Lab Assignment 6

Theodore Janson

260868223

ECSE 403

November 9th 2020

1. Write the definition of phase margin and gain margin. Explain their importance as a design criteria.[10 marks]

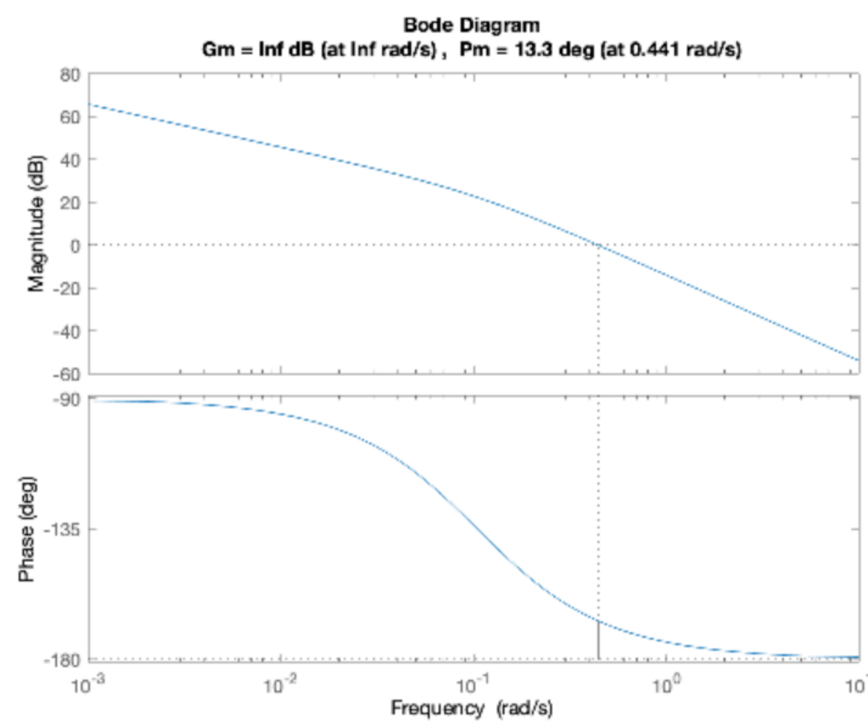
Gain margin is defined as the amount of change in open-loop gain at a phase of 180 degrees which would make the system under closed-loop conditions unstable. Similarly, the phase margin is the change in open loop phase (or the phase shift required) under a gain of 1 which would make the closed loop system unstable. Their importance as design criteria lies in their ability to quantify how 'near' as system is to instability – systems with small phase margins are most susceptible to becoming unstable due to input changes. Similarly, systems with small gain margins are more susceptible to oscillation, no matter the disturbance or change in input.

2. Using the transfer function θ(s)/v(s) in Lab 1, find the gain margin and phase margin of the system.[5 marks]Hint: You may use commands bode or margin.

```
B = [0.2];

A = [1 0.104 0]

G1 = tf(B,A)

margin(G1)
```

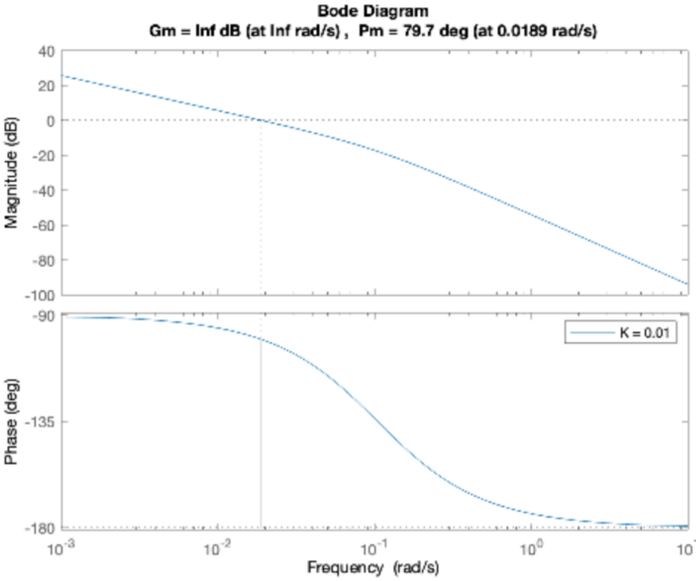
As seen by the margin plot, the gain margin is infinite and the phase margin is 13.3 degrees.

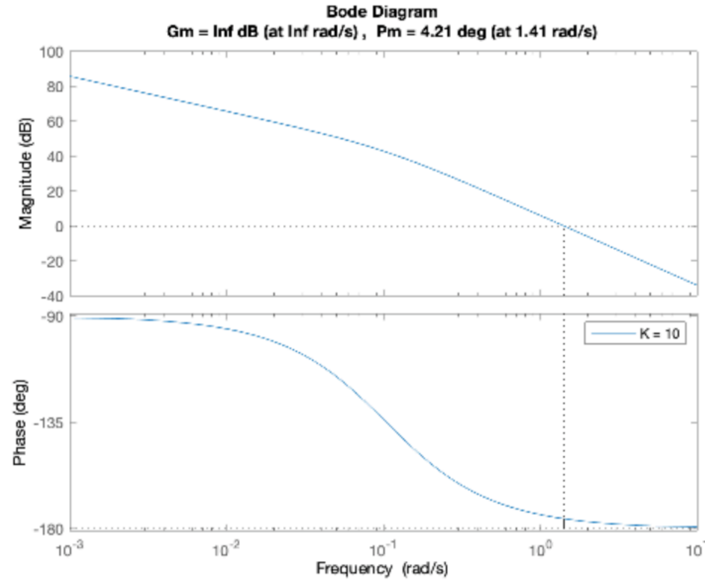
```
G1 =

      0.2
  -------------
  s^2 + 0.104 s

Continuous-time transfer
```

