A Vine Copula Panel Model for Day-Ahead Electricity Prices

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Agenda

Background
Modelling Day-Ahead Prices

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- Prices are determined from the same information set $\implies$ panel data
- Common to consider base prices, i.e. daily means

Our Goals

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- Simulate the day-ahead price distribution from the full model

Recap on Copulas
Definition

Definition (Copula)

A *d-dimensional copula* is a function $C : [0,1]^d \to [0,1]$ such that

1. $C(u_1, \dots, u_d) = 0$ if $u_j = 0$ for at least one $j \in \{1, \dots, d\}$
2. $C\big(1, \dots, 1, u_j, 1, \dots, 1\big) = u_j$ for all $j \in \{1, \dots, d\}$
3. $V_C\big((\boldsymbol{a}, \boldsymbol{b}]\big) \geq 0$ for every d-box $(\boldsymbol{a}, \boldsymbol{b}] \subseteq [0,1]^d$

where $(\boldsymbol{a}, \boldsymbol{b}] \coloneqq (a_1, b_1] \times \cdots (a_d, b_d]$ and

$$V_C\big((\boldsymbol{a}, \boldsymbol{b}]\big) \coloneqq \sum_{\boldsymbol{v} \in \mathrm{ver}(\boldsymbol{a}, \boldsymbol{b}]} \mathrm{sign}(\boldsymbol{v})\, C(\boldsymbol{v})\,;$$

$$\mathrm{ver}\,(\boldsymbol{a}, \boldsymbol{b}] \coloneqq \{a_1, b_1\} \times \cdots \times \{a_d, b_d\}\,,$$

$$\mathrm{sign}(\boldsymbol{v}) \coloneqq \begin{cases} 1, & \text{if } v_j = a_j \text{ for an even number of indices,} \\ -1, & \text{otherwise.} \end{cases}$$

Recap on Copulas

Sklar's Theorem

Theorem (Sklar's Theorem)

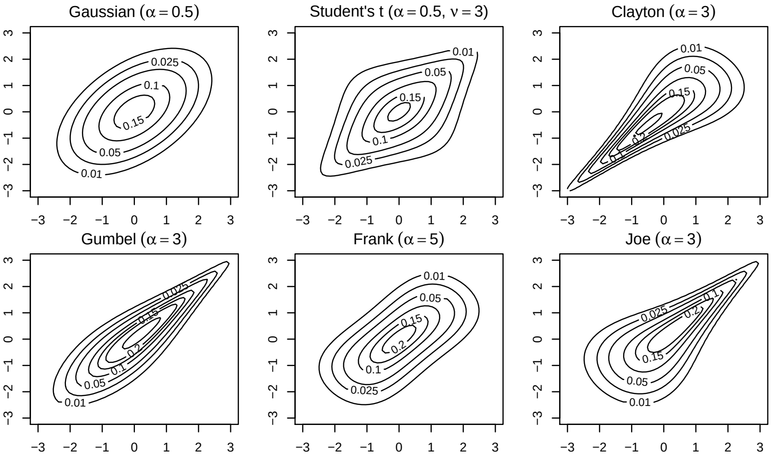
Let $\boldsymbol{X}$ be a d-dimensional random vector with joint distribution function F and marginals $F_1, \ldots, F_d$. Then there exists a d-copula C such that

$$F(\boldsymbol{x}) = C\big(F_1(x_1), \ldots, F_d(x_d)\big), \quad \forall \boldsymbol{x} \in \mathbb{R}^d$$

and if the marginals are all continous, then C is uniquely defined.

Recap on Copulas

Families of Copulas ($d = 2$)



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Basic Idea Behind Vine Copulas

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  - Marginals are nodes in the first tree
  - Pair-copulas between the marginals are edges in the first tree
  - Edges from the previous tree becomes nodes in the next
  - Edges in subsequent trees are conditional pair-copulas
  - Nodes in a tree can be joined by an edge, if the nodes share a node from the previous tree

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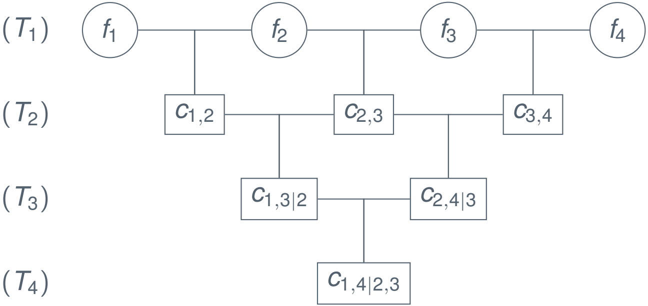
  - Nodes in a tree can be joined by an edge, if the nodes share a node from the previous tree
- The chosen pair-copulas need not be the same

