OPTIMAL SYNTHESIS OF PLANAR FIVE-LINK MECHANISMS FOR THE PRODUCTION OF NONLINEAR MECHANICAL ADVANTAGE

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Optimal Synthesis of Planar Five-link Mechanisms for the Production of Nonlinear Mechanical Advantage

Ricardo C. Blackett

(Abstract)

This thesis presents a technique for the optimal synthesis of planar five-link mechanisms that produce a desired mechanical advantage function over a specified path. Since a five-bar linkage has two degrees of freedom, small deviations from the specified path are possible without significantly altering the mechanical advantage function. The research shows one potential application, the design of strength machines, where it is important to control force while allowing the user freedom of motion.

In the past, closed-form analytical synthesis techniques have been used to design mechanical-advantage-generating linkages. This method is time consuming and case specific. However, optimal synthesis techniques apply to the general case and present a robust solution procedure. This thesis uses the non-linear pattern search technique of Hooke and Jeeves to synthesize five-bar linkages. The search technique matches user strength curves and mechanism resistance curves to produce a five-link mechanism. This mechanism produces the desired mechanical-advantage function and serves as the basis for strength training machines. Unlike analytical synthesis, optimization allows direct incorporation of a greater number of design constraints, thus resulting in solutions that are more practical. The pattern search technique aims to minimize a given objective function that depends primarily on the force generating capabilities and kinematic constraints on of the linkage.

For my beautiful, intelligent, and humorous daughter Tenee. In your three short years, you have given me the strength and focus to accomplish my goals. You have been my beacon of light when things seemed darkest, and my biggest fan when others vanished.

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Merriam-Webster describes a journey as something suggesting travel or passage from one place to another. Six years ago a journey began, and now ends with the completion of this thesis. Many people helped me reach the end of my passage successfully. First, and foremost, I want to thank God, for He made a way when all roads seemed blocked. I want to thank my daughter for inspiring me with her innocence to aim higher, work harder, and reach for the stars. Many thanks due to my mother for her words of encouragement and inspiration. I want to thank my father for instilling in me the thirst for education and for whetting my appetite for engineering. My immediate and extended families deserve many thanks, for they helped me to keep my eyes on the prize.

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CONTENTS

FIGURES

TABLES

NOMENCLATURE

X^0	Initial design vector
W	Load weight
w	Objective function weighting factor
v	Weighting factor
t	Quadratic variable
q	Current optimization variable
P	Magnitude of P
l_r	Length of weight arm (right side of mechanism)
l_n	Length of *n*
l_l	Length of weight arm (left side of mechanism)
k	Vector direction, objective function weighting factor
j	Euler index, imaginary axis
i	$\sqrt{-1}$, real axis
F_n	Force in link n
F_i	Targeted force
F_{actual}	Optimization computed force
e	Exponential constant, Euler notation
A	Distance of first fixed pivot from origin
Δx	Optimization step size
θ_r	Angle of weight arm (right side of mechanism)
θ_P	Angle of P
θ_n	Link angle n
θ_l	Angle of weight arm (left side of mechanism)
θ_A	Orientation of first fixed pivot
β	Arbitrary force angle
Ω	Objective function weighting factor
Π	Objective function weighting factor
Θ	Direction of Force

CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1 Background and Motivation

The research presented in this thesis uses optimization techniques to design planar five-link mechanisms that generate a programmed nonlinear mechanical advantage. These mechanisms can serve as the basis for strength training machines. Most machines today, like the Nautilus machine shown in Figure 1.1, use wrapping cams or four-bar linkages to generate a nonlinear mechanical advantage. A properly designed machine matches the nonlinear mechanical advantage of the mechanism and the strength potential of the user throughout the range of motion. In other words, the user should feel a uniformly difficult resistance at every point in the motion cycle. Unfortunately, current machines provide resistance, but at the expense of freedom of motion. The user must follow the simple path of motion, usually a straight line or circular arc, programmed into the machine.

Figure 1-1. Nautilus shoulder press machine. The machine is based on the four-bar linkage drawn in the figure.

Due to an added degree of freedom, five-link mechanisms, such as the one pictured in Figure 1-2, offer the distinct advantage of increased mobility over four link mechanisms. Unlike four-link mechanisms, the five-bar requires two weights to produce the desired mechanical advantage. The user inputs a continuously varying force over a user-defined path of motion. For a given position of the user input, the kinematics of the mechanism and the weight added to the grounded links determine the magnitude and direction of the force experienced by the end user.

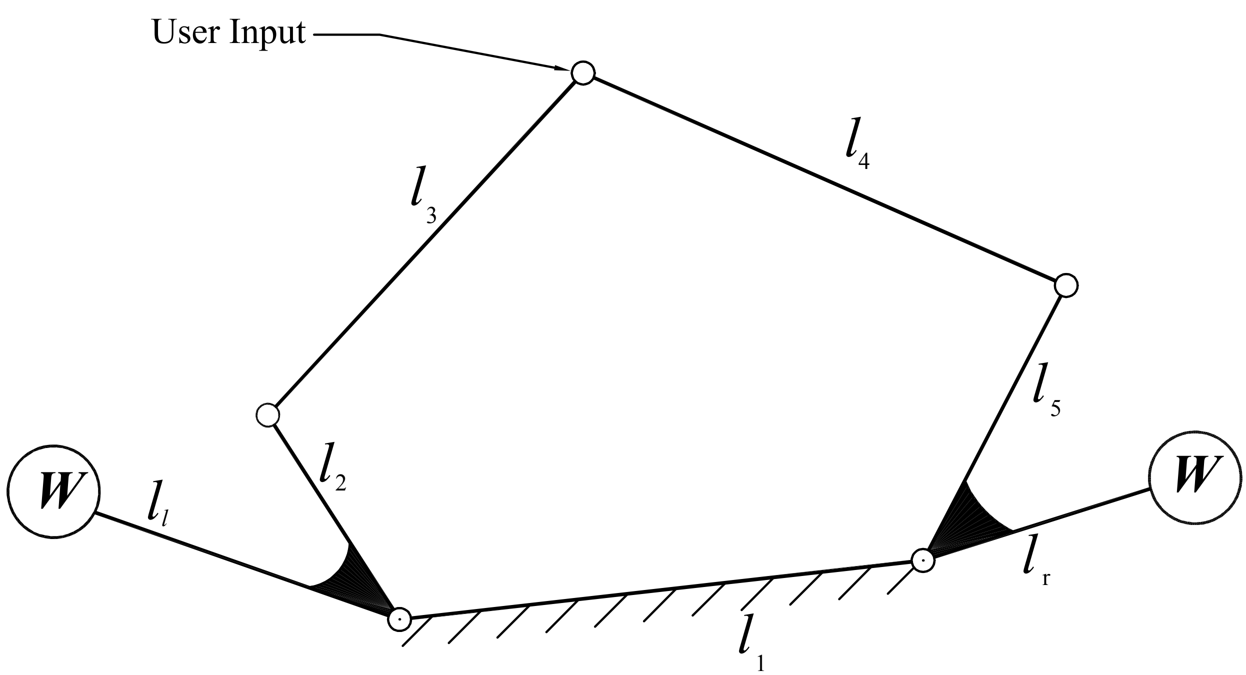

Figure 1-2. Planar five-link mechanism.

A four-bar linkage consists of four rigid members attached to one another using revolute joints. The four rigid members are as follows: the fixed member, the crank, the follower and the coupler. When analyzed using the Grubler mobility equation, the four-bar linkage has one degree of freedom [Mabie, 1987]. Here, a degree of freedom is defined as the number of independent inputs required to determine the position of all links of a mechanism. Since most mechanism tasks require transferring a single input to a single output, the four-bar linkage is used often [Erdman, 1984]. Consequently, machines based on a four-bar linkage restrict the user to perform a single exercise along a predefined path. The linkage provides a variable resistance along the path, which greatly increases the user's work on each repetition.

Five-link mechanisms require an additional revolute joint and an additional rigid member. Use of the Grubler mobility equation, shows that such a mechanism posses two degrees of freedom, and this increased mobility allows the user to vary the path of motion of the exercise and possibly perform two exercises with a single machine.

Two different training methods exists to produce the well documented benefits of strength training. However, the two broadly divided schools of thought cannot agree on the best method to achieve the benefits. One school advocates the use of free weights, such as barbells and dumbbells to provide resistance. This free-weight approach emphasizes freedom of motion and the proper balance of the weights. The second school advocates the use of resistance training machines such as those produced by Nautilus and Hammer. The machine-based approach emphasizes safety and controlled resistance along the path of motion. Theoretically, strength machines provide a more efficient workout than using free weights. Work is defined as the area under the force versus displacement curve as shown in Figure 1-3. Using a constant resistance like a free weight, leads to a constant amount of work throughout the range of motion, or stroke, of the exercise. The weakest point of the user along the path will always limit the magnitude of the resistance. On the other hand, the resistance provided by exercise machines varies over the stroke of the exercise, thus increasing the amount of work compared to free weights. From Figure 1-3, the hatched area represents the increased work that an exercise machine requires. This increase in work translates into more energy expended by the user in a shorter time. Those who advocate free weights do so because of the freedom and the control of the weight during concentric and eccentric phases of the exercises. On other hand, those who advocate using strength machines believe that the user performs more useful work with a machine as opposed to the magnitude of the weight lifted. This thesis proposes a new type of mechanism that combines the advantages of both free weights and traditional machines. Exercise machines based on five-link mechanisms give the user both control and variable resistance, thus combining the two feuding schools of thought.

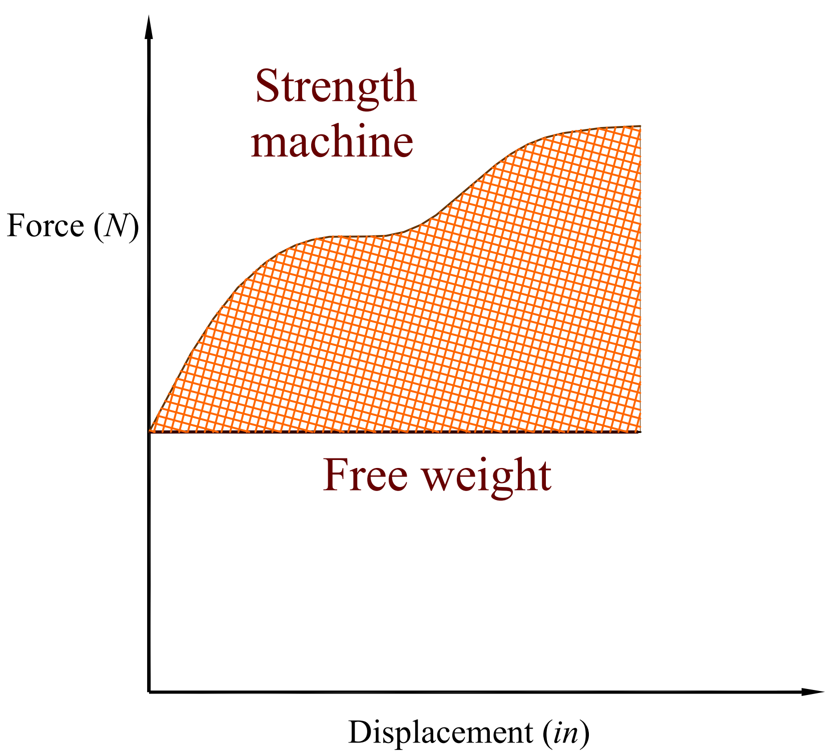

Figure 1-3. Force versus Displacement Curve, i.e. Work Curve

1.2 Research Scope

This thesis focuses on the optimal design of five-bar mechanisms that produce a nonlinear mechanical advantage. The aim is to develop and demonstrate a tool that will accurately synthesize a multi-degree of freedom mechanism that provides a non-linear mechanical advantage. The research uses the magnitude and direction of force and position to synthesize the linkage. Optimal design can be considered as synthesis by the repeated analysis and comparison of potential design alternatives. In this case, analysis refers to the calculation of required user input force as a function of position and load weight. Synthesis is the inverse of analysis, and finds the dimensions of the linkage that produces the desired mechanical advantage. Dimensional synthesis best describes the problem of this thesis, and Hartenberg [1964] defines it as the determination of parts, lengths, and angles, necessary to create a mechanism that will affect a desired motion transformation. This thesis uses optimization to synthesize the physical dimensions of the five-link mechanism through repeated analysis. The optimization algorithm uses an objective function that assigns a numerical value to each solution; the objective function consists of the structural error between the force data, and constraints such as maximum and minimum link lengths. Chapter two presents an in-depth discussion of the objective function and the components.

