FINAL PROJECT - ECSE-507 - 2022

PART 1

Write your own sub-programs implementing the following optimization algorithms as if they were to be included in a user-friendly optimization toolbox.

(a) The steepest descent algorithm;

(b) The Armijo step size rule selection;

(c) The conjugate gradient algorithm;

(d) The secant algorithm;

(e) Provide versions of the sub-programs of (a), (b) and (d) that employ finite difference approximations of the gradient;

(f) The penalty and barrier function algorithms for equality and inequality constrained optimization problems (see Lecture 21);

(g) The augmented Lagrangian algorithm (see Lecture 21);

(h) The Lagrange-Newton algorithm (see Lecture 22).

General guidelines

(i) Use either MATLAB or Python;

(ii) Provide your "software documentation" in the form of comments within the source code of the programs. The software documentation should be written in a way permitting its deployment by other less experienced users. To facilitate understanding of the documentation, always include the following (in the form of comment lines within the code) :

(a) description of the function of the program or sub-program,

(b) description of all parameters and variables that are needed to call and execute the program,

(c) example values for the parameters required.

(iii) Test your unconstrained optimization sub-programs on the examples provided below.

(iv) Seek out and apply MATLAB Optimization Toolbox functions that can solve the same problems. Compare the results obtained using your own software with that delivered by MATLAB Optimization Toolbox. Discuss differences and challenges encountered.

EXAMPLE PROBLEMS

The objective is to minimize the following cost functions $V(x)$:

$$
\begin{aligned}
&\text{(A)}\;\; V(x) = 5 + \begin{bmatrix} 1 & 4 & 5 & 4 & 2 & 1 \end{bmatrix} x + x^T \begin{bmatrix} 9 & 1 & 7 & 5 & 4 & 7 \\ 1 & 11 & 4 & 2 & 7 & 5 \\ 7 & 4 & 13 & 5 & 0 & 7 \\ 5 & 2 & 5 & 17 & 1 & 9 \\ 4 & 7 & 0 & 1 & 21 & 15 \\ 7 & 5 & 7 & 9 & 15 & 27 \end{bmatrix} x; \;\; x \in \mathbb{R}^6 \\
&\text{(B)}\;\; V(x) = -\frac{\sqrt{(x_1^2+1)(2x_2^2+1)}}{x_1^2+x_2^2+0.5}; \;\; x \in \mathbb{R}^2 \\
&\text{(C)}\;\; V(x) = 1 + \begin{bmatrix} 1 & 2 \end{bmatrix} x + \frac{1}{2} x^T \begin{bmatrix} 12 & 3 \\ 3 & 10 \end{bmatrix} x \\
&\qquad + 10\ln(1+x_1^4)\sin(100x_1) + 10\ln(1+x_2^4)\cos(100x_2); \;\; x \in \mathbb{R}^2
\end{aligned} \tag{1}
$$

PART 2

Using your own software solve the following constrained problems:

(1)

$$
\begin{aligned}
&\text{minimize } V(x) = |x_1 - 1| + |x_2 - 2|; \;\; x = [x_1\; x_2] \in \mathbb{R}^2 \\
&\text{subject to :} \\
&\qquad h_1(x) = x_1 - x_2^2 \geq 0 \\
&\qquad h_2(x) = x_1^2 + x_2^2 - 1 = 0
\end{aligned}
$$

(2)

$$
\begin{aligned}
&\text{minimize } V(x) = -x_1 x_2; \;\; x \in \mathbb{R}^2 \\
&\text{subject to :} \\
&\qquad h_1(x) = -x_1 - x_2^2 + 1 \geq 0 \\
&\qquad h_2(x) = x_1 + x_2 \geq 0
\end{aligned}
$$

(3)

$$
\begin{aligned}
&\text{minimize } V(x) = \ln(x_1) - x_2; \;\; x \in \mathbb{R}^2 \\
&\text{subject to :} \\
&\qquad h_1(x) = x_1 - 1 \geq 0 \\
&\qquad h_2(x) = x_1^2 + x_2^2 - 4 = 0
\end{aligned}
$$

Also try using the MATLAB Optimization Toolbox to compare or confirm the results obtained by using your own programs.

Project submission guidelines :

- Submit your project report for evaluation by uploading it on myCourses as part of a zipped folder called by your name and student number;

- Your report should have a title page : "Project Report" with your name and student number, course name and number, and date.
- Your report should be typed using the Latex software.
- Your zipped folder should contain the following :
  - (i) a pdf file of your report;
  - (ii) a sub-folder with all your Latex source code, style files and figures used in the production of your report;
  - (iii) text files with a listing of all your source code in your optimization toolbox together with master programs for the solution of the problems of Parts 1 and 2 of the project;
- The report should contain a description of your work including:
  - (a) the statement of the optimization problems solved ;
  - (b) all numerical solutions and their comparisons (wherever possible illustrate the convergence of your algorithms graphically);
  - (c) discussion of possible challenges encountered during project execution.

Tentative deadline for submission : 6 days after the last day of lectures.