

*Term paper*

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# Acronyms

**CNC** Computerized Numerical Control

**SPS** Speicherprogrammierbare Steuerung



# 1. Hints

Please, use **biber.exe**.

If you want to create a symbol directory, call makeindex:  
**makeindex %.nlo -s nomencl.ist**



## 2. Bézier-Curves

### 2.1. Introduction

Bézier curves are parameter curves that can be used to represent free-form curves. They are named after the French engineer Pierre Bézier, who developed them in the 1960s at Renault to design car body shapes. During the same period, French physicist and mathematician Paul de Casteljau developed these curves independently of Pierre Bézier at Citroën. Paul de Casteljau's results were available earlier, but they were not published. This is the reason why Pierre Bézier is the namesake of these parameter curves.[Far02]

The Bézier curves are the basis for computer-aided design of models. This is referred to either as Computer Aided Design (CAD) or, when the emphasis is more on a mathematical view, Computer Aided Geometric Design (CAGD). [BN11] Computer requires a mathematical description of shapes to represent them. The most suitable description method for this is the use of parametric curves and surfaces. Here Bézier curves play the central role, because they are the most numerically stable polynomial bases used in CAD/CAGD software.[Far02]

Typography on the computer also uses Bézier curves. There are two ways of representing type with the computer. The simplest way is to save each individual letter with a fixed resolution and size as a bitmap and copy it to the memory area of the screen as needed. These so-called bitmap fonts can be displayed quickly but require a lot of memory if the characters are to be available in different sizes. In this case, an extra bitmap must be created for each size. The quality also decreases when these bitmap fonts are scaled in size and resolution. Vector fonts are an alternative. Their name is derived from the fact that they use curves defined in a two-dimensional vector space to represent the characters. The advantage here is that the characters displayed in this way can be scaled without loss of quality. Two standards have developed for vector fonts. One is the TrueType font and the other is the PostScript font. The TrueType fonts use square Bézier curves while the PostScript fonts use cubic Bézier curves. The cubic Bézier curves have more control points which leads to a better quality.

Another field of application is the control of machine tools. When moving to a corner, the axis must be braked to a standstill at the corner point and then accelerated again after direction correction. This procedure ensures that it is not possible to move at a constant speed, which has negative consequences for the cycle time and quality. A possible solution to this problem is to place a curve between the two sections instead of a sharp corner, which can be run at a constant speed. Bézier curves are suitable for this purpose because only two points and two tangents are needed to form them. In the case of a corner, the points as well as the tangents would lie on the sections that form the corner. The resulting error would be analytically controllable and would allow an adjustment of the curve tolerance.[SS14]

## 2.2. Allgemeine mathematische Beschreibung Bézier-Kurve

Bézier curves are formed with the help of Bernstein polynomials. If at least two points and two tangents are known, control points can be determined with which a control polygon is formed. The course of the curve is oriented to this control polygon. The number of control points depends on the degree of the Bézier curve. A Bézier curve of degree  $n$  has  $n+1$  control points. The Bézier curve is calculated using the De Casteljau algorithm.

**Definition.** In the interval  $[0; 1]$  the **Bernstein polynomial of  $n^{\text{th}}$  degree** is defined by:

$$b_{i,n}(\lambda) = \binom{n}{i} (1 - \lambda)^{n-i} \lambda^i, \quad \lambda \in [0, 1], \quad i = 0, \dots, n \quad (2.1)$$

**Definition.** The binomial coefficient is defined by:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}, \quad i = 0, \dots, n \quad (2.2)$$

Here the Bernstein polynomials  $b_{i,n}$  form a basis of the vector space  $\mathbb{P}^n(I)$  of polynomials of degree at most  $n$  over  $I$ . Thus every polynomial of degree at most  $n$  can be written uniquely as a linear combination. [Far02]

**Beispiel.** Determination of Bernstein polynomials for  $n = 3$  holds:

$$b_{3,0}(\lambda) = \binom{3}{0} (1 - \lambda)^{3-0} \lambda^0 = (1 - \lambda)^3$$

$$b_{3,1}(\lambda) = \binom{3}{1} (1 - \lambda)^{3-1} \lambda^1 = 3\lambda(1 - \lambda)^2$$

$$b_{3,2}(\lambda) = \binom{3}{2} (1 - \lambda)^{3-2} \lambda^2 = 3\lambda^2(1 - \lambda)$$

$$b_{3,3}(\lambda) = \binom{3}{3} (1 - \lambda)^{3-3} \lambda^3 = \lambda^3$$

$b_{3,i}(\lambda)$  with  $i = 0, 1, 2, 3$  are the cubic Bernstein polynomials of degree 3.



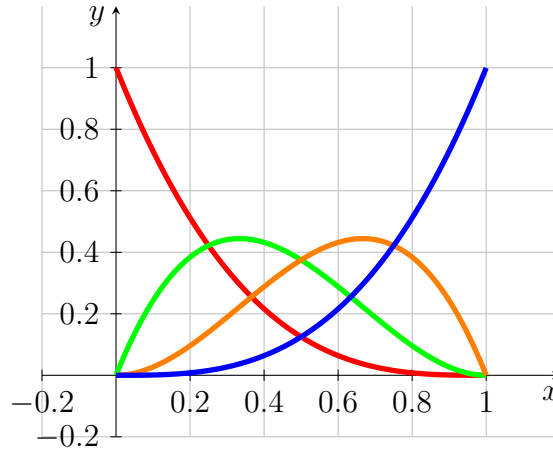


Figure 2.1.: Bernstein polynomial of degree 3

**Definition.** Given are the points

$$Q_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \quad \text{mit} \quad Q_i \in \mathbb{R}^2, \quad i = 0, 1, \dots, n$$

A **Bézier curve** is then defined by

$$C(\lambda) = \sum_{i=0}^n Q_i \cdot b_{i,n}(\lambda) \quad (2.3)$$

The points  $Q_i, i = 0, \dots, n$  are called **control points**.

The control points of a Bézier curve form the so-called control polygon.

**Bemerkung.** Let the starting point  $P_0$  and the end point  $P_1$ , as well as the tangents  $\vec{t}_0$  and  $\vec{t}_1$  be given. The tangents are not necessarily normalised.

The control points  $Q_0, Q_1, Q_2$  and  $Q_3$  of the associated Bézier curve are given by the following equations:

$$Q_0 = P_0, \quad Q_1 = P_0 + \lambda_0 \vec{t}_0, \quad Q_2 = P_1 - \lambda_1 \vec{t}_1, \quad Q_3 = P_1 \quad (2.4)$$

[Jak+10]

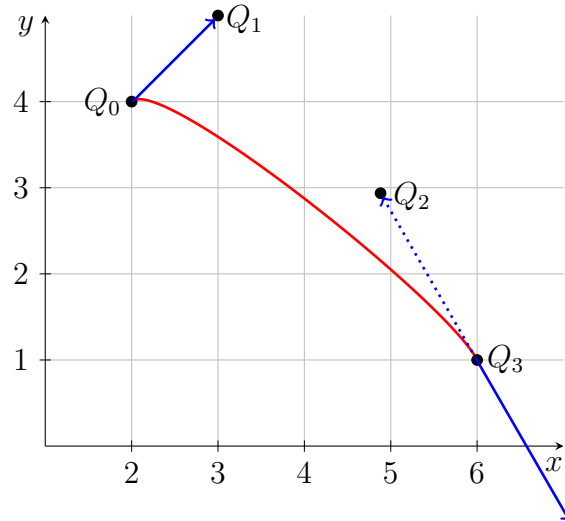


Figure 2.2.: Bézier curve for example 2.2

**Beispiel.** Given are

$$P_0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad \vec{t}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{und} \quad P_1 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \quad \vec{t}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Then, according to theorem ?? for control points of the associated Bézier curve results:

$$Q_0 = P_0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad Q_1 = P_0 + \vec{t}_0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix},$$

$$Q_2 = P_1 - \vec{t}_1 = \begin{pmatrix} 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \quad Q_3 = P_1 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

The Bézier curve and its control points are shown in the figure ??.



## 3. Verrundung mit einem Kreisbogen

### 3.1. Gleichungen

Blending with an arc is a simple variant of smoothing corners. This variant enables the processing and the generation of files according to DIN 66025. For smoothing, three points  $P_0$  and  $S$  and  $P_1$  are always considered, thus a symmetric Hermite problem results. According to theorem ?? it is assumed to be in the standard form  $HP(L, \alpha)$ .