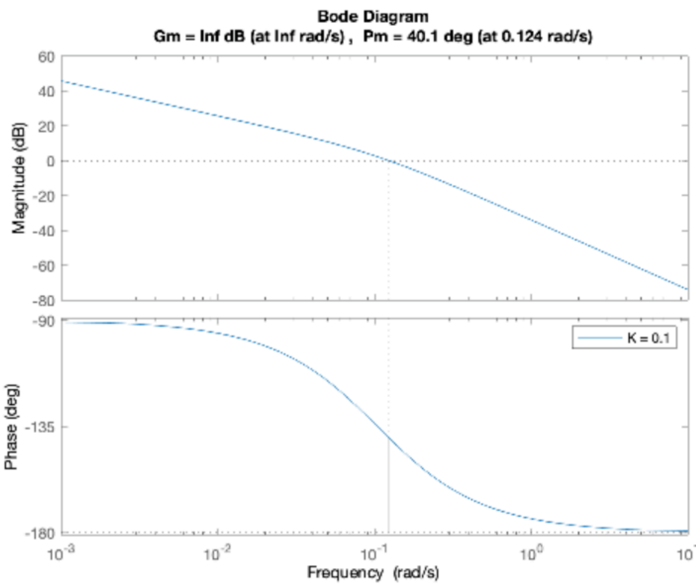


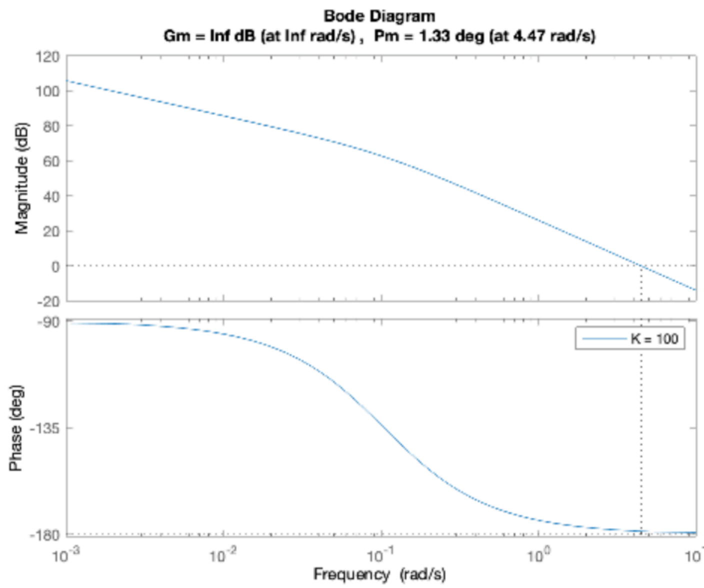
3. Using a proportional gain (Kp), find the gain margin and phase margin of (Kpθ(s)/v(s)) for different Kp. Explain the connection between overshoot and phase margin using your observations. Explain how proportional gain is changing bode diagram and how it affects phase and gain margin.[10 marks]

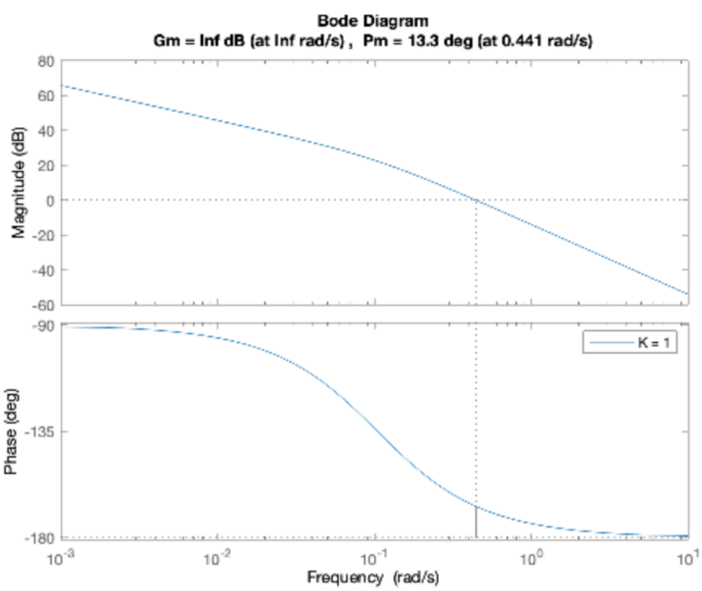
```
Kp = [0.01 0.1 1 10 100 1000];



margin(Kp(1,1)*G1)

legend('K = 0.01')


margin(Kp(1,2)*G1)

legend('K = 0.1')


margin(Kp(1,3)*G1)

legend('K = 1')


margin(Kp(1,4)*G1)

legend('K = 10')


margin(Kp(1,5)*G1)

legend('K = 100')


margin(Kp(1,6)*G1)

legend('K = 1000')
```

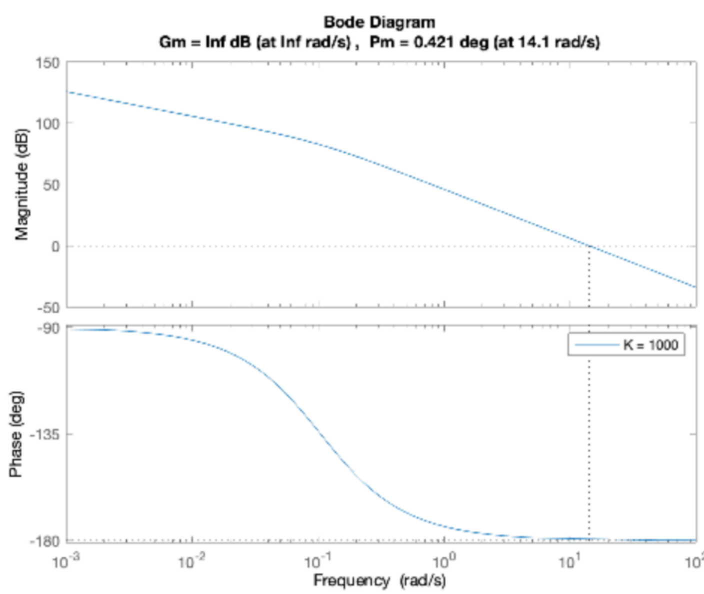

Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 79.7 deg (at 0.0189 rad/s)
Magnitude (dB)
Phase (deg)
K = 0.01
Frequency (rad/s)


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 4.21 deg (at 1.41 rad/s)
Magnitude (dB)
Phase (deg)
K = 10
Frequency (rad/s)


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 40.1 deg (at 0.124 rad/s)
Magnitude (dB)
Phase (deg)
K = 0.1
Frequency (rad/s)


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 1.33 deg (at 4.47 rad/s)
Magnitude (dB)
Phase (deg)
K = 100
Frequency (rad/s)

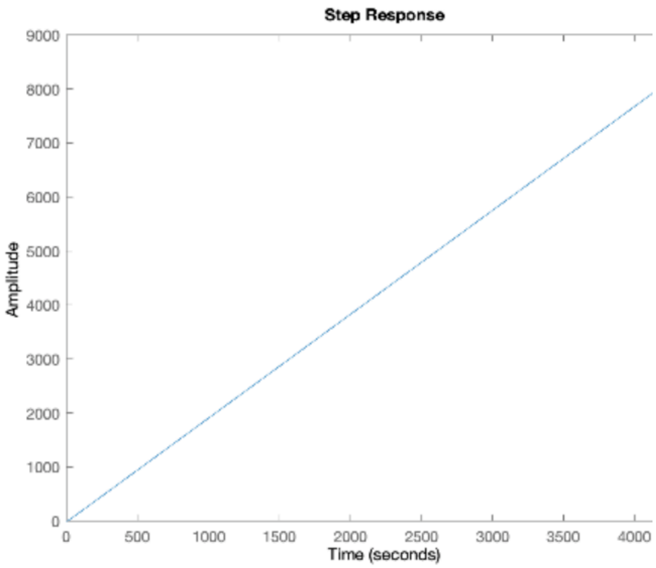

Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 13.3 deg (at 0.441 rad/s)
Magnitude (dB)
Phase (deg)
K = 1
Frequency (rad/s)

