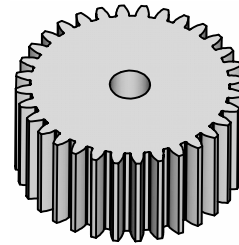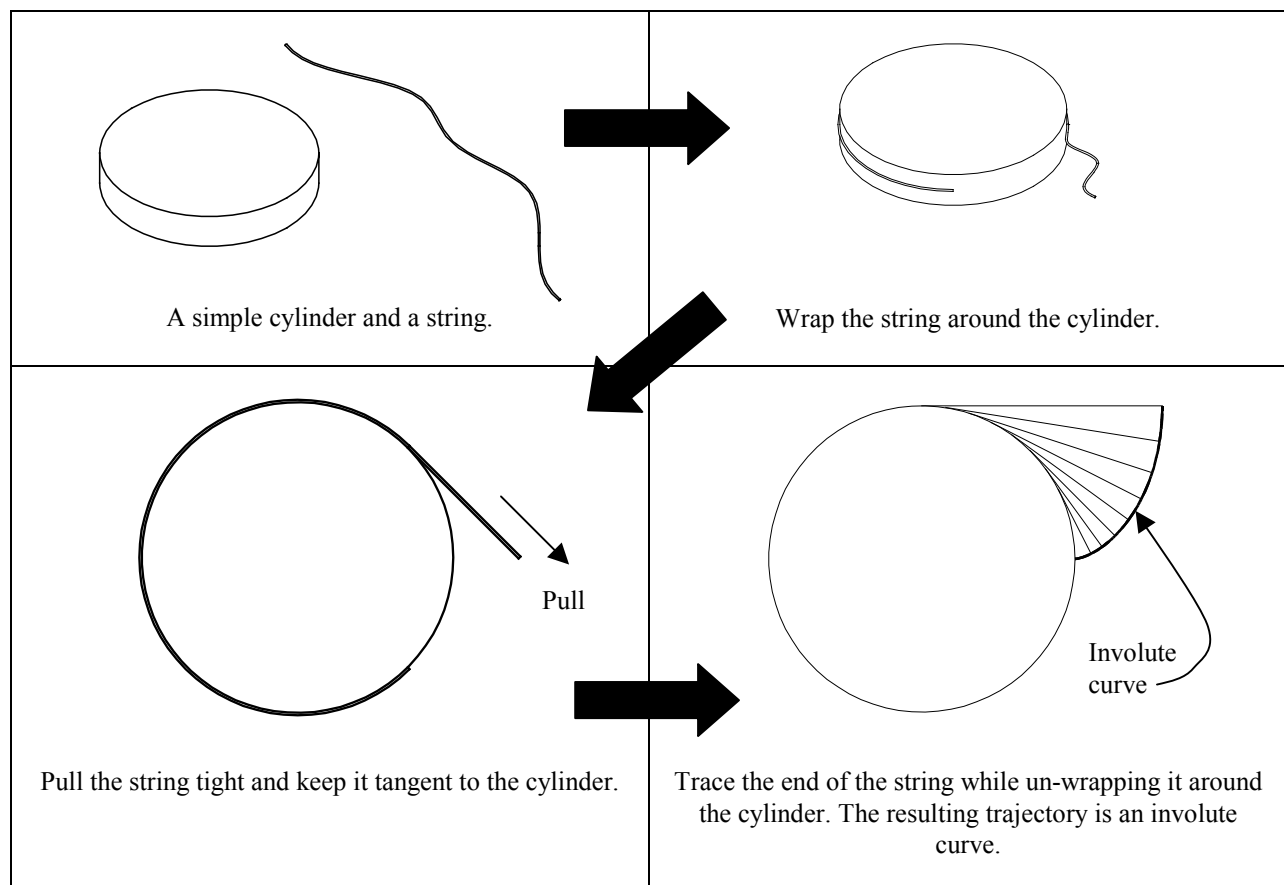# *Involute Gears*

Introduction

This lesson covers the development of 'involute' gears. An involute gear is based on an involute curve, which is a mathematical shape. To understand what an involute is, consider a simple cylinder and a string as shown below. Wrap the string around the cylinder. While maintaining tension on the string, trace the path that the end of the string makes while un-wrapping it around the cylinder. This path is an involute curve.

