

PERFORMANCE EVALUATION OF TRADITIONAL AND ADAPTIVE LIFTING BASED WAVELETS WITH SPIHT FOR LOSSY IMAGE COMPRESSION

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Abstract- Nowadays wavelet transform has been one of the most effective transform means in the realm of image processing, especially the biorthogonal 9/7 wavelet filters proposed by Daubechies, which have good performance in image compression. Hence, in this paper an attempt has been made to analyse traditional and adaptive lifting based wavelet techniques for image compression. The original image is transformed using adaptive lifting based CDF 9/7 wavelet transform and traditional CDF 9/7 followed by it is compressed using Set Partitioning In Hierarchical Tree algorithm (SPIHT) and the performance was compared with the popular traditional CDF9/7 wavelet transform. The performance metric Peak Signal to Noise Ratio (PSNR) for the reconstructed image was computed. The proposed adaptive lifting algorithm give better performance than traditional CDF9/7 wavelet, the most popular wavelet transforms. Lifting allows us to incorporate adaptivity and nonlinear operators into the transform. The proposed methods efficiently represent the edges and appear promising for image compression. The proposed adaptive methods reduce edge artifacts and ringing and give improved PSNR of 4.69 to 6.09 dB than the traditional CDF 9/7 for edge dominated 2D images.

I. INTRODUCTION

As a rapidly developing branch of applied mathematics began in the late 1980s, wavelet transform is certainly a milestone in the history of traditional Fourier analysis. Meanwhile, it has become a powerful tool in the realm of digital image compression. The main advantage of wavelet transform over discrete cosine transform (DCT) is that it has both time and frequency localization ability, which result in better performance in image compression. Thus, researchers have paid much attention to wavelet construction and proposed some well-known wavelet bases. Especially, the CDF (Cohen, Daubechies and Feauveau) 9/7 biorthogonal wavelet, with outstanding transform properties, have been widely used in many areas including the new generation of static image compression standard JPEG2000.

The wavelet lifting scheme[6], proposed by Sweldens, is a simple construction of second generation wavelets. By transforming the signal in space domain, it successfully

completes the task of frequency-domain signal analysis. Compared to the traditional Mallat algorithm based on convolution computation, the lifting algorithm has advantages of simpler operation procedure, lower storage demand, as well as consistent positive transformation and reverse transformation etc, thus it leads to a faster, in-place calculation of wavelet transform. However, according to the adaptive lifting structure, it still requires substantial time-consuming floating-point operations to calculate wavelet coefficients, which increases the difficulties in structure realization of wavelet transform and affects the efficiency of whole compression algorithm.

To reduce the complexity, Caypo and Sweldens [16] proposed a lifting scheme for implementing CDF 9/7-tap wavelet. Since the coefficients of the CDF 9/7-tap wavelet are all irrational numbers, [4] suggested approximating the coefficients of the CDF 9/7-tap wavelet by using binary fraction, in which 26 integer additions and 18 shifts are needed per two wavelet coefficients. The purpose of this paper is to develop a modified structure L with new simple binary fraction for adaptive lifting based CDF 9/7, in which the compression performance is better than existing [1][3] lifting methods and new proposed CDF 9/7-tap wavelet can be considered as a very good alternative to existing lifting wavelets for 2 to 8 lifting levels with 0.1 to 8.0 bpp bit rate.

II. TRADITIONAL CDF 9/7

The Cohen-Daubechies-Feauveau (CDF) 9/7 biorthogonal wavelet is a more complex wavelet than the CDF 5/3 wavelet. It also the Cohen-Daubechies-Feauveau (CDF) 9/7 biorthogonal wavelet is a more complex wavelet than the CDF 5/3 wavelet. It also has two sets of scaling and wavelet functions for analysis and synthesis, however, they are nearly identical and therefore more orthonormal than the CDF 5/3. The CDF 9/7 wavelet has a 9-tap low-pass analysis filter $h(z)$ and 7-tap high-pass analysis filter $g(z)$. The CDF 9/7 also has a 7-tap low-pass synthesis filter $h(z)$ and 9-tap high-pass synthesis filter $g(z)$. The CDF 9/7 analysis and synthesis wavelets has two sets of scaling and wavelet functions for analysis and synthesis, however, they are nearly identical and

therefore more orthonormal than the CDF 5/3. The CDF 9/7 wavelet has a 9-tap low-pass analysis filter $h(z)$ and 7-tap high-pass analysis filter $g(z)$. The CDF 9/7 also has a 7-tap low-pass synthesis filter $h(z)$ and 9-tap high-pass synthesis filter $g(z)$. An example of a transformed image for two levels of decomposition is shown in Figure 1. This nested four-subband representation, or dyadic decomposition, of a transformed image is commonly referred to as a “quad-tree”.