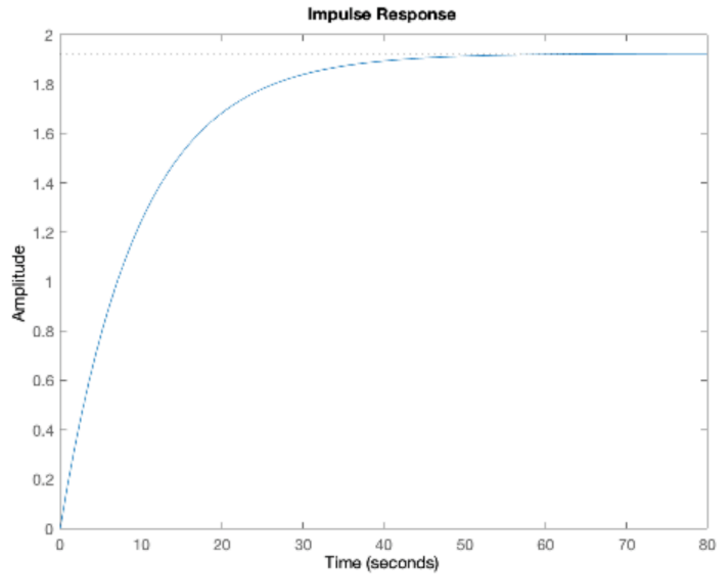

Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 0.421 deg (at 14.1 rad/s)
Magnitude (dB)
Phase (deg)
K = 1000
Frequency (rad/s)

As the proportional gain increases, so does overshoot, and phase margin as evidenced by the plots above. As overshoot increases, the relative instability of the system increases as the response oscillates with greater amplitude, so the system becomes more susceptible to becoming unstable given a disturbance. The phase margin quantifies this change. This is shown in the magnitude plot of the bode plot – the plot 'shifts up' , so the phase margin decreases. Conversely, the increase in proportional gain has no affect on the phase plot, and by extent the gain margin.

4. Consider unity feedback closed loop system with open loop plant θ(s)/v(s). Find the steady state error to step and ramp using final value theorem. Verify your answers by plotting the responses in MATLAB.[5 marks]

```
s = tf('s')
stepplot(G1)


impulse(G1)


syms x
Va = 1/s;
G = 0.2 / (x^2 + 0.104*x);
Kv = double(limit(G*x, x, 0))


ss_ramp_error = 1/Kv


%steady state ramp error is infinite
```






```
Kv = 1.9231
ss_ramp_error = 0.5200
```

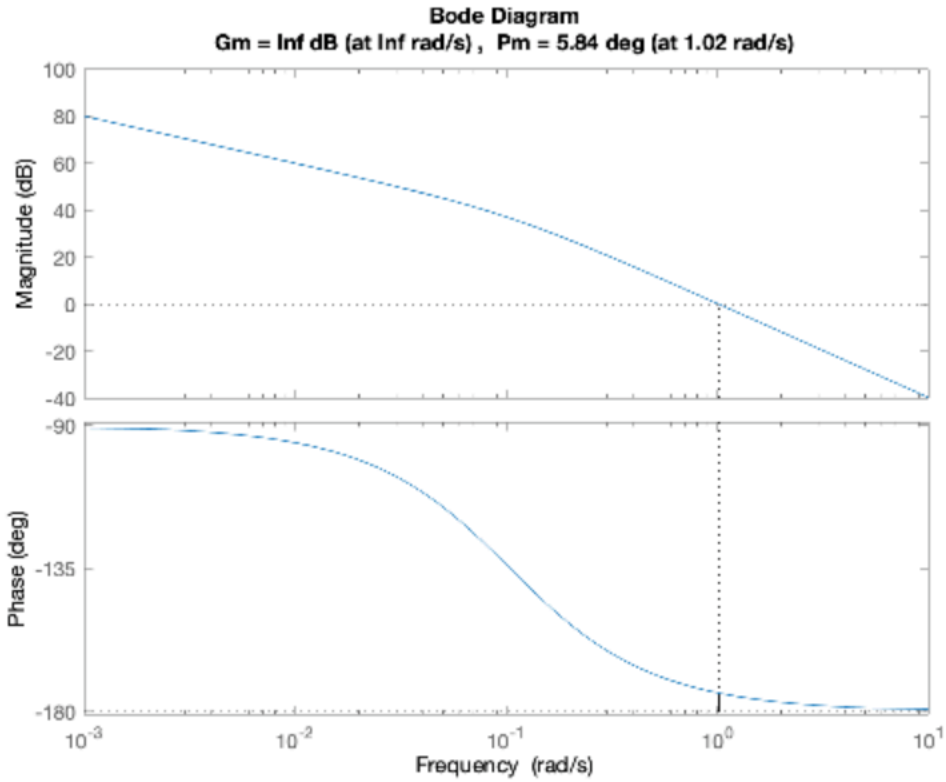
5. Design a lead controller which reduces the steady state error to a ramp signal to 0.1 and increases the phase margin of the system to 60. [20 marks]

```
de = 0.1;
p_margin = 60;

K = (1/de)/Kv

[GM, PM] = margin(G1*K)
```

```
K = 5.2000  PM = 5.8381
GM = Inf
```

```
max_phase = 10 + p_margin - PM
a = (1+sind(max_phase))/(1-sind(max_phase))

gain_wmax = -20*log10(sqrt(a))
margin(G1*K)
```

```
a = 19.0058
gain_wmax = -12.7889
```

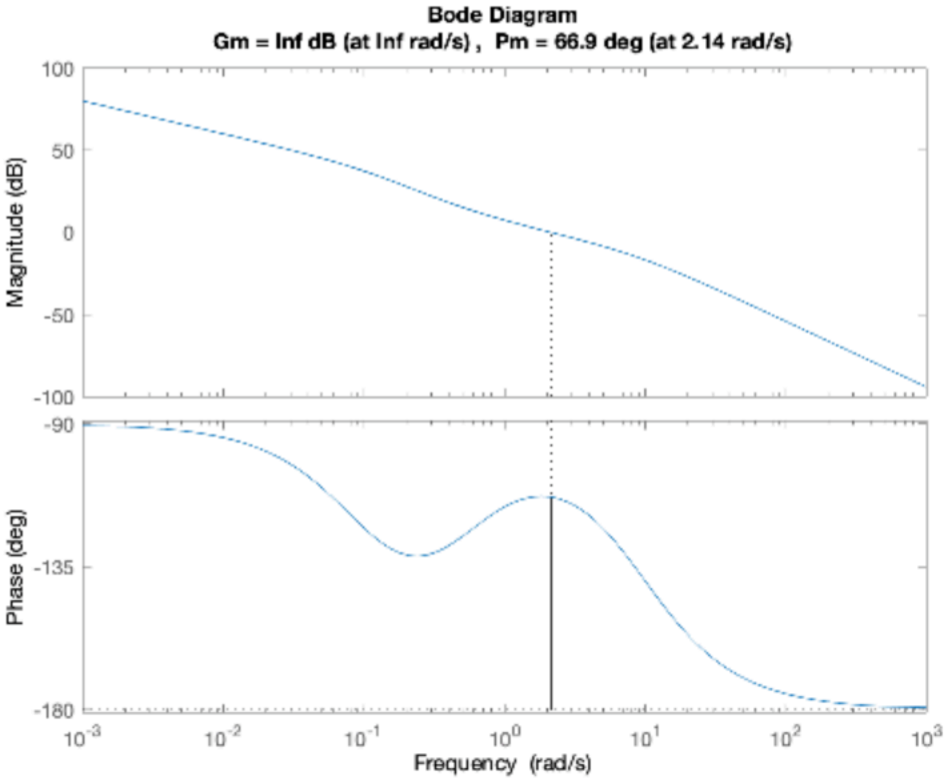


By inspection of the bode plot, w where Gain = -12.78

```
wn = 2.11;

T = 1 / (wn * sqrt(a))


lead_contoller = K * (T*a*s + 1) / (T*s + 1)
```

```
T = 0.1087
lead_contoller =

  10.74 s + 5.2
  -------------
  0.1087 s + 1
```

```
margin(G1*lead_contoller)

[GM2, PM2] = margin(G1*lead_contoller)
```




```
GM2 = Inf
PM2 = 66.9376
```

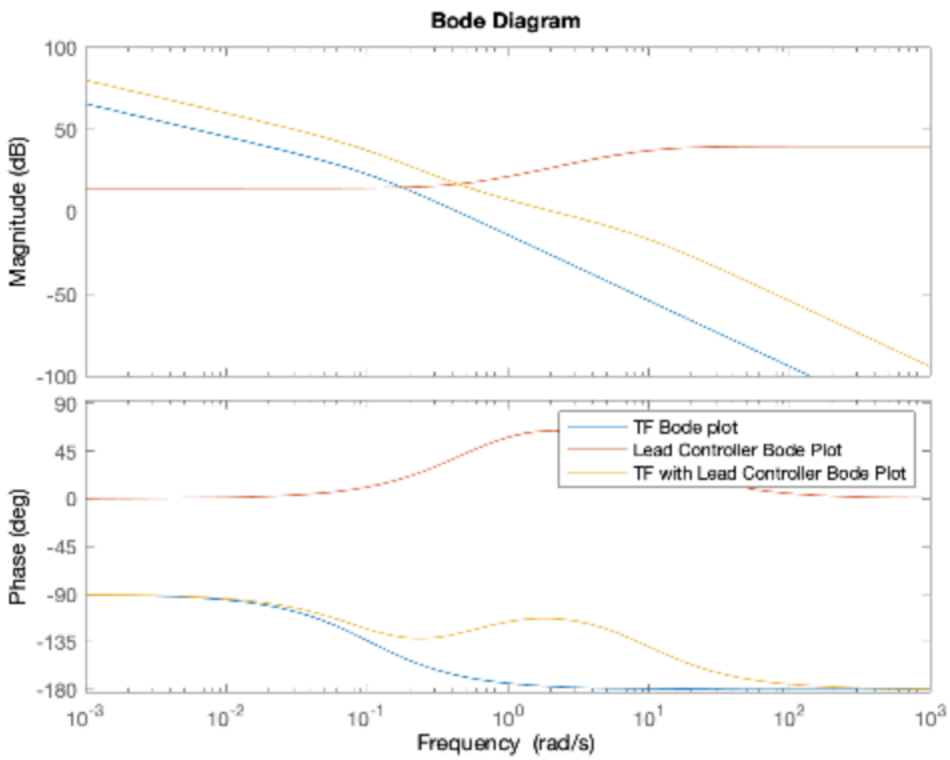
6. Is there a lead compensator which provides steady state error of 0 to ramp input? Justify your answer mathematically. [10 marks]

In practice, zero steady state error would be impossible. However, mathematically, this is possible as an infinite velocity constant (Kv) would give a zero steady state error for a ramp input because error = 1 / Kv. The lead compensator would place a pole at the origin, but this would also affect the transient response of the system, as the root locus plot would shift.

7. Plot the bode diagram of uncompensated, compensator and compensated systems in one diagram. Describe the relation between these diagrams. [5 marks]

```
figure(1)
bode(G1)
hold on
bode(lead_contoller)
hold on
bode(G1*lead_contoller)
```

```
legend('TF Bode plot', 'Lead Controller Bode Plot', 'TF with Lead Controller Bode Plot')
hold off
```



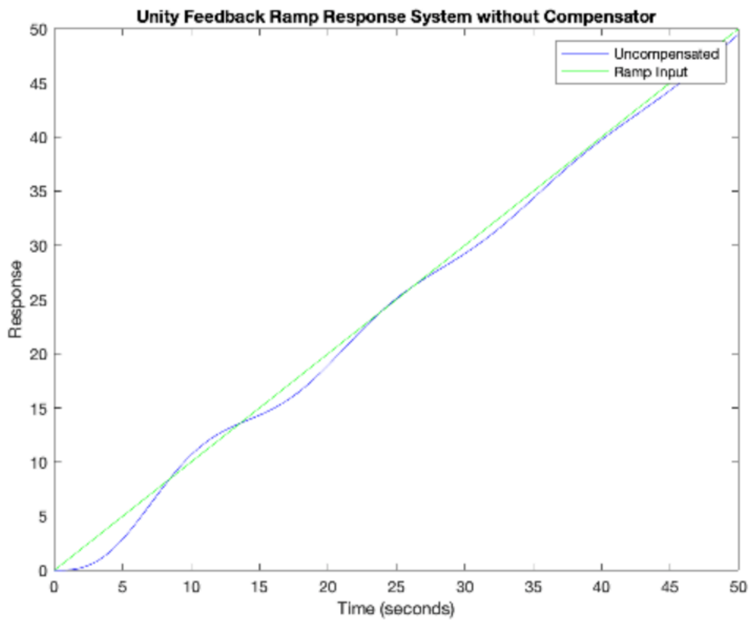
The original system's open loop transfer function, the compensator transfer function and the compensated system's transfer function are all superimposed on the bode plot above. clearly, the controller increases the phase and the gain of the original systems. The effect of varying slopes can clearly be seen as well.

8. Plot the ramp response of unity feedback closed loop system of compensated and uncompensated system, verify the steady state error is within the required region.[5 marks]

```
G2 = feedback(G1, 1)


T = 0:0.01:50;
[y,t] = lsim(G2, T, T);
```

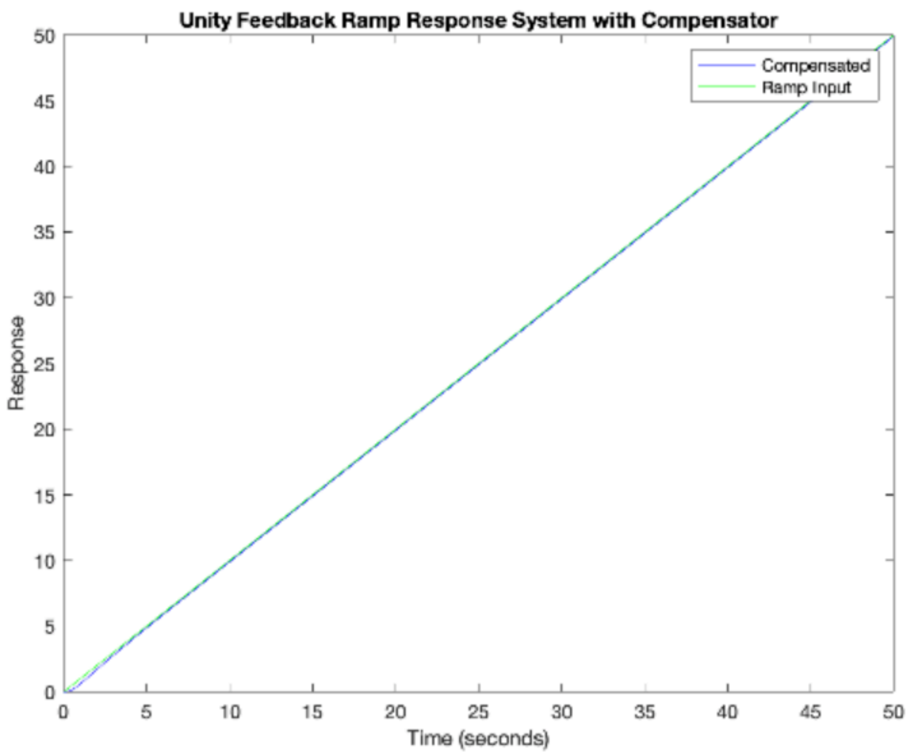
```
plot(t,y,'b',t,T,'g')
xlabel('Time (seconds)')
ylabel('Response')

title('Unity Feedback Ramp Response System without Compensator')
legend('Uncompensated', 'Ramp Input')
```

```
G2 =

         0.2
  -------------------
  s^2 + 0.104 s + 0.2
```




```
G3 = feedback(G1*lead_contoller, 1);

T = 0:0.01:50;
[y,t] = lsim(G3, T, T);
plot(t,y,'b',t,T,'g')
xlabel('Time (seconds)')
ylabel('Response')
```

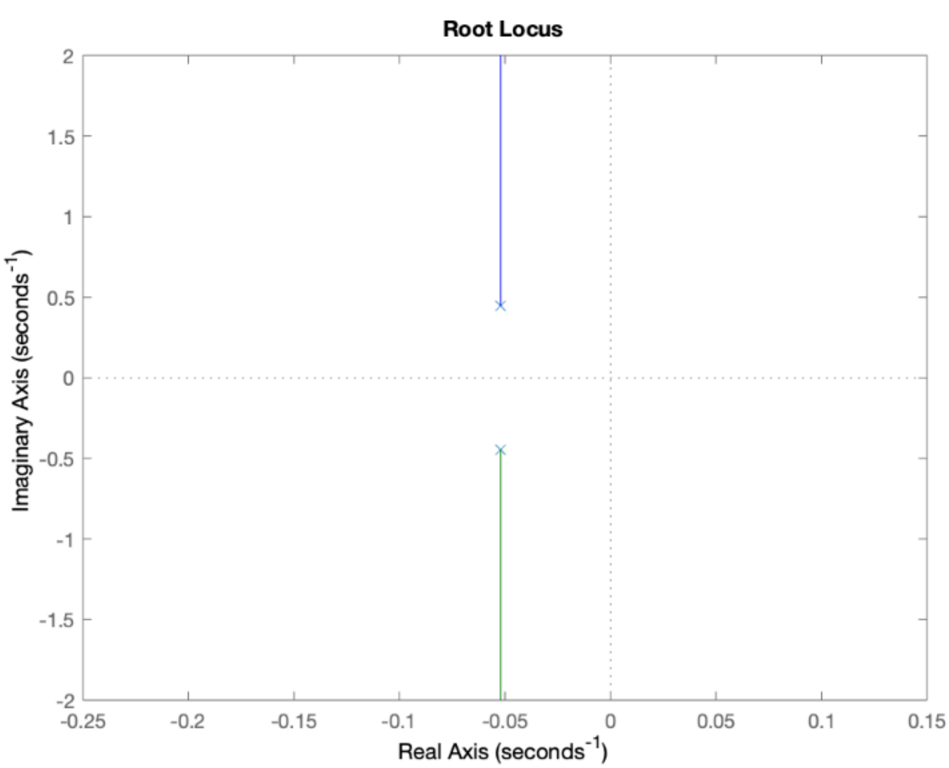
```
title('Unity Feedback Ramp Response System with Compensator')
legend('Compensated', 'Ramp Input')
```




```
plot(t,y,'b',t,T,'g')
 axis([29.9,30.1,29.9,30.1])
 title('Required Region of Steady State Error')
```

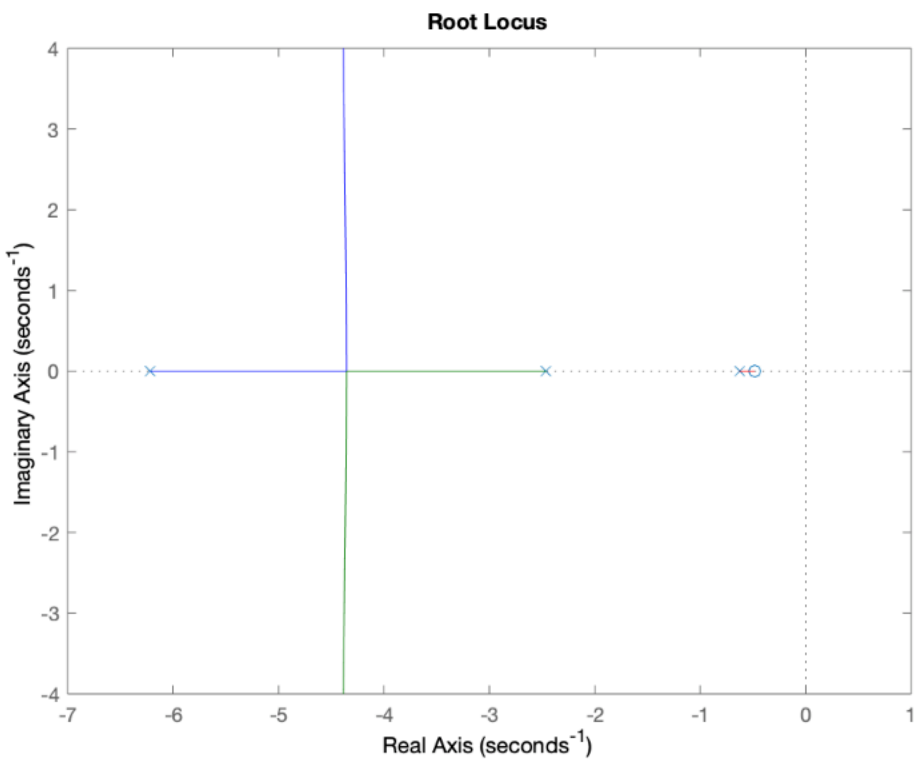
9. Plot the root locus diagram of both compensated and uncompensated systems and describe the effect of a lead controller on poles of the system.[5 marks]

```
rlocus(G2)

[p1,z1] = pzmap(G2)
```



```
p1 = 2×1 complex
   -0.0520 + 0.4442i
   -0.0520 - 0.4442i
z1 =

  0×1 empty double column vector
```

```
rlocus(G3)
[p2,z2] = pzmap(G3)
```