Recap on Copulas

Basic Idea Behind Vine Copulas: Example

Example in $d = 4$



Recap on Copulas

Estimating Vine Copulas

Sequential estimation

Maximum likelihood estimation

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- Estimate tree structure with maximum spanning tree algorithm, using e.g. Kendall's τ as edge weights

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Maximum likelihood estimation

- Full MLE using numerical optimisation methods
- Can use sequential estimation results as starting values

Data

German Day-Ahead Electricity Prices

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$\Longrightarrow$ 24 daily time series of 1668 observations

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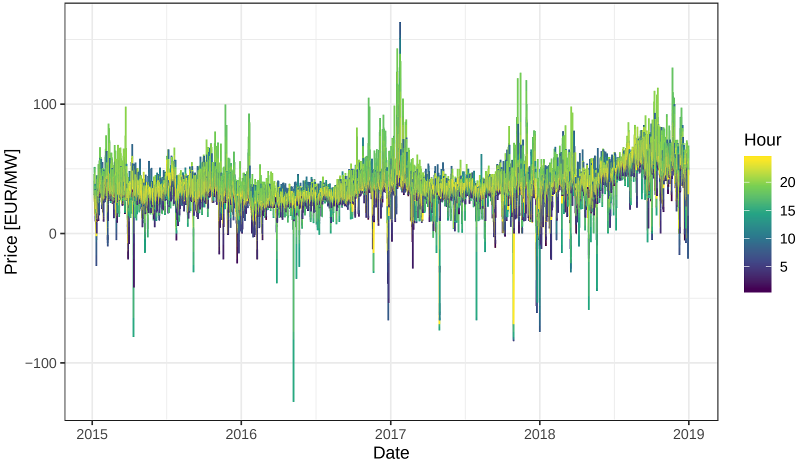

Marginal Models
Overview

With $t \in \{1, \ldots, 1668\}$ and $h \in \{1, \ldots, 24\}$,

$$p_{t,h} := \log\big(P_{t,h} + K\big) = s_{t,h} + X_{t,h},$$

where

- $K = 500$ (chosen as such from bid restrictions)
- $s_{t,h}$ is a deterministic component for capturing season
- $X_{t,h}$ is a stochastic component for capturing serial behaviour

Marginal Models
Seasonal Component

Seasonal model specification

$$s_{t,h} = \alpha_{0,h} + \alpha_{1,h} \cdot t + \sum_{\phi \in \Phi} \left(\beta_{1,\phi,h} \sin(2\pi t \phi) + \beta_{2,\phi,h} \cos(2\pi t \phi) \right) + \gamma_{w,h} w_t,$$

where

- $\Phi = \{1/365, 2/365\}$ is a set of frequencies
- w_t is a factor variable for weekdays

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Estimated with a linear model, residuals used for serial model

Marginal Models
Serial Component

Serial model specification

Each model is chosen among a panel of ARMA-GARCH models:

- ARMA orders: $p, q \in \{0, 1\}$
- GARCH orders: $p, q \in \{0, 1\}$ such that at least one is nonzero
- GARCH types: GJR-GARCH and EGARCH
- Conditional distributions: Student's t-distribution and its skewed variant

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$\implies$ 48 different models for each series;
estimated with MLE, model chosen with BIC

Marginal Models
Serial Component

Hour	Model
1	ARMA(1,1)-GJR-GARCH(1,1)-st
2	ARMA(1,1)-GJR-GARCH(1,1)-st
3	ARMA(1,1)-E-GARCH(1,1)-st
4	ARMA(1,1)-GJR-GARCH(1,1)-st
5	ARMA(1,1)-E-GARCH(1,1)-st
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9	ARMA(1,1)-E-GARCH(1,1)-st
10	ARMA(1,1)-E-GARCH(1,1)-st
11	ARMA(1,1)-E-GARCH(1,1)-st
12	ARMA(1,1)-E-GARCH(1,1)-st

Hour	Model
13	ARMA(1,1)-GJR-GARCH(1,0)-st
14	ARMA(1,1)-GJR-GARCH(1,0)-st
15	AR(1)-GJR-GARCH(1,0)-st
16	ARMA(1,1)-GJR-GARCH(1,0)-st
17	ARMA(1,1)-GJR-GARCH(1,1)-st
18	ARMA(1,1)-E-GARCH(1,1)-st
19	ARMA(1,1)-GJR-GARCH(1,1)-t
20	ARMA(1,1)-GJR-GARCH(1,1)-st
21	ARMA(1,1)-GJR-GARCH(1,1)-st
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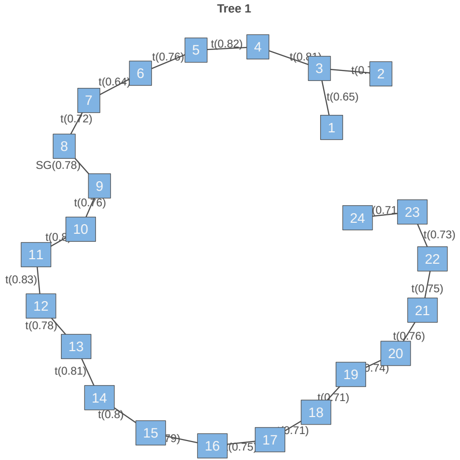
3. Reestimate with MLE