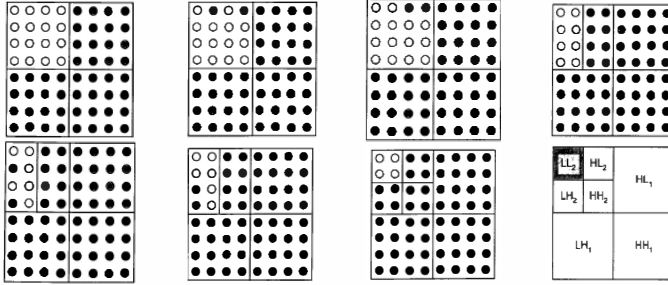


Figure 1: Forward 2-D DWT Row and Column Processing of Image for Multiple Levels of Decomposition

The wavelet used for the irreversible lossy DWT source encoder is the CDF 9/7 biorthogonal wavelet mentioned earlier. Due to the floating point nature of the CDF 9/7 it is only used as an irreversible transform but due to its higher energy compaction than the Spline 5/3 it provides significantly better compression ratios.

The traditional convolution-based CDF 9/7 DWT [3] followed by spilt [11] is applied to the Lena and Barbara image for 1 to 8 levels of decomposition. With 0.25 to 6 bit rate. Next the MATLAB model of the adaptive lifting based DWT using the actual full precision lifting coefficients and modified structure coefficients $k1$ and $k2$ for the CDF 9/7 wavelet are applied to the Lena image and Barbara image for 1 to 8 levels of decomposition. The resulting images are found in implementation section and table II and III shows the PSNR and percent difference measurements.

III. SECOND GENERATION WAVELETS

Since the lifting scheme does not use Fourier analysis to compute the DWT, it can be used in situations where translation and dilation is impossible. One example would be near boundaries of a finite signal where normal Fourier techniques would provide border distortion or artifacts.

Lifting Step Extraction: As mentioned above the lifting scheme is an alternative technique for performing the DWT using biorthogonal wavelets. In order to perform the DWT using the lifting scheme the corresponding lifting and scaling steps must be derived from the biorthogonal wavelets. The analysis filters of the particular wavelet are first written in polyphase matrix form shown below.

$$P(z) = \begin{bmatrix} h_{\text{even}}(z) & g_{\text{even}}(z) \\ h_{\text{odd}}(z) & g_{\text{odd}}(z) \end{bmatrix} \quad (1)$$

The polyphase matrix is a 2×2 matrix containing the analysis low-pass and high-pass filters each split up into their even and odd polynomial coefficients and normalized. From here the

matrix is factored into a series of 2×2 upper and lower triangular matrices each with diagonal entries equal to 1. The upper triangular matrices contain the coefficients for the predict steps and the lower triangular matrices contain the coefficients for the update steps. A matrix consisting of all O's with the exception of the diagonal values may be extracted to derive the scaling step coefficients. The polyphase matrix is factored into the form shown in the equation below, a is the coefficient for the predict step and p is the coefficient for the update step.

$$P(z) = \begin{bmatrix} 1 & a(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b(1+z) & 1 \end{bmatrix} \quad (2)$$

An example of a more complicated extraction having multiple predict and update steps as well as scaling steps is shown below; a is the coefficient for the first predict step, p is the coefficient for the first update step, A , is the coefficient for the second predict step, 5 is the coefficient for the second update step, $k1$ is the odd sample scaling coefficient, and $k2$ is the even sample scaling coefficient.

$$P(z) = \begin{bmatrix} 1 & a(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ b(1+z) & 1 \end{bmatrix} \begin{bmatrix} 1 & c(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d(1+z) & 1 \end{bmatrix} \begin{bmatrix} k1 & 0 \\ 0 & k2 \end{bmatrix} \quad (3)$$

According to matrix theory, any matrix having polynomial entries and a determinant of 1 can be factored as described above. Therefore every FTR wavelet or filter bank can be decomposed into a series of lifting and scaling steps. Daubechies and Sweldens discuss lifting step extraction in further detail. [6] The lifting step extraction for the CDF9/7 biorthogonal wavelets is shown below.

