18.06 RECITATION 2

Exercise 1. Show that

$$\begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -\lambda & 1 \end{bmatrix}.$$

Exercise 2. Consider the equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & \alpha & 0 \\ 1 & -2 & -1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

If we apply the Gaussian elimination, which values of α lead to a row exchange, and which values of α lead to a missing pivot? In the singular case, find a non-zero solution $\mathbf{x}$.

Exercise 3. Consider the following *tridiagonal* matrix:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 4 & -2 & 0 \\ 0 & 0 & -2 & 5 & -2 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}.$$

(1) Compute the LU decomposition of A. Do you notice the pattern of nonzero entries of the resulting matrices?
(2) Compute the third column of A^{-1} without inverting any matrix. Do you notice the pattern of nonzero entries of the resulting vector? What lessons do you learn?

Exercise 4. True or false (give an explanation if true, or a counterexample if false):
(1) If A is square and invertible, then $(A^{\mathsf{T}})^{-1} = (A^{-1})^{\mathsf{T}}$.
(2) If Q is a square orthogonal matrix, then so is Q^{T}.[1]
(3) If Q is an orthogonal matrix, then so is Q^{T}.
(4) If Q_1 and Q_2 are orthogonal matrices, then so is $Q_1 Q_2$.

Exercise 5. How many ways are there to fill in the question marks in the following matrix to make it orthogonal:

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & ? \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & ? \\ 0 & -\frac{1}{\sqrt{3}} & ? \end{bmatrix}.$$

Date: February 10, 2026.
[1]A (not necessarily square) matrix Q is said to be *orthogonal* if $Q^{\mathsf{T}}Q = I$.