Linköping University

TDDC17 -Artificial Intelligence

Lab 3

Bayesian Networks

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Q&A

Q: ***What is the risk of melt-down in the power plant during a day if no observations have been made? What if there is icy weather?***

A: The risk melt-down if no prior observation have been made is$0,02578$ and the risk melt-down if icy weather has been observed $0,03472$

Q: ***Suppose that both warning sensors indicate failure. What is the risk of a meltdown in that case? Compare this result with the risk of a melt-down when there is an actual pump failure and water leak. What is the difference? The answers must be expressed as conditional probabilities of the observed variables,*** $P(Meltdown \mid ...)$***?***

A: We came upp with the following.

$$P(M \mid PFWarning = T \wedge WLWarning = T) = 0,14535$$

$$\begin{aligned} P(M \mid PFWarning, WLWarning, PFailure = T, WLeak = T) &= \\ P(M \mid PFailure = T, WLeak = T) &= 0,2000 \end{aligned}$$

That is given a PumpFailure and WaterLeak, Meltdown i conditionally independent of PumpFailureWarning and WaterLeakWarning???. Intuitively we view this as if we know about actual pump failing or actual water leakage then indication if such things on some kind of panel doesn't increase the probability of meltdown.

Q: The conditional probabilities for the stochastic variables are often estimated by repeated experiments or observations. Why is it sometimes very difficult to get accurate numbers for these? What conditional probabilites in the model of the plant do you think are difficult or impossible to estimate?

A: Icy weather is hard to predict because weather in general is dependent on a lot of parametser which makes the scale of predicting weather very large. All the parameter cant fit in the model which is a simplifictation of the real world. (Maybe something about other nodes as well usually we have statistical data to support certain behaviour these sometimes require a lot of reppeated experiments which might not be possible to simulate bla bla bla etc..l.). Meltdown also seems difficult to predict. How would you do an experiment of a full scale meltdown of a powerplant?

Q: Assume that the "IcyWeather" variable is changed to a more accurate "Temperature" variable instead (don't change your model). What are the different alternatives for the domain of this variable? What will happen with the probability distribution of P(WaterLeak | Temperature) in each alternative?

A: Currently the domain of temperature (icyWeather)consist of $\{true, false\}$ which given lectures notation is a boolean domain.One alternative would be using discrete domain. An example of a discrete domain could be $\{icy, cold, mild, hot\}$. One way to implement this would be dividing a continuous temperature range into a fixed set of intervals (Course book uses the term discretization). For instance we could limit the total range to be $-60°C \leq temprature < 50°C$ could be divided int four ranges where by icy we would mean $-60°C \leq temprature < 0°C$, cold could be $0°C \leq temprature < 10°C$ and so on.Doing this would require change the conditional probability table P(WaterLeak | Temperature) so that we have in this case four rows, one for each discrete variable represent the different atomic events. Note such an approach could lead to large conditional probability Tables, as effect of adding discrete variables in parent node ripples downwards in network.

Another approach is using continuous domain. Essentially a continuous range possible infinite. This would imply a function to calculate the distribution of P(WaterLeak | Temperature) in some sensible way. The book also discusses way through threshold functions as a way to make it possible to have continuous parents and discrete child nodes.

Q: What does a probability table in a Bayesian network represent?.

A: They represent the different possible probabilities (degrees of believes) of event occurring given current nodes parent nodes. Put another they are sets of conditional probability distributions which "tell" how parent nodes effect their child nodes. Mathematically the entries in such a table, could be described by

$$P(X_i \mid Parents(X_i))$$

Where X_i is a random variable represent event of node i (the node of current interest) and $Parents(X_i)$ are it's events represented by parent nodes.

Q: What is a joint probability distribution? Using the chain rule on the structure of the Bayesian network to rewrite the joint distribution as a product of P(child | parent) expressions, calculate manually the particular entry in the joint distribution of P(Meltdown=F, PumpFailureWarning=F, PumpFailure=F, WaterLeakWaring=F, WaterLeak=F, IcyWeather=F). Is this a common state for the nuclear plant to be in?