The basic approach to exercise machine synthesis matches user strength curves and mechanism resistance curves [Soper, 1995]. Strength curve normalization accounts for the difference in strength magnitude among users, because the shape of the strength curve remains the same regardless of the maximum value. Then, the problem becomes one of generating a specified mechanical advantage curve based on the human strength curve for a given exercise. The magnitude of the weight added to the machine determines the magnitude of the resistance. If the mechanism resistance curve and user strength curve match closely, the user will experience a rigorous workout.

Conservation of energy principles and techniques from traditional kinematics serve as the basis of mechanism synthesis. The mechanical advantage of a machine is the ratio of output force to input force. Kinematic properties and conservation of energy principles accurately expresses the mechanical advantage of the linkage. These same properties contribute to the analytical synthesis of the linkage along with the desired user strength curve.

The five chapters of this thesis deal with different aspects of the research. The first chapter outlines previous work in the field and gives the necessary background to proceed throughout the thesis. The second chapter defines the problem, outlines the solution procedure, and discusses design constraints. The third chapter discusses the different types of optimization-based synthesis and the Hooke and Jeeves pattern search technique. Chapter 4 outlines the solution procedure, and discusses illustrative examples using the same pattern search technique. Chapter 5 offers practical design recommendations for future work and further advances in mechanism optimization.

1.3 Literature Review

Current analytical synthesis of force-generating mechanisms focuses on four-bars, and some of the same techniques apply to five-link mechanisms; the literature review aims to highlight the applicable techniques. In addition, many of the analytical and optimal synthesis techniques developed for four-link mechanisms apply to five-link mechanisms. Most references use synthesis to satisfy position requirements rather than force requirements. In the past, the majority of designers used precision point synthesis (a type of analytical synthesis) for determining the dimensions of four-link mechanisms. A smaller set of designers used optimization based techniques for synthesis. Both optimization and analytical precision point techniques apply to five-link mechanisms; however, only analytical precision point techniques have been used to synthesize five-link mechanisms.

Precision Point Synthesis. Analytical synthesis techniques rely on precision points, which prescribe successive locations of the output. Norton [1992] points out that the number of equations limits the number precision points for a given linkage. Currently, seven serves as the upper limit for precision point synthesis using a four-link mechanism. Synthesis of more than four points involves solving a set of non-linear simultaneous systems of equations. These analytical synthesis techniques guarantee that a solution will reach each of the precision points. However, analytical synthesis does not provide information about the mechanism path between these precision points. In the case of exercise machines, the path between each point determines the effectiveness of the machine. Force synthesis serves as one method of extracting precision points from the user strength curve.

Four-Link Mechanism Force Synthesis. Force synthesis, a type of precision point synthesis, extracts precision points from force relationships. Soper et al. [1995] used force synthesis to obtain a four-link mechanism that gives the prescribed non-linear force output. Using the principle of virtual work, Soper et al. showed the commonality between force synthesis and velocity synthesis. This commonality, determined a relationship between the mechanical advantage and velocity ratio, and this served as the basis of the synthesis techniques. Simply put, synthesizing for mechanical advantage requires prescribed linkage velocities.

Soper et al.[1995] used four precision points to synthesize force generating four-link mechanisms. Examining Burmester curves (graphical representation of an infinite number of solution pairs of the mechanism) allowed Soper to find a number of different solutions. After the synthesis, he analyzed each mechanism based on a number of constraints. The chosen mechanism was safe for the user and allowed the weights to remain relatively close to the ground throughout the range of motion.

Multi-Degree of Freedom Analytical Synthesis. Soper et al. [1998] used force synthesis techniques to design multiple-degree-of-freedom mechanisms. They found that five-bar linkages produced a tailored non-linear resistance along a specified path. In addition, Soper et al proved that isolation of two-force members within the mechanism ensured the validity of the synthesis technique, because the linkage did not posses redundant links.

At each precision point, the force generated by the five-bar linkage must match the direction of input force. As a result, force synthesis becomes more difficult as compared with four-link precision point synthesis, but the Burmester theory remains the same. The additional force constraint led the team to look at specific linkage configurations to obtain a final solution. Soper et al. [1998] found that synthesizing a force generating five-link mechanism required designer intuition in order to satisfy the force constraint. Limiting the solution to a specific configuration, forced Soper et al to use Burmester curves to synthesize the linkage.

In this case, the mechanism can satisfy four precision points. However, the motion between these points remains unknown until the linkage analysis. The motion between the precision points should match the user strength curve at each point to ensure maximum benefit from the exercise machine.

Optimal Synthesis. Optimization, defined as the act of finding the best result under a given set of constraints [Reinholtz, 1983]. It is the act of finding the minimum of a function or maximum of desired benefit. In this case, optimization uses the minimum difference between the desired user strength curve and the mechanism resistance curve, i.e. structural error, to generate a linkage. Optimization uses an objective function, based on a set of design variables and constraints, assigns a numerical value to the desired benefit or required effort. The design variables stem from the physical constraints of the problem. Moreover, the objective function incorporates user-defined limits on design variables and problem constraints. Along with the numerical algorithm, the objective function plays a big role in the effectiveness of the optimization problem.

Four-link Optimization. In the past, optimal techniques used to design planar four-link mechanisms proved effective. Sardinia [1996] used analysis to derive relationships between the crank and coupler links. Then, he obtained discrete data points for the user strength curves using the static force measurements from a force transducer.

The objective function included a structural error term, defined as the summed-squared error between the force curve and strength curve at a finite number of points spanning the input range. Scardina considered other design considerations, such as performance, aesthetics, and practicality, and used Hooke and Jeeves pattern search method of optimization to find the best solution to the problem. Hooke and Jeeves, used extensively for mechanism synthesis, because of its robustness, simplicity, and versatility. Optimal synthesis requires formulation of an objective function and a user provided initial guess of design variables.

Scardina presented a detailed methodology of optimal techniques to synthesize a compound row exercise machine. After a number of successful optimization trials each finding a different local minimum, he had several linkages from which to choose. One can see the power of optimization; because Scardina supplied different initial link length guesses and obtained different solutions that, all satisfied the force objective and design constraints.

Technique Comparison. When compared with optimal synthesis, analytical synthesis has distinct advantages and disadvantages. Analytical techniques result in a well-defined solution space, whereas optimal techniques minimize the given objective function. The minimum value obtained depends upon the initial guess put into the algorithm. The analytical methods do not require an initial guess, but optimal methods require an initial guess that effects how fast the solution converges [Scardina, 1996]. Intuition does not play a crucial role when using optimal techniques, because the algorithm will converge to a local minimum regardless of the validity of the initial guess.

However, a good initial guess greatly increases the chances of finding a global minimum. Optimization finds only local minimums, not global minimums, using the objective function; however, with numerous iterations and design intuition the chances of finding a global minimum are increased.

Based on a review of the literature, no research concerning optimal synthesis of five-link linkages has been done. However, the same optimization techniques apply directly to both four and five link mechanisms. The goal of the literature review was to survey the field and find information that pertained to the problem of this thesis. Using optimization allows the designer more design constraints and parameters. Optimization also affords the designer flexibility and offers practical linkages without a great deal of analysis.

1.4 Optimization Theory

Optimization, described as any process, which seeks to find the best possible solution to a problem. Mechanism optimization is the repeated analysis of randomly determined mechanisms to find the best design [Scardina, 1996]. In this thesis, optimization is the process of finding the best possible five-link mechanism based on an objective function subject to a number of constraints.

The best solution will effectively satisfy the design constraints and produce the minimum value for the objective function. When multiple or conflicting constraints enter the problem, the process of finding the best solution becomes more difficult. A weighting procedure considers conflicting design constraints. The relative importance of each constraint is specified in the objective function. Depending on the weighting process, the designer can tailor the final solutions. Optimization yields mathematically correct solutions, but these solutions may posses mechanical defects, thus designer's job is to sort through all the solutions to find the best possible solution.

There are four basic elements to the mechanism optimization problem [Reinholtz, 1983]. These elements are listed below and explained further in the paragraphs to follow.

1. The conceptual design.
2. The development of a model that represents the physical system from which the design variables and governing equations can be extracted.
3. A scalar function (called the objective function) of the design variables to measure the overall effectiveness of the system.
4. Finding a set of values that produce the best value for part (3) and satisfies all design constraints.

The first listed item is the conceptual design. The designer must choose which type of mechanism to use and the specific configuration. Cams, linkages, or gears are just a few of the mechanisms the at the designer's disposal. Configuration covers two areas in the design. The first area is the orientation of the mechanism and the second deals with the specific arrangement. For example, if the chosen mechanism is a linkage, the specific configuration may either be spatial or planar with four or five-links. For this thesis, the chosen mechanism is a five-bar linkage operating in the plane.

The second element is developing an accurate model of the linkage and obtaining design variables. The model is based on the mathematical equations that govern the system. Once the mathematical model is developed, the design variables can be obtained from the model. In this case, basic kinematic mechanism theory is applied to perform position and force analysis. There can be an infinite number of design variables obtained from the model, in this case the design variables will be the link lengths and the angles of the attached weights.

The third element is developing the objective function and design constraints. The objective function is a scalar function of the design variables whose numerical values reflect the quality of the linkage being designed. This objective function is often defined as the sum-squared error between points at a finite number of positions. As an example, the objective function could be the sum-squared error of the deviation between the actual and desired positions of a point on the linkage. As mentioned earlier, when multiple or conflicting constraints are used, a weighted combination of the different constraints can be built into the objective function. Developing a poor objective function will lead to poor solutions, for this reason the objective function plays a key role in optimization.

The last element is simply a matter of repetition. This step requires the designer to input different starting values into the algorithm and inspecting the final linkage. If the algorithm does not produce practical linkages, the designer has the ability to change the objective functions or design constraints instantaneously to obtain feasible results. One of the powerful advantages of optimization is the ability to change constraints to obtain feasible solutions.

CHAPTER 2: PROBLEM DEFINITION AND CONSTRAINTS

This chapter presents an in-depth discussion of the design process and a detailed description of the kinematic model. The first step develops a conceptual design. With the conceptual design in hand, the designer constructs the kinematic model. Finally, the kinematic model aids the designer develop the objective function.

As mentioned earlier, the designer determined the specific configuration of the mechanism. Here, the designer used a five-link planar mechanism, but other mechanism configurations produced similar results. However, the added complexity outweighed the potential benefits from their use. Next, the designer chose the type of joints needed to connect the links. Revolute, rolling, and cam joints present the designer with a multitude of choices. The designer needs intuition and a thorough understanding of the available options in order to choose the appropriate configuration, because models tend to become complicated quickly. Simply put, a complex model translates into a complex analysis. Increasing the design parameters affords the designer more flexibility, while requiring additional constraints. The conceptual design accurately describes the mechanism while making the analysis straightforward.

2.1 Conceptual Design

Selecting five links and five joints, the linkage possesses two degrees of freedom. Figure 2-1 shows the specific configuration of the linkage. All angles are measured in a positive right hand sense (counterclockwise).

A distance, l_1, commonly called the ground link, separates the two fixed pivots. Links 2 through 5, l_2, l_3, l_4, l_5, are each defined by angles measured from the horizontal axis in a counterclockwise fashion. These links represent the distance and orientation between the joints, or moving pivots, in the mechanism. The weight arms, which provide the resistance, are rigidly attached to links l_2 and l_5. Rigidly attached means that the weight arms and reference link act as one unit. For simplicity, both weights will have the same for value for this algorithm. As mentioned earlier, five link mechanisms use two weight stacks to produce the non-linear mechanical advantage. Unlike the other angles in the model, θ_l and θ_r are measured from links l_2 and l_5. The user applies a force at the junction of links 4 and 5. Four additional parameters are used in the description of the mechanism to uniquely specify the location of force application. These four additional parameters give the designer more flexibility, while increasing problem complexity. The location of the first fixed pivot is defined by a distance A and angle θ_A. Point P specifies the magnitude and direction of user input, and θ_p describes the orientation.