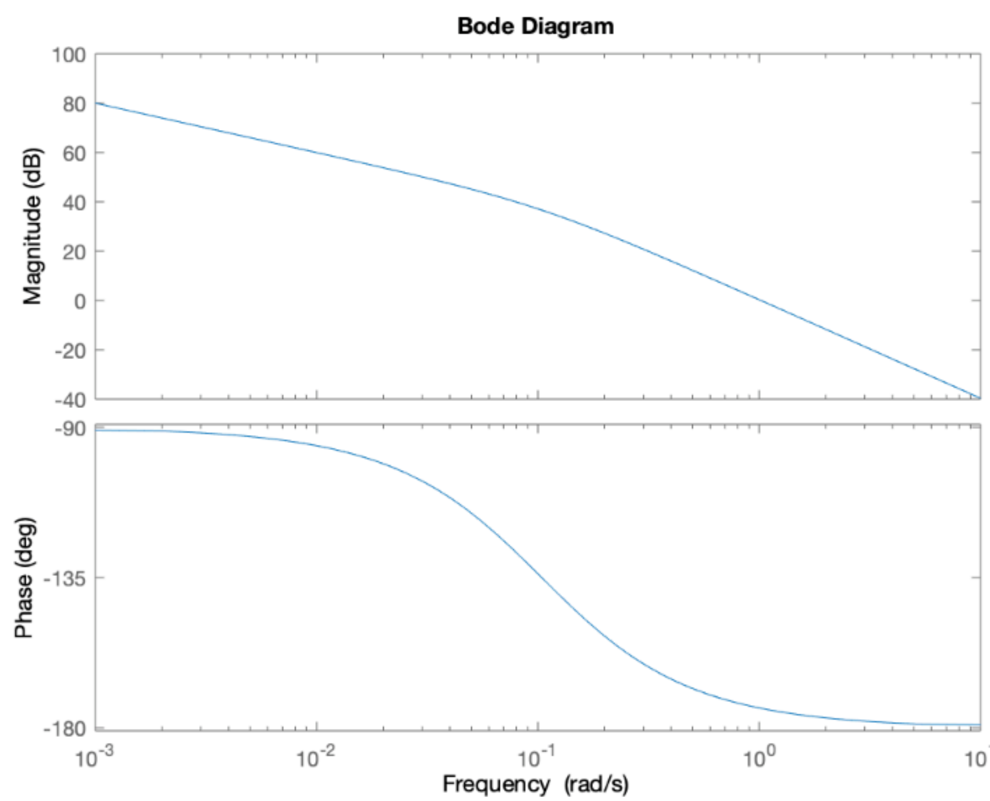



```
p2 = 3×1
    -6.2169
    -2.4603
    -0.6255
z2 = -0.4840
```

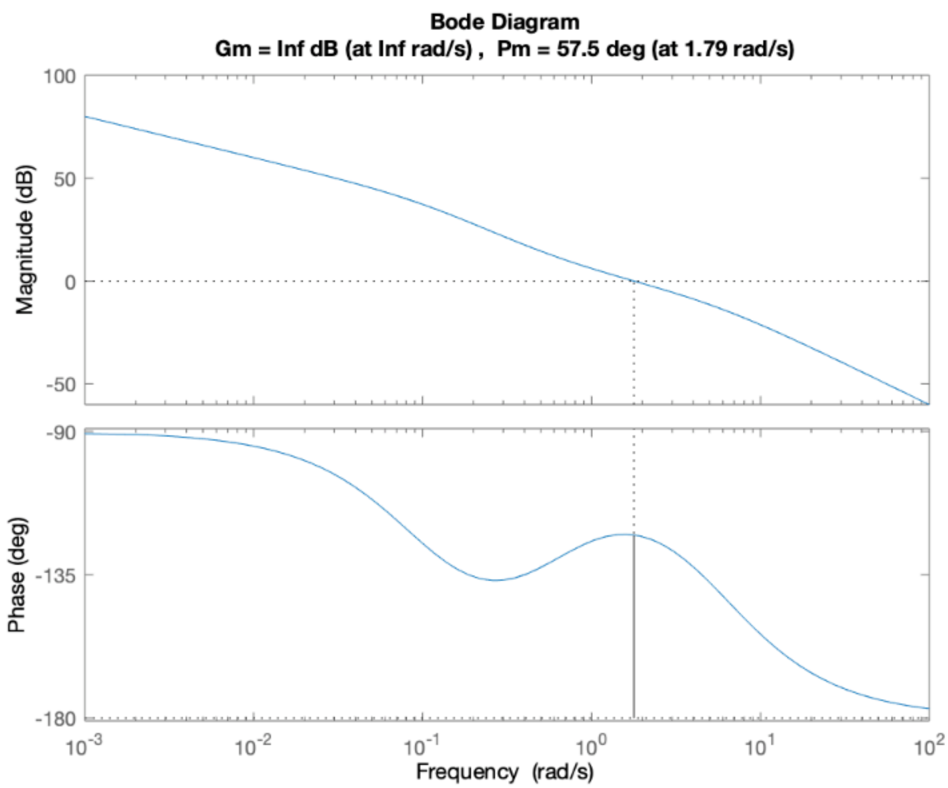
10. Repeat steps of Question 5 for phase margins [50○, 60○, 70○], and find corresponding compensator. Plotting step response of unity feedback closed loop compensated systems, describe the effect of different phase margins on the step responses.[15 marks]

```
phase_margins = [50,60,70];


[GM, PM] = margin(G1*K);


max_phase = phase_margins(1,1) - PM + 10

a = (1+sind(max_phase))/(1-sind(max_phase));

gain_wmax = -20*log10(sqrt(a))

bode(G1*K)

%By inspection of Bode plot, we can get the natural frequency at
which the gain is -9.8063

wn = 1.8;

t = 1/(wn * sqrt(a));

controller_3 = K * (a * t * s + 1)/(t * s + 1)


[GM3, PM3] = margin(G1*controller_3)

margin(G1*controller_3)
```

```
max_phase = 54.1619
gain_wmax = -9.8063
```



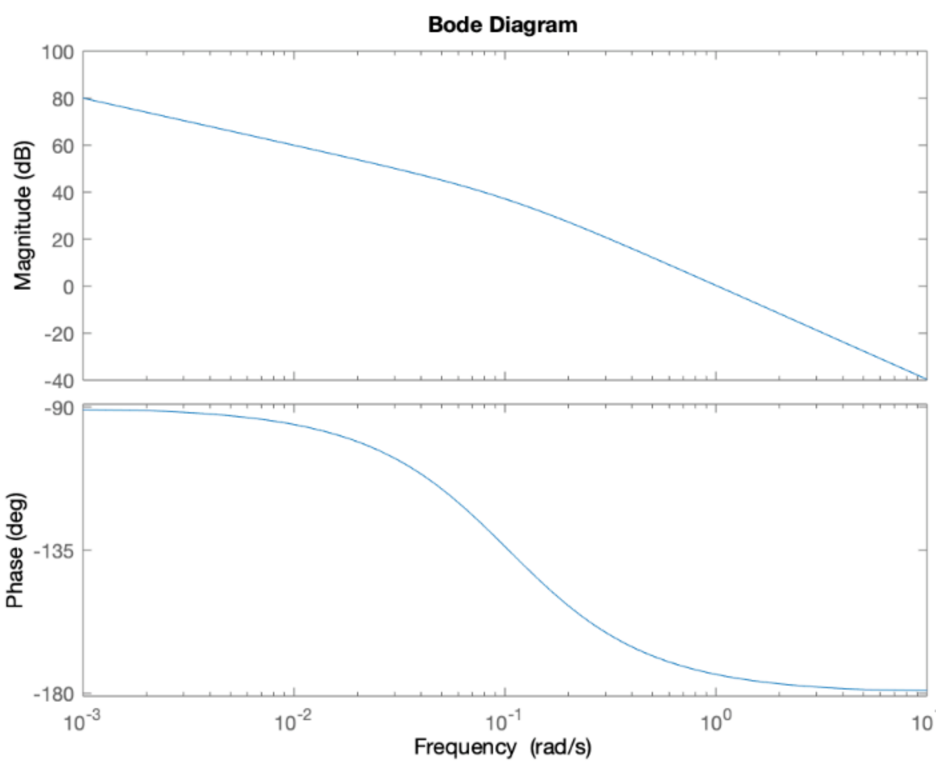
```
controller_3 =

  8.934 s + 5.2
  -------------
  0.1796 s + 1

Continuous-time transfer function.
GM3 = Inf
PM3 = 57.4930
```



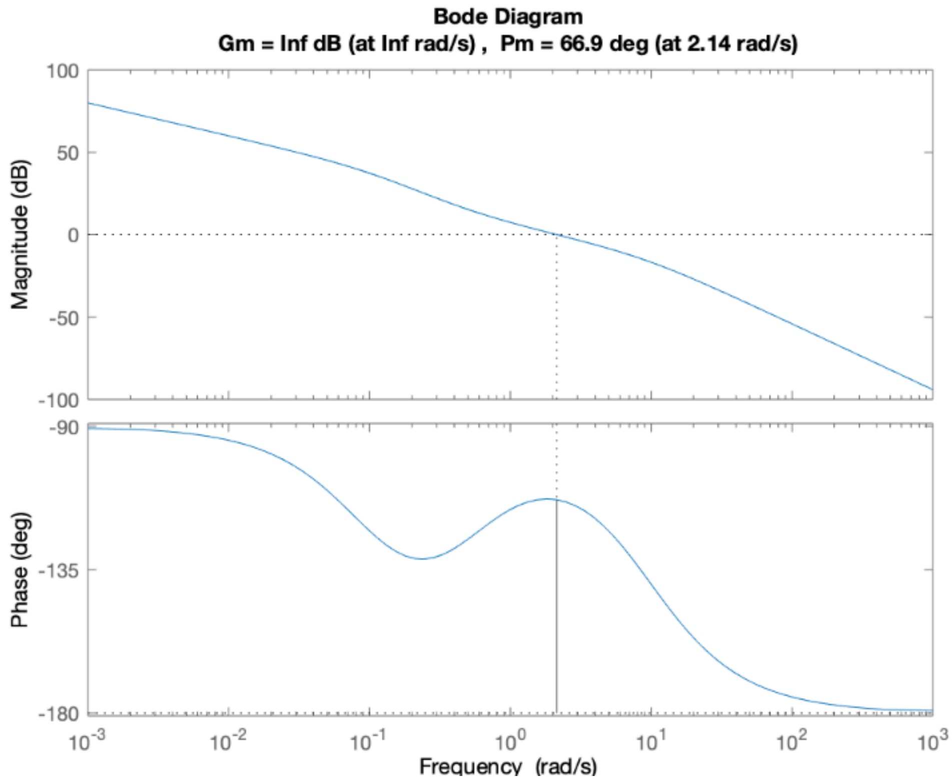

```
max_phase = phase_margins(1,2) - PM + 10
a = (1+sind(max_phase))/(1-sind(max_phase));
gain_wmax = -20*log10(sqrt(a))
bode(G1*K)
```

```
%By inspection of Bode plot, we can get the natural frequency at
which the gain is -12.789

wn = 2.11;

t = 1/(wn * sqrt(a));

controller_4 = K * (a * t * s + 1)/(t * s + 1)


[GM4, PM4] = margin(G1*controller_4)

margin(G1*controller_4)
```

```
max_phase = 64.1619
gain_wmax = -12.7889
```

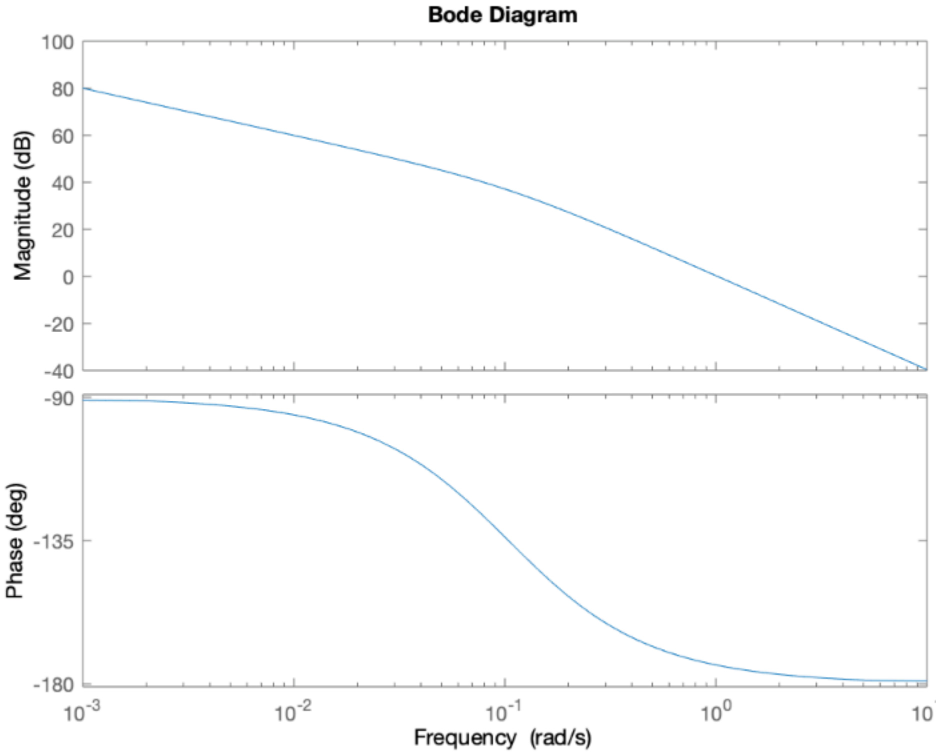



```
controller_4 =

  10.74 s + 5.2
  -------------
  0.1087 s + 1

Continuous-time transfer function.
GM4 = Inf
PM4 = 66.9376
```