TABLE-I

Adaptive Lifting parameters of modified structure of CDF 9/7

Parameters	values
a	-1.5861343420693648
b	-0.0529801185718856
c	0.8829110755411875
d	0.4435068520511142
k1,k2 (proposed)	0.230174104914126 and 1.6257861322319229

A total of four lift steps are required, two predict and two update steps, to perform the proposed modified structure of CDF 9/7 DWT by adapting new scaling values of scaling coefficients of $k1$ and $k2$. The coefficients for predict and update steps are enlisted in table 1:

The predict and update equations for the CDF 9/7 filter are shown below.

$$\begin{aligned} \text{Predict1: } odd_{\text{new}} &= odd_{\text{old}} + [a(even_{\text{left}} + even_{\text{right}})] \\ \text{Update1: } even_{\text{new}} &= even_{\text{old}} + [b(odd_{\text{left}} + odd_{\text{right}})] \\ \text{Predict2: } odd_{\text{new}} &= odd_{\text{old}} + [c(even_{\text{left}} + even_{\text{right}})] \\ \text{Update2: } even_{\text{new}} &= even_{\text{old}} + [d(odd_{\text{left}} + odd_{\text{right}})] \\ \text{Scale odd: } odd_{\text{new}} &= [k1 \times odd_{\text{old}}] \\ \text{Scale even: } even_{\text{new}} &= [k2 \times even_{\text{old}}] \end{aligned} \quad (4)$$

The floor function is used for all the predict, update and scale equations to provide an integer-to integer transform. The forward CDF 9/7 DWT using the lifting scheme is shown in Figure 2.

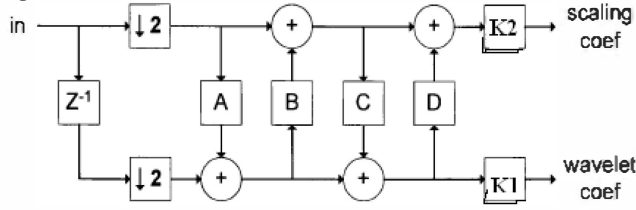


Figure 2: Forward CDF 9/7 DWT using adaptive Lifting Scheme

The CDF 9/7 DWT consists of four lifting steps and two scaling steps. The first lifting step (predict step 1) is applied to the original row of samples and the results then safely overwrite the odd samples in the original signal for use in the next lifting step.

$odd_{new} = odd_{old} + [a(even_{left} + even_{right})]$ and $a = -1.5861343420693648$

Row of samples : 4 7 3 5 9 6
Lifting step 1 results: -5 -15 -23
New row of samples: 4 5 3 -15 9 -23

The second lifting step (update step 1) is applied to the results from the first lifting step and the remaining even samples of the original signal. The results then safely overwrite the even samples in the signal

$even_{new} = even_{old} + [b(odd_{left} + odd_{right})]$ and $b = -0.0529801185718856$

Row of samples : 4 5 3 -15 9 -23
Lifting step 2 results: 4 4 11
New row of samples: 4 5 4 -15 11 -23

The third lifting step (predict step 2) is applied to the results from the first and second lifting steps. The results then safely overwrite the results from the first lifting step.

$odd_{new} = odd_{old} + [c(even_{left} + even_{right})]$ and $c = 0.8829110755411875$

Row of samples : 4 5 4 -15 11 -23
Lifting step 3 results: 2 -2 -4
New row of samples: 4 2 4 -2 11 -4

The fourth lifting step (update step 2) is applied to the results from the second and third lifting steps. The results then safely overwrite the results from the second lifting step.

$even_{new} = even_{old} + [d(odd_{left} + odd_{right})]$ and $d = 0.4435068520511142$

Row of samples : 4 2 4 -2 11 -4
Lifting step 4 results: 5 4 8
New row of samples: 5 2 4 -2 8 -4

The first scaling step is applied to the results from the third lifting step. The results then safely overwrite the results from the third lifting step. The results from this step are the wavelet coefficients.

$odd_{new} = [k1 \times odd_{old}]$ and $k1 = 0.230174104914126$

Row of samples : 5 2 4 -2 8 -4
Lifting step 4 results: 2 -3 -5
New row of samples: 5 2 4 -3 8 -5

Wavelet coefficients: 2 -3 -5

The second scaling step is applied to the results from the fourth lifting step. The results then safely overwrite the results from the fourth lifting step. The results from this step are the scaling coefficients.

$even_{new} = [k2 \times even_{old}]$ and $k2 = 1.6257861322319229$

Row of samples : 5 2 4 -3 8 -5
Lifting step 4 results: 4 3 6
New row of samples: 4 2 3 -3 6 -5
Scaling coefficients: 4 3 6

IV. SPIHT CODING SCHEME

When the decomposition image is obtained, we try to find a way how to code the wavelet coefficients into an efficient result, taking redundancy and storage space into consideration. SPIHT [2] is one of the most advanced schemes available, even outperforming the state-of-the-art JPEG 2000 in some situations.

