Statistical Inference Course Project 1

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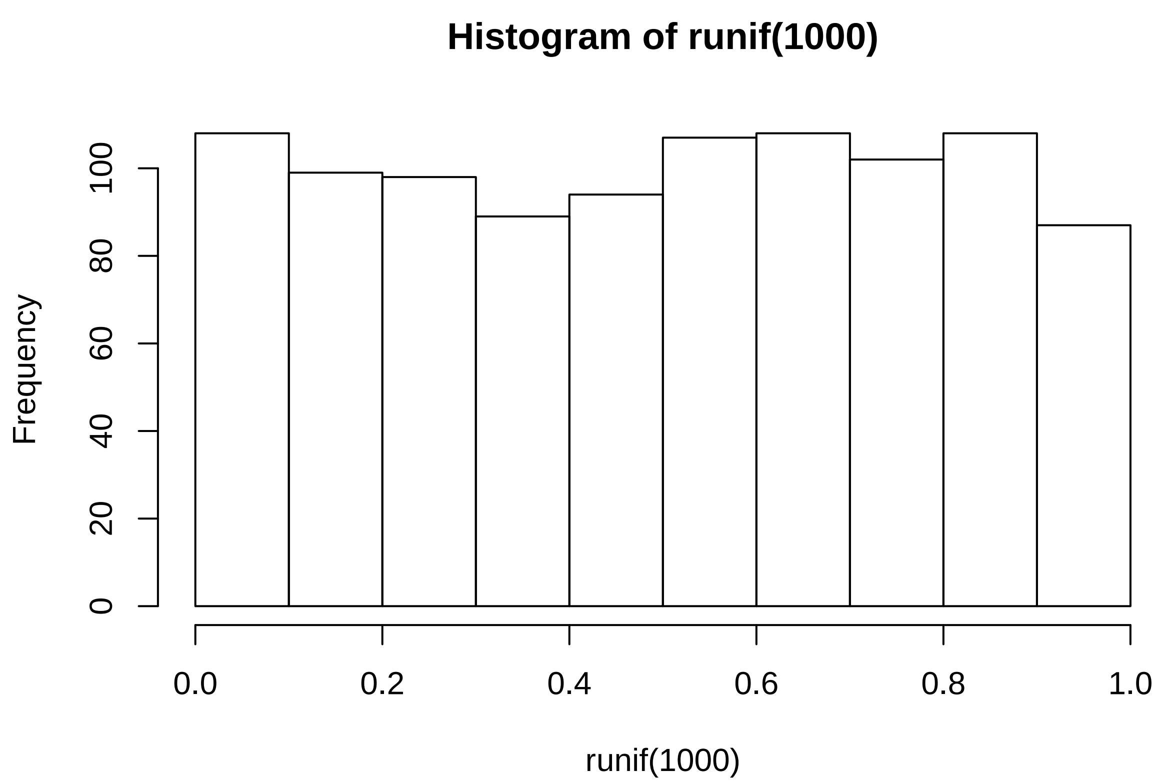
In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should 1. Show the sample mean and compare it to the theoretical mean of the distribution. 2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution. 3. Show that the distribution is approximately normal.

In point 3, focus on the difference between the distribution of a large collection of random exponentials and the distribution of a large collection of averages of 40 exponentials.