A simple cylinder and a string.

Wrap the string around the cylinder.

Pull

Pull the string tight and keep it tangent to the cylinder.

Involute curve

Trace the end of the string while un-wrapping it around the cylinder. The resulting trajectory is an involute curve.

There are two key parameters that control the involute curve: the diameter of the cylinder and the angle that the string is un-wrapped around the cylinder. In the example above, the string is un-wrapped 90º around the cylinder.

The Involute Curve

Development of the mathematical equations for the involute curve uses simple trigonometry. We will develop the equations for 90º of one full involute. In other words, we are un-wrapping the string 90º around the cylinder in a counterclockwise direction.

The circumference of a circle is defined by: $c = 2 * \pi * r$

One quarter (90º) of the circumference of a circle is then: $c = \pi * r / 2$

In the figure below, the line '**s**' is the same length as the part of the circumference the string has been un-wrapped (**s'**).

We are trying to define this curve by finding equations for x and y as θ changes
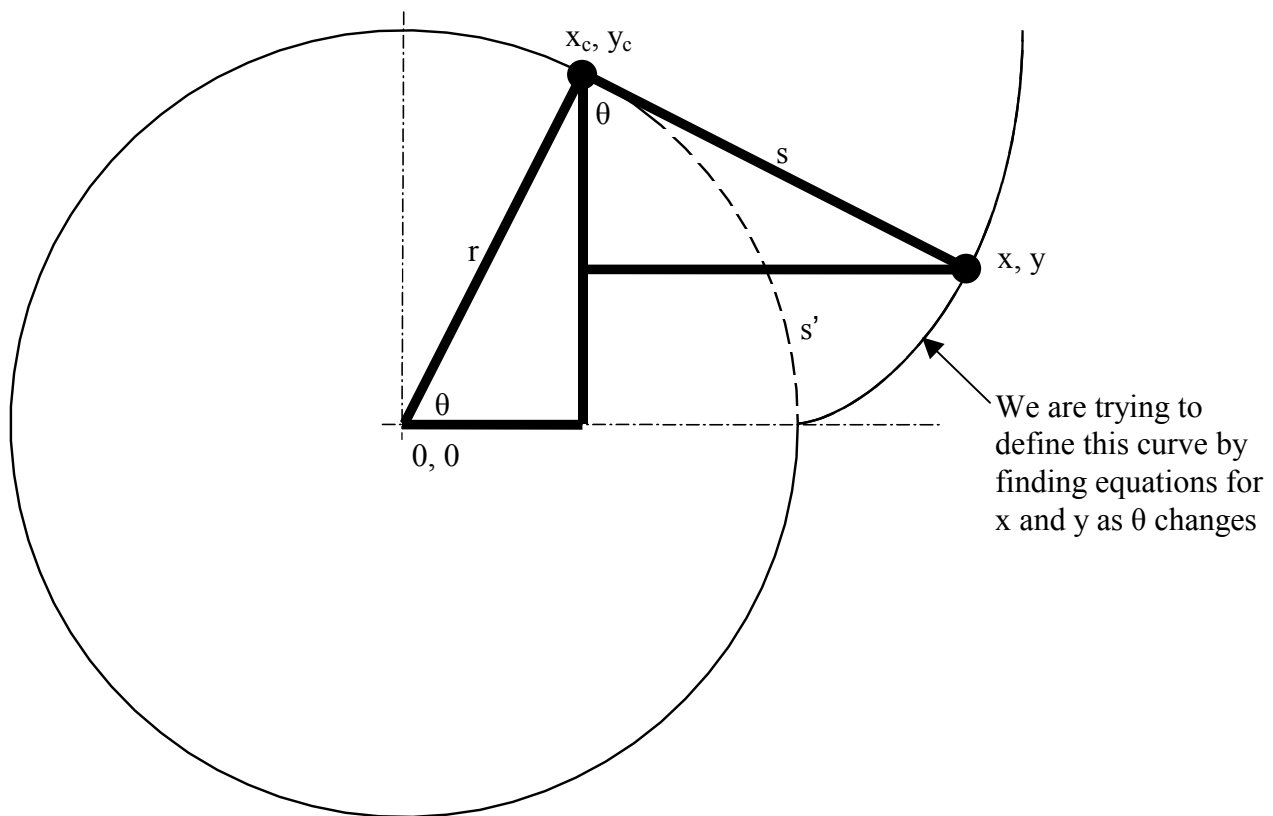
Using trigonometry, we find that:

$$x_c = r * \cos \theta \qquad\qquad x = x_c + (s * \sin \theta)$$
$$y_c = r * \sin \theta \qquad\qquad y = y_c - (s * \cos \theta)$$

