The signs in the original equation have been changed in order to make a neater weak form:

$$\frac{\partial c_{M1}}{\partial t} = \nabla\left(D_{M1}\nabla c_{M1}\right) + \nabla\left(C_{M1}\nabla g_{TNF}\right) - A_{M1}c_{M1} - F5c_{M1} - Y_{M1}c_{M1} \tag{1a}$$

$$\frac{\partial c_{M2}}{\partial t} = \nabla\left(D_{M2}\nabla c_{M2}\right) - A_{M2}c_{M2} - F5'c_{M1} - Y_{M2}c_{M2} \tag{1b}$$

$$\frac{\partial g_{TNF}}{\partial t} = \nabla\left(D_{TNF}\nabla g_{TNF}\right) - E_{TNF}c_{M1} - H_{TNF}g_{TNF} \tag{1c}$$

$$\frac{\partial g_{IL4}}{\partial t} = \nabla\left(D_{IL4}\nabla g_{IL4}\right) - E_{IL4}c_{M2} - H_{IL4}g_{IL4} \tag{1d}$$

$$\frac{\partial m_d}{\partial t} = \nabla\left(D_d\nabla m_d\right) - d_d c_{M1} - d_d c_{M2} \tag{1e}$$

where

$$D_i = f\left(m\right) = \frac{D_{mi}m}{K_{mi,D}^2 + m^2} \quad \text{for} \quad i = M1, M2 \tag{2a}$$

$$C_{M1} = f\left(g_{TNF}, m\right) = -\frac{C_{TNFM1}g_{TNF}}{K_{TNFM1,C}^2 + g_{TNF}^2}\frac{C_{mM1}m}{K_{mM1,C}^2 + m^2}c_{M1} \tag{2b}$$

$$A_i = f\left(s, g_j\right) = -\frac{A_{ji}g_j}{K_{ji,A}^2 + g_j^2}\frac{A_{si}s}{K_{si,A}^2 + s^2} \quad \text{for} \quad (i,j) = (M1, TNF), (M2, IL4) \tag{2c}$$

$$F5 = f\left(g_{IL4}, g_{TNF}\right) = \frac{I_{IL4M1}g_{IL4}^2}{K_{IL4M1,I}^2 + g_{IL4}^2}\frac{I_{TNFM1}g_{TNF}^2}{K_{TNFM1,I}^2 + g_{TNF}^2} \tag{2d}$$

$$F5' = -F5 \tag{2e}$$

$$Y_{M1} = f\left(g_{TNF}\right) = Y_{M1,h} - \frac{Y_{M1,l}g_{TNF}}{K_{TNFM1,Y}^2 + g_{TNF}^2} \tag{2f}$$

$$Y_{M2} = Y_{M2,h} \tag{2g}$$

$$E_{TNF} = f\left(m_d, g_{TNF}\right) = -\frac{E_{TNF,h}m_d}{K_{TNF}^2 + g_{TNF}^2} \tag{2h}$$

$$E_{IL4} = f\left(g_{IL4}\right) = -\frac{E_{IL4,h}}{K_{IL4}^2 + g_{IL4}^2} \tag{2i}$$

$$H_i = f\left(c_{M1}, c_{M2}\right) = d_i\left(1 + c_{M1} + c_{M2}\right) \quad \text{for} \quad i = TNF, IL4 \tag{2j}$$

and the weak form without boundary terms after moving everything to the left and applying integration by parts:

$$\begin{aligned}\left(\frac{\partial c_{M1}}{\partial t}, \varphi_{c_{M1}}\right) &+ \left(D_{M1}\nabla c_{M1}, \nabla\varphi_{c_{M1}}\right) + \left(C_{M1}\nabla g_{TNF}, \nabla\varphi_{c_{M1}}\right)\\ &+ \left(A_{M1}c_{M1}, \varphi_{c_{M1}}\right) + \left(F5c_{M1}, \varphi_{c_{M1}}\right) + \left(Y_{M1}c_{M1}, \varphi_{c_{M1}}\right) = 0\end{aligned} \tag{3a}$$

$$\left(\frac{\partial c_{M2}}{\partial t}, \varphi_{c_{M2}}\right) + \left(D_{M2}\nabla c_{M2}, \nabla\varphi_{c_{M2}}\right) + \left(A_{M2}c_{M2}, \varphi_{c_{M2}}\right) + \left(F5'c_{M1}, \varphi_{c_{M2}}\right) + \left(Y_{M2}c_{M2}, \varphi_{c_{M2}}\right) = 0 \tag{3b}$$

$$\left(\frac{\partial g_{TNF}}{\partial t}, \varphi_{g_{TNF}}\right) + \left(D_{TNF}\nabla g_{TNF}, \nabla\varphi_{g_{TNF}}\right) + \left(E_{TNF}c_{M1}, \varphi_{g_{TNF}}\right) + \left(H_{TNF}g_{TNF}, \varphi_{g_{TNF}}\right) = 0 \tag{3c}$$

$$\left(\frac{\partial g_{IL4}}{\partial t}, \varphi_{g_{IL4}}\right) + \left(D_{IL4}\nabla g_{IL4}, \nabla\varphi_{g_{IL4}}\right) + \left(E_{IL4}c_{M2}, \varphi_{g_{IL4}}\right) + \left(H_{IL4}g_{IL4}, \varphi_{g_{IL4}}\right) = 0 \tag{3d}$$

$$\left(\frac{\partial m_d}{\partial t}, \varphi_{m_d}\right) + \left(D_d\nabla m_d, \nabla\varphi_{m_d}\right) + \left(d_d c_{M1}, \varphi_{m_d}\right) + \left(d_d c_{M2}, \varphi_{m_d}\right) = 0 \tag{3e}$$