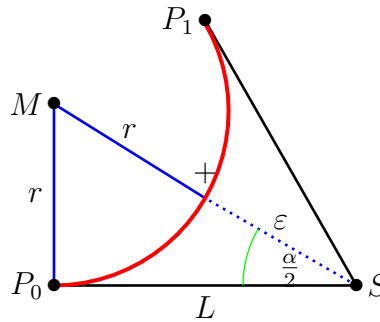


Figure 3.1.: Smoothing a corner with the help of an arc - triangle

The figure ?? represents the situation. The points  $P_0$ ,  $S$  and  $P_1$  are given. The included angle  $\alpha = \angle(P_0; S; P_1)$  as well as the distance  $L = \|S - P_0\| = \|P_1 - S\|$  are shown in the graph. The red arc is the desired result. The distance of its centre  $M$  to  $S$  is then  $r + \varepsilon$ , where  $r$  is the radius of the arc and  $\varepsilon$  is the given tolerance. Thus we obtain a right triangle  $P_0SM$  whose edge lengths are  $L$ ,  $r + \varepsilon$  and  $r$ .

According to the definition of the sine and the tangent, it follows:

$$\sin\left(\frac{\alpha}{2}\right) = \frac{L}{r + \varepsilon} \quad \text{und} \quad \tan\left(\frac{\alpha}{2}\right) = \frac{L}{r}$$

$$\Leftrightarrow \varepsilon = \frac{L}{\sin\left(\frac{\alpha}{2}\right)} - r \quad \text{und} \quad r = \cot\left(\frac{\alpha}{2}\right) L$$

The two equations can be combined to give the following conditions:

$$\varepsilon = L \cdot \frac{1 - \cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \quad \text{bzw.} \quad L = \varepsilon \cdot \frac{\sin\left(\frac{\alpha}{2}\right)}{1 - \cos\left(\frac{\alpha}{2}\right)}$$

This gives the equation for the radius:

$$r = L \cdot \cot\left(\frac{\alpha}{2}\right) = L \cdot \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}} = L \cdot \frac{\sin(\alpha)}{1 + \cos(\alpha)} = L \cdot \frac{P_{1,x}}{P_{1,y}}$$

The factor for converting  $L$  and  $\varepsilon$  can be simplified using trigonometric transformations.

$$\frac{1 - \cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = \frac{1 - \sqrt{\frac{1 - \cos(\alpha)}{2}}}{\sqrt{\frac{1 + \cos(\alpha)}{2}}} = \frac{\sqrt{2} - \sqrt{1 - \cos(\alpha)}}{\sqrt{1 + \cos(\alpha)}} = \frac{\sqrt{1 + \cos(\alpha)} - \sin(\alpha)}{1 + \cos(\alpha)}$$

By comparing this with the symmetric Hermite problem in standard form, the following notation is then obtained:

$$\frac{1 - \cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = \frac{\sqrt{P_{1,x} - P_{1,y}}}{P_{1,x}}$$

The previous considerations are now summarised in the following sentence.

**Satz.** Let a symmetric Hermite problem  $(P_0, \vec{t}_0, P_1, \vec{t}_1, S, L)$  be given. Without restriction of generality, it is in the standard form

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} L + L \cdot \cos(\alpha) \\ L \cdot \sin(\alpha) \end{pmatrix}, \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}, \begin{pmatrix} L \\ 0 \end{pmatrix}, L \right\}$$

with  $L \in \mathbb{R}^{>0}$  and  $\alpha \in (-\pi; \pi]$ .

Then an arc can be found that connects the points  $P_0$  and  $P_1$ , has the same tangent directions at the two points.

For the circular arcs applies:

$$r = L \cdot \left| \frac{P_{1,x}}{P_{1,y}} \right|; \quad \phi_0 = -\text{sign}(\alpha) \cdot \frac{\pi}{2}; \quad \phi_1 - \phi_0 = \text{sign}(\alpha) \cdot \pi - \alpha = \beta;$$

$$M = P_0 + \text{sign}(\alpha) \cdot r \cdot \vec{t}_0^\perp = \text{sign}(\alpha) \cdot r \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

From the specification of the distance  $L$ , the maximum error can be calculated, as shown in theorem ?? . According to the derivation, it is also possible to specify the maximum error  $\varepsilon$  and determine the maximum distance  $L$  from it.

**Satz.** Let a symmetric Hermite problem  $(P_0, \vec{t}_0, P_1, \vec{t}_1, S, L)$  be given. Without restriction of generality, it is in the standard form

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} L + L \cdot \cos(\alpha) \\ L \cdot \sin(\alpha) \end{pmatrix}, \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}, \begin{pmatrix} L \\ 0 \end{pmatrix}, L \right\}$$

with  $L \in \mathbb{R}^{>0}$  and  $\alpha \in (-\pi; \pi]$ .

- a) Let the maximum error  $\varepsilon$  be given. The maximum distance  $L$  for which an arc exists according to the theorem ?? that takes the error into account is given by:

$$L(\varepsilon, \alpha) = \varepsilon \cdot \frac{P_{1,x}}{\sqrt{P_{1,x} - P_{1,y}}} = \varepsilon \cdot \frac{L + L \cdot \cos(\alpha)}{\sqrt{L + L \cdot \cos(\alpha)} - L + L \cdot \cos(\alpha)}$$

- b) When the distance  $L$  is specified, the following maximum error results:

$$\varepsilon(L, \alpha) = L \cdot \frac{\sqrt{P_{1,x} - P_{1,y}}}{P_{1,x}} = L \cdot \frac{\sqrt{L + L \cdot \cos(\alpha)} - L + L \cdot \cos(\alpha)}{L + L \cdot \cos(\alpha)}$$

These considerations result in limitations for the use of this strategy, which are summarised in the following comment.

**Bemerkung.** The following conditions for applying the strategy of a circle must be fulfilled:

- a)

$$P_0 \neq P_1$$

- b)

$$\vec{t}_0 = -\vec{t}_1 \Leftrightarrow \alpha = 0$$

- c)

$$\vec{t}_0 = \vec{t}_1 \Leftrightarrow \alpha = \pi$$

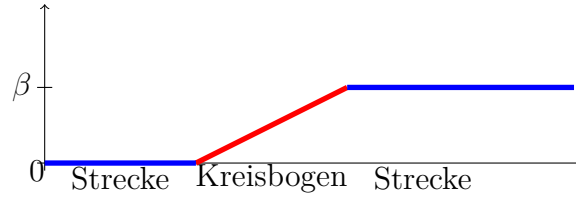


Figure 3.2.: Angular change with blending arc

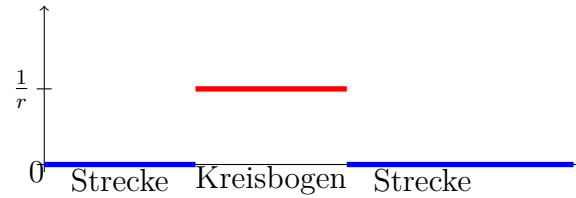


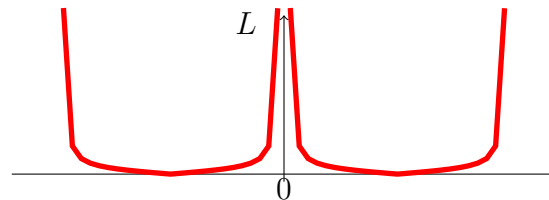
Figure 3.3.: Curvature progression for a blending arc

### 3.2. Bewertung

Rounding by means of a circular arc leads to a continuous course of the angle change. The figure ?? illustrates this.

Due to the rounding with a circular arc, the curvature of the curve is not continuous. The figure ?? shows that the curvature is constant in the range of distances 0 and on the circular arc with the value  $\frac{1}{r}$ , where  $r$  is the radius of the circular arc used. Due to the formula  $a = \frac{v^2}{r}$  for a movement on a circular arc, the acceleration course exhibits continuity jumps at the transitions for a constant path velocity.

Depending on the angle change  $\beta$  at the corner  $S$  and the tolerance  $\varepsilon$ , the maximum length of the shortening of the lines can be calculated. The figure ?? shows the corresponding diagram, where the tolerance  $\varepsilon$  is set to the value 1.