Datum Curves From Equation

In Pro/ENGINEER, datum curves can be defined using mathematical equations. When using this feature, a system variable '**t**' is used, that varies from 0 to 1 over the length of the curve. For the involute curve, let the angle (ang) be equal to '**t**' times 90. This means that the variable 'ang' will vary from zero to 90º.

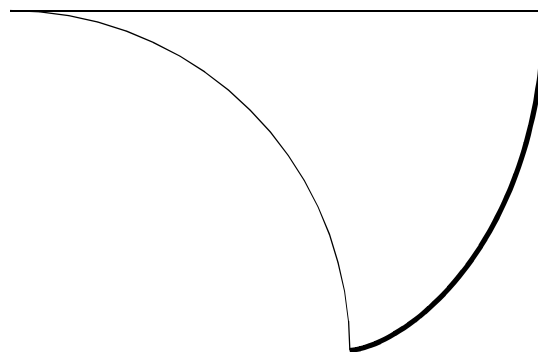To define '**s**', we combine the circumference equation and '**t**' to form:

$$s = (\pi * r / 2) * t$$

In this example, the diameter of the cylinder is 1**.**500. This is the *base diameter* in gear design terminology. The radius (r) of the cylinder is 0**.**75 inches.

The equations for Pro/ENGINEER are:

$$r = 0.75$$
$$ang = t * 90$$
$$s = (PI * r * t) / 2$$
$$x_c = 0.75 * \cos(ang)$$
$$y_c = 0.75 * \sin(ang)$$
$$x = x_c + (s * \sin(ang))$$
$$y = y_c - (s * \cos(ang))$$
$$z = 0$$

The resulting datum curve is shown in bold below.

Creating the Gear

In Pro/ENGINEER, the gear is started by modeling a cylinder at the outside diameter of the gear, see Figure 1. Several sketches are created to define the root diameter, base diameter, and pitch diameter of the gear, see Figure 2. Next, the involute shaped datum curve is created and moved into the correct position to define one side of one gear tooth, see Figure 3. The curve is mirrored and the gear tooth is cut into the cylinder using the edge of the involute datum curves, see Figure 4. The cut is patterned and rounds are added at the base of the gear. The completed gear is shown in Figures 5 and 6.
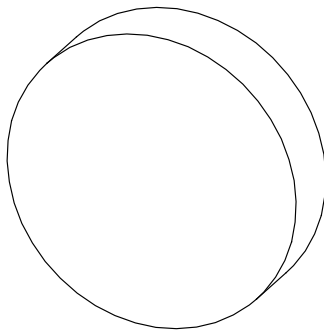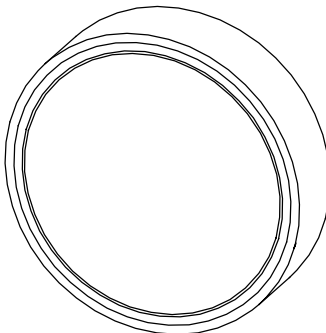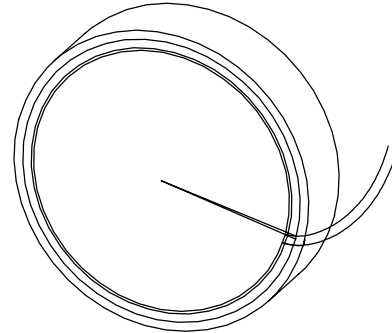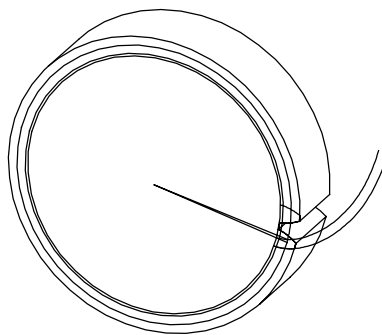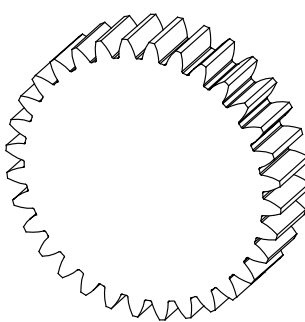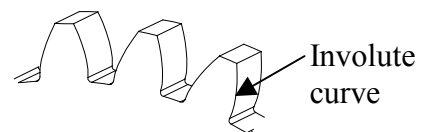
**Figure 1**   **Figure 2**   **Figure 3**

