HW2 January Shen

P1a-1. $Tr(AB^T) = \sum_{i=1}^{n} A_i \cdot B_i^T = \sum_{i=1}^{n} B_i^T \cdot A_i = Tr(B^T A)$ ※

Let A_i = i-th row of A

P1a-2 $Tr(\Sigma) = Tr(\frac{1}{n} X^T \cdot X) = \frac{1}{n}\sum_{i=1}^{n} X_i^2 = \frac{1}{n}\sum_{i=1}^{n} \|X_i\|_2^2$ ※

$= Tr(\frac{1}{n} X \cdot X^T)$

$Tr(\Sigma) = Tr(\frac{1}{n} X \cdot X^T)$ = sum of the eigenvalues.

∵ dimension of $Tr(\frac{1}{n} X \cdot X^T) = d \times d$

∴ $Tr(\frac{1}{n} X \cdot X^T) = \sum_{i=1}^{d} \lambda_i$ ※

P3 a

$\varepsilon_{iu} = 2(r_{iu} - q_i \cdot p_u)$

$q_i = q_i + \eta(\varepsilon_{iu} \cdot p_u - 2\lambda \cdot q_i)$

$p_u = p_u + \eta(\varepsilon_{iu} \cdot q_i - 2\lambda \cdot p_u)$

P4 a. $T = R \cdot R^T$

$T_{ii} = P_{ii}$ = degree of user node i. How many items that user i likes.

$T_{ij}, i \neq j$ = the number of items that both i & j like.

P4 b. $R^T \cdot R$ = matrix of item i · item j

Let C_i = i-th column of R, Let $R^T \cdot R = M$, $M_{ij} = C_i \cdot C_j$, $i, j \in [1, n]$

$Q_{rc}^{-\frac{1}{2}} = \frac{1}{\sqrt{Q_{rc}}}$, since Q is a diagonal matrix, $\frac{1}{\sqrt{Q_i}} = \frac{1}{\|Q_i\|_2}$. Let Q_i = i-th column of Q

Let $Q^{-\frac{1}{2}} \cdot R^T \cdot R \cdot Q^{-\frac{1}{2}} = Z$, then $Z_{ij} = \frac{C_i \cdot C_j}{\|C_i\|_2 \|C_j\|_2}$, $Z = S_I$ ※

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p4b. (cont.)

Let $S_u = U$, $U_{ij} = \frac{User_i \cdot User_j}{\|User_i\| \cdot \|User_j\|}$

$R \cdot R^T$ = matrix of $user_i \cdot user_j \overset{Let}{=} M$.

Let C_i = ith row of R., $M_{ij} = C_i \cdot C_j$

Since P is a diagonal matrix, $P_i^{-\frac{1}{2}} = \frac{1}{\sqrt{P_i}}$, let P_i = ith row of P

$\Rightarrow$ user similarity matrix $S_u = P^{-\frac{1}{2}} \cdot R \cdot R^T \cdot P^{-\frac{1}{2}}$ ※

p4c.

Let Γ_I = the recommendation matrix for item-item case.

$$\Gamma_I = R \cdot \underbrace{Q^{-\frac{1}{2}} \cdot R^T \cdot R \cdot Q^{-\frac{1}{2}}}_{=}$$

cosine similarity of items, where the ith column means every other item's similarity to item-i

Let Γ_{IK} = K-th row of Γ_I.

Γ_{IK} means that K-user's potential preference of each item.

$\Gamma_{IK} = R_K \cdot$ cosine similarity of items.

↳ R's kth row

Γ_{Iij} = user i's score of item j based on i's preference in R. ※

Similarly, Let Γ_U = the recommendation matrix for user-user case.

$\Gamma_U = \underbrace{P^{-\frac{1}{2}} \cdot R \cdot R^T \cdot P^{-\frac{1}{2}}}_{\text{cosine similarity of users}} \cdot R$, Γ_{Uij} = score of item j based on everyother user's rating on j and user i's prefercence compared to other users. ※