

## Advanced Games Engine Creation

### Introduction to Mathematics for 3D Programming

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a) The identity transformation 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

b) A scaling by a factor of 3 in all dimensions

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

c) A scaling by a factor of 3 in x, 2 in y and 1.5 in the z direction

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

d) A rotation of 30 degrees about the z axis

$$\begin{bmatrix} \cos(30) & -\sin(30) & 0 & 0 \\ \sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

e) A rotation of  $\pi$  radians about the z axis

$$\begin{bmatrix} \cos(180) & -\sin(180) & 0 & 0 \\ \sin(180) & \cos(180) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- f) A translation of -3 units in the x direction, +2 units in the y direction, and 0 units in the z direction

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -3 \\ 2 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Back to pen and paper

- 5,6. Sketch the result you would expect if the following transformations were applied to the triangle of question 1:

- a) Scaling by a factor of 3 in x, 2 in y and 1.5 in the z direction, followed by a rotation of 30 degrees about the z axis.

$$\begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.6 & -1 & 0 & 0 \\ 1.5 & 1.732 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2.6 & -1 & 0 & 0 \\ 1.5 & 1.732 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2.6 & -1 \\ 0 & 1.5 & 1.732 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- b) Rotating by 30 degrees about the z axis, followed by scaling by a factor of 3 in x, 2 in y and 1.5 in the z direction

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.6 & -1.5 & 0 & 0 \\ 1.0 & 1.732 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2.6 & -1.5 & 0 & 0 \\ 1.0 & 1.732 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2.6 & -1.5 \\ 0 & 1.0 & 1.732 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- c) Scaling by a factor of 3 in x, 2 in y and 1.5 in the z direction, followed by a translation of -3 units in the x direction, +2 units in the y direction, and 0 units in the z direction

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & -3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & -3 \\ 2 & 2 & 4 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- d) A translation of -3 units in the x direction, +2 units in the y direction, and 0 units in the z direction, followed by scaling by a factor of 3 in x, 2 in y and 1.5 in the z direction

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & -9 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -9 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -6 & -9 \\ 4 & 4 & 6 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1.3 & -0.75 & 0 & 0.9 \\ 0.75 & 1.33 & 0 & 0.076 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1.3 & -0.75 & 0 & 0.9 \\ 0.75 & 1.33 & 0 & 0.076 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ = \begin{bmatrix} -1.7 & 2.2 & 0.7 & -3.2 \\ -1.424 & 0.826 & 3.486 & 1.236 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$