A: Joint probability distribution is simply a set(table) of probabilities of all possible combinations of **given** Random variables. Note full joint probability distribution is then defined joint distribution of **all** random variables.

$$\begin{aligned} P(\neg MD, \neg PFWarn, \neg PFail, \neg WLWarn, \neg WLeak, \neg IW) &= \\ P(\neg MD \mid \neg PFail, \neg WLeak) * P(\neg WLWarn \mid \neg WLeak) * \\ P(\neg PFWarn \mid \neg PFail) * P(\neg WLeak \mid \neg IW) * P(\neg PFail) * P(\neg IW) &= \\ 0,999 * 0,95 * 0,95 * 0,9 * 0,9 * 0,95 &= 0,693779276 \end{aligned}$$

Q: ***What is the probability of a meltdown if you know that there is both a water leak and a pump failure? Would knowing the state of any other variable matter? Explain your reasoning!***

A: The probability taken from the applet was 0.2 . In this model the other random variables won't effect the outcome . This is because the nodes that effects meltdown directly are PumpFailure and WaterLeak which in this scenario we have observed (both)to be true, hence our belief in Pump failing and water leaking is a 100%. This won't change even thou we get more information from other node. The only thing then that remains is the fact of how these two nodes effect chances of a meltdown (need to explain mathematically)

Q: ***Calculate manually the probability of a meltdown when you happen to know that PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F and IcyWeather=F but you are not really sure about a pump failure. Hint: Use the Exact Inference formula near the end of the slides, or in sec. 14.4.1 in the book. This formula includes both conditioning on the variables you know (evidence) and marginalizing (summing) over the variable(s) you do not know (often called unobserved or hidden). You need to calculate this both for*** $P(Meltdown = T \mid ...)$ ***and*** $P(Meltdown = F \mid ..)$ ***and normalize them so that they sum to 1. This normalization factor is the alpha symbol in the equation. With this formula you could answer any query in the network, though it will be impractical for cases with many unobserved variables. A suggestion is to move the terms that do not involve the pump failure variable out of the sum over the two states pump failure can be in (T/F). You may use inference in the applet for verification purposes, but small differences is expected due to rounding errors.***

A: The given network is depict in the following picture

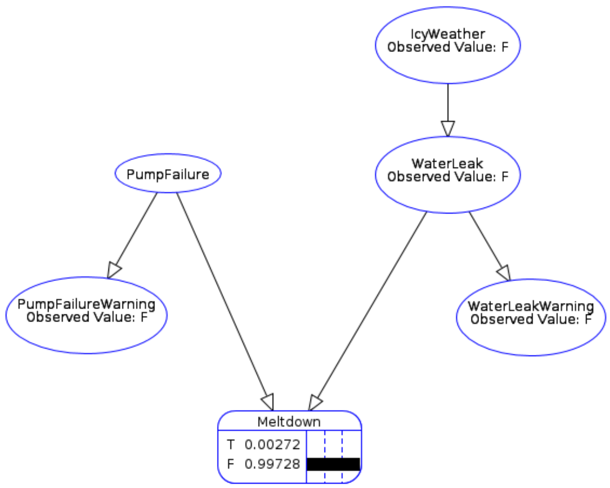

Figure 1: powerplant network with given observations

The suggested formula looks as follows.

$$\mathbf{P}(X \mid e) = \alpha \mathbf{P}(X, e) = \alpha * \sum_{y} \mathbf{P}(X, e, y) \tag{1}$$

In which X is the query variable, e are observed values on evidence variables, y are variable/value pairs on remaining unobserved variables and α as mentioned above is the normalization factor. So in this assignment we have.

$$\begin{aligned}
X &= \{M\} \\
E &= \{PFW, WL, WLW, IW\} \\
e &= \{\neg pFW, \neg wL, \neg wLW, \neg iW\} \\
Y &= \{PF\} \\
y &= \{\{pF, \neg pF\}\}
\end{aligned}$$