Figure 4   **Figure 5**   **Figure 6**

Involute curve

Gear Parameters

The following are geometric parameters in gear design:

Pitch Diameter	Diametral Pitch = Pitch Diameter + (2 * Addendum)
Root Diameter	Circular Pitch = π / Pitch Diameter
Outside Diameter	Number of Teeth = Diametrical Pitch * Pitch Diameter
Base Diameter	Pressure Angle
Addendum	
Dedendum	
Tooth Thickness	
Tooth Thickness Angle	
Fillet Size	

The figure describes some of the gear parameters.
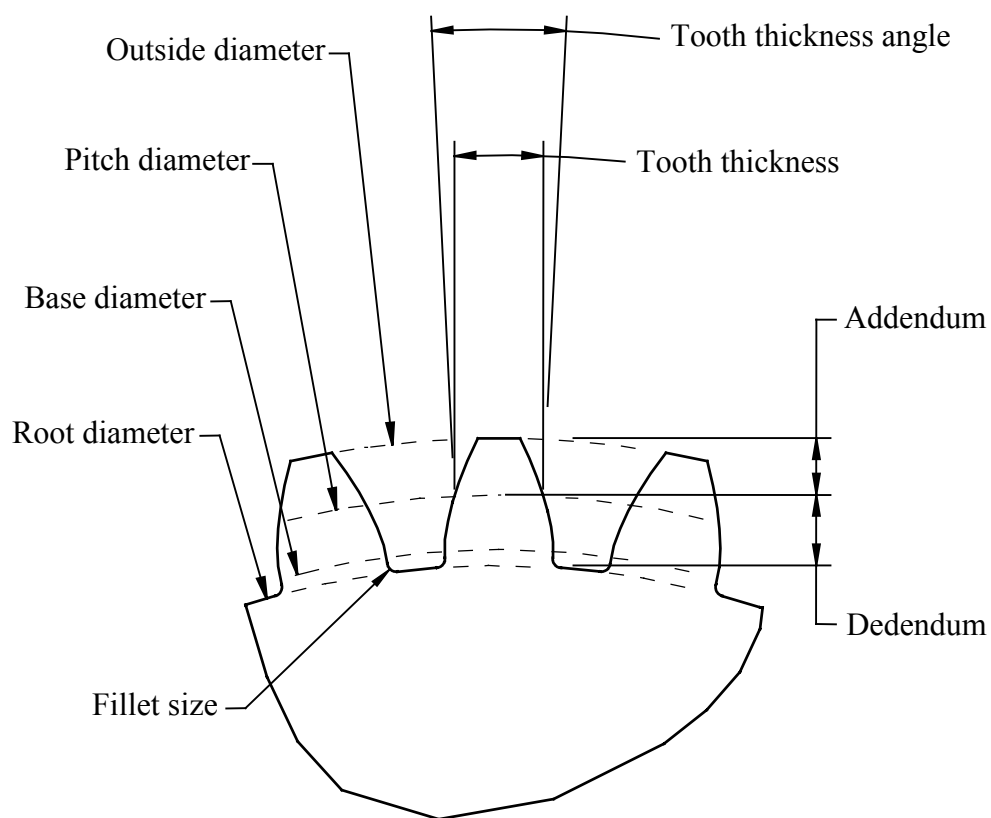
Gear Parameters

EXERCISE 4 – INVOLUTE GEARS

Task 1:   Create a datum curve driven by an equation.