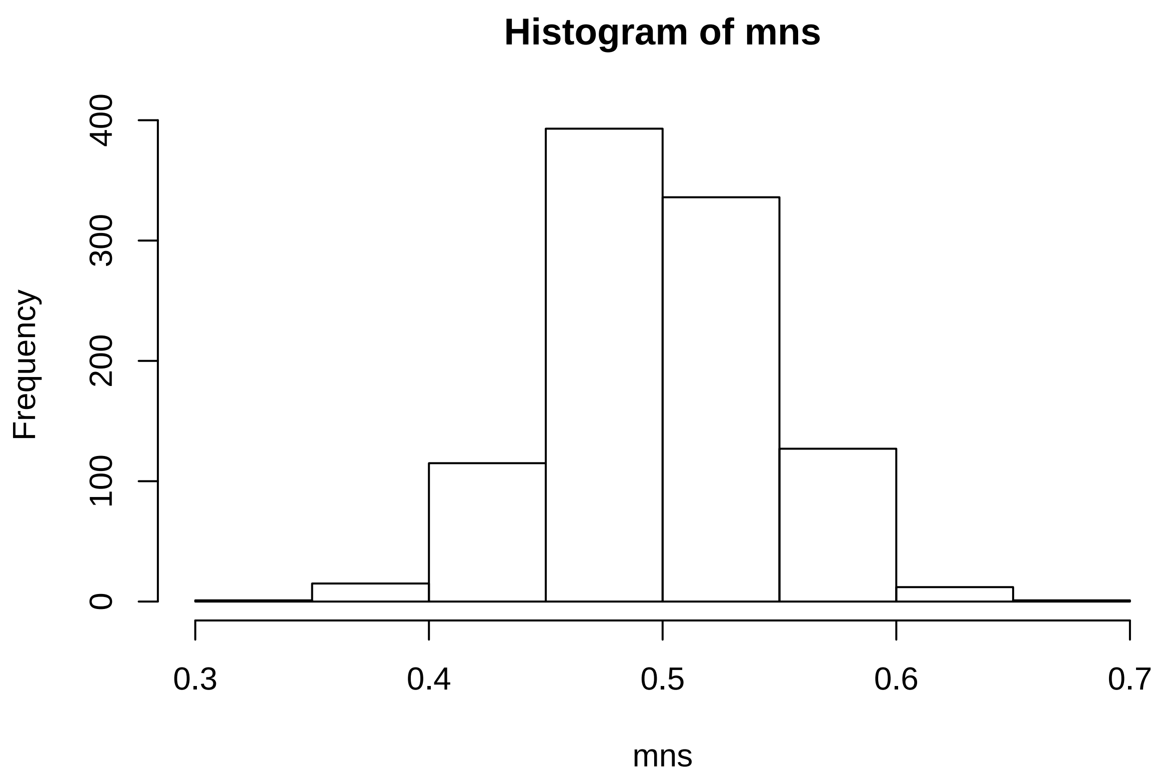
As a motivating example, compare the distribution of 1000 random uniforms

```r
hist(runif(1000))
```



and the distribution of 1000 averages of 40 random uniforms

```r
mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(runif(40)))
hist(mns)
```



This distribution looks far more Gaussian than the original uniform distribution!

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also also 1/lambda. Set lambda = 0.2 for all of the simulations. In this simulation, you will investigate the distribution of averages of 40 exponential(0.2)s. Note that you will need to do a thousand or so simulated averages of 40 exponentials.

```r
# set lambda to 0.2
lambda <- 0.2

# 40 samples
n <- 40

# 1000 simulations
simulations <- 1000

# simulate
simulated_exponentials <- replicate(simulations, rexp(n, lambda))

# calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```

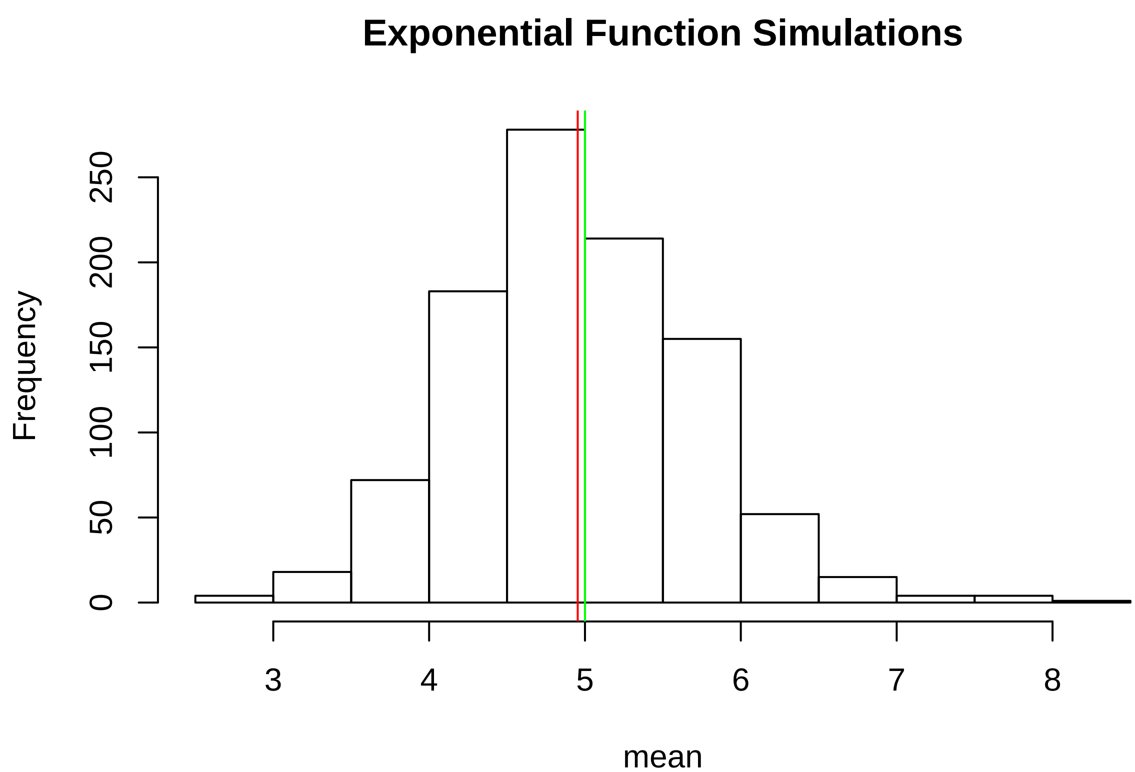
Question 1: Show where the distribution is centered at and compare it to the theoretical center of the distribution.