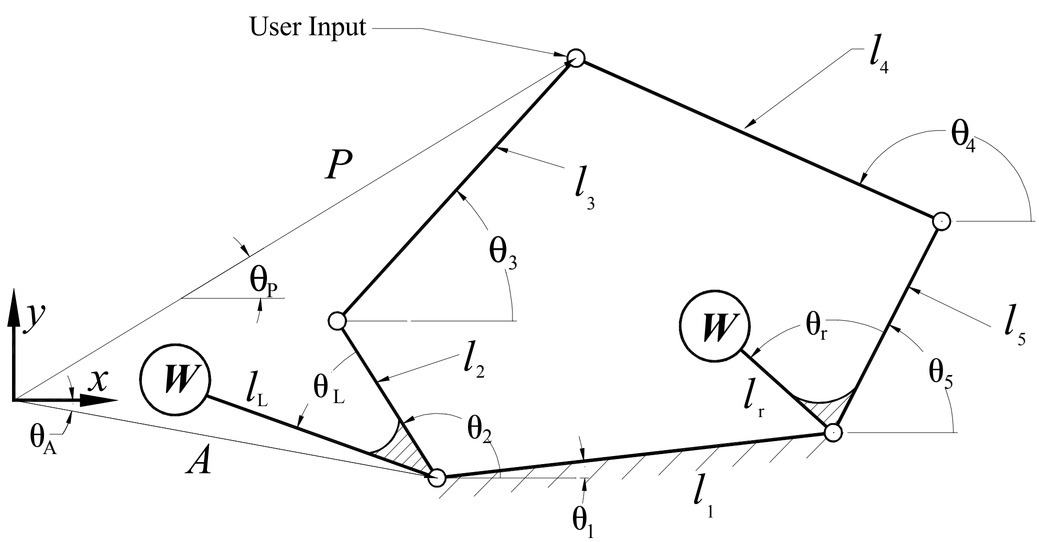

Figure 2-1. Conceptual design for the five-link mechanism. All angles are measured in counterclockwise sense from horizontal.

It is important to stress that the arbitrary nature of the chosen configuration, the designer determined the placement of each link and weight. After determining the conceptual layout of the mechanism, the analysis of the mechanism using kinematic principles can be done.

2.2 Kinematic Model

The governing equations of the kinematic model came from position analysis. Position analysis solved the angles of all links in the mechanism as a function of input angles. There are several approaches to solve the kinematic position analysis problem; all require insights and manipulations to obtain the desired output as a function of the input angle [Mabie, 1987]. Loop closure methods, used in this research, described vectors of closed loops obtained from the linkage orientation. Link orientation determined the vector direction, and the link length determined the magnitude of the vector.

Two loops described the motion of the mechanism based on the user input **P**. As mentioned earlier, position analysis requires insights and manipulations to obtain the output angles. In order to make the analysis simple the designer used two loops. From Figure 2-1, one loop is defined by

$$\mathbf{A} + \mathbf{l}_2 + \mathbf{l}_3 = \mathbf{P} \tag{2.1}$$

where $\mathbf{l_2}$ and $\mathbf{l_3}$ are vectors describing links 2 and 3 of the mechanism; **A** is the vector describing the magnitude and direction of the fixed pivot from the origin; **P** is the vector describing the magnitude and direction between the origin and the point of application of the force.

Using complex notation, equation 2.1 is rewritten as follows:

$$Ae^{j\theta_A} + l_2 e^{j\theta_2} + l_3 e^{j\theta_3} = Pe^{j\theta_P} \tag{2.2}$$

where l_2 is the length of the second link; θ_2 is the direction of the link 2; θ_A is the direction of A; l_3 and θ_3 are the magnitude and direction of link 3; P and θ_P are the magnitude and direction of the vector describing the location of the user input.

Equation 2.2 represents one two-dimensional vector equation with two unknowns. A vector equation consists of two scalar equations. Therefore, breaking equation 2.2 into two scalar equations with two unknowns yields a simple system of equations. The angles, θ_2 and θ_3, are embedded in transcendental functions, i.e. trigonometric functions of sine and cosine. Extracting these two angles proves difficult, and one approach multiplied the transcendental function by its complex conjugate; thus allowing isolation of the unknown angles. The complex conjugate of equation 2.2 is given by

$$Ae^{-j\theta_A} + l_2 e^{-j\theta_2} + l_3 e^{-j\theta_3} = Pe^{-j\theta_P} \tag{2.3}$$

Multiplying equation 2.2 by equation 2.3 gives the following equation:

$$\begin{aligned} l_3^{\,2} = l_2^{\,2} + P^2 + A^2 - AP\left(e^{j(\theta_P-\theta_A)} + e^{j(\theta_A-\theta_P)}\right) - Pl_2\left(e^{j(\theta_P-\theta_2)} + e^{j(\theta_2-\theta_P)}\right) \\ + Al_2\left(e^{j(\theta_A-\theta_2)} + e^{j(\theta_2-\theta_A)}\right) \end{aligned} \tag{2.4}$$

In order to continue with the approach, equation 2.4 needs to be simplified using the following Euler identity:

$$e^{j(\beta)} = (\cos\beta - j\sin\beta) \tag{2.5}$$

where e is an exponential constant used by Euler; j is a coordinate notation for the imaginary axis; β is an angle used for illustrative purposes.

Upon substituting equation 2.5 into 2.4 an expression in terms of sines and cosines is found. The resulting equation is as follows:

$$l_3^2 = l_2^2 + P^2 + A^2 - 2AP(c\theta_P c\theta_A + s\theta_P s\theta_A) - 2Pl_2(c\theta_2 c\theta_P + s\theta_P s\theta_2) + 2Al_2(c\theta_2 c\theta_A + s\theta_2 s\theta_A) \tag{2.6}$$

where $c\theta_p$ is a substitution for the expression $\cos(\theta_p)$; similar substitutions are made for the other transcendental functions.

In equation 2.6, the only unknowns are θ_2 and θ_A, since every link length and θ_P is given. To simplify equation 2.6, replace the sine and cosine terms with the following identities:

$$\cos(\beta) = \frac{1-t^2}{1+t^2} \tag{2.7}$$

$$\sin(\beta) = \frac{2t}{1+t^2} \tag{2.8}$$

where $\boldsymbol{t} = \tan\frac{\beta}{2}$.

Multiplying through by $1+t^2$, yields a quadratic equation given by

$$At^2 + Bt + C = 0 \tag{2.9}$$

where A,B, and C represent strings of polynomial terms. Expressions for the polynomials are as follows:

$$A = P^2 + l_2^2 + A^2 + 2AP[c\theta_P c\theta_A + s\theta_P s\theta_A] - l_3^2 + 2Pl_2 c\theta_P - 2Al_2 c\theta_A,$$

$$B = 4Al_2 s\theta_A - 4Pl_2 s\theta_P,$$

$$C = P^2 + l_2^2 + A^2 + 2AP[c\theta_P c\theta_A + s\theta_P s\theta_A] - l_3^2 - 2Pl_2 c\theta_P + 2Al_2 c\theta_A$$

The quadratic formula is used to solve for t. The solution of t is given by

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{2.10}$$

where A, B, and C are the string of polynomial terms in equation 2.9.

Once t is obtained, θ_2 is found from the following equation:

$$\theta_2 = \mathbf{tan}^{-1}(2\mathbf{t}), \tag{2.11}$$

Finally an expression for θ_3 is given by

$$\theta_3 = \mathbf{cos}^{-1}\left(\frac{\mathbf{P}\,\mathbf{cos}(\theta_{\mathbf{P}}) - \mathbf{A}\,\mathbf{cos}(\theta_{\mathbf{A}}) - \mathbf{l}_2\,\mathbf{cos}(\theta_2)}{\mathbf{l}_3}\right). \tag{2.12}$$

A similar procedure solved the second loop of the mechanism for angles θ_4 and θ_5.

The discriminant, the radical term in the quadratic equation, had three possible solutions; and gave the designer an idea of the relative configuration of the linkage. A nil discriminant represents a mechanism singularity or limit. A negative discriminant denotes a negative square root, and the linkage cannot assemble at the desired position. The final case, a positive discriminant signals that the linkage reaches the desired position in two ways, given by the two roots. These two possible roots, real or complex, stem from the radical term in the equation. Figure 2-2, shows an example of the two possible closures for a four-bar linkage, which the positive root and the negative root reached the same goal position. A five bar mechanism reaches the same end position in four different configurations. For this optimization algorithm, the designer used one set of roots throughout the entire analysis.

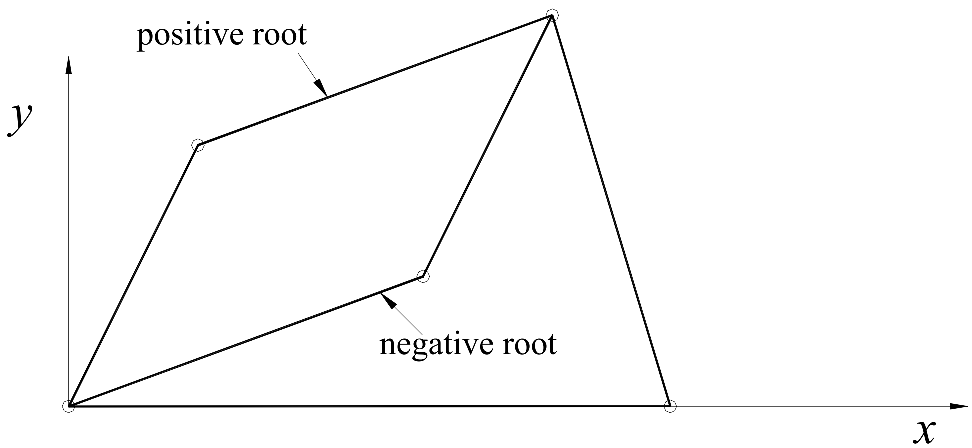

Figure 2-2. Four bar linkage showing both possible closures.

2.3 Optimization Design Variables

Optimization involves adjusting a given set of design variables to find a solution. Twelve design variables, taken directly from the kinematic model, completely described the linkage. The design variables include the length of each link and weight arm, and the angle of each weight arm. The objective function incorporates each design variable and finds a locally optimal solution based on the design constraints. The variables are combined into a single vector known as the design vector, $\boldsymbol{X^0}$, with components x_1 through x_{12}. Table 2-1 lists each of the design variables along with a brief description of each.

Table 2-1
Optimization Design Variables

Parameter	Description
l_1	Link between the two fixed pivots
l_2	Link between first fixed pivot and first moving pivot
l_3	Link between first and second moving pivot
l_4	Link between second and third moving pivot
l_5	Link between third moving pivot and second fixed pivot
l_L	Link describing weight arm attached to link 2
θ_L	Angle between weight arm and link 2
l_r	Link describing weight arm attached to link 5
θ_r	Angle between weight arm and link 5
A	Distance describing first moving pivot
θ_A	Orientation of moving pivot
θ_1	Orientation of the ground link

2.4 Objective Function

The objective function proves valuable to the optimization algorithm. The objective function assigns a numerical value to each linkage configuration; where the optimal solution retains the lowest value. Many different methods and procedures exist to construct the objective function; this thesis uses penalty functions to develop the objective function.

For each set of length links, the objective function solved the position analysis and computed the resistive forces. Then, the objective function computed the structural error between the mechanism resistance curve and user strength curve. The structural error is the square root of the summed-squared error between the desired force curve and the resistance curve. Larger error propagates at a faster rate than smaller errors, because the summed-squared error penalizes negative and positive error equally. This allowed the objective function to converge very efficiently [Scardina, 1996].

Force errors at each position of the mechanism contributed to the value of the objective function. Equation 2.13 shows the mathematical form of the unconstrained objective function. The unconstrained objective function is given by

$$OF = f(X, P) = \sum_{i=1}^{n} w_i \left[F_i^{2} - F_{actual_i}^{2} \right] \tag{2.13}$$

where w is a scaling factor; F is the intended force obtained from the optimization algorithm; F_{actual} is the actual force taken from the user strength curve.

Penalty functions and constraints. Since the unconstrained objective function cannot inherently account for all design criteria, the designer used penalty functions. Optimization found solutions that closely matched the set of force data, without regard to link lengths, weight angles, or other physical constraints, and penalty functions presented the author with a method to implement these additional constraints. For example, a solution that placed the weights at angles close to 180 degrees proves unsafe. In order to eliminate such solutions, penalty functions increased the value of the objective function when constraint violations occur. As a result, the algorithm considered solutions with reasonable angles in the final answer.

This research used penalty functions to enforce proportionality constraints on the resulting mechanism. A proportionality constraint compared the relative size of each link in the mechanism, because disproportionate length links makes mechanism fabrication and testing difficult. For instance, one cannot easily fabricate a link length of 0.1 inches. A possible link constraint is as follows:

$$\frac{\mathbf{l}_1}{\mathbf{l}_5} \leq 10 \tag{2.14}$$

where l_1 and l_5 are the link lengths of the ground and fifth link of the mechanism. Such a constraint forced the link dimensions to have reasonable relative proportions. The designer decided on the value of the inequality, and used similar inequalities for each link length in the mechanism.