- Pick **File, New**, then enter < involute_gear > for the name of the new part

- Pick **OK** in the **New** dialog box

- Pick **Insert, Model Datum, Curve, From Equation, Done**

- Select the default coordinate system, then pick **Cartesian**

- In the Notepad window, enter the following equations:

    $r = 2.819$
    $ang = t * 90$
    $s = (PI * r * t) / 2$
    $xc = r * \cos(ang)$
    $yc = r * \sin(ang)$
    $x = xc + (s * \sin(ang))$
    $y = yc - (s * \cos(ang))$
    $z = 0$

- Pick **File, Save** and **File, Exit** in Notepad

- Pick **OK** in the dialog box

- The result is shown here

- This curve will be used to create an involute gear
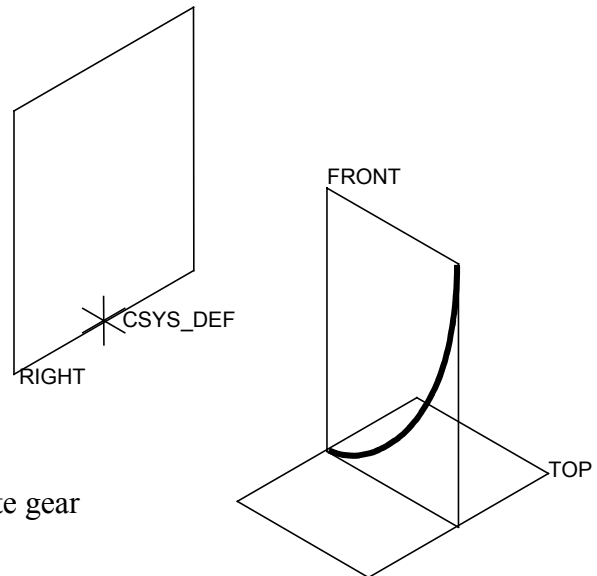
Task 2:   Create a cylinder.

- Select the FRONT datum plane

- Pick the **Extrude** icon ⬚

- Press and hold the right mouse button and select **Define Internal Sketch**

- Pick **Flip** in the **Sketch** dialog box

- Pick **Sketch** in the **Sketch** dialog box

- Sketch a circle centered on the references, with a diameter of 6**.**500, to represent the *outside diameter* of the gear (see the figure on the previous page)

- Pick the checkmark to complete the sketch ✔

- Enter < 1**.**750 > for the depth then pick the checkmark in the dashboard tool

- Pick **View, Orientation, Standard Orientation**

- The result is shown below, be sure the curve is in front of the cylinder as shown
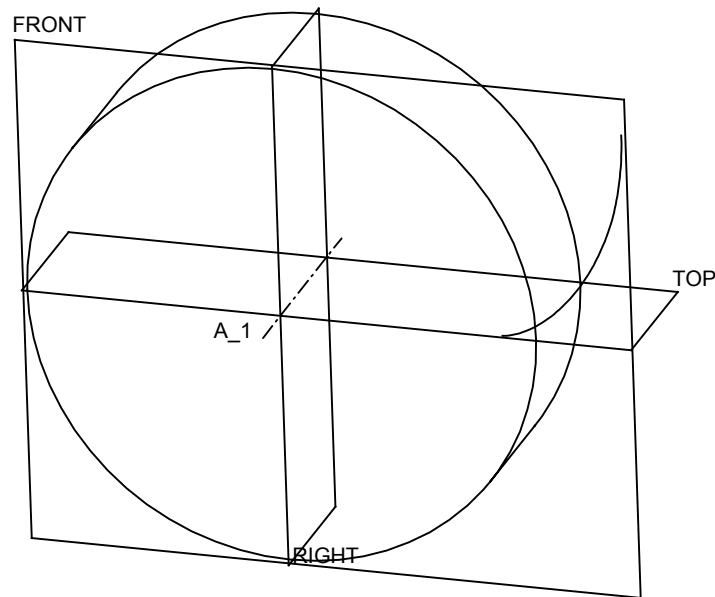
Task 3:   Create three circles representing the major diameters.