The basic principle is the same; a progressive coding is applied, processing the image respectively to a lowering threshold. The difference is in the concept of zero trees (spatial orientation trees in SPIHT). This is an idea that takes bounds between coefficients across subbands in different levels into consideration. The first idea is always the same: if there is an coefficient in the highest level of transform in a particular subband considered insignificant against a particular threshold, it is very probable that its descendants in lower levels will be insignificant too, so we can code quite a large group of coefficients with one symbol. SPIHT makes use of three lists namely the List of Significant Pixels (LSP), List of Insignificant Pixels (LIP) and List of Insignificant Sets (LIS). These are coefficient location lists that contain their coordinates. After the initialization, the algorithm takes two stages for each level of threshold – the sorting pass (in which lists are organized) and the refinement pass (which does the actual progressive coding transmission). The result is in the form of a bit stream.

The algorithm has several advantages. The first one is an intensive progressive capability we can interrupt the decoding (or coding) at any time and a result of maximum possible detail can be reconstructed with one-bit precision. Second advantage is a very compact output bitstream with large bit variability no additional entropy coding or scrambling has to be applied.

V. EXPERIMENTAL RESULTS

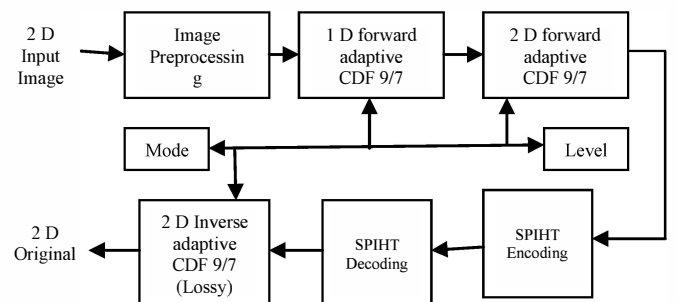


Figure 3 block dia. For proposed 2 D adaptive CDF 9/7 wavelet with spiht

We were proposed as shown in figure 3 a two dimensional (2D) adaptive Lifting of CDF 9/7 with modified structure i.e. scaling coefficient $L=[k_1, k_2]$ with 0.230174104914126 and 1.6257861322319229 for lossy Image compression. Furthermore, by introducing the adaptive lifting to the modified structure, the 2D proposed structure was realized and applied along with SPIHT (Set Partitioning in Hierarchical Trees) into lossy image compression. SPIHT not only has longer wavelet zero tree but also can more efficiently cluster wavelet zero coefficients and will outperforms it by a PSNR amount in the range from 0.2 to 0.9 dB.

In the following we focus on the comparison of compression performance. In the simulation, we were try the following four configures:

- CDF-SP-L: traditional CDF 9/7-tap [3] wavelet is used in both encoder and decoder followed by SPIHT codec for 2D Lena Image;
- PAL-CDF-SP-L: Proposed adaptive lifting based CDF 9/7-tap wavelet is used in both encoder and decoder followed by SPIHT codec as shown in figure 3 for 2D Lena Image;
- CDF-SP-B: traditional CDF 9/7-tap wavelet[3] is used in both encoder and decoder followed by SPIHT codec for 2D Barbara Image;
- PAL-CDF-SP-B: Proposed adaptive lifting based CDF 9/7-tap wavelet is used in both encoder and decoder followed by SPIHT codec as shown in figure 3 for 2D Barbara Image;

The traditional and proposed lifting algorithms are implemented in MATLAB. The barbara and lena images experimental results are enlised in table I and table 2 respectively. Figure 4-a~b and 5-a~b gives the visual quality of the reconstructed Barbara image and lena for the traditional and the proposed algorithms.

The objective coding results (PSNR in dB) for standard 256 X 256 Lena and Barbara test images are tabulated in Table 1~2. It is easily to see that the performance of PAL-CDF-SP-L/B is better than CDF-SP-L/B from level 4 to 8. The visual quality of their reconstructed images is also exactly the same, as demonstrated in Fig. 4-a~b and 5-a~b. Table II seems that proposed structure work efficiently from level 4 to level 8 and maximum PSNR for barbara becomes 70.92dB and it is observed that about 4.69 dB gains in PSNR is obtained with the proposed algorithm compared to traditional CDF 9/7for same bit rate. Further, Table II seems that proposed structure work efficiently from level 4 to level 8 and maximum PSNR for Lena becomes 72.33 dB and it is observed that about 6.09 dB gains in PSNR is obtained with the proposed algorithm compared to traditional CDF 9/7for same bit rate.