Figure 3.4.: Maximum range for a blending arc of a circle -  $L(\varepsilon = \text{const}, \alpha)$

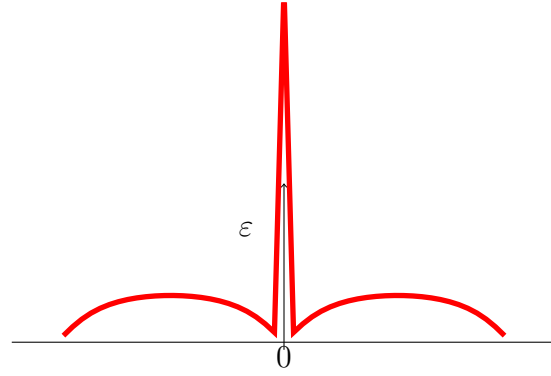


Figure 3.5.: Deviation when specifying the distance  $L$  when rounding with a circular arc

The area provided can also be specified. Depending on the distance  $L$  from the corner point  $S$ , the deviation  $\varepsilon$  can be calculated. The figure ?? represents the function as a function of  $L$  and the angle  $\alpha$ .

The figures ?? and ?? show the curves of the length as a function of the angle change for a fixed tolerance and the error as a function of the angle change for a given length. If the angle change is minimal, then the error or length  $L$  is extreme. In this case, rounding by means of an arc is not possible. In order to develop a sensible strategy, a minimum angle change must be specified from which a blending arc is permitted. Likewise, a maximum angle change must also be defined. For an angular change of  $\pm\pi$  is a bend. In this case, a blending is also not possible.



### 3.3. Examples

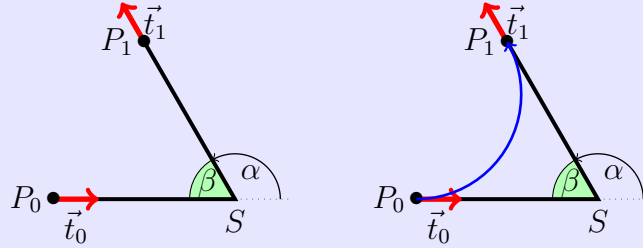
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(\frac{2}{3}\pi\right) \\ 6.0 \cdot \sin\left(\frac{2}{3}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(\frac{2}{3}\pi\right) \\ \sin\left(\frac{2}{3}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = \frac{2}{3}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ 3,4641 \end{pmatrix}; \quad r = 3,46412; \quad \phi_0 = -\frac{\pi}{2}; \quad \alpha = \frac{2}{3}\pi$$



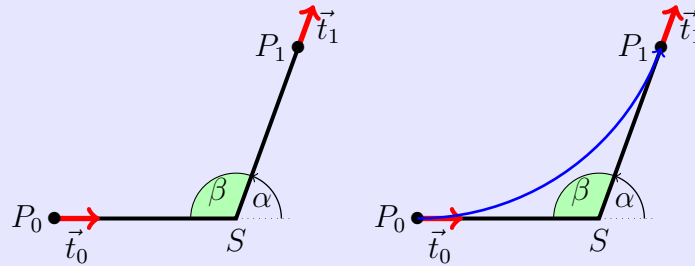
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(\frac{7}{18}\pi\right) \\ 6.0 \cdot \sin\left(\frac{7}{18}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(\frac{7}{18}\pi\right) \\ \sin\left(\frac{7}{18}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = \frac{7}{18}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ 8,5689 \end{pmatrix}; \quad r = 8,5689; \quad \phi_0 = -\frac{\pi}{2}; \quad \alpha = \frac{7}{18}\pi$$



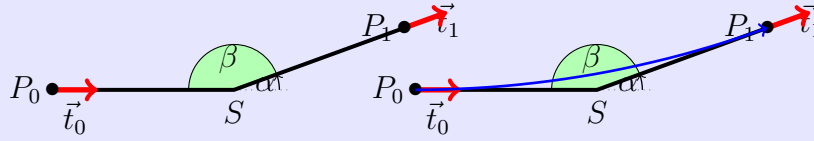
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(\frac{1}{9}\pi\right) \\ 6.0 \cdot \sin\left(\frac{1}{9}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(\frac{1}{9}\pi\right) \\ \sin\left(\frac{1}{9}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = \frac{1}{9}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ 34,0277 \end{pmatrix}; \quad r = 34,0277; \quad \phi_0 = -\frac{\pi}{2}; \quad \alpha = \frac{1}{9}\pi$$

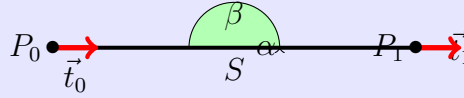


**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 12.0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = 0$ .

There is no blending arc here.



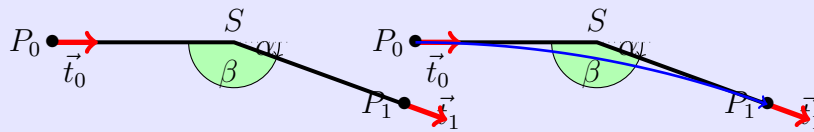
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(-\frac{1}{9}\pi\right) \\ 6.0 \cdot \sin\left(-\frac{1}{9}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(-\frac{1}{9}\pi\right) \\ \sin\left(-\frac{1}{9}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = -\frac{1}{9}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ -34,0277 \end{pmatrix}; \quad r = 34,0277; \quad \phi_0 = \frac{\pi}{2}; \quad \alpha = \frac{1}{9}\pi$$



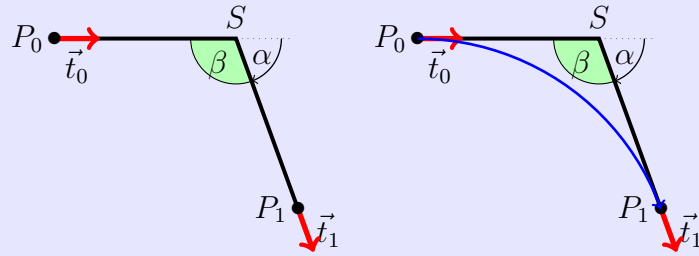
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(-\frac{7}{18}\pi\right) \\ 6.0 \cdot \sin\left(-\frac{7}{18}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(-\frac{7}{18}\pi\right) \\ \sin\left(-\frac{7}{18}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = -\frac{7}{18}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ -8,5689 \end{pmatrix}; \quad r = 8,5689; \quad \phi_0 = \frac{\pi}{2}; \quad \alpha = -\frac{7}{18}\pi$$



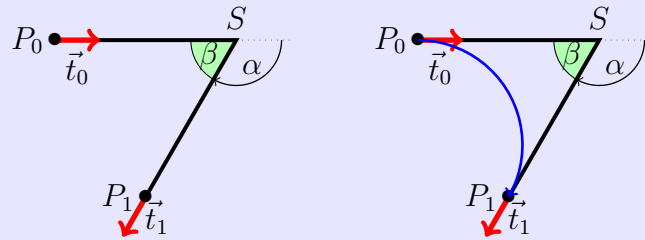
**Beispiel.** Let it be the symmetric Hermite problem

$$\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.0 + 6.0 \cdot \cos\left(-\frac{2}{3}\pi\right) \\ 6.0 \cdot \sin\left(-\frac{2}{3}\pi\right) \end{pmatrix}, \begin{pmatrix} \cos\left(-\frac{2}{3}\pi\right) \\ \sin\left(-\frac{2}{3}\pi\right) \end{pmatrix}, \begin{pmatrix} 6.0 \\ 0 \end{pmatrix}, 6.0 \right\}$$

with  $L = 6$  and  $\alpha = -\frac{2}{3}\pi$ .

For the blending arc follows:

$$M = \begin{pmatrix} 0 \\ -3,4641 \end{pmatrix}; \quad r = 3,4641; \quad \phi_0 = \frac{\pi}{2}; \quad \alpha = -\frac{2}{3}\pi$$



## 4. Maple files

[Wat17c; Wat17d; Wat17b; Wat17e; Wat17a; Wat17f]

### 4.1. Geometries with Maple

The algorithms presented can be well represented using geometries. Two aspects can be investigated. On the one hand, the effort for an implementation, the computational accuracy and the stability of an algorithm are interesting; on the other hand, the methods are to be evaluated and compared with regard to their usefulness. For this purpose, modules for the formula manipulation system Maple [Wat17c] were developed.