- Create a circular sketch on the FRONT datum plane with a diameter of 5.376 to represent the *root diameter* of the gear

- Create a circular sketch on the FRONT datum plane with a diameter of 5.638 to represent the *base diameter* of the gear

- Create a circular sketch on the FRONT datum plane with a diameter of 6.000 to represent the *pitch diameter* of the gear
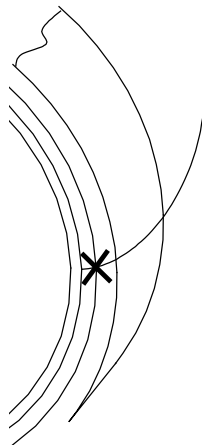
- The result is shown below
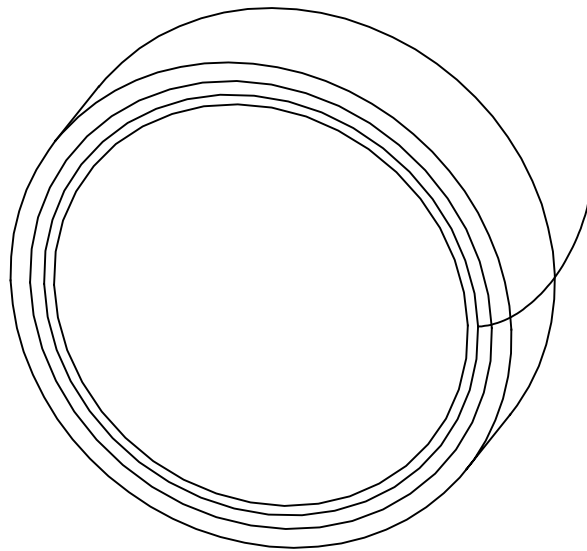
Task 4:   Create a datum point.

- Set the selection filter to *Datums*, pick the involute curve *and* the 6.000 diameter circular sketch, then pick the **Point** icon and then pick **OK** to create a datum point as shown below
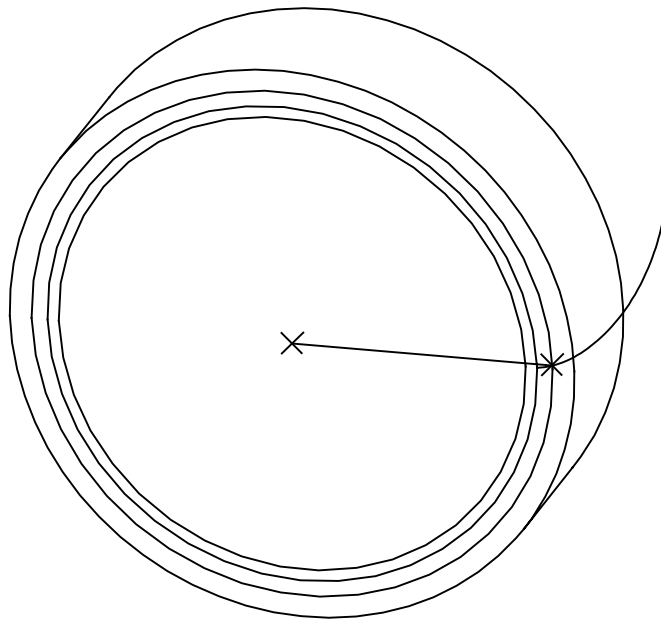

Task 5:   **Create another datum point and a datum curve.**

- Be sure the selection filter is set to ***Datums***

- Pick RIGHT, TOP, and FRONT then pick the Point icon [⊠] then pick **OK**

- Pick **Insert, Model Datum, Curve, Done**

- Pick the two datum points then pick **Done** and **OK**

- The result is shown below

Task 6:   **Create a datum evaluate feature.**


- Pick **Insert, Model Datum, Evaluate**

- Enter < measure > for the name of the feature

- Pick **Create**, then enter < angle > for the name of the measurement parameter

- Pick **Angle**, then pick the TOP datum plane and the curve you just created through the points

- Pick **Done** then read the message window and look at the model tree

- **Save** the part

Task 7:   Copy the involute curve.