Penalty functions also checked to ensure the resulting linkage assembled in each of the desired positions. As mentioned earlier, the discriminant yields three possible linkage outcomes, and using penalty functions allowed the algorithm to check each possible outcome. A penalty function checked the sign of the discriminant to ensure the linkage assembled in each of the desired positions, and avoided incorrect linkages when searching for the optimal solution. Combining all the elements into one function yields the constrained objective function given by

$$\mathrm{OF} = f(X, P) = \sum_{i=1}^{n} w\left[F_i^{\,2} - F_{\mathrm{actual}\ i}^{\,2}\right] + v * \Omega_i + k * \Pi_i \tag{2.15}$$

where *w,v*, and *k* are scaling factors to penalize the solutions for violating constraints; Ω imposed link length constraints; Π checked to ensure the linkage assembles in all positions.

2.5 Force Analysis

Superposition and the matrix method gave the designer two common methods for analyzing the forces. In this case, the simple linkage lends itself to the superstition method. Using this method, the algorithm performed a separate analysis for each moving link considering inertial forces, external forces, and external torques acting on that link alone. For *n* moving links in a mechanism *n* separate analysis need to be done. The results of the analyses are summed together to determine the total forces acting on the mechanism [Mabie, 1987]. In this case, no inertial forces or external torques acted on the linkage, thus making the analysis straightforward. Figure 2-3 shows a free body diagram for one half of the mechanism. Two force members; links 3 and 4, experienced the forces applied by the user.

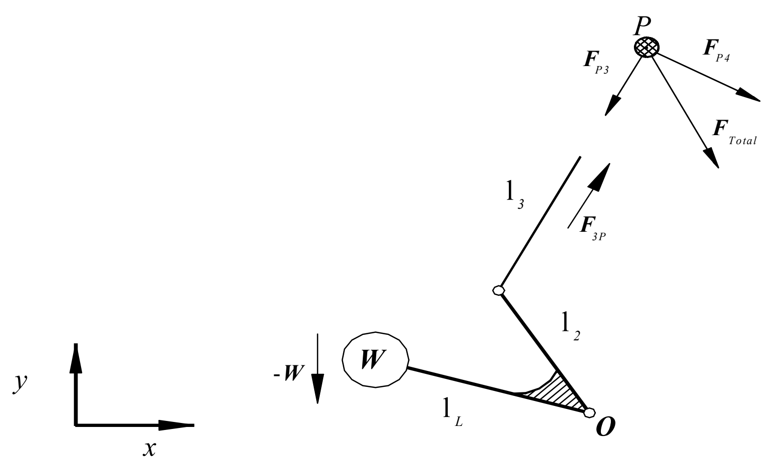

Figure 2-3. Free body diagram for a portion of the linkage. The moments are summed about the point O.

Summing the moments about the fixed pivot O, yields the following equation:

$$\sum M_O = l_L e^{j(\theta_2+\theta_4)} \times -jW_L + l_2 e^{j\theta_2} \times F_3 e^{j\theta_3} = 0 \qquad (2.16)$$

where l_L and θ_L describe the placement of the weight stack attached to the second link l_2; F_3 is the force in link 3; θ_3 is the orientation of the third link; *j* is the symbol to denote an imaginary number.

Expanding the cross product and simplifying yields:

$$\left(l_L c(\alpha)i + l_L s(\alpha)j\right)\times\left(-jW\right)+\left(l_2 c(\theta_2)i + l_2 s(\theta_2)j\right)\times\left(F_3 c(\theta_3)i + F_3 s(\theta_3)j\right)=0 \tag{2.17}$$

where $\alpha = \theta_L + \theta_2$

When similar terms are collected equation 2.17 is given by

$$-Wl_L\cos(\alpha)k + F_3 l_2\cos(\theta_2)\sin(\theta_3)k - F_3 l_2\sin(\theta_2)\cos(\theta_3)k = 0 \tag{2.18}$$

The expression for F_3 as given by

$$F_3 = \frac{Wl_L\cos(\alpha)}{l_2\left[\cos(\theta_2)\sin(\theta_3)-\sin(\theta_2)\cos(\theta_3)\right]} \tag{2.19}$$

Following a similar reasoning, the force applied in link 4 can be computed.

$$F_4 = \frac{Wl_r\cos(\omega)}{l_5\left[\cos(\theta_5)\sin(\theta_4)-\sin(\theta_5)\cos(\theta_4)\right]} \tag{2.20}$$

where l_r and θ_r describe the placement of the weight stack attached to the link l_5; F_4 is the force in link 4; θ_4 is the orientation of the link l_4; $\omega = \theta_r + \theta_5$.

Equations 2.19 and 2.20 give expressions for the magnitude of the forces in links 3 and 4, whereas kinematic position analysis gives the direction. The computed magnitude and direction of force at each position yields the total force acting at point P. For example, if F_3 were applied at some angle β and F_4 was applied at some angle α. The total force in each link is given by

$$F_3 = F_3\cos(\beta)i + F_3\sin(\beta)j \tag{2.21}$$

$$F_4 = F_4\cos(\alpha)i + F_4\sin(\alpha)j \tag{2.22}$$

where i represents the x-component of force and j represents the y-component of force. The magnitude of composite force is simply the summed squared of the force components given by

$$\left|F_{total}\right| = \sqrt{\left[\left(F_3 c\beta + F_4 c\alpha\right)^2 + \left(F_3 s\beta + F_4 s\alpha\right)^2\right]}. \tag{2.23}$$

where $c\beta$ is a substitution for $\cos(\beta)$; and $s\beta$ is a substitution for $\sin(\beta)$.

The direction of the force is found from the tangent, denoted by

$$\Theta = \tan \frac{F_3 s\beta + F_4 s\alpha}{F_3 c\beta + F_4 c\alpha} \tag{2.24}$$

The algorithm computes the magnitude and direction of the composite force, and compares the computed force to the user-targeted force. This term, the structural error, makes up the unconstrained objective function.

2.6 Strength Data Discussion

Strength curves plot the maximum force or torque exerted during a particular exercise as a function of position. Typically, the designer normalizes the strength curve with a maximum force of unity, in order to compare the curves for a variety of individuals [Scardina, 1996]. Although absolute strength of each person varies, the general shape of the normalized strength curve remains the same for the vast majority of users. Isokinetic machines, one application for the mechanisms designed in this research, intend for slow near-static movements [Powers, 1997]. A governing assumption of the thesis ignored dynamic effects of the linkage.

Figure 2-4 shows the actual range of motion for a leg extension exercise. The test subject exerted the maximum force at a series of stationary positions throughout the range of motion. A cable attached to a load cell applies the resisting toque and the load cell measured static force at a discrete number of positions in time. In order to compensate for fatigue from repeated maximal exertions of the same muscle group, the experimenter spaced the test in time and varied the test sequence.

Figure 2-4. Leg Extension Strength Machine.

The mechanism in Figure 2-4 is a single degree of freedom strength machine based on a wrapping cam mechanism. A cable wrapped around the cam created the nonlinear mechanical advantage for the leg extension exercise.

While performing knee extensions, cheating and improper technique introduce side forces. In order to minimize the effects of side forces the tester stabilizes the subject to prevent lifting of the buttocks or hip rotation. The subject initially sits on a bench with a 40 degree angle between the thigh and calf. As the knee flexes, the lower leg raises and the angle increases to 180 degrees. The data is usually taken at discrete positions throughout the range of motion and the plotted on a scatter plot. Figure in 2-5 is a best fit curve to the data.

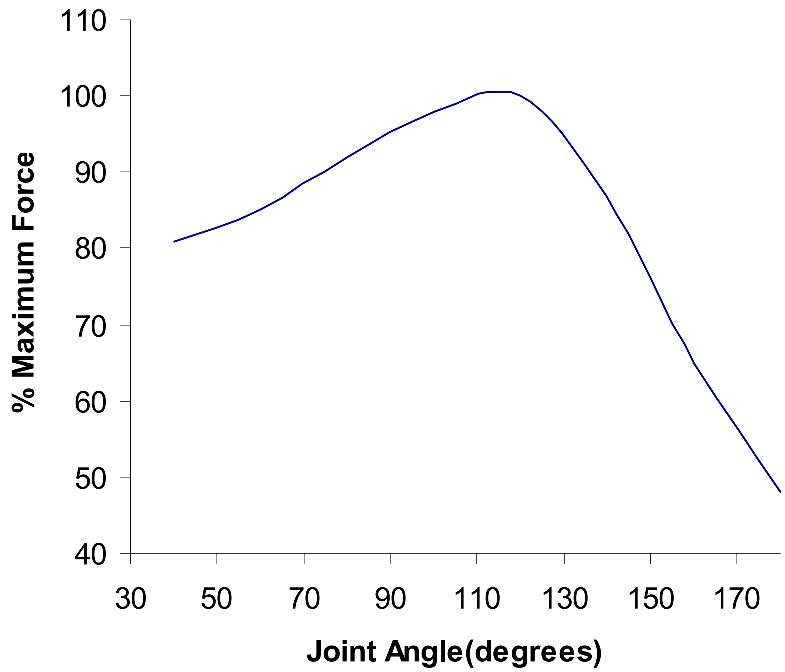

Figure 2-5. Strength curve for leg extension exercise [Powers, 1997].

The method of collecting data for human strength curves is not an exact science. The human body, a complex machine, can adjust to produce the required torque or power for any movement or exercise.

The angle convention for the testing procedure differs from the sign convention presented in this research. In the case of the leg extension, the angle is measured between the quadriceps and hamstring muscles. In this thesis, angles are measured in a positive right hand sense from the horizontal. In order to use the strength data in the model, the angles must be converted to the positive right hand notation. Table 2-2 shows the actual data from Figure 2-4, the corrected angle, and the normalized force. The corrected angle and the normalized force are used in the objective function.

Table 2-2
Strength Data for Leg Extension

Joint Angle	Right--Hand Convention	% Max Force
40	220	81
60	240	85
80	260	92
100	280	98
120	300	100
140	320	87
160	340	65
180	360	48

As mentioned earlier the user strength curve is normalized and in this case the % Max Force represents the normalized data. Using superposition, the algorithm computed the forces in the mechanism for a specified number of points along the range of motion. However, the algorithm required the correct joint orientation to compute the mechanism resistance forces. The designer had to make a similar table for each exercise, in order to synthesize a corresponding linkage.

CHAPTER 3: OPTIMIZATION-BASED SYNTHESIS

This section discusses techniques of non-linear optimization, and highlights techniques that apply to the mechanism synthesis problem. A brief explanation of constrained and unconstrained non-linear optimization techniques welcomes the reader. Then, the chapter describes the Hooke and Jeeves optimization technique used in this thesis.

3.1 Constrained Nonlinear Optimization

Within non-linear optimization two groups of techniques exists, constrained versus unconstrained techniques. Constrained optimization techniques use traditional calculus to solve problems; however, the complexity of the linkage synthesis problem renders this approach impractical. For this reason, iterative solution methods better solve the constrained nonlinear linkage synthesis problem.

Table 3-1 shows the two groups of iterative methods, the differences lie in the manner in which the algorithm handles constraints. Direct methods deal with the constraints explicitly. Indirect methods, often used because of their versatility, transform the constrained problem into an unconstrained problem. This transformation allows unconstrained nonlinear techniques to solve the constrained problem. The following section discusses unconstrained non-linear techniques.

Table 3-1
Constrained Nonlinear Optimization Techniques

Direct Methods	Indirect Methods
Heuristic search	Transformation of variables
Constraint approximation	Penalty functions
Feasible directions	

3.2 Unconstrained Nonlinear Optimization

The majority of numerical optimization algorithms solve unconstrained non-linear problems. Unconstrained problems constitute the most general case, and by using simple transformations, the same techniques yield solutions for constrained problems as well. Table 3-2 shows the two groups of unconstrained minimization methods, direct search methods and gradient methods. Descent methods often referred to as first order or gradient methods, require analytical or numerical derivatives of the objective function with respect to the design variables. On the other hand, direct search methods require only a search algorithm.

Table 3-2
Unconstrained Nonlinear Optimization Techniques

Direct Search Methods	Descent Methods
Random search	Steepest descent
Grid search	Conjugate gradient (Fletcher-Reeves)
Univariate search	Newton’s method
Pattern search(Hooke and Jeeve’s)	Variable metric
Simplex method	

Eason and Fenton [1974] made an extensive comparison of the numerical optimization techniques for engineering design. The study used different minimization techniques for a given objective function to find the best optimization technique. Direct search algorithms yielded the best results, whereas none of the gradient methods performed better than average. Use of the secant derivative approximation caused the poor showing by the gradient methods, because these approximations introduce errors in the solution. Direct search methods performed better than gradient methods, with pattern search and the polyhedral simplex algorithms producing the best results within that category.