```
## [1] 4.95345
```

```
# analytical mean
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

```
# visualization
hist(means_exponentials, xlab = "mean", main = "Exponential Function Simulations")
abline(v = analytical_mean, col = "red")
abline(v = theory_mean, col = "green")
```



Answer 1: The analytics mean is 4.993867 the theoretical mean 5. The center of distribution of averages of 40 exponentials is very close to the theoretical center of the distribution.

Question 2: Show how variable it is and compare it to the theoretical variance of the distribution.

```
# standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
```

```
## [1] 0.764475
```

```
# variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
```

```
## [1] 0.584422
```

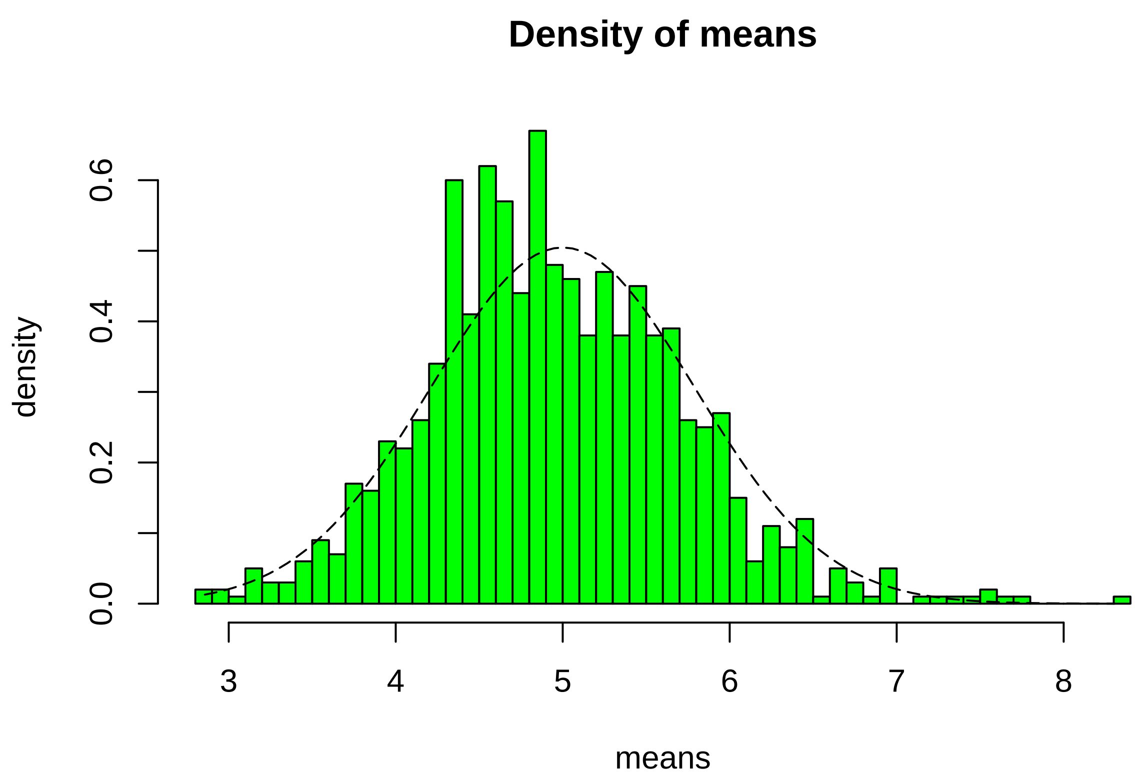
```
# variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
```

```
## [1] 0.625
```

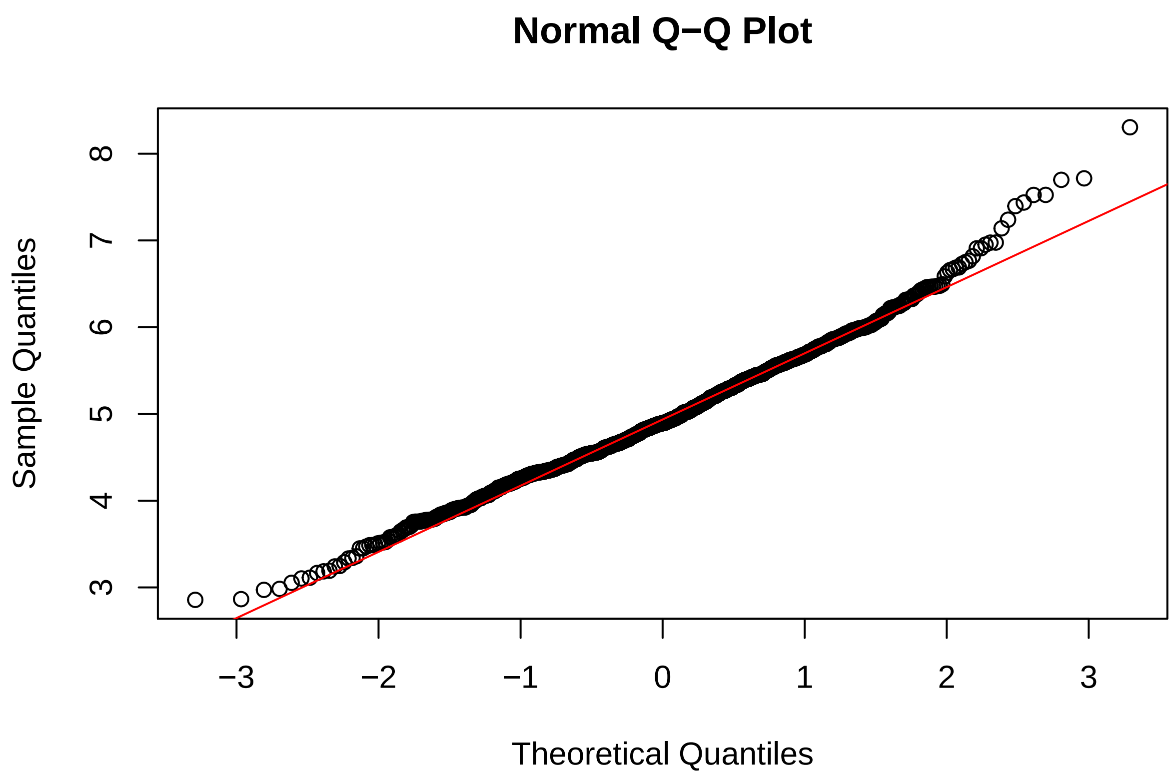
Answer 2: Standard Deviation of the distribution is 0.7931608 with the theoretical SD calculated as 0.7905694. The Theoretical variance is calculated as ((1/lambda)*(1/sqrt(n)))^2 = 0.625. The actual variance of the distribution is 0.6291041

Question 3: Show that the distribution is approximately normal.

```
xfit <- seq(min(means_exponentials), max(means_exponentials), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(n)))
hist(means_exponentials,breaks=n,prob=T,col="green",xlab = "means",main="Density of means",ylab="densit
lines(xfit, yfit, pch=22, col="black", lty=5)
```



```r
# compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_exponentials)
qqline(means_exponentials, col = 2)
```



Answer 3: Due to Due to the central limit theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.