- Set the selection filter to *Features*

- Pick the involute curve then pick **Edit, Copy**

- Pick **Edit, Paste Special**

- Add the checkmark next to **Apply Move/Rotate transformations to copies** in the dialog box then pick **OK**

- Pick **Rotate** in the right mouse button popup menu

- Pick the axis in the center of the cylinder then enter < -30 > for the rotation angle

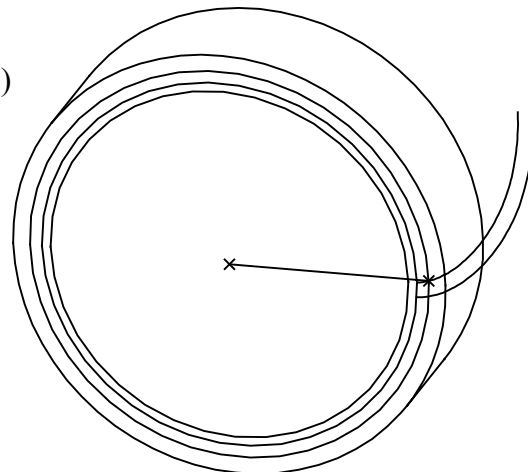- Pick the checkmark in the dashboard tool

- The result is shown
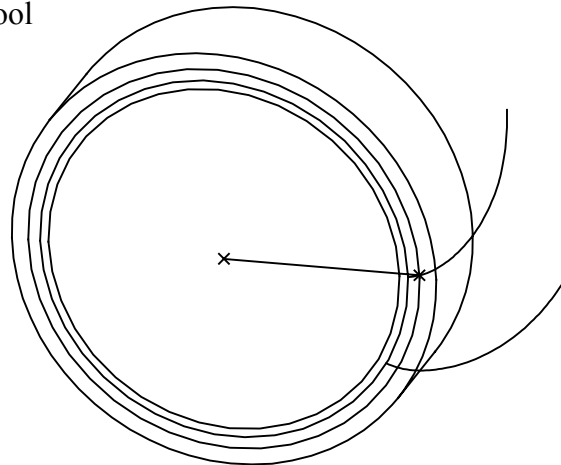
Task 8:   Add a relation and regenerate the part.

- Double pick the copied involute curve (the one created in task 7)

- Double pick the 30 dimension

- In the small text box, carefully enter:

  angle:FID_MEASURE + (7.5 / 2)

- Pick **Yes** to add the relation
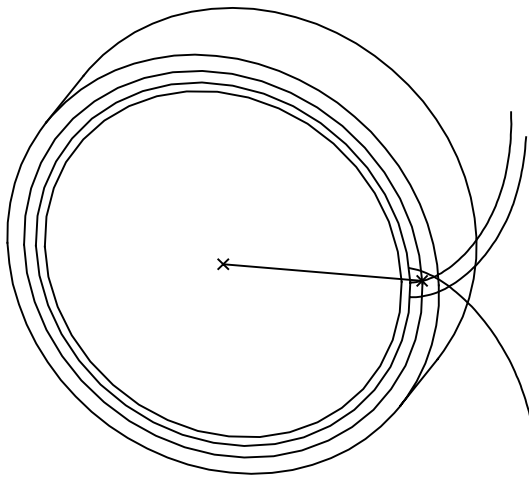
- Pick **Edit, Regenerate**

- The result is shown

Task 9:   Mirror the copied involute curve.

- Pick the copied involute curve (the one created in task 7) then pick **Edit, Mirror**

- Pick the TOP datum plane then pick the checkmark in the dashboard tool
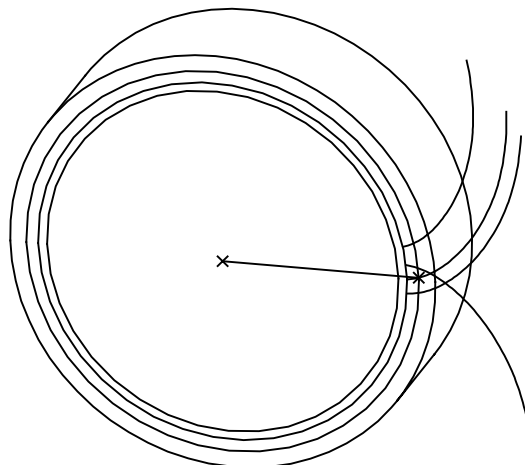
- The result is shown

Task 10:   Copy the curve again.

- Pick the copied involute curve (the one created in task 7) then pick **Edit, Copy**

- Pick **Edit, Paste Special**

- Add the checkmark next to **Apply Move/Rotate transformations to copies** in the dialog box then pick **OK**

- Pick **Rotate** in the right mouse button popup menu

- Pick the axis in the center of the cylinder then enter < -15 > for the rotation angle

- Pick the checkmark in the dashboard tool

- The result is shown
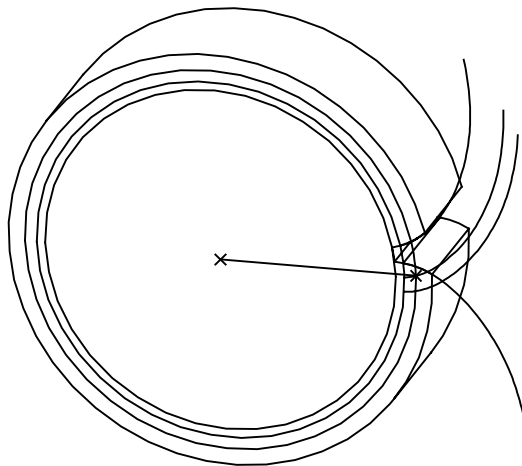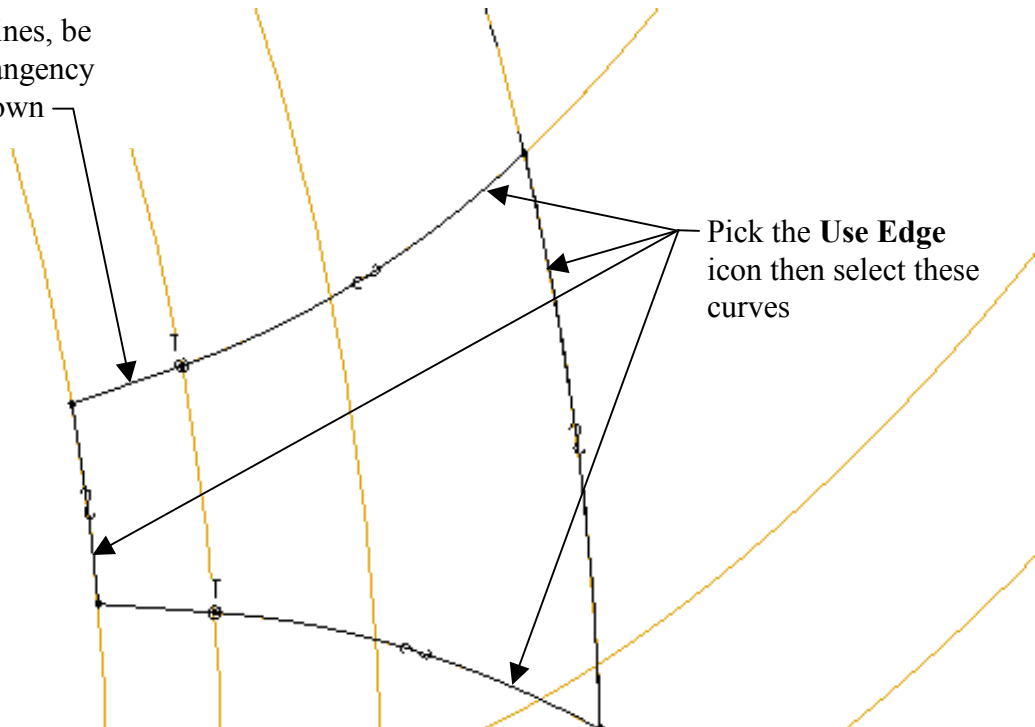
Task 11:   Create an extruded cut.

- Create an extruded (through all) cut using the edges of the curves as shown below
- Trim the sketch as shown below
- The result is shown at the bottom of the page

Add these two lines, be
sure to get the tangency
constraint as shown

Pick the **Use Edge**
icon then select these
curves

Task 12:   Pattern the cut.

- Set the selection filter to *Features* then pick the cut you just created
- Pick **Edit, Pattern**, then set the type of pattern to **Axis** in the dashboard tool
- Select the axis in the center of the cylinder
- Enter < 24 > for the number of instances in the pattern
- Enter < 15 > for the increment angle
- Pick the checkmark in the dashboard tool to complete the pattern

Task 13:   Finish the part.

- Create a 0.020 round at the base of the gear tooth
- Pattern the round
- Create a coaxial hole with a diameter of 2.000
- Use **Layers** to hide the datum curves and sketches
- The completed part is shown below
- **Save** the part and **Close** the window