Joint Model
Summary of Results



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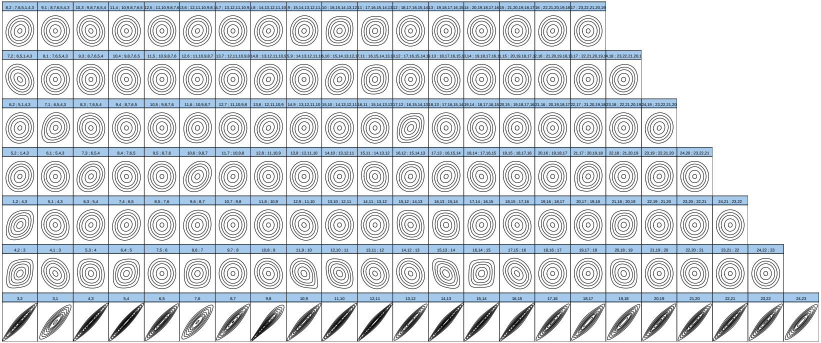

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- Many (but but not all) *t*-copulas in first tree
- Strong, positive dependence in first tree, including tail dependence
- A lot of independence or "near-independence" in later trees, in particular tree 19, 21, 22, 23, and 24
  - For tree 5 and later, over half of the edges are independence copulas
  - In total, about 53% of the copulas are independence copulas
  - On the other hand, dependence persists until tree 20

Simulation of Price Distribution

Prediction Framework

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5. Prices in EUR/MW are then $P_{n,h} = \exp p_{n,h} - K$

Simulation of Price Distribution

Example

Predicting the payoff distribution of a forward contract

Let $F(t, t_1, t_2)$ be a price in EUR/MW determined today (at time t) for delivery of power in the period $[t_1, t_2]$. Payoff on a long position is then

$$\sum_{s=t_1}^{t_2} \sum_{h \in H(s)} \left(P_{s,h} - F(t, t_1, t_2) \right),$$

where $H(s) \subseteq \{1, \ldots, 24\}$ are the hours on day s for which the forward is in effect.

Simulation of Price Distribution

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Example simulation

1. Prices simulated for the period 2019-01-01 – 2019-07-31
   $\Longrightarrow$ 212 observations $\times$ 24 hours.
2. Payoff calculated for some contract $F(t, t_1, t_2)$
3. Steps 1–2 repeated 500 times
   $\Longrightarrow$ simulated payoff distribution

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For example, if the delivery interval is February 2019, on hours 2–4 and 16–18, for 40 EUR/MW, the simulated mean payoff is **14.10**, while the payoff using actual prices is **5.05**.

Simulation of Price Distribution

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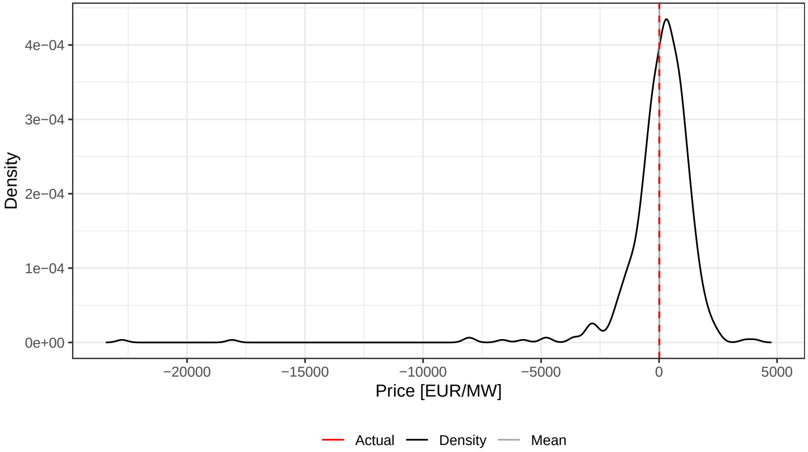

Simulation of Price Distribution

Example of Single Draw

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- Majority of pair-copulas in first tree are *t*-copulas—model could be compared to a 24-dimensional *t*-copula
- Simulation occasionally yields infinite values; about 2.5% of samples contained infinite values, and out of those, these values comprised about 0.5% of the sample—could be kinks in the code or parts of the model itself

Thanks for listening!