TABLE II

PSNR evaluation for Barbara, in dB

Rate (bpp)	level	2	4	6	8
0.25	Cdf 9/7+spiht	15.18	29.98	28.44	25.81
	PAL+cdf 9/7+sphit	11.47	31.86	33.46	33.48
0.5	Cdf 9/7+spiht	16.24	31.49	31.76	29.98
	PAL+cdf 9/7+sphit	13.28	35.36	36.23	36.25

1.0	Cdf 9/7+spiht	26.09	36.29	35.38	33.29
	PAL+cdf 9/7+sphit	22.36	40.07	40.64	40.66
2.0	Cdf 9/7+spiht	33.93	44.81	43.77	42.24
	PAL+cdf 9/7+sphit	32.17	46.92	47.42	47.44
4.0	Cdf 9/7+spiht	46.64	55.86	54.95	53.90
	PAL+cdf 9/7+sphit	46.17	58.29	58.83	58.85
6.0	Cdf 9/7+spiht	57.75	67.49	66.86	66.23
	PAL+cdf 9/7+sphit	57.46	70.19	70.89	70.92



Level=1,rate=0.5 PSNR=14.89 Level=2,rate=1 PSNR=26.09 Level=4,rate=2 PSNR=44.81 Level=8,rate=6 PSNR=66.23

Figure 4-a reconstructed Barbara image for traditional CDF 9/7 +SPIHT



Level=1,rate=0.5 PSNR=11.27 Level=2,rate=1 PSNR=22.36 Level=4,rate=2 PSNR=46.92 Level=8,rate=6 PSNR=70.92

Figure 4-b reconstructed Barbara image for Proposed Adaptive Lifting based CDF 9/7 +SPIHT

TABLE-III

PSNR evaluation for Lena, in dB

Rate	Level	2	4	6	8
0.25	Cdf 9/7+spiht	15.75	30.24	28.41	27.44
	PAL+cdf 9/7+sphit	11.08	31.98	34.05	34.12
0.5	Cdf 9/7+spiht	16.97	34.56	32.79	30.27
	PAL+cdf 9/7+sphit	12.82	36.77	37.82	37.88
1.0	Cdf 9/7+spiht	25.90	40.42	38.63	36.92
	PAL+cdf 9/7+sphit	22.44	42.41	43.15	43.16
2.0	Cdf 9/7+spiht	34.53	44.57	44.65	42.93
	PAL+cdf 9/7+sphit	32.58	49.28	49.75	49.77
4.0	Cdf 9/7+spiht	48.70	57.96	56.16	54.52
	PAL+cdf 9/7+sphit	47.56	60.12	60.46	60.48
6.0	Cdf 9/7+spiht	59.74	69.74	68.50	66.24
	PAL+cdf 9/7+sphit	58.64	71.98	72.32	72.33



Level=1,rate=0.5 PSNR=14.36 Level=2,rate=1 PSNR=25.90 Level=4,rate=2 PSNR=46.57 Level=8,rate=6 p PSNR=66.24

Figure 5-a reconstructed Lena image for Proposed Adaptive Lifting based CDF 9/7 +SPIHT



Level=1,rate=0.5 Level=2 ,rate=1 Level=4,rate=2 Level=8,rate=6
PSNR=11.51 PSNR=22.44 PSNR=49.28 PSNR=72.33

Figure 5-b reconstructed Lena image for Proposed Adaptive Lifting based CDF 9/7 +SPIHT

VI. CONCLUSIONS and FUTURE SCOPE

In this paper, traditional and proposed adaptive lifting based wavelet transforms followed by SPIHT are compared and has been applied to 256x256 8 bit images. This adaptive lifting transform with SPIHT appears promising for image compression. It reduces edge artifacts and ringing and gives improved PSNR of 4.69dB than the traditional for edge dominated images like Barbara. For smooth images like Lena, cdf9/7 transform with SPIHT gives much better performance of 6.09 dB than traditional.

Further, all this can be achieved without extra cost on coding the filter decisions. This proposed method does not require any side information regarding to the filter selection to be sent to the backward transform for perfect reconstruction

From the above discussion, it is evident that the adaptive lifting based wavelets outperform the traditional wavelets. The future scope of the presented work can be summarized as follows.

- The techniques can be used along with modified SPIHT for improved image compression.
- The techniques can be extended for video compression.
- The techniques can be extended for any other image processing applications for better results.

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