So using formula (1) we get

$$\begin{aligned}
\mathbf{P}(M \mid \neg pFW, \neg wLW, \neg wL, \neg iW) &= \alpha \mathbf{P}(M, \neg pFW, \neg wLW \neg wL, \neg iW) \\
&= \alpha * \sum_{pF} \mathbf{P}(M, \neg pFW, PF = pF, \neg wLW, \neg wL, \neg iW)
\end{aligned}$$

Which can be expressed in terms CPT-entries $\left(P\left(x_1, .., x_n\right) = \prod_{i=1}^{n} P\left(x_i \mid parents\left(X_i\right)\right)\right)$

$$\begin{aligned}
&\alpha * \sum_{pF} P(M \mid \neg wlw, PF = pF) P(\neg pFW \mid PF = pF) * P(PF = pF) * P(\neg wLW \mid \neg wL) * \\
&\qquad P(\neg wL \mid \neg iW) * P(\neg lW) = \\
&\alpha * P(\neg wLW \mid \neg wL) * P(\neg wL \mid \neg iW) * P(\neg lW) * \sum_{pF} P(M \mid \neg wlw, PF = pF) * \\
&\qquad P(\neg pFW \mid PF = pF) * P(PF = pF)
\end{aligned}$$

The blue part of course is constant for both $P(m)$ and $P(\neg)$ so i'm gonna go ahead and plug in the numbers (from probability tables), calculate it and call it C.

$$C = P(\neg wLW \mid \neg wL) * P(\neg wL \mid \neg iW) * P(\neg lW) = 0.95 * 0.9 * 0.95 = 0.812250$$

Next we brake down the calculation in cases for true and false given for meltdown ($P(m \mid \neg pFW, \neg wLW, \neg wL, \neg iW)$ and $P(\neg \mid \neg pFW, \neg wLW, \neg wL, \neg iW)$)

$$\alpha * C * \sum_{pF} P(m \mid PF = pF\neg wlw, PF = pF) * P(\neg pFW \mid PF = pF) * P(PF = pF) \quad (2)$$

$$\alpha * C * \sum_{pF} P(\neg m \mid PF = pF\neg wlw, PF = pF) * P(\neg pFW \mid PF = pF) * P(PF = pF) \quad (3)$$

Expanding the sum (2) , that is the true case and plugging in some numbers gives us. Bellow T is the sum for true case.

$$\begin{aligned} &\alpha * C * T = \\ \alpha * C * (P(m \mid pF = true, \neg wlw, pF = true) * P(\neg pFW \mid pF = true) * P(pF = true) &+ \\ P(m \mid pF = pF\neg wlw, pF = false) * P(\neg pFW \mid pF = false) * P(pF = false)) &= \\ \alpha * C * ((0.15 * 0.1 * 0.1) + (0.001 * 0.95 * 0.9)) &= \\ \alpha * 0.812250 * 0.002355 &= \\ \alpha * 0.001912848750 & \end{aligned}$$

And same way with False case (3) the is called F bellow.

$$\begin{aligned} &\alpha * C * \alpha * F = \\ C * (P(\neg m \mid pF = true, \neg wlw, pF = true) * P(\neg pFW \mid pF = true) * P(pF = true) &+ \\ P(\neg m \mid pF = false\neg wlw, pF = false) * P(\neg pFW \mid pF = false) * P(pF = false)) &= \\ \alpha * C * ((0.85 * 0.1 * 0.1) + (0.999 * 0.95 * 0.9)) &= \\ \alpha * 0.812250 * 0.862645 &= \\ \alpha * 0.70068340125 & \end{aligned}$$

The only thing that remains to do is to figure out α. The true and false case have to sum up to one so.

$$\begin{aligned} \alpha * (0.001912848750 + 0.70068340125 = 1 \quad &\Leftrightarrow \quad \alpha = \frac{1}{(0.001912848750 + 0.70068340125)} \\ &\Leftrightarrow \quad \alpha = \frac{1}{0.70259625} \end{aligned}$$

And finally, if you may drum roles, please :-).