Maple offers the environment to implement algorithms easily and quickly. Furthermore, graphical representation is possible with simple means. However, a simple workbook of Maple is not designed for very large software projects. Here, one must resort to the possibility of creating and using libraries. On the one hand, Maple offers the possibility of creating one's own libraries, so-called modules. In this way, one remains within the environment and syntax of Maple. This path will be pursued further here. Another possibility is the use of libraries created by means of a higher programming language, e.g. C++, the so-called DLLs.

In the following, the creation and use of a test environment with the help of modules is described. First, the geometry elements that are stored in various modules and their use are described. The structure and use of a module in Maple is then explained so that own extensions and additions are possible.

### 4.2. Geometry elements used

This is a test environment for the algorithms presented, only geometries that have been described are also used.

- points (**MPoint**)
- lines (**MLine**)
- arcs (**MArc**)
- Bézier curves (**Bezier**)

- polygon courses (**MPolygon**)
- Geometry List (**MGeoList**)
- Hermite problems (**MHermiteProblem**)
- Symmetric Hermite Problems (**MHermiteProblemSym**)

For each geometry element a corresponding module has been created, the name of which is given in the list above. The list of which functions are available is presented in another section.

When implementing such a project, it quickly becomes clear that when using several geometry elements, a clear data structure and well-defined access to the data is essential. For example, when representing straight lines, the decision must be made whether the representation should be point-directional or by means of two points. The choice is generally made depending on the application. Here, the path is taken that, due to a well-defined access to the data, the use of both representations is possible.

### 4.3. Building the data structure

The idea is to use a data structure that is as uniform and simple as possible. Maple does offer the possibility to define objects. However, it is difficult to use within a procedural environment. Therefore, all data is basically represented as lists. The first list element always contains an identifier of the element `??`. This is followed by the data, which in turn can be geometry elements. It should be noted that direct access to the data cannot be prevented; here the user is responsible.

Access to the data should be exclusively via procedures of a module. The following is an example of the data structure for points:

```
[MVPOINT, [x, y]]
```

The creation of a point then takes place via a procedure **New**:

```
NeuerPunkt := MPoint:-New(10,15);
```

When called up in the test file, the following is then output:

```
TestP0 := ["Point", [10,15]]
```

These data structures are used for the calculation of geometries. They are used in functions or procedures. Unlike variables, the data structures and procedures are defined globally and not locally. Under the command **export** the procedures are declared at the beginning of the module and can thus also be used outside the module. If, for example, a data structure from **MPoint** is to be used in another module or outside the modules, it is called as follows:

```
P0 := MPoint:-New(x,y);
```

In addition to the modules for the geometries, there is a module (MConstant) for saving constants and names. There, for **MVPOINT**, for example. „Point “ or e.g. a constant for the comparison to zero for real numbers.

Finally there is the test file, which is not a module, in which the modules are loaded, called and tested in their function. More about this file later in „1.4 Maple test file “.

## 4.4. Structure of a module

The following section deals with the purpose of modules and their structure.

The project is about capturing the above geometries in a data structure and representing them in Maple. However, the calculations and formulas that have to be used are a hindrance to the final representation. Modules are very well suited for summarising and hiding the calculations that take place in functions. This way, the important functions can be accessed in Maple without seeing what is in them.

Example:

The function **Angle** from the module **MPoint**: Function to calculate an angle between location vector and x-axis:

```
Angle := proc(P)
  local alpha, x, y;
  x := GetX(P);
  y := GetY(P);
  alpha := 0;
  if abs(x) < 0.00001
  then
    alpha := 3*Pi/2;
  else
    alpha := Pi/2;
  end if;
else
  alpha := arctan(y/x);
  if x < 0
  then
    alpha := alpha + Pi;
  else
    if y < 0
```

```
        then
            alpha := alpha + 2*Pi;
        end if;
    end if;
end if;
return factor(alpha);
end proc;
```

This function is long, but it only represents a small part of the programme for the representation in question. That is why it is in the module and can thus be called externally with a single command:

```
Winkel := MPoint:-Angle(P)
```

The following section describes the structure of a module. Since Maple offers a wide range of possibilities, a restriction is made. Only the structure of the modules used is described, here on the basis of the module `MPoint`. For more detailed possibilities, please refer to the Maple manual [Wat17c].

structure:

The module must first be started. This is done with the name (here always a capital M and the geometry) of the module and the following command:

```
MPoint := module()
```

Then, similar to procedures, the variables and functions must be defined. You can declare them either locally or globally. If the variables or procedures are only used and changed in the module, the declaration is made with the command `local` as follows:

```
local Variable names, with, comma, separated;
```

If the variables or procedures are also to be usable outside the module, this is defined with `export`:

```
export Function names, with, comma, separated;
```

This is followed by the specification of the options:

```
option package;
```

the description of the module:

```
description "Self-selected module description, e.g. module for points ";
```

and the initialisation of the module:

```
ModuleLoad := proc
  MVPPOINT:=MConstant:-GetPoint();
  print("Module MPoint is loaded");
end proc;
```

From here on, programming is done as usual in Maple. The previously defined procedure and variable names are used and programming is done with commands that are also used outside of a module in Maple. It is important to know that with modules, each function can be called constantly, so the order of the functions is irrelevant.

To finally end the module, use the following command:

```
end module;
```

#### 4.4.1. General structure of a module

In summary, the modules have the following structure:

```
Modulname := module()

  local Names, with, comma, separated;

  export Names, with, comma, separated;

  global Names, with, comma, separated;1

  option package;

  description „ Self-selected module description“;

  ModuleLoad := proc()2
    MVPPOINT:=MConstant:-GetPoint();
    print("Modul MPoint is loaded");
  end proc;

  Procedure1 := proc (Passing parameters)
    inhalt;
  end proc;

  Procedure2 := proc (Passing parameters)
```

---

<sup>1</sup>Possible, but not used in the modules

<sup>2</sup>Example **MPoint**



```
        content;  
    end proc;
```

```
    ...
```

```
end module;
```

### 4.4.2. Saving a module

The management of modules is done automatically by Maple. However, since the modules are passed on and have to be edited individually, some settings have to be made. Therefore, the following prefix is used for each Maple file of the project:

```
1 restart;
2 with(LibraryTools);
3 lib := "C:/FH/Tools/Maple/MyLibs/Blending.mla";
4 march('open', lib);
5 ThisModule := 'MArc';
```

The first line initialises the system. The next 3 lines enable working with archives. In the second line, the tools are loaded so that the variable `lib` can be occupied. All modules are stored in an archive; in this example it is the file `Blending.mla` in the directory `C:/FH/Tools/Maple/MyLibs/`. The command `ThisModule := 'MArc';` contains the name of the current module.

A module is then saved using the command `savelib('ModuleName')`. A file `ModuleName.mla` is then created. Depending on the configuration of Maple, the set path where the file is automatically created may not be writable. Then you can configure the system so that the mla file is stored in the current directory or in a directory of your choice.

If the above prefix is used for a Maple file, all that is required to save the module is

```
savelib(ThisModule, lib);
```

### 4.4.3. Verwendung eines Moduls

A module that has been saved can now be used in other Maple worksheets. To do this, the command

```
with(ModuleName)
```

is used. If you have used a special path, Maple cannot find the file `ModuleName.mla`. Then the path must be made known, e.g.:

```
savelibname := "c:/Maple/MyLibs";, savelibname;
```

Now the procedures of the module can be accessed. An overview of all exported procedures is provided by the command

```
Describe(ModuleName)
```

is displayed. A list of the names including the names of the transfer parameters is displayed. If a procedure is equipped with the field `description`, this text is also displayed.

#### 4.4.4. Creating a Maple module

Creating your own module is done quickly if you use the framework above. However, one should make some considerations beforehand and maintain a standard.

Before creating an own module, the data model and the corresponding procedures should be worked out. A programme flow chart is useful here. The central task of the module is usually quickly determined. In addition, however, the following rules should be observed.

**Rule 1** Basically, both the module and each procedure receive a description. This is not limited to the optional argument **description**, but is placed in front of each procedure. The description basically contains the description of the task. The prerequisite is also mentioned or an error handling is described. Then follows the description of all input parameters, their function and their data structure. The return value is then described.

**rule 2** Each module receives a procedure **Version()**, which returns the current version number.

**rule 3** Data is not global. To access data, procedures **Set\*** and **Get\*** are provided. These procedures are also used within the procedures of a module.

**rule 4** At least one test function is written for each procedure. The test function can be used to illustrate the use and any special features.

### 4.5. Functions of the modules

In the following, the individual modules are listed. The data structure of each module and all functions with associated tasks are listed.

#### 4.5.1. Module **MPoint**

**MPoint** is a module for points and works with a data structure and with procedures/functions. The data structure **New** for a point is a list, which is structured as follows:

**[MVPOINT, [x,y]]**

Its first element is a name to identify the module. Your second element is again a list containing the elements of the geometry. In this case the name is "Point" and the elements are the x and y coordinates for a point. No distinction is made here between point and vector. A point can also be seen as a vector from the coordinate origin to the point.

Via the Get functions **GetX** and **GetY** one can capture the coordinates of the point and use them in other functions. The other functions can then be used to calculate with the points/vectors.

### **MPoint**

---

E	<b>New</b>	Data structure for a point
E	<b>GetX</b>	Reading the x-coordinate
E	<b>GetY</b>	reading the y-coordinate
E	<b>Angle</b>	calculating the angle between the x-axis and the point
E	<b>Add</b>	Calculates a linear combination of two points/vectors
E	<b>Sub</b>	Calculates the difference of two points/vectors
E	<b>Cos</b>	Calculates the cosine between two vectors
E	<b>Sin</b>	Calculates the sine between two vectors
E	<b>Scale</b>	Scales a vector with a factor
E	<b>Perp</b>	Calculates a vector that is orthogonal to the given vector
L	<b>IllustrateXY</b>	Plot function to illustrate a blue point
E	<b>Illustrate</b>	Plot of a blue point
E	<b>Plot</b>	Plot of a green point
E	<b>Plot2D</b>	Plot of a point with own options
E	<b>Length</b>	Calculates the distance of the point to the coordinate origin
E	<b>Uniform</b>	Normalises the vector to length 1
E	<b>LinetoVector</b>	Calculates vector from a distance
E	<b>Distance</b>	Calculates the distance between two points

#### 4.5.2. Module **MLine**

**MLine** is a module for lines and works with a data structure and with procedures/functions. The data structure **New** for a line is a list which is structured as follows:

```
[MVLINE, [P0,P1]]
```

Its first element is a name to identify the module. Its second element is again a list containing the elements of the geometry. In this case the name is „Line“ and the elements are the start and end points for a line.

The data structure **NewPointVerctor** is also a data structure for a line, but here the line is defined by a start point and a direction vector.

The Get functions **StartPoint** and **EndPoint** can be used to capture the start and end points of the route and use them in other functions. The other functions can then be used to calculate with the routes.

---

**MLine**


---

E	<b>New</b>	Data structure for a line (two-point form)
E	<b>NewPointVector</b>	creation of a route (point-direction form)
E	<b>StartPoint</b>	reading the starting point
E	<b>EndPoint</b>	reading the end point
E	<b>Position</b>	Calculate a point that lies on the line
E	<b>Plot2D</b>	Plot a part of the route starting from the starting point
E	<b>Plot2DTangent</b>	Plot of a point with tangent
E	<b>Plot2DTangentArrow</b>	Plot of a point with tangent (as arrow)
E	<b>LineLine</b>	Calculation of the intersection of two straight lines.
E	<b>AngleLine</b>	Calculation of the angle between two straight lines

### 4.5.3. Module **MArc**

**MArc** is a module for circular arcs and works with a data structure and with procedures/functions. The data structure **New** for an arc is a list that is structured as follows:

```
[MVARC, [mx,my,r,phi,alpha]]
```

Its first element is a name to identify the module. Your second element is again a list containing the elements of the geometry. In this case the name is "Arc" and the elements are the x- and y-coordinate for the centre, the radius, the start angle and the angle change for an arc.

The Get functions **GetM**, **GetMX**, **GetMY**, **GetR**, **GetPhi**, and **GetAlpha** can be used to collect the data for an arc and use it in other functions. The other functions can then be used to calculate with the arcs.

---

**MArc**


---

E	<b>New</b>	Data structure for an arc
E	<b>GetMX</b>	Reading the x-coordinate of the centre point.
E	<b>GetMY</b>	read the y-coordinate of the centre point
E	<b>GetR</b>	read the radius
E	<b>GetPhi</b>	reading the start angle
E	<b>GetAlpha</b>	Reading the change of angle
E	<b>GetM</b>	reading the centre point
E	<b>Position</b>	Calculating a point on the arc
E	<b>Plot2D</b>	Plot a part of the arc starting from the start angle
E	<b>Blend</b>	calculating an arc from a symmetric Hermite problem

### 4.5.4. module **MBezier**

The **New** data structure for a polygon is a list constructed as follows:

**[MVBEZIER, [PointList]]**

Within the module **MBezier** exist the procedures listed in the following table.

### **MBezier**

E	<b>New</b>	Manual input of control points for a Bézier curve
E	<b>Version</b>	Output of the versions
E	<b>BlendCurvature</b>	Determination of control points from symmetric Hermite problem
E	<b>BlendCurvatureEpsilon</b>	determination of control points from symmetric Hermite problem with given error
E	<b>Position</b>	position on the Bézier curve
E	<b>GetTheta</b>	Reading the angle
E	<b>GetEpsilon</b>	reading the tolerance
E	<b>GetControlPoint</b>	Read the control points
E	<b>Plot2D</b>	Read the Bézier curve
E	<b>PlotControlPoints</b>	representation of all control points

### 4.5.5. Module **MPolygon**

**MPolygon** is a module for polygons and works with a data structure and with procedures/functions. The data structure **New** for a polygon is a list that is structured as follows:

**[MVPOLYGON, [PointList]]**

Its first element is a name to identify the module. Your second element is again a list containing the elements of the geometry. In this case the name is "Polygon" and the elements are any number of points.

Using the Get functions **GetPoint** and **GetN** you can get the number of points and the points themselves and use them in other functions. The other functions can then be used to calculate with the points or the polygon course.

### **MPolygon**

E	<b>New</b>	Data structure for a list of points
E	<b>GetPoint</b>	Reading the ith point from the point list
E	<b>GetN</b>	Determine the number of points in the point list
E	<b>Length</b>	Determination of the Euclidean length of the polygon
E	<b>Position</b>	Calculation of a point on the polygon course
E	<b>Tangents</b>	calculation of the tangent
L	<b>Plot2DAll</b>	Plot list of all points
E	<b>Plot2D</b>	Plot of all points as polygonal plots.
E	<b>Plot2DTangent</b>	representation of the polygon course with tangent
E	<b>PlotPoints</b>	representation of all points

#### 4.5.6. module **MGeoList**

**MeoList** is a module for a geometry list and works with a data structure and with procedures/functions. The data structure **New** for a geometry list is a list that is structured as follows:

```
[MVGEOLIST, []]
```

Its first element is a name to identify the module. Its second element is again a list containing the elements of the geometry. In this case the name is „GeoList“ and the elements are any number of individual geometries.

Using the Get functions **GeoGeo** and **GetN**, the i-th element of the list and the number of elements in the list can be captured and used in other functions. The other functions can then be used to calculate with the geometries or the list. In the functions **Length**, **Plot2DAll**, **Plot2D** and **Position** the functions are called in themselves. This is possible because the functions work independently of each other.

##### **MGeoList**

E	<b>New</b>	Data structure for a geometry list
E	<b>Append</b>	Append a geometry element to the data structure
E	<b>Prepend</b>	Inserting a geometry element as the first element of the list
E	<b>Replace</b>	Replace a geometry element with another one.
E	<b>GetN</b>	Determine the number of geometry elements in the list
E	<b>GeoGeo</b>	Reading the i-th geometry element
E	<b>Length</b>	Calculate the Euclidean length of the geometry elements
E	<b>Position</b>	Calculates a point on the geometry
L	<b>Plot2DAll</b>	Plot function for plotting the geometry elements
E	<b>Plot2D</b>	Plot a part of the geometry list starting from the first element

#### 4.5.7. Module **MHermiteProblem**

**MHermiteProblem** is a module for Hermite problems and works with a data structure and with procedures/functions. The data structure **New** for a route is a list, which is structured as follows:

```
[MVHERMITEPROBLEM, [P0,T0n,P1,T1n]]
```

Your first element is a name to identify the module. Its second element is again a list containing the elements of the geometry. In this case the name is „HermiteProb“ and the elements are two points with associated tangents (lines).

Using the Get functions **StartPoint**, **EndPoint**, **StartTangent** and **EndTangent**, the points and their tangents can be captured and used in other functions. The other functions can then be used to calculate with the data for the Hermite problem.

### **MHermiteProblem**

---

E	<b>New</b>	Data structure for a Hermite problem
E	<b>StartPoint</b>	reading the start point
E	<b>EndPoint</b>	Reading the end point command
E	<b>StartTangent</b>	reading the start tangent
E	<b>EndTangent</b>	Reading the end tangent
E	<b>Plot2D</b>	Plotting the Hermite problem

#### **4.5.8. Module MHermiteProblemSym**

**MHermiteProblemSym** is a module for symmetric Hermite problems and works with a data structure and with procedures/functions. The data structure **New** for a symmetric Hermite problem is a list built as follows:

**[MVHERMITEPROBLEMSYM, [P0,T0n,P1,T1n,S,L]]**

Your first element is a name to identify the module. Its second element is again a list containing the elements of the geometry. In this case the name is „SymHermiteProb“ and the elements are two points with associated tangents, their intersection, and the distance of the points to the intersection.

Via the Get functions **StartPoint**, **EndPoint**, **StartTangent**, **EndTangent**, **CrossPoint** and **ParameterL** one can acquire the data and use it in other functions. The other functions can then be used to calculate with the data for the symmetric Hermite problem.

### **MHermiteProblemSym**

---

E	<b>New</b>	Data structure for a symmetric Hermite problem
E	<b>StartPoint</b>	reading the start point
E	<b>EndPoint</b>	Reading the end point command
E	<b>StartTangent</b>	reading the start tangent
E	<b>EndTangent</b>	Reading the end tangent
E	<b>ParameterL</b>	reading of the distance
E	<b>CrossPoint</b>	reading of the intersection point
E	<b>Plot2D</b>	Plotting of the symmetrical Hermite problem
E	<b>Create</b>	creation of a Sym. Hermite problem by 3 points
E	<b>BlendArc</b>	rounding of the corner point by an arc



#### 4.5.9. Module **MConstant**

**MConstant** is a module for storing fixed constants and names to identify data structures. In the locally declared functions, starting with CV, the constants for declaring the different geometries are defined. These are names for recognising the geometry elements in the test file. In the globally declared functions, starting with Get, the names for identifying the geometry elements are returned.

##### **MConstant**

L	<b>NULLEPS</b>	constant to compare to zero
L	<b>CVPOINT</b>	constant for geometry elements: Point
L	<b>CVLINE</b>	constant for geometry elements: Line
L	<b>CVARC</b>	constant for geometry elements: Arc
L	<b>CVPOLYGON</b>	constant for geometry elements: polygon
L	<b>CVGEOLIST</b>	constant for geometry elements: GeoList
L	<b>CVHERMITEPROBLEM</b>	constant for geometry elements: HermiteProb
L	<b>CVHERMITEPROBLEMSYMMETRIC</b>	Const. for geometry elements: SymHermiteProb
L	<b>CVBIARC</b>	Const. for geometry elements: Biarc
E	<b>GetNullePs</b>	return of the zero comparison command.
E	<b>GetPoint</b>	identifier for points
E	<b>GetLine</b>	Identifier for lines
E	<b>GetArc</b>	identifier for circular arcs
E	<b>GetPolygon</b>	identifier for polygons
E	<b>GetGeoList</b>	Identifier for geometry lists
E	<b>GetHermiteProblemSymmetric</b>	Identifier for symmetric Hermite problems
E	<b>GetHermiteProblem</b>	ID for Hermite problems
E	<b>GetBiarc</b>	identifier for Biarcs

#### 4.5.10. Module **MGeneralMath**

**MGeneralMath** is a module for general mathematical functions and works with the data structures and procedures.

### **MenerealMath**

---

E	<b>MPoint</b>	Data structure for a point
E	<b>MPointX</b>	Reading of the x-coordinate for a point
E	<b>MPointY</b>	reading the y-coordinate for one point
L	<b>MPointIllustrateXY</b>	plot structure for a blue point
E	<b>MPointIllustrate</b>	plot structure for a blue point
E	<b>MPointPlot</b>	Illustration of a green point
E	<b>MLine</b>	Data structure for a line
E	<b>MLineStartPoint</b>	Reading of the start point for a draw frame
E	<b>MLineEndPoint</b>	Reading the end point for a line
E	<b>MPointOnLine</b>	Calculation of a point on the line
E	<b>MLinePlot2D</b>	Plot the part of a line starting at the starting point
. E	<b>MLineLine</b>	calculating the intersection of two linesline

#### **4.5.11. Module Biarc**

##### **Data Structure**

Requirements and specifications:

The Biarc is to be used to solve a Hermite problem. A Hermite problem is defined as follows:

Given two points  $P_0$  and  $P_1$  with associated normalised tangents  $\vec{t}_0$  and  $\vec{t}_1$ . The biarc must connect the points such that the tangents of the start and end points of the biarc coincide with the tangents of the Hermites problem.

Data structure:

The data structure **New** for a biarc must contain two arcs.

So the data structure for one arc is needed: **MArc:-New**.

This in turn contains the coordinates for the centre, the radius, the start angle and the angle change.

#### **4.5.12. Module MBiarc**

**MBiarc** is a module for Biarcs and works with a data structure and with procedures/functions. The data structure **New** for a Biarc is a list which is structured as follows:

**[MVBIARC, [Arc0, Arc1]]**

Its first element is a name to identify the module. Your second element is again a list containing the elements of the geometry. In this case the name is „Biarc“ and the elements are two arcs.

Using the Get functions **GetArc0** and **GetArc1** one can capture the two arcs for the biarc. The functions listed below can be used to calculate the data for the biarc.

---

**MBiarc**


---

E	<b>New</b>	Data structure for a biarc
E	<b>GetArc0</b>	Reading of the first arc
E	<b>GetArc1</b>	Reading the second arc
E	<b>Circle</b>	calculating the circle $K_J$ from the Hermite problem data.
E	<b>Plot2DCircle</b>	plot of the circle $K_J$
E	<b>angle</b>	Calculate the angle from the centre of the circle to points on the circle
E	<b>Plot2D</b>	Plot of the biarc
E	<b>ConnectionPoint</b>	Calculation of the connection point J (Equal Chord)
E	<b>TangentTj</b>	Calculation of the tangent to J
E	<b>Tangent Biarc</b>	rotation of Tj for the biarc.
E	<b>BiarcCenter</b>	Calculate the centres of the arcs of the biarc
E	<b>Biarc</b>	calculate the centre of the biarc.
E	<b>Position</b>	Determine a point on the biarc
E	<b>Blend</b>	Calculate the biarc only from the Hermite problem

## 4.6. Programme Flowchart

### 4.6.1. Overall Flow

### 4.6.2. Verrundung der Kurve

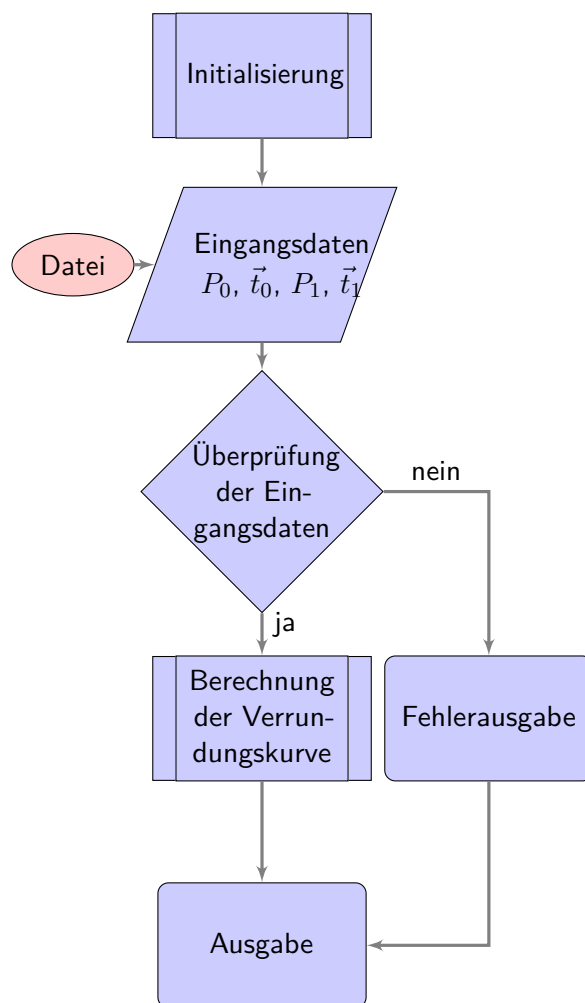


Figure 4.1.: Programme flow chart „Corner rounding“

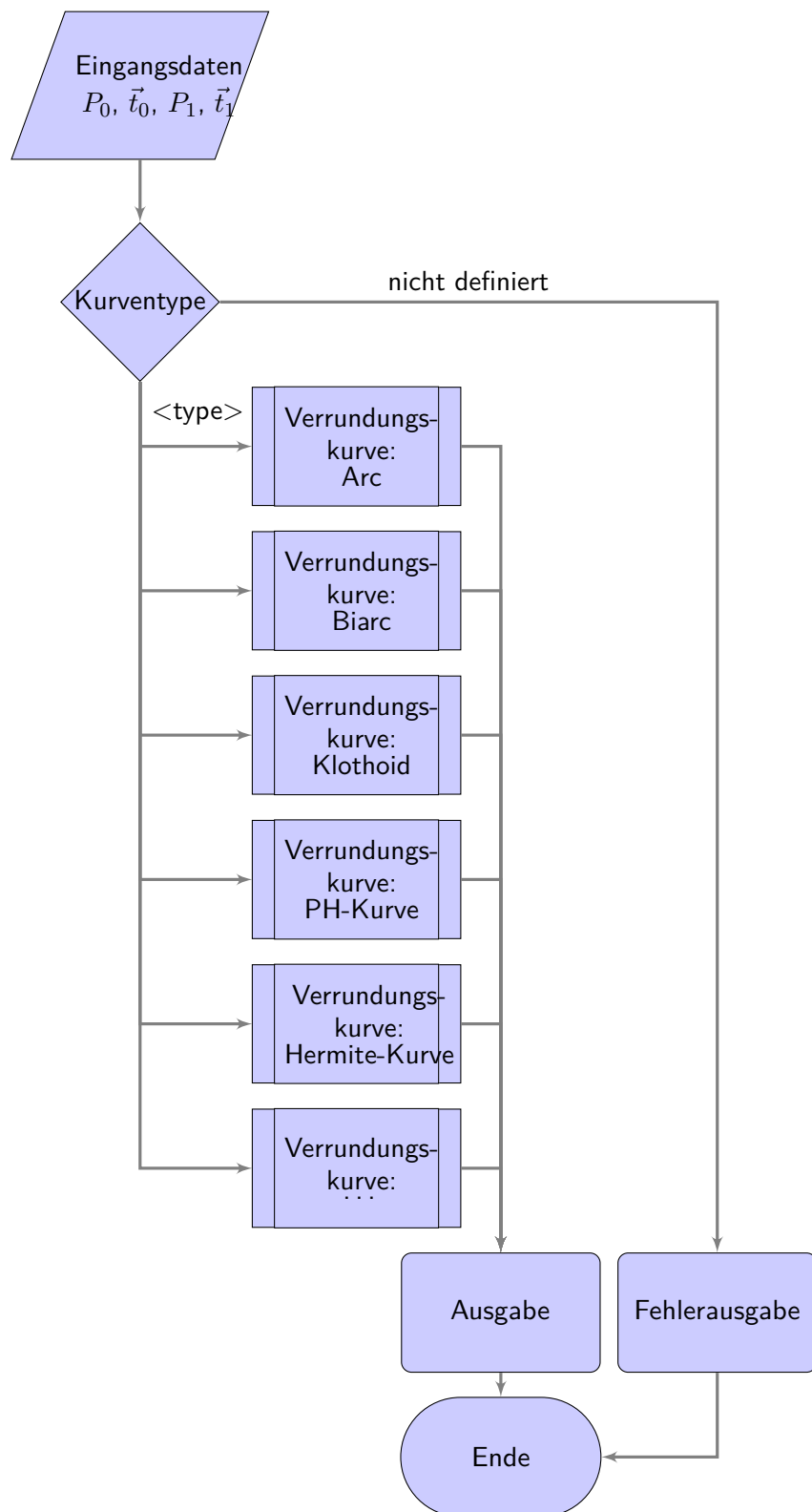


Figure 4.2.: Programme flowchart „Selection of the rounding strategies“

## 5. Representation of Python Programs

It is possible to integrate a part into the document, see 5.1. This is the most elegant method and is also preferable. Individual lines and line ranges can also be selected in the list of options. If a file is integrated, it is always as up-to-date as the document.

It may also be useful to integrate program lines, see 5.2.

The integration of programs with the help of images is pointless.

```
# Hello World for microcontroller boards
import pyb

redLED = pyb.LED(1) # built-in red LED
greenLED = pyb.LED(2) # built-in green LED
blueLED = pyb.LED(3) # built-in blue LED
while True:
    # Turns on the red LED
    redLED.on()
    # Makes the script wait for 1 second (1000 milliseconds)
    pyb.delay(1000)
    # Turns off the red LED
    redLED.off()
    pyb.delay(1000)
    greenLED.on()
    pyb.delay(1000)
    greenLED.off()
    pyb.delay(1000)
    blueLED.on()
    pyb.delay(1000)
    blueLED.off()
    pyb.delay(1000)
```

Listing 5.1.: The program “Hello World” in Python for microcontroller boards is inserted from the file [Blink.py](#).

```
# Hello World for microcontroller boards
import pyb

redLED = pyb.LED(1) # built-in red LED
greenLED = pyb.LED(2) # built-in green LED
blueLED = pyb.LED(3) # built-in blue LED
while True:
    # Turns on the red LED
    redLED.on()
    # Makes the script wait for 1 second (1000 miliseconds)
    pyb.delay(1000)
    # Turns off the red LED
    redLED.off()
    pyb.delay(1000)
    greenLED.on()
    pyb.delay(1000)
    greenLED.off()
    pyb.delay(1000)
    blueLED.on()
    pyb.delay(1000)
    blueLED.off()
    pyb.delay(1000)
```

Listing 5.2.: The program “Hello World” in Python for microcontroller boards has been inserted directly into the L<sup>A</sup>T<sub>E</sub>X file..





## 6. First Chapter

...

A Speicherprogrammierbare Steuerung (SPS) is ...

A Computerized Numerical Control (CNC) needs a SPS (PLC) to ...



## 7. CAGD

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

The book CAGD by Gerald Farin is a classic on splines. [Far02]

The standard 66025 for programming CNC machines is also a classic; however, it does not deal with splines. [DIN66025-2]

Mr. F. Farouki has dealt with both CNC machine programming and splines. His article<sup>1</sup> on a real-time interpolator also shows this. [FS17]

A new dimension of machine tools have emerged with the invention of 3D printers. [Rus+07]

Another aspect of automation technology is communication. Another milestone has been reached with 5G technology. another milestone has been reached.<sup>2</sup>

---

<sup>1</sup>co-author is J. Srinathu

<sup>2</sup>Zaf+20.



## A. drawings with tikz

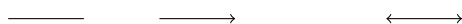
The package `tikz` is a powerful tool for creating graphics. Many introductions exist. Here only the first steps are shown, so that you can easily create flowcharts.

Drawing a line and arrows

```
\begin{tikzpicture}
  \draw (0,0) — (1,0);

  \draw[->] (2,0) — (3,0);

  \draw[<->] (5,0) — (6,0);
\end{tikzpicture}
```



Drawing a thick blue line

```
\begin{tikzpicture}
  \draw[line width=2pt, blue] (0,0) — (1,0);

  \draw[line width=2pt, red, dotted] (2,0) — (3,0);

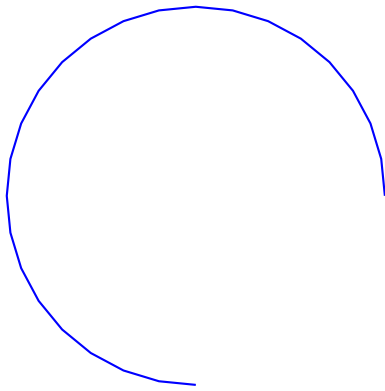
  \draw[line width=2pt, dashed, green] (4,0) — (5,0);

  \draw [thick, dash dot] (0,1) — (5,1);
  \draw [thick, dash pattern={on 7pt off 2pt on 1pt off 3pt}] (0,2) — (5,2);
\end{tikzpicture}
```



Drawing an arc

```
\begin{tikzpicture}
  \draw [blue, thick, domain=0:270] plot ({5+2.5*cos(\x)}, {1+2.5*sin(\x)});
\end{tikzpicture}
```



Draw a function

```
\begin{tikzpicture}[
  declare function={%
    F(\x)                =3-2*pow(2.7979,-0.8*\x);
  }
]

% Zeichnen der Funktion
\draw[red,line width=2pt,domain=0:4] plot({\x},{F(\x)});

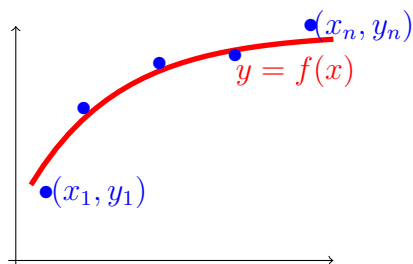
% Bezeichnung
\node[red] (O) at (3.5,2.5) {$y=f(x)$};

% Punkte
\node[blue] (P1n) at (0.9,0.9) {$(x_1,y_1)$};
\node[blue] (P1) at (0.2,0.9) {$\bullet$};

\node[blue] (P2) at (2.7,2.7) {$\bullet$};
\node[blue] (P3) at (1.7,2.6) {$\bullet$};
\node[blue] (P4) at (0.7,2) {$\bullet$};

\node[blue] (P5n) at (4.4,3.1) {$(x_n,y_n)$};
\node[blue] (P5) at (3.7,3.1) {$\bullet$};

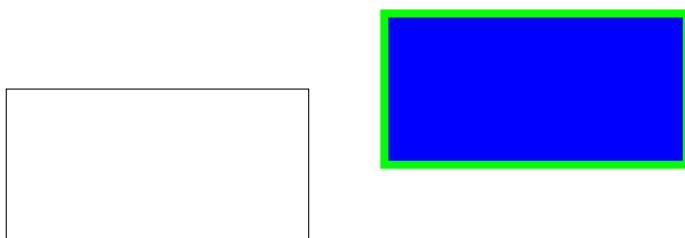
% Koordinatensystem
\draw[color=black,->] (-0.2,-0.1) — (-0.2,3.1);
\draw[color=black,->] (-0.3,0) — (4,0);
\end{tikzpicture}
```



Drawing rectangles and moving objects

```
\begin{tikzpicture}
  \draw (0,0) — (4,0) — (4,2) — (0,2) — cycle;

  \begin{scope}[shift={ (5,1) }]
    \draw[green,fill=blue,line width=3pt] (0,0) — (4,0)
    — (4,2) — (0,2) — cycle;
  \end{scope}
\end{tikzpicture}
```



Use of variables

```
\begin{tikzpicture}
\pgfmathsetmacro{\PHI}{-15}
% Now use \PHI anywhere you want -15 to appear,
% can also be used in calculations like 2*\PHI
\def\x{10};

\draw[red] (0,4) — (1-\PHI*0.5,4);
\draw[green] (0,2) — (1+1/\x,2);
\draw[blue] (0,0) — ({1+3*(\x/5+1)},0);
\end{tikzpicture}

\bigskip

%\usetikzlibrary{math} %needed tikz library

\begin{tikzpicture}
\pgfmathsetmacro{\drehpunkt}{11.63815573}
```



---

```

%Variables must be declared in a tikzmath environment but
% can be used outside
\tikzmath{\x1 = 1; \y1 =1;
%computations are also possible
\x2 = \x1 + 1; \y2 =\y1 +3; }
\draw[>-] (\x1, \y1)--(\x2, \y2);
\end{tikzpicture}

```

---



Use of points

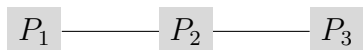
```
%\usetikzlibrary{backgrounds} % is needed
```

```

\begin{tikzpicture}
  \node [fill=gray!30] (P1) at (0,0) { $P_1$ };
  \node [fill=gray!30] (P2) at (2,0) { $P_2$ };
  \node [fill=gray!30] (P3) at (4,0) { $P_3$ };

  \begin{scope}[on background layer]
    \draw (P1) — (P3);
  \end{scope}
\end{tikzpicture}

```



Use of nodes

```

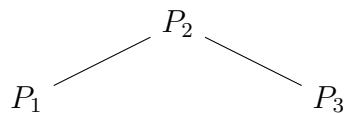
\begin{tikzpicture}
  \node (P1) at (0,0){ $P_1$ };
  \node (P2) at (2,1){ $P_2$ };
  \node (P3) at (4,0){ $P_3$ };

```

```

\begin{scope}
  \draw (P1) — (P2) — (P3);
\end{scope}
\end{tikzpicture}

```

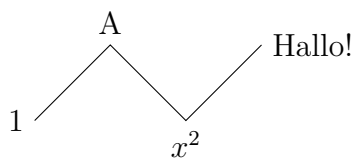


Use of nodes

```

\begin{tikzpicture}
  \draw (0, 0) node[left]{1}
    — ++(1, 1) node[above]{A}
    — ++(1,-1) node[below]{$x^2$}
    — ++(1, 1) node[right]{Hallo!};
\end{tikzpicture}

```





## B. Criteria for a good L<sup>A</sup>T<sub>E</sub>X project

A good report is not only characterised by good content, but also fulfils formal aspects. The following list should help to comply with basic rules. Before submitting, all points should be checked and ticked off.

- ☐ Is a suitable directory structure used?
- ☐ Is an appropriate division into files used?
- ☐ Are meaningful directory and file names used?
  - ☐ Are directory and file names such as report or term paper avoided?
  - ☐ Are meaningful names used for images and not „image1“?
  - ☐ Are directory and file names not too long?
  - ☐ Are special characters and spaces avoided?
- ☐ Are commands like `\newline` and `\\` avoided?
- ☐ Are the L<sup>A</sup>T<sub>E</sub>X files clearly laid out?
  - ☐ Are indentations used in the L<sup>A</sup>T<sub>E</sub>X files?
  - ☐ Are free lines inserted?
  - ☐ Is the project mentioned in the header of the files?
  - ☐ Does the header of the files mention the main sources?
  - ☐ Is the author mentioned in the header of the files?
- ☐ Are all necessary files given?
- ☐ Are the temporary files deleted?
- ☐ Are graphics created by yourself?
- ☐ Are the sources of the images given?
- ☐ Are citations made with the command `\cite`?
- ☐ Is a bib file used?

- ☐ Are the entries of the bib-file clearly arranged?
- ☐ Are the entries of the bib-file edited to show them correctly?
- ☐ Are the correct types used for the bib entries?
- ☐ Are meaningful keys used in the bib files?

Unfortunately, the list cannot be complete, but it provides some clues.

## C. Model Card

- Model Details - Basic information about the model
  - Person or organization developing model
  - Model date
  - Model version
  - Model type
  - Information about training algorithms, parameters, fairness constraints or other applied approaches, and features
  - Paper or other resource for more information
  - Citation details
  - License
  - Where to send questions or comments about the model
- Intended Use. Use cases that were envisioned during development.
  - Primary intended uses
  - Out-of-scope use cases
- Factors: Factors could include demographic or phenotypic groups, environmental conditions, technical attributes, or others
  - Relevant factors
  - Evaluation factors
- Metrics: Metrics should be chosen to reflect potential real world impacts of the model.
  - Model performance measures
  - Decision thresholds
  - Variation approaches
- Evaluation Data: Details on the dataset(s) used for the quantitative analyses in the card.
  - Datasets
  - Motivation
  - Preprocessing

- Training Data: May not be possible to provide in practice
  - When possible, this section should mirror Evaluation Data. If such detail is not possible, minimal allowable information should be provided here, such as details of the distribution over various factors in the training datasets.
- Quantitative Analyses
  - Unitary results
  - Intersectional results
- Ethical Considerations
- Caveats and Recommendation

## **D. List of Material**





## E. SBOM

`requirements.txt`



## F. Methodology

Domain Knowledge:

- Part Domain, e.g. HW, application
- Part technologies, e.g. algorithms
- Part Tools, e.g. IDE



## **G. doxygen - Example**



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