The Eason and Fenton [1974] study produced a number of conclusions about choosing an optimization method. These are as follows:

1. Codes that require analytical or numerical derivatives should be avoided, if possible, in order to permit application to general design problems.
2. Pattern and simplex search methods are better than gradient methods using analytical approximations.
3. Built-in problem scaling increases the generality and efficiency of the optimization method.

3.3 Hooke and Jeeves Nonlinear Optimization

Hooke and Jeeves developed one of the most powerful and robust optimization algorithms. The algorithm uses a sequential stepping technique that consists of alternating exploratory and pattern moves. The exploratory search takes small steps around the initial starting point using the given step size. Then, the objective function evaluates surrounding points and the optimal value determines the direction of the pattern move. Larger pattern moves in the direction of steepest descent continue as the value of the objective function becomes smaller. Essentially, this pattern search method checks for a gradient around the starting point and moves in that direction and stops at a local minimum. Selection of an accurate starting vector helps determine the effectiveness of the Hooke and Jeeves algorithm. In this case, linkage dimension and angles compose the starting vector. However, no steadfast rule exists for choosing a starting vector. Figure 3.1 shows a complete flow chart of the Hooke and Jeeves optimization technique. Application of the Hooke and Jeeves procedure is as follows.

First, an initial starting point (design vector) drops the user into the design space for the mechanisms. As mentioned earlier in chapter 1, optimization maps a larger design space than closed form solution techniques. The starting vector has a length equal to the number of design variables, X^0. The user also sets the step size and the minimum step size for the algorithm, Δx and $\Delta x_{\min}$ for each design variable. Once the initial guess and the step sizes are determined, the exploratory search for the first design variable is made. The move is done about the starting point by the specified by Δx. The objective function is evaluated at the new point and compared to the previous value of the objective function. If the value improves, the new point is stored as the new base point. If not a negative step, $-\Delta x$, is made and the objective function is re-evaluated. Again, if the value of the objective function has improved the new point is stored as the new base point. If neither is successful, no move is made in the design space and the step size is decreased, and the exploratory search begins again. The exploratory procedure is performed for each of the design parameters to get the local behavior of the objective function.

The next step is to repeat the exploratory search in all the design variables using the most recent retained starting point. This is the pattern move; the new point is retained if the value of objective function is improved compared to the previous value. The pattern move is repeated until the value of the objective function does not improve. However, when the pattern move fails, the exploratory move begins again with the first design variable and the last stored point.

Finally, this process of making the combination of exploratory and pattern moves is repeated until all the step values fall below the step tolerance, $\Delta x_{\min}$. Once iteration has ended, a local minimum has been reached. There is no assurance that a convergence constitutes an acceptable solution. The optimization procedure locates local minima and cannot guarantee a global minimum. Performing the optimization with different initial design vector will produce a set of solutions that can be evaluated and compared to obtain the best possible solution.

This pattern search technique is very methodical, in that a properly structured objective function will yield a local minimum. One drawback is that Hooke and Jeeves cannot guarantee a global minimum of the solution. For each different set of design vectors chosen, Hooke and Jeeves will produce a different local minimum. One viable method of finding a global minimum is to perform the optimization algorithm and obtain several local minimums. This method uses the different local minima, as initial starting vectors for the optimization algorithm. Through repeated iterations a global minimum can be reached.

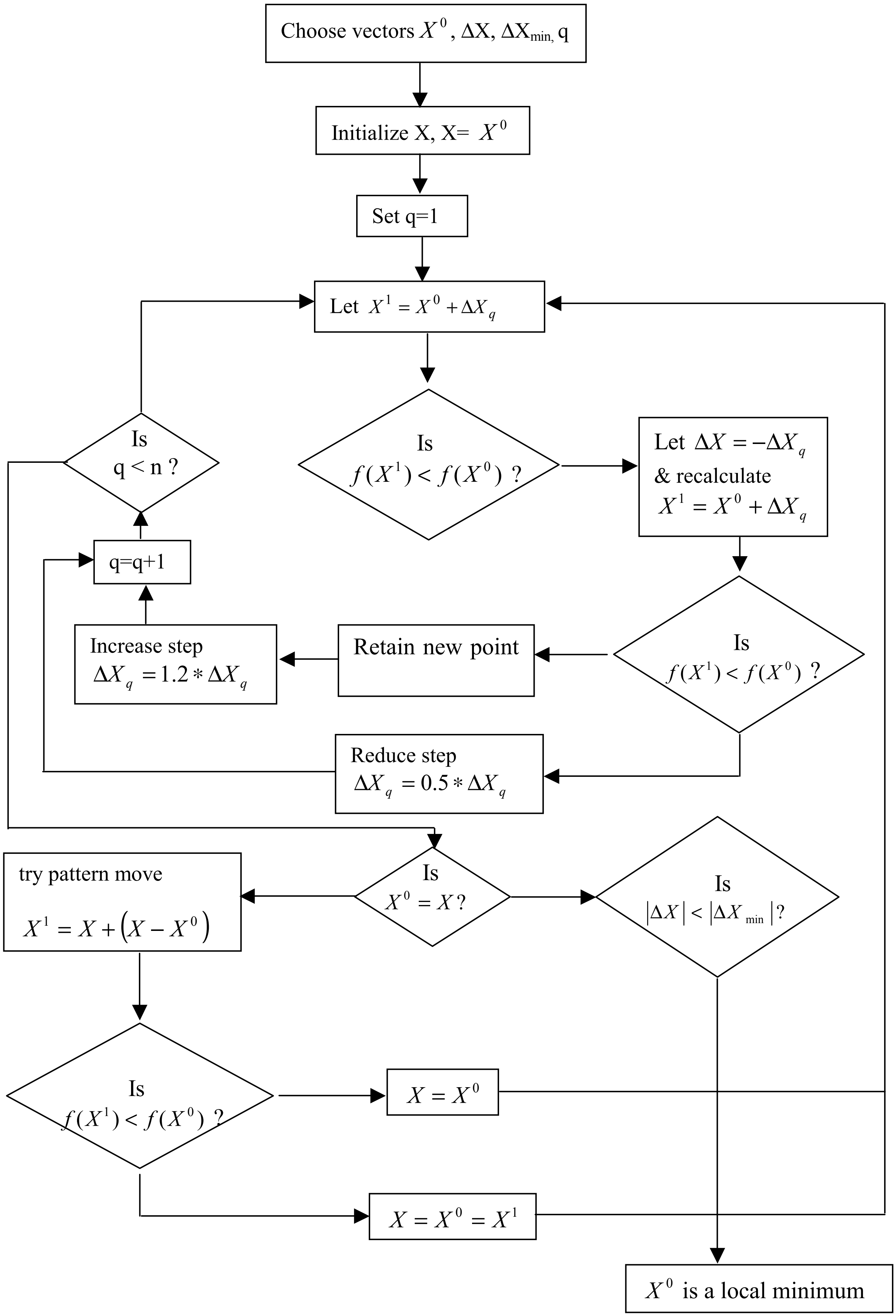

Figure 3-3: Hooke and Jeeves Pattern Search Technique [Reinholtz,1983].

CHAPTER 4: DESIGN PROCEDURE AND EXAMPLES

This chapter discusses the general procedure for designing a mechanism that produces a specified non-linear mechanical advantage. In addition, several examples show the capabilities and limitations of the algorithm.

4.1 Procedure

The procedure involved many parts and inputs. The designer first identified the problem, and then determined the known versus unknown information. Before putting any information into the algorithm, the designer determined a linkage configuration, developed a kinematic model, and specified the design variables.

After selecting the curve, the designer decided how many precision points the mechanism will attempt to match. Traditionally, four precision points have been the capacity for mechanism synthesis; however, this research presents results with three, four, and five precision points. Generally, the designer picked points that represent the general shape of the curve.

After obtaining relevant data from the strength curve, the designer decided upon the placement of **P** for each of the desired positions. The next step involved choosing an initial design vector, which is put into the algorithm. No rule exists for selecting an initial design vector. For instance, one would not use negative link lengths or small links in the initial guess. With practice, the designer gained intuition in selecting a starting vector. Section 4.3 gives a specific hierocracy for the algorithm.

At this point, the designer allowed the algorithm to complete the synthesis, the designer input the strength data, number of precision points, location of **P**, necessary tolerances and step sizes, and an initial design vector.

After completion of the previous steps, the algorithm performed the kinematic position analysis. As discussed in section 2.2, position analysis solved for the set of output angles for a given set of link lengths at each position. For this portion of the problem, all link lengths, ***P*,** and ***A*** are known from the initial design vector, $\boldsymbol{X^0}$. The designer gave all link dimensions in inches and all angles in degrees. The computer code then solved the position analysis at a discrete set of positions for **P**. The algorithm outputted the mechanism angles at each input position. Using the set of newly computed angles, the algorithm computed the linkage forces using the principle of superposition.

A vector of twelve numbers, the output vector, uniquely described the dimensions of the five-bar linkage. A separate analysis routine examined link lengths and other important factors, and drafting software such as AutoCAD allowed the designer to quickly analyze numerous linkage geometries. As mentioned earlier, the algorithm disregarded linkages due to disproportionate link lengths, or because of poor weight placement. The optimization algorithm supplied the designer with a reasonable set of kinematic linkage solutions; the designer needed to sift through the solutions and determine a safe and practical linkage for the end user.

4.2 Example

The biceps curl served as an excellent example to illustrate the optimization algorithm, because of the simple parabolic strength curve. Figure 4-1 shows the strength curve for the biceps curl exercise obtained from Clarke et al.

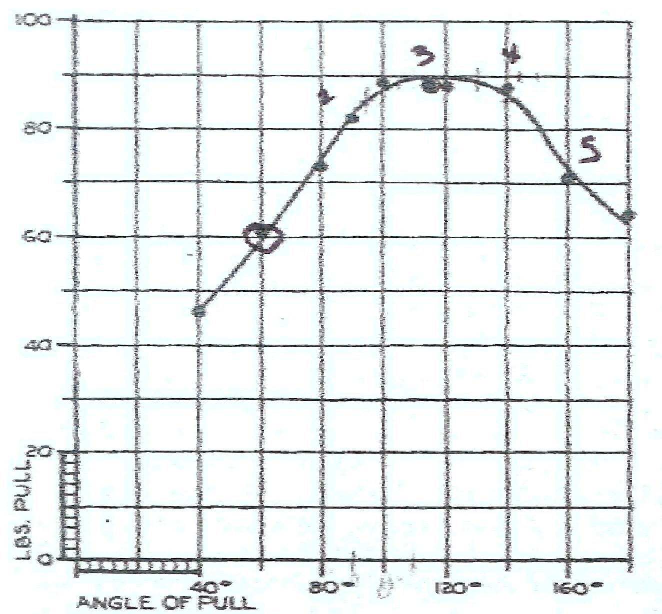

Figure 4-1. Biceps Curl Strength Curve [Clarke et al, 1950]. The user exerts maximum effort at a series of discrete location spaced in time.

The range of motion for the biceps curl exercise resembles an arc, with all user forces perpendicular to the weight used in the exercise. Figure 4-2 shows the range of motion for the biceps curl; the machine pictured represents one type of strength machine used today. The strength machine is based upon a single degree of freedom wrapping cam. A belt, attached to the cam, provides the nonlinear mechanical advantage for the exercise. When seated, using a machine similar to that picture in Figure 4-2, the maximum occurs when the forearm is perpendicular to the padded armrest.

Figure 4-2. Biceps Curl strength machine. The machine is based on a wrapping cam.

Table 4-1 shows the data points from the user strength curve in Figure 4-1. The algorithm used the normalized magnitude and the angle from the curve to ultimately compute the value of the objective function for a given linkage configuration. As mentioned in chapter 2, the algorithm also used a different angle convention and required normalized strength data. Here percent maximum represented the normalized data. In this thesis, the designer measured angles in a positive counterclockwise sense, whereas Clarke et al, measured the angles from a vertical line clockwise to the link.

Table 4-1
Biceps Curl Strength Data

Angle of Pull, ϕ	**Percent Maximum, S(%)**
40	51.11
60	66.67
80	0.80
90	91.11
100	97.78
110	100.00
120	100.00
140	96.67
160	78.89
180	71.11

For the biceps curl, the forearm length determined the magnitude of **P**, and the direction acted perpendicular to the applied force. The examples used three, four, and five positions to synthesize the resulting linkage. By setting the tolerance to 0.01 and the step size to 0.1, the computing time of algorithm was decreased. With all the necessary information available, the designer performed numerous iterations to find the optimal mechanism.

By using a proportionality constraint, the constrained objective function limited the link lengths to 50 inches, and if any link length exceeded 50 inches, in any iteration, the function got penalized 100 times the excess value. In addition, the objective function computed the structural error and penalized the objective function 1000 units for any difference in force. Moreover, the objective function checked the discriminant to ensure that the linkage assembles in each of the desired positions.

4.3 Optimization Algorithm

Three different subroutines comprise the optimization algorithm. One subroutine, used for the actual Hooke and Jeeves pattern search algorithm, also contains the tolerance and step size used throughout the program. A second subroutine computed the actual objective function. The main program, the third subroutine, allowed the user to input an initial vector and the required information from the user strength curve. These three programs worked in harmony to produce a set of final values that uniquely described the linkage and the angles of each of the links at each position. The user, prompted by the program, entered the initial design vector of twelve variables. The cost function subroutine used these values and performed all the kinematic and static analysis, checks for linkage closures, and returns a value. Hooke and Jeeves algorithm used this value to determine the optimal direction to step and determine the step size for the pattern move. The process continued until successive values of the objective function fall within required tolerance or when the algorithm reached the maximum number of iterations. Appendix A and B give the specifics of the optimization algorithm.

4.4 Mechanical Design

After completion of the optimization algorithm, the designer performed a subjective mechanical design analysis. Since the designer does not fabricate the mechanism, this may prove difficult. Thus, the designer looked for key aspects that govern the behavior of the mechanism. These key aspects can give an accurate account of the necessary time and energy to actually build the mechanism and later build the strength machine. The designer focused on three areas when analyzing a solution, center of gravity location, distance between fixed pivots, and pinch points.

The first area concerns the location of the mechanism's center of gravity, ideally center of gravity remains close to the ground. Since the links are assumed massless, the vertical distance between the weights and the ground link determines the location of the center of gravity. A large vertical distance between the weights and l_1 could cause a relatively high center of gravity, and could provide obstacles in designing a user friendly, safe strength machine.

Another key point addresses the distance between the two fixed pivots. This distance direct relates to the frame of the resulting strength machine, thus a large frame translates into a large machine. This may prove advantageous in some instances, but this research aims to find mechanisms with a short distance between fixed pivots.

The final idea addresses pinch points, because pinch points often lead to end user injuries. By definition, pinch points are points where links in the mechanism cross, and during motion these points to become very dangerous. For this reason, the designer wishes to find mechanism with few or no pinch points.

4.5 Results

Three precision point synthesis. Figure 4-2 shows one possible mechanism solution for a biceps curl exercise using three input positions. Table 4-2 shows the link dimensions, Figure 4-3 shows the mechanism resistance curve and user strength curve. An initial guess of 2.5 inches for all link lengths, 2.5 degrees for all angles, and a 50 pound weight produced the resistance curve shown in Figure 4-3. Two pinch points exists where l_3 and l_5 cross, and where l_2 and l_r, cross. A large vertical distance between the ground pivots and weights contributes to a relatively high mechanism center of gravity, creating a potentially unstable mechanism. The two dangerous pinch points represent the only drawbacks of the mechanism.

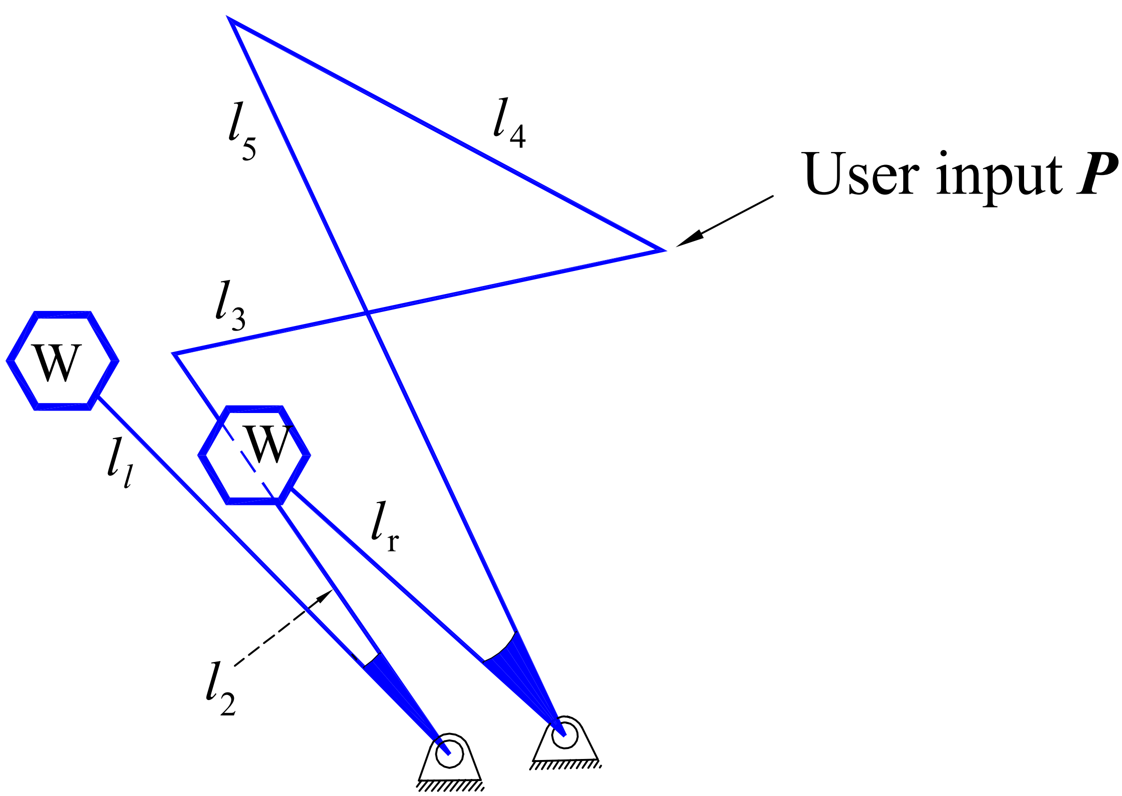

Figure 4-3. Biceps curl mechanism. The mechanism is drawn at the third position out of a possible three positions

Table 4-2 shows the actual linkage dimensions for the mechanism in Figure 4-2. The link length penalty function causes proportionate link lengths; however, the weight arms experienced no such penalty function. None of the links in Figure 4-2 came close to maximum link length of 50 inches.

Table 4-2
Bicep Curl Mechanism Design Values (3 positions)

Parameter	Description	Optimal value
l_1	Link between the two fixed pivots	2.3”
l_2	Link between first fixed pivot and first moving pivot	9.5"
l_3	Link between first and second moving pivot	9.8"
l_4	Link between second and third moving pivot	9.6"
l_5	Link between third moving pivot and second fixed pivot	10.3"
l_L	Link describing weight arm attached to link 2	10.5"
θ_L	Angle between weight arm and link 2	10°
l_r	Link describing weight arm attached to link 5	7.9"
θ_r	Angle between weight arm and link 5	13.4°
A	Distance describing first moving pivot	9.0"
θ_A	Orientation of moving pivot	10.7°
θ_1	Orientation of the ground link	11.5°

Figure 4-3 shows the user strength curve, mechanism resistance curve, and the three accurately matched positions. Moreover, the figure shows the effectiveness of the algorithm when using three positions. The optimization algorithm used three representative values from the user strength curve, because the points span a range of the user strength curve. Three positions produced a sound linkage that matched each of the three points effectively.

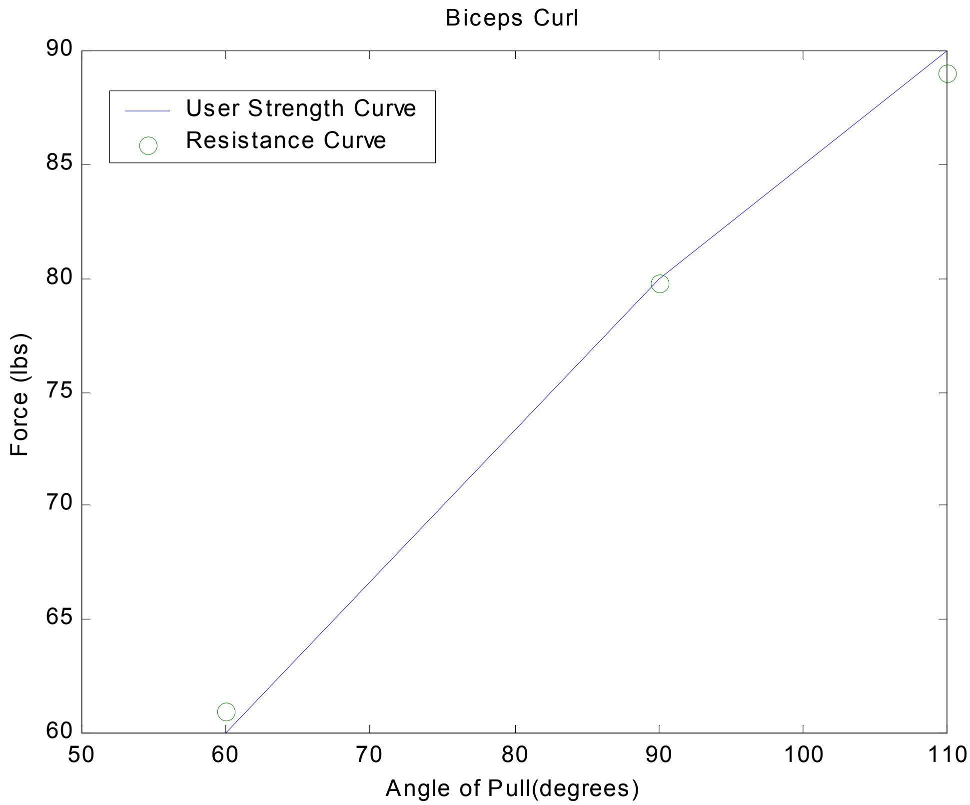

Figure 4-4. Bicep curl mechanism resistance data and user strength data

Four precision point synthesis. An initial configuration with 10 inch link lengths and 25 degree angles produced the mechanism shown in Figure 4-4. Again, due to the proportionality constraints, the algorithm produced links with similar dimensions.

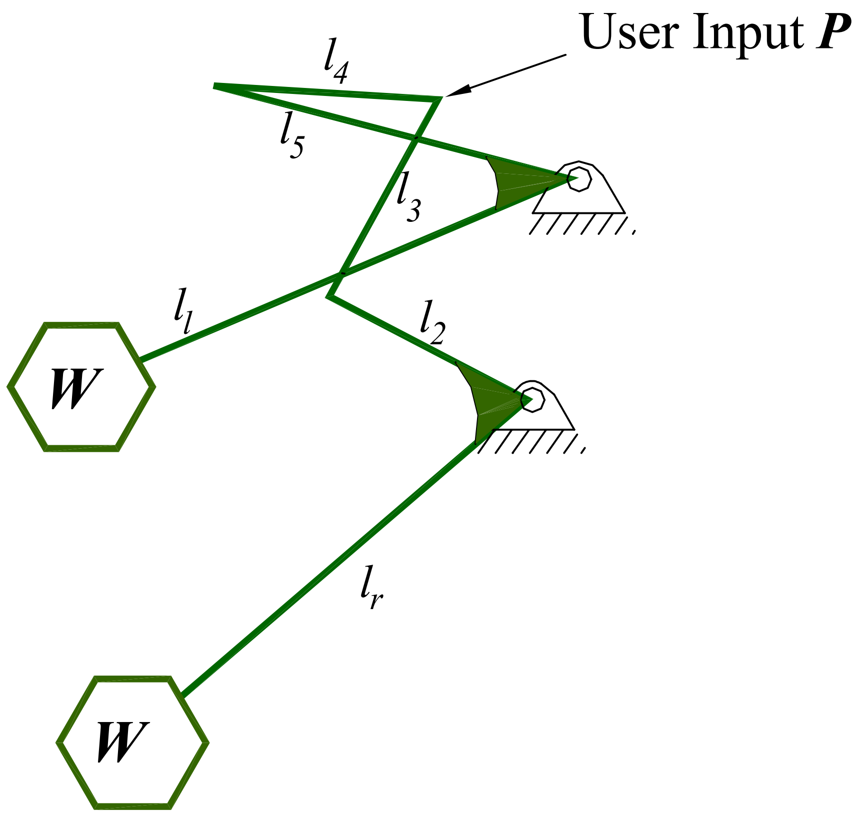

Figure 4-5. Biceps curl mechanism. This is drawn in the first position out of four possible positions.

Unlike, the mechanism produced with three positions, several linkage dimensions reach the 50 inch limit. However, the extremely long weight arms, twice the size of the next longest link length, make the mechanism unattractive. Moreover, the mechanism's center of gravity sits high due to the position of the weights in relation to the ground link. In addition, a large distance between the fixed pivots gives the mechanism a large frame. Three links cross one another, representing potentially dangerous pinch points. In addition, the links of over 4 feet in length and the weight arms over 9 feet in length, lead to a big, cumbersome strength machine. Using a weight of 40 pounds produced the results shown in Table 4-3.

Table 4-3
Bicep Curl Mechanism Design Values (4 positions)

Parameter	Description	Optimal value
l_1	Link between the two fixed pivots	50"
l_2	Link between first fixed pivot and first moving pivot	49"
l_3	Link between first and second moving pivot	50"
l_4	Link between second and third moving pivot	50"
l_5	Link between third moving pivot and second fixed pivot	35.7"
l_L	Link describing weight arm attached to link 2	115.4"
θ_L	Angle between weight arm and link 2	68°
l_r	Link describing weight arm attached to link 5	118"
θ_r	Angle between weight arm and link 5	50.3°
A	Distance describing first moving pivot	78.9"
θ_A	Orientation of moving pivot	292.9°
θ_1	Orientation of the ground link	86°

Figure 4-5 shows the produced resistance at the desired locations; however, the algorithm only matched one position within reason. The circles represent the mechanism resistance data; the data does not follow the general shape of the user strength curve.

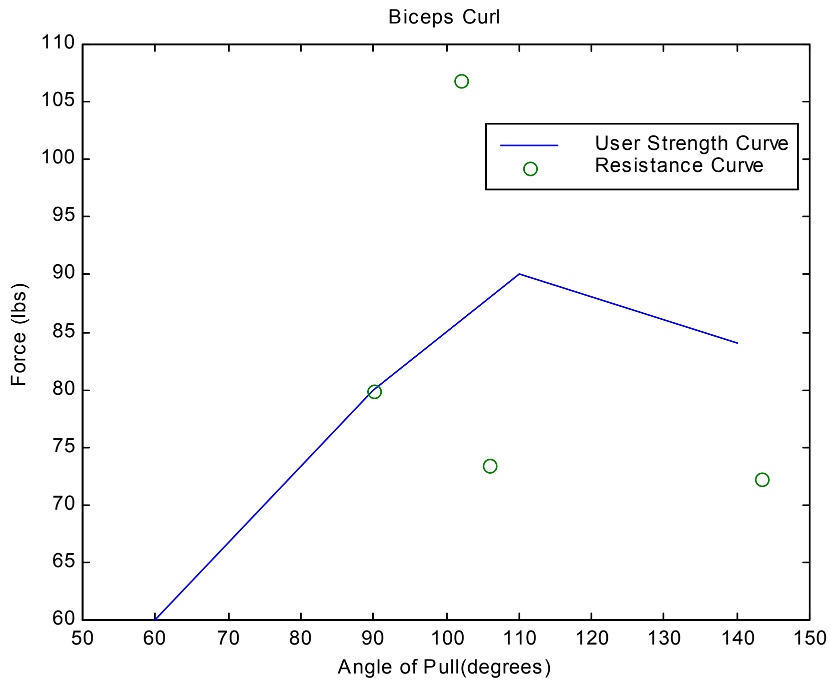

Figure 4-6. Bicep curl resistance data and strength data

Five Position Synthesis. The resulting linkage in Figure 4-6 used five positions; however, the algorithm matched only three precision points. A single pinch point, where l_r and l_2 cross, and a large frame set this linkage apart from the previous two. Moreover, a low mechanism center of gravity appeals to the designer and end user. Due to the appealing attributes of the physical linkage, it proves the best of the three. However, the resistance data and user strength curve do not match very accurately.

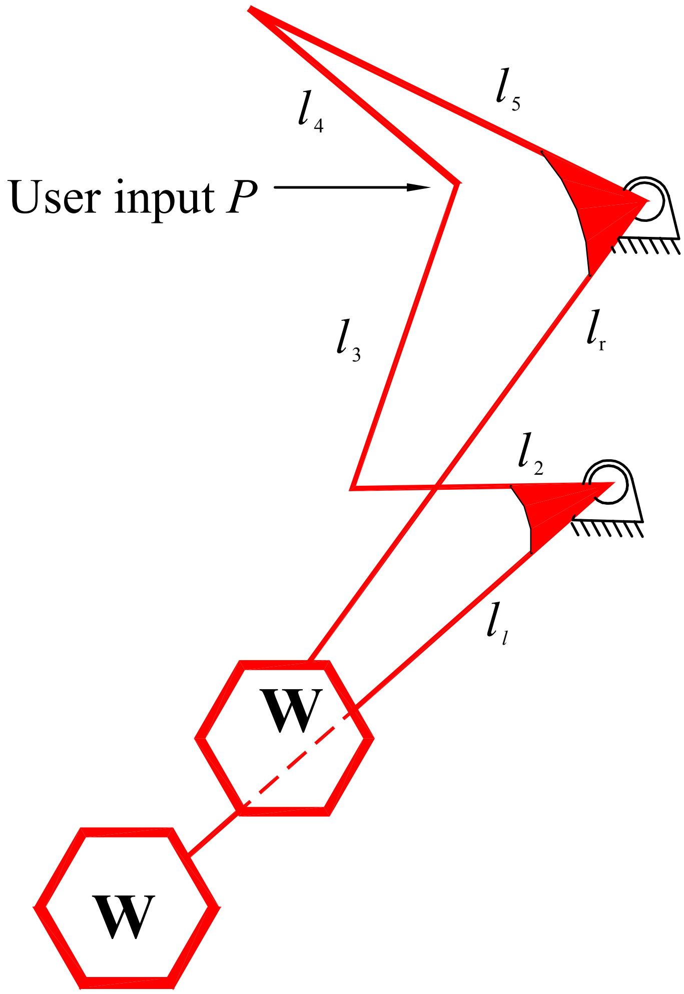

Figure 4-7. Biceps curl mechanism. This is drawn at the third position out of a possible five positions.

Table 4-4 shows similar to the data in Table 4-3; however, with a completely different linkage configuration, the major difference stemming from the smaller weight angles. The initial guess assigned a value of 10 inches to all link lenths and 45 degrees to all angles and 40 pounds for the weight. As with the linkage in Figure 4-5, the mechanism possess large but proportionate link lengths.

Table 4-4
Bicep Curl Mechanism Design Values (5 positions)

Parameter	Description	Optimal value
l_1	Link between the two fixed pivots	57.5"
l_2	Link between first fixed pivot and first moving pivot	51.7"
l_3	Link between first and second moving pivot	65.1"
l_4	Link between second and third moving pivot	54.7"
l_5	Link between third moving pivot and second fixed pivot	40.9"
l_L	Link describing weight arm attached to link 2	121.2"
θ_L	Angle between weight arm and link 2	40.6°
l_r	Link describing weight arm attached to link 5	124"
θ_r	Angle between weight arm and link 5	53.1°
A	Distance describing first moving pivot	82.9"
θ_A	Orientation of moving pivot	295.4°
θ_1	Orientation of the ground link	86°

Figure 4-7 shows the user strength curve and the mechanism resistance curve. The algorithm matched three random points accurately, while the other two points did not match well. The three positions matched seemed to be purely random as with the four precision point example. The following chapter discusses reasons why the mechanism resistance data and user strength curve do not accurately match.

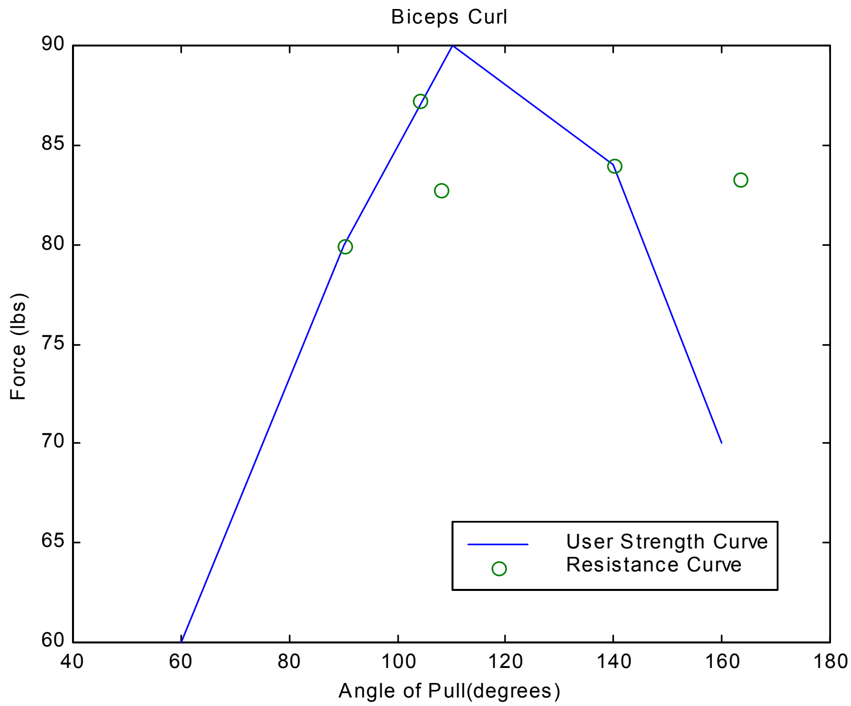

Figure 4-8. Bicep curl resistance data and strength data (5 precision points).

The three example mechanisms presented give the reader a feel for the capabilities and limitation of the chosen optimization algorithm. Optimization produced kinematically sound linkages; the designer had to wade through the linkages a find a suitable mechanism. In order to find a suitable mechanism, the designer examined the mechanical design aspects of each mechanism.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This research developed and demonstrated the optimization theory and tools needed to synthesize a planar multi-degree of freedom mechanism that produces a given non-linear mechanical advantage. Past synthesis techniques, matched only the forces produced by function generating mechanisms, whereas this research used both position and force objectives to synthesize the mechanism.

The optimization algorithm produced the best results when using three positions; however, from a mechanical design perspective the linkage proved unattractive. Neither four nor five position synthesis produced good solutions or accurate matching of force data. There exist several reasons for the poor linkages produced using optimal synthesis. For one, proportionality link length constraints and inequalities limited the solution space of the optimization algorithm, thus reducing the number of possible solutions available to the algorithm. Often, solutions with disproportionate link lengths are found, and these solutions are not desirable for the reasons given in Section 2.4. The algorithm also assigned the same value to each load weight, thus restricting the design space and subtracting two design variables from the problem. As mentioned earlier, this guaranteed user-friendly mechanisms in the end.

Since the algorithm only matched three positions, it is safe to say that the algorithm needed more design variables. Twelve design variables did not give the algorithm enough freedom to match four or five positions consistently, as evidenced by the linkage dimensions from the resulting linkages. Three position synthesis yielded relatively small dimensions compared to the mechanisms for four and five positions. The linkage dimensions for those mechanisms fell victim to the link ratio constraint of 50 inches. The algorithm produced mechanisms with potentially dangerous pinch points, and large cumbersome dimensions. Despite the relatively poor linkages, the research goals were met. The thesis developed a tool and demonstrated the effectiveness of the tool to produce a five bar mechanical advantage producing mechanism.

Previously emphasized, the mechanism produced governs the movement of the resistance machine, but another arduous process actually designs the strength machine. This mechanism serves as the beginning step in the process of fabricating a new type of resistance training machine that combines the best attributes of strength machines and free weights.

5.2 Recommendations for Future Advancement

As mentioned earlier, this research showed promising results and offered a glimpse of the potential for designing force-generating multiple degree of freedom linkages. Using more than one optimization technique to synthesize such a linkage could lead to further advancements in this area. One may want to use a hybrid optimization technique that combines the pros of both constrained and unconstrained non-linear optimization techniques. Each optimization technique offers unique advantages, but finding the one technique that offers the optimal performance given the constraints of this type of problem may prove difficult.

As mentioned earlier, the research presented needed more design variable to match four or even five positions. Therefore, one needs to examine ways to introduce additional parameters, and allow the optimization algorithm to map out a larger design space. Allowing the optimization algorithm to independently change the value of both weights is one way to introduce two additional parameters.

The force analysis in this research ignored dynamic effects, due to the intended use for the mechanism; however, a dynamic analysis proves advantageous. As part of the model, future researchers may want to include a dynamic analysis. A dynamic model would introduce added constraints and possibly increasing the efficiency of the constrained objective function.

In addition, the multi degree of freedom mechanism may lend itself to spatial mechanisms better than their planar counterparts. Spatial mechanism afford the designed more parameters to change, and possibly match more positions.

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APPENDIX A

MATLAB OBJECTIVE FUNCTION CODE

```
%  Ricardo Blackett -- Master's Thesis
%  Objective funtion for Hooke and Jeeves
%       The value of the objective function is passed to Hooke and Jeeves subroutine


function[value]=Cost(param)
global F_target F_dir_tar check theta_2 theta_3 F_dir P value
      F_mag theta_4 theta_5 F_i F_j

%  'Param' is input from the main program
%  Define the position of the fixed pivots
   A = param(10);
   theta_a = param(11)*pi/180;
   l_1 = param(1);
   theta_1 = param(12)*pi/180;

%  Define the link lengths of the five bar in inches
   l_2 = param(2);
   l_3 = param(3);
   l_4 = param(4);
   l_5 = param(5);

%  Define the length of the weight arms and amount of weight.
%  Weight arms are defined w.r.t link in a positive ccw direction
   l_l = param(6);
   theta_l = param(8)*pi/180;
   w_l = 40;
   l_r = param(7);
   theta_r = param(9)*pi/180;
   w_r = 40;

%  Number of precision points to be matched
   n=5;

%  P and theta_p are hard coded into the objective function
%  Defines the position vectors and angles
   P = [15 15 15 15 15];

%  Direction of each position
   theta_p = [30 0 -20]*pi/180 -50 -70]*pi/180;

%  Creates aliases for kinematic calcuations
   c_a = cos(theta_a);
   s_a = sin(theta_a);
   c_1 = cos(theta_1);
   s_1 = sin(theta_1);
   c_p = cos(theta_p);
   s_p = sin(theta_p);
   c_l = cos(theta_l);
   s_l = sin(theta_l);
   c_r = cos(theta_r);
   s_r = sin(theta_r);

%  Defines the DESIRED FORCE vectors obtained from Strength curve
%  Defines the magnitude and normalizes
   F_target = [60 80  90 84 70]./90;
```

```
%  Defines the direction
   F_dir_tar = [-60 -90 -110 -140 -160]*pi/180 ;
 % quadratic formula to solve for theta_2
   D = P.^2+l_2^2+A^2-2*A*P.*(c_p*c_a+s_p*s_a)-l_3^2+2*c_p.*P.*l_2-
2*l_2*A*c_a;
   E = -4*l_2.*P.*s_p+4*l_2*A*s_a;
   F = P.^2+l_2^2+A^2-2*A.*P.*(c_p*c_a+s_p*s_a)-l_3^2-
2*c_p.*P.*l_2+2*l_2*A*c_a;
 % Check to see if discriminant is negative
   for i=1:n
      check(i)= (E(i)^2-4*D(i)*F(i));
      if check(i) < 0
         of_a(i) = 1000*check(i)^2;
      else
         of_a(i)=0;
      end
   end

 % Finds theta_2 using Atan2 command to remedy quandrant problem
   num = (-E+sqrt(E.^2-4.*D.*F));
   den = 2.*D;
   theta_2 = 2*atan2(num,den);
   c_2 = cos(theta_2);
   s_2 = sin(theta_2);
   theta_3 = acos((P.*c_p-A*c_a-l_2*c_2)/l_3);
   c_3 = cos(theta_3);
   s_3 = sin(theta_3);

%  Solves second loop to find other two angles
%  Creates alias for a component of the likage synthesis
           Z = P.^2+A^2+l_5^2+l_1^2-2*l_1.*P.*(c_1*c_p+s_1*s_p)-

2*A.*P.*(c_a*c_p+s_a*s_p)+2*l_1*A*(c_a*c_1+s_1*s_a);

   G = Z-l_4^2-2*A*l_5*c_a-2*l_1*l_5*c_1+2*l_5.*P.*c_p;
   H = 4*l_1*l_5*s_1-4*s_p*l_5.*P+4*s_a*A*l_5;
   I = Z-l_4^2-2*l_5.*P.*c_p+2*l_1*l_5*c_1+2*A*l_5*c_a;

% Check to see if discriminant is zero
   n = 3;
   for i = 1:n
   check_2(i)= (G(i)^2-4*H(i)*I(i));
      if check_2(i) < 0
         of_b(i) = 1000*check_2(i)^2;
      else
         of_b(i) = 0;
      end
   end

%  Quadratic Formula for second loop
   num_1 = (-H-sqrt(H.^2-4.*G.*I));
   den_1 = 2.*G;
   theta_5 = 2*atan2(num_1,den_1);
   c_5 = cos(theta_5);
   s_5 = sin(theta_5);
   theta_4 = acos( (P.*c_p-l_5*c_5-l_1*c_1-A*c_a)/l_4);
```

```
   c_4 = cos(theta_4);
   s_4 = sin(theta_4);
   c_l_a =  cos(theta_l+theta_2);
   c_r_a = cos(theta_r+theta_5);
%  Solve for the forces in the linkage
   F_3 = -((w_l*l_l*c_l_a)./(l_2*(c_2.*s_3-s_2.*c_3)));
   F_4 = -((w_r*l_r*c_r_a)./(l_5*(c_5.*s_4-s_5.*c_4)));

%  Computes the total force at each point
   F_i = (F_3).*c_3+(F_4).*c_4;
   F_j = (F_3).*s_3+(F_4).*s_4;

%  Magnitude and direction of optimum force
   F_mag = sqrt(F_i.^2+F_j.^2);
   F_dir = atan2(F_j,F_i);

%  Structural Error between force
   error_x = (F_mag.*cos(F_dir)-F_target.*cos(F_dir_tar));
   error_y = (F_mag.*sin(F_dir)-F_target.*sin(F_dir_tar));
   error= (sqrt(error_x.^2+error_y.^2));

 % Computes difference in direction between desired and actual
forces
   dir_error=(F_dir-F_dir_tar).^2;
   for i=1:n
         if dir_error(i)>0.02
         of_d(i)=100*dir_error(i);
         else
         of_d(i)=0;
         end
   end

% Computes the difference in magnitude between desired and actual
forces
   mag_error=(F_mag-F_target).^2;
   for i=1:n
      if mag_error(i)>10
         of_e(i)=10*mag_error(i);
      else
         of_e(i)=0;
      end
   end

%  Penalty functions
%  Negative Link Length Constraints
   if l_1<0
      of_1=1000*abs(l_1)^2;
   elseif l_1>60
      of_1=1000*l_1^3;
   else
      of_1=0;
   end

   if l_2<0
      of_2=100*abs(l_2)^2;
   elseif l_2>60
      of_2=100*l_2^3;
```

```
   else
      of_2=0;
   end
   if l_3<0
      of_3=100*abs(l_3)^2;
   elseif l_3>60
      of_3=100*l_3^3;
   else
      of_3=0;
   end

   if l_4<0
      of_4=100*abs(l_4)^2;
   elseif l_4>60
      of_4=100*l_4^3;
   else
      of_4=0;
   end
   if l_5<0
      of_5=100*abs(l_5)^2;
   elseif l_5>60
      of_5=100*l_5^3;
   else
     of_5=0;
   end

%  Computes the cost function using a composite of the smaller cost
functions
```

```
value=sum(error)+sum(of_a)+sum(of_b)+100*sum(of_1)+sum(of_2)+sum(of_3)+su
          m(of_4)+sum(of_5);
```

Appendix B

MATLAB HOOKE AND JEEVES CODE

```
%  Ricardo Blackett -- Master's Thesis
%  Optimization of a five-link force generating mechanism
%  Hook and Jeeves optimization code
%  Defines Hooke and Jeeves subrountine that calls the objective
function

%  'paramer' is the initial design vector is passed to the
subroutine by the main program

%  'value' is the output of the function
   function[value]=feedme(paramer);

%  Declare global variables
   global F_target check theta_2 theta_3 F_dir value F_mag theta_4
      theta_5 P F_i F_j F_dir_tar

%  Clears the counter for the loop
   clear count

%  Number of parameters in optimization loop i.e. the size of
'param'
   close all;
   q=12;

%  Print every incr'th value
   incr=100;

%  Maximum number of iterations
   countmax=50000;

%  Sets the tolerances for Hooke and Jeeves
   cost_diff = 0.01;
   StepCheck =0.001*ones(1,q) ;

%  Intializes the size of design vectors depending on the value of
'q'
%  'Param' and 'Newparam' is used in Hooke and Jeeves
   param = zeros(1,q);
   newparam = zeros(1,q);

%  Opens file for data and overwrites
   fid = fopen('optimization_file.txt','a+');

%  Assigns the values to initial design vector to begin optimization
loop
   param = paramer;

%  Sets the step size for optimization
   StepSize = 0.01*ones(1,q);
   OStepSize = StepSize;

%  Initializes to take inside the optimization loop
   [CurrentCost] = Cost(param);
   OldCost = 1000000;
```

```
%  Intialize the counter
   count=0;

%  Start Optimization Loop
%  Compares the two cost functions, the step sizes and the counter
%  If all conditions are satisfied then the loop will run
while (abs(OldCost-CurrentCost) > cost_diff |
sum(StepCheck<StepSize)>0) & count< countmax

%  Assigns new parameters to begin the optimization
   newparam = param;
   OldCost = CurrentCost;

%  Starts counter to compare with the max number of iteration
   count = count+1;

%  Exploratory Search
   Direction = ones(1,q);

% Check (n-1) directions
   for j = 1:q
      newparam(j) = param(j)+StepSize(j)*Direction(j);
      [newcost] = Cost(newparam);
      if newcost > CurrentCost
      Direction(j) = -Direction(j);
         newparam(j) = param(j)+StepSize(j)*Direction(j);
         [newcost] = Cost(newparam);
      if newcost > CurrentCost
         Direction(j) = 0;
         StepSize(j) = StepSize(j)/2;
      end
      end
   end

% Prints the incr'th value of the cost function to the screen
  if count/incr == fix(count/incr)
     newcost
  end

%  End of the Exploratory Search
%  After the exploratory serch is done and stored a move is made and
saved
   move = StepSize;

%  Start the Pattern Move
   newparam = param + StepSize.*Direction;
   [newcost] = Cost(newparam);
   while newcost < CurrentCost
      param = param + move.*Direction;
      CurrentCost = newcost;
      move = move*1.25;
      newparam = param + move.*Direction;
      [newcost] = Cost(newparam);
   end
end

%  Print optimized variables in data file
```

```
   fprintf(fid,'%3.1f %3.1f %3.1f %3.1f %3.1f %3.1f %3.1f %3.1f
                %3.1f %3.1f %3.1f %3.1f\n\n' , param);
%  Print output angles depending on number of precision points used
   fprintf(fid,'%3.1f %3.1f %3.1f  %3.1f %3.1f\n',theta_2.*180/pi);
   fprintf(fid,'%3.1f %3.1f %3.1f  %3.1f %3.1f\n',theta_3.*180/pi);
   fprintf(fid,'%3.1f %3.1f %3.1f  %3.1f %3.1f\n',theta_4.*180/pi);
   fprintf(fid,'%3.1f %3.1f %3.1f  %3.1f
%3.1f\n\n\n',theta_5.*180/pi);

%  Close the file
   fclose(fid);

% Plots force curve and resistance data for comparing
   figure(1)

% Negative sign added for plotting purposes to match the strength
curve from the data

  plot((-F_dir_tar)*180/pi,F_target,'-',(-
    F_dir_tar)*180/pi,F_mag,'o');
   xlabel('Angle of Pull(degrees)');
   ylabel('Force (lbs)');
   title('Biceps Curl');
   legend('User Strength Curve','Resistance Curve');
```

VITA

Barbara Thomas and Orville Blackett welcomed Ricardo Corey Blackett into the world on March 8, 1977 in Baltimore, Maryland. In the spring of 1995, he graduated from Baltimore Polytechnic Institute with an A-course Diploma. In the summer of 1995, he set foot on the University of Delaware campus, as a Resource to Insure Successful Engineers (RISE) mechanical engineering student. He spent his summers working for Boeing Defense and Space and Frito-Lay, Inc. as an intern. During the winter of 1999, he had the opportunity to study abroad in Pretoria, South Africa. In the spring 1999, he received his Bachelor of Mechanical Engineering from the University of Delaware. After four grueling years of engineering, he wanted more, so in the Fall of 1999, he used his National Consortium for Graduate Degrees for Minorities in Engineering and Science, Inc (GEM) fellowship to pursue a masters degree at Virginia Polytechnic Institute and State University in scenic Blacksburg, Virginia. During the summer following his first year graduate studies, he interned at Air Products and Chemicals, Inc. In the spring of 2001, he will receive his Master of Science in Mechanical Engineering from Virginia Polytechnic Institute and State University.