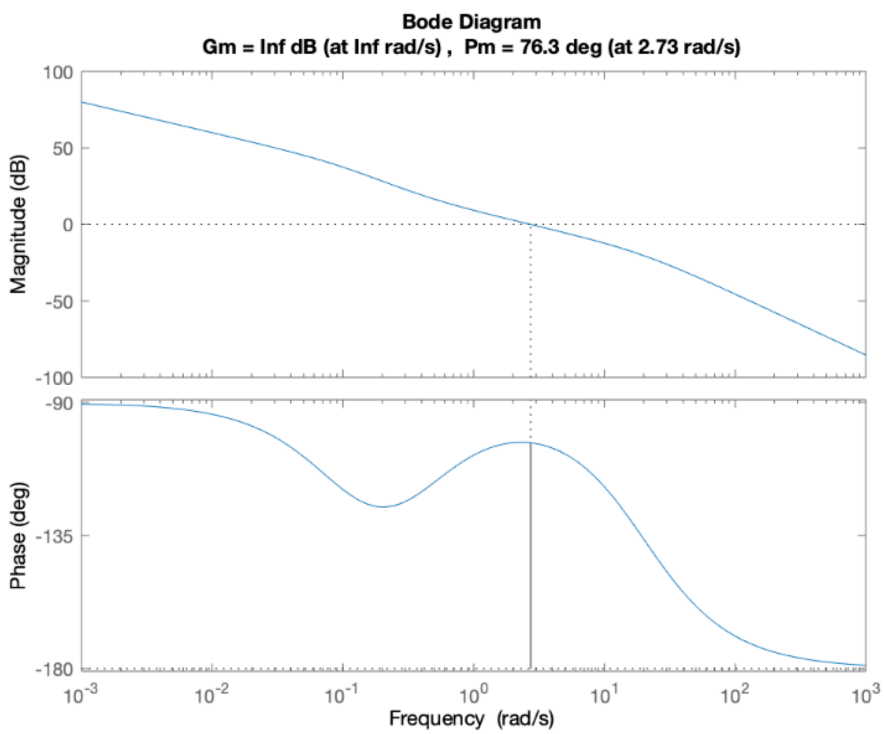



```
max_phase = phase_margins(1,3) - PM + 10
a = (1+sind(max_phase))/(1-sind(max_phase));
gain_wmax = -20*log10(sqrt(a))
bode(G1*K)
%By inspection of Bode plot, we can get the natural frequency at
which the gain is -17.1334
wn = 2.74;
t = 1/(wn * sqrt(a));
controller_5 = K * (a * t * s + 1)/(t * s + 1)


[GM5, PM5] = margin(G1*controller_5)
margin(G1*controller_5)
```

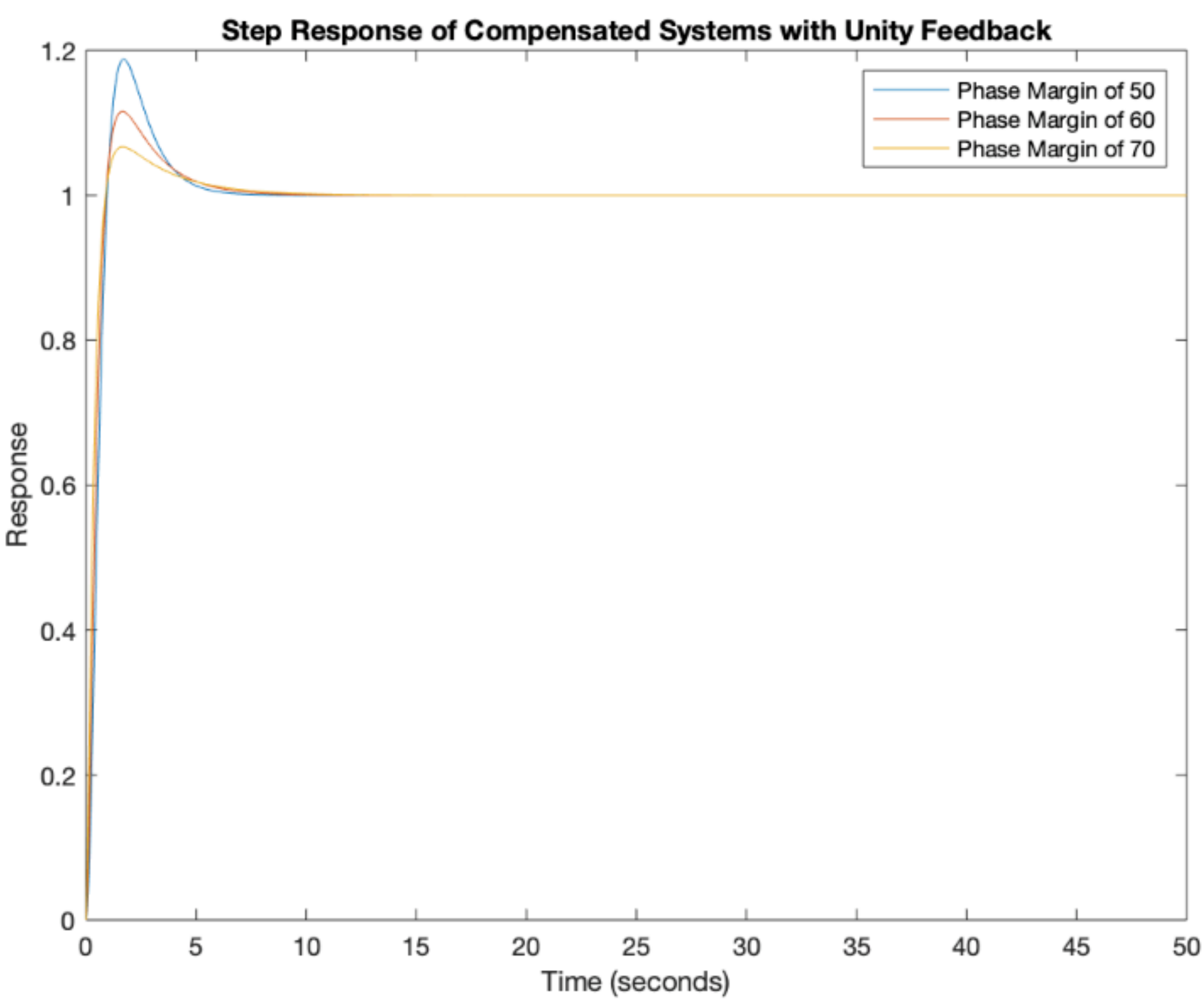
```
max_phase = 74.1619
gain_wmax = -17.1334
```




```
controller_5 =

  13.64 s + 5.2
  -------------
  0.05077 s + 1
```



```
G3 = feedback(G1*controller_3, 1);
G4 = feedback(G1*controller_4, 1);
G5 = feedback(G1*controller_5, 1);

t = 0:0.01:50;
[y3,t3] = step(G3, t);
[y4,t4] = step(G4, t);
[y5,t5] = step(G5, t);

plot(t3, y3, t4, y4, t5, y5)
xlabel('Time (seconds)')
ylabel('Response')
title('Step Response of Compensated Systems with Unity Feedback')
legend('Phase Margin of 50','Phase Margin of 60','Phase Margin of 
70')
```



As shown by the plots, as phase margin is increased, the overshoot of the time response of the system decreases.

11. Find crossover frequencies of compensated systems in Question 10. Explain the effect of crossover frequency on time domain response of the system. [10 marks]

```
[GM,PM,Wcg3,Wcp3] = margin(G1*controller_3);
Wcg3
Wcp3
```

```
Wcg3 = Inf
Wcp3 = 1.7864
```

```
[GM,PM,Wcg4,Wcp4] = margin(G1*controller_4);
Wcg4
Wcp4
```

```
Wcg4 = Inf
Wcp4 = 2.1429
```

```
[GM,PM,Wcg5,Wcp5] = margin(G1*controller_5);
Wcg5
Wcp5
```

```
Wcg5 = Inf
Wcp5 = 2.7272
```

As shown by the results above, as the phase crossover frequency is increased with increasing phase margins. In the time domain, the crossover frequency represents where the gain is 0, and the system becomes unstable.