $[-2-i, -1+i])$
*The complex, non-conjugate poles cannot steer the real state to another real state.*

*Finally for appropriate A3, C = 3x3 Identity matrix and pole placement at* $[-1-i, -1+i, -10]$
*( Matlab results for K4) (Q6)*



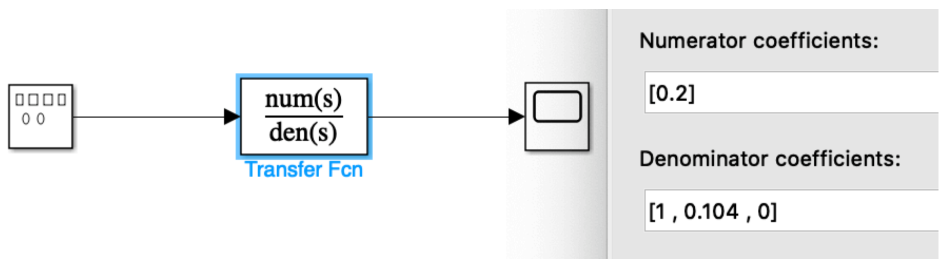
## 3.2 Linearity of the system

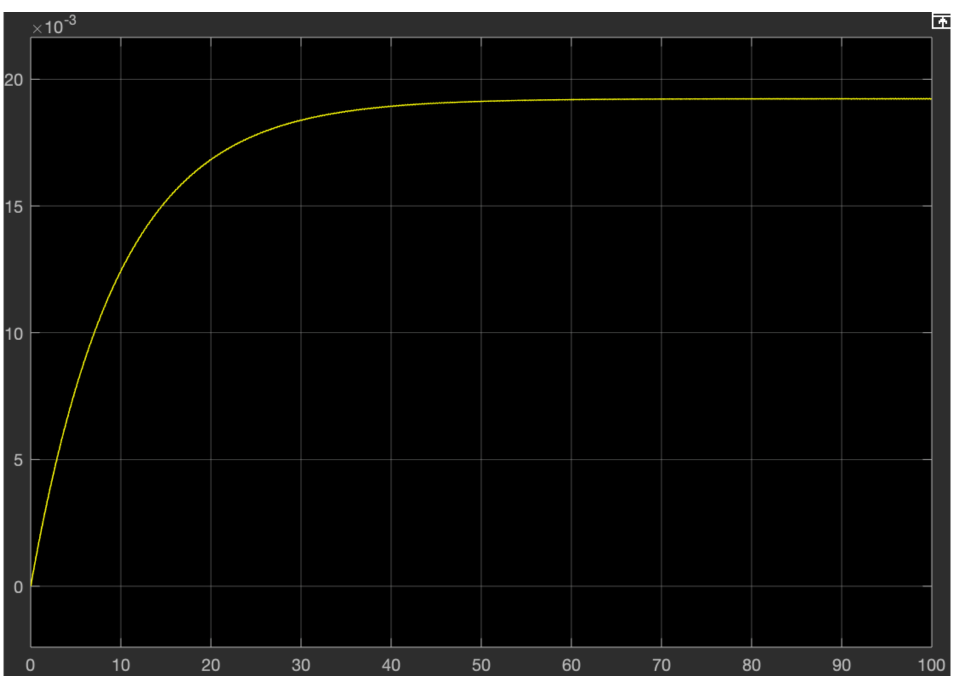
12. Choose two arbitrary input signals x1(t),x2(t) in the Simulink of any form (such as sine waves, saw-tooth waves, square waves, unit steps...). Apply x1(t),x2(t) to the θ(s)/v(s) (in Lab 1) model separately and record the output data, denote them by H{x1(t)} and H{x2(t)}. Then using two signal generators, gains and sum blocks, apply the input ax1(t) + bx2(t) where a, b are arbitrary coefficients. Record the data and denote it by H{ax1(t) + bx2(t)}. Using a MATLAB code verify the fundamental linearity property of the systems that:

H{ax1(t) + bx2(t)} = aH{x1(t)} + bH{x2(t)}
Hint: In order to export the data from Simulink environment to MATLAB workspace, you can use To Works space block. [15 marks]

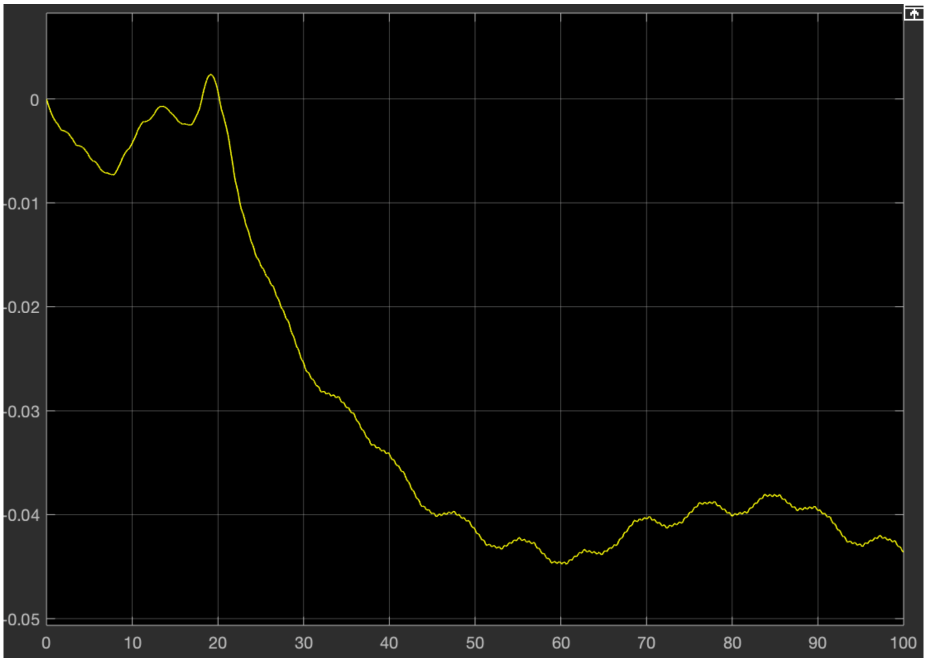
Suppose the previous question was performed in the lab instead of Simulink, do you expect to observe same answers? Explain possible sources of non-linearity in the physical system. [10 marks]

*H{x1(t)} with sinusoidal input of frequency 100 rad/s and amplitude 1.*

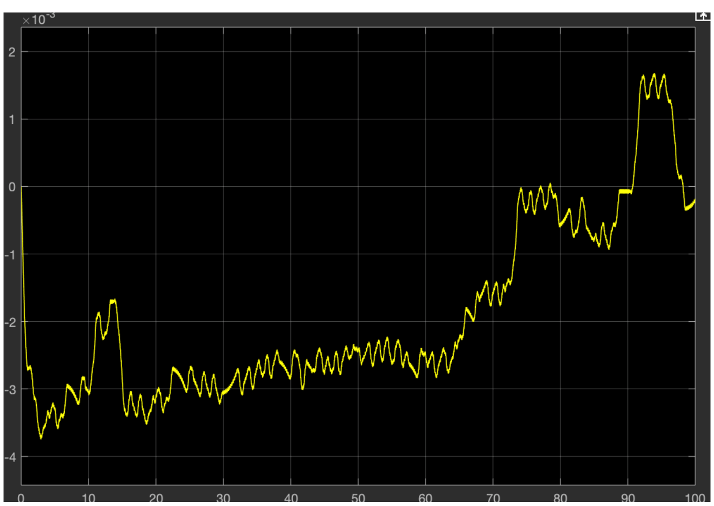


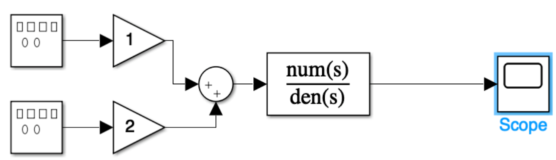



*H{x2(t)} with square wave input of frequency 100 rad/s and amplitude 1.*

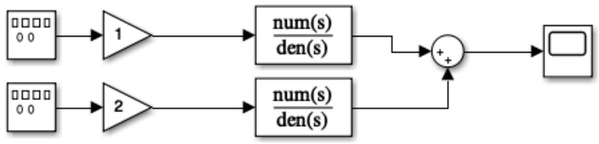


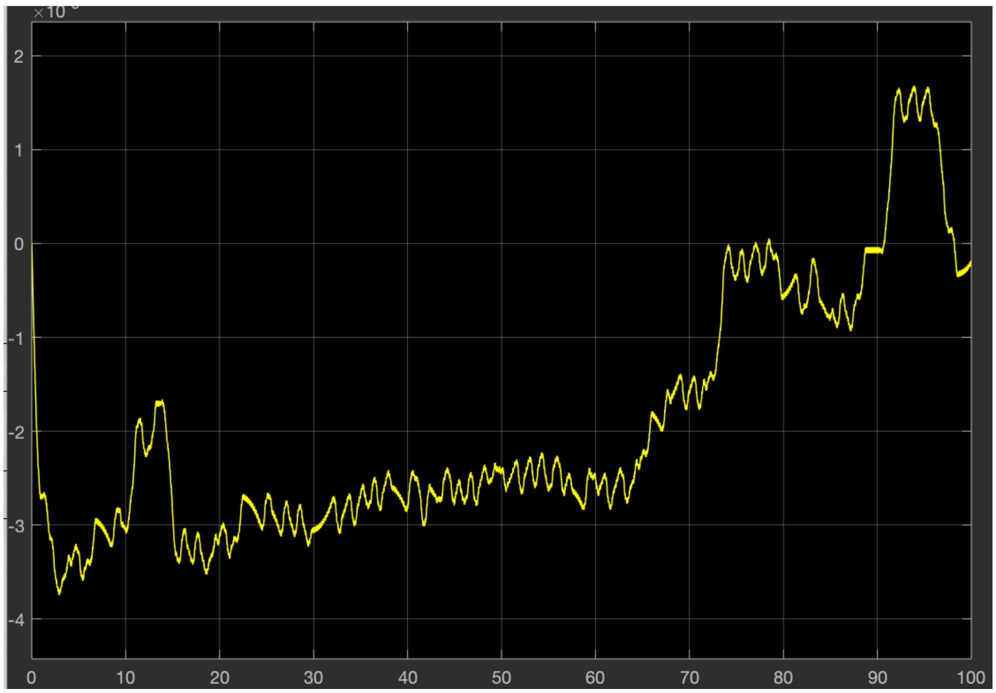

*H{ax1(t) + bx2(t)}, where a = 1 (gain on sine function) and b = 2 (gain on square wave)*



*Checking for linearity of state trajectories: aH{x1(t)} + bH{x2(t)}*



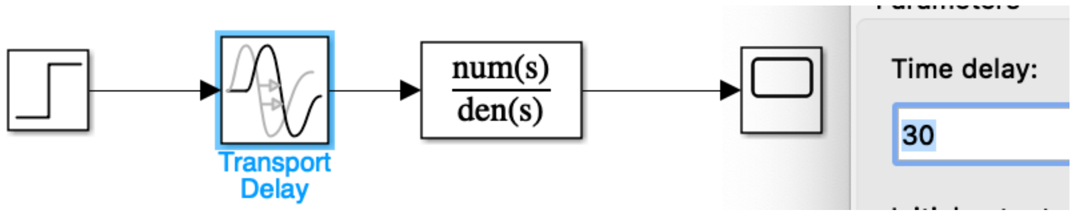



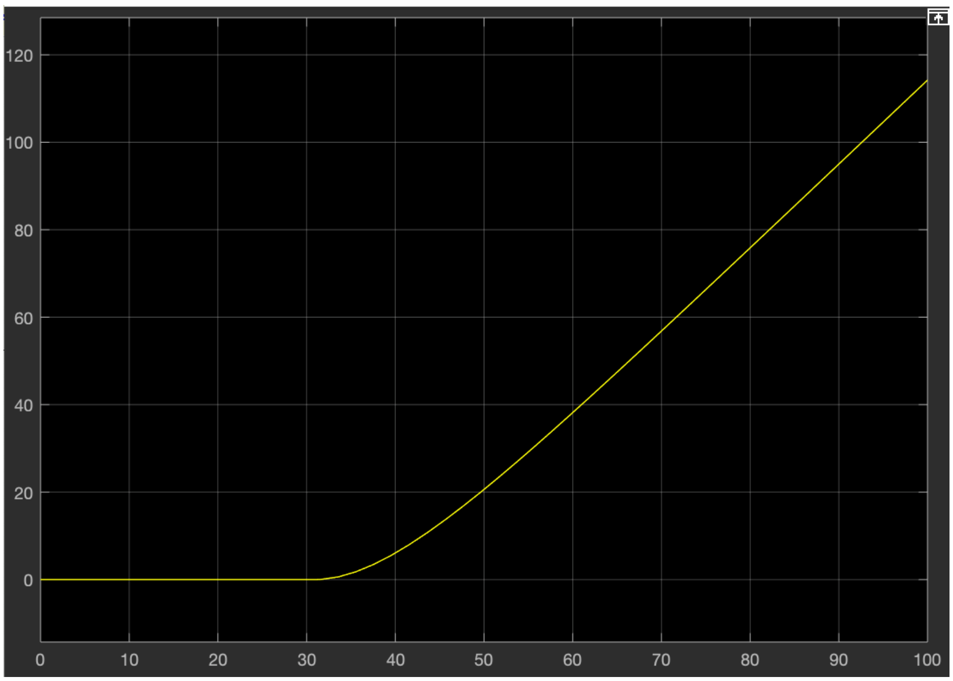
*Clearly, the state trajectories are the same.*

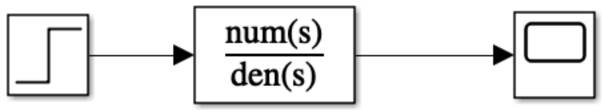
13. Suppose the previous question was performed in the lab instead of Simulink, do you expect to observe same answers? Explain possible sources of non-linearity in the physical system. [10 marks]

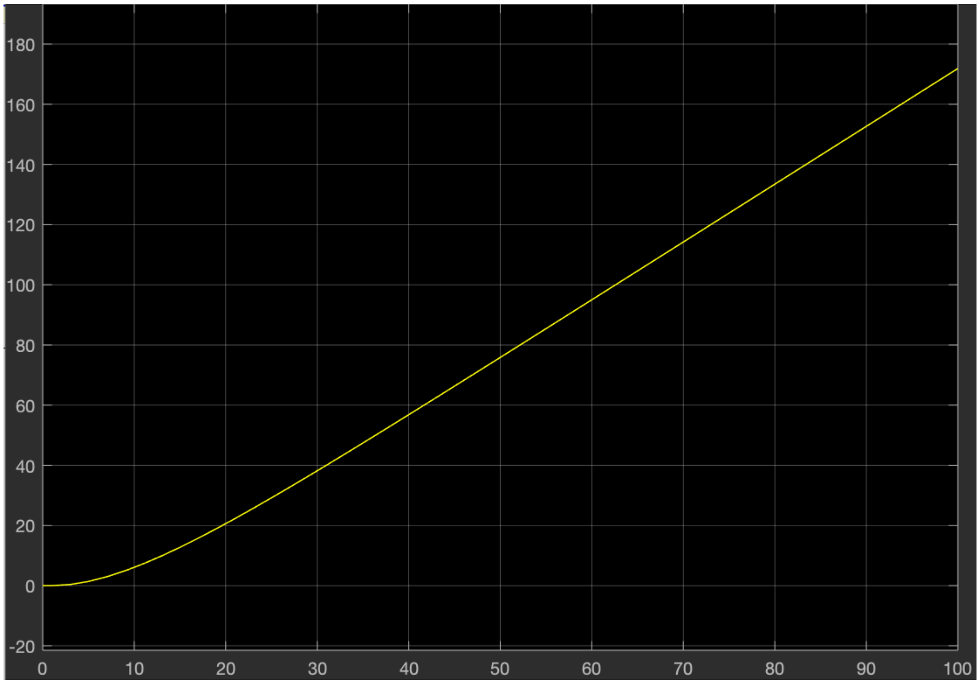
*The state trajectories would be similar, but vary slightly due to non-linearity sources introduced by experimental conditions. Such sources might be due to the presence of noise in electrical machinery, the presence of friction of the motor which may have increased due to age and is no longer be correctly accounted for by the machine's computer.*

14. Design and perform an experiment similar to Q12, which shows the time invariancy of the the system. [10 marks]






Clearly, the system is time invariant. In the first case, the step input is applied after 30 seconds, and in the second case the input is applied immediately. The state trajectory of the system with delayed input is the same as the system without, only delayed by 30 seconds. (ie: state with no initial conditions or input for 30 seconds remains at origin for 30 seconds).