$$\begin{aligned} \mathbf{P}(M \mid \neg pFW, \neg wLW, \neg wL, \neg iW) &= \alpha * \langle 0.00191284875, 0.70068340125 \rangle \\ &= \frac{1}{0.70259625} * \langle 0.00191284875, 0.70068340125 \rangle \\ &= < 0.002722543352601, 0.997277456647399 > \end{aligned}$$

Which if take a look in the network seems to be accurate. Part III: Extending a network Extending the nuclear power plant with car and bicycle ended up with the following Bayesian network (application uses the term belief network)

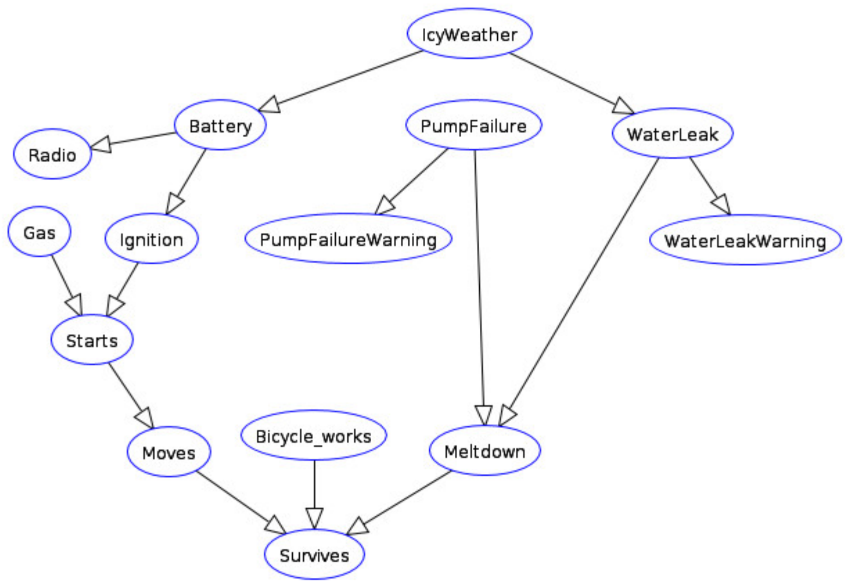

Figure 2: power plant extended with car an bicycle

Q:* *During the lunch break, the owner tries to show off for his employees by demonstrating the many features of his car stereo. To everyone's disappointment, it doesn't work. How did the owner's chances of surviving the day change after this observation?

A: It decreases chances of survival slightly. Here are the probabilities we came up with in our model.

$$\mathbf{P}(Survival) = \langle 0.99041, 0.00959 \rangle$$
$$\mathbf{P}(Survival \mid \neg radio) = \langle 0.98338, 0.01662 \rangle$$

Q:* *The owner buys a new bicycle that he brings to work every day. The bicycle has the following properties:

$$P(bicycle\_works) = 0.9$$
$$P(survives \mid \neg moves \wedge meltdown \wedge bicycle\_works) = 0.6$$
$$P(survives \mid moves \wedge meltdown \wedge bicycle\_works) = 0.9$$

How does the bicycle change the owner's chances of survival?

A: Probability of survival will increase slightly given no knowledge about condition of bicycle. Given bicycle works it survival chances will increase a bit more.

$$\mathbf{P}(Survival) = \langle 0.99522, 0.00478\rangle$$
$$\mathbf{P}(Survival \mid \neg radio) = \langle 0.99213, 0.00787\rangle$$
$$\mathbf{P}(Survival \mid bicycle\_works) = \langle 0.99576, 0.00424\rangle$$

Q: It is possible to model any function in propositional logic with Bayesian Networks. What does this fact say about the complexity of exact inference in Bayesian Networks? What alternatives are there to exact inference?

A: In the case were networks are multiply connected (intuitively this means networks that include nodes which can be reached through more then one path from some given node) exact inference in Bayesian Networks are NP-hard which means that they in to large degree are intractable. This should be interpreted as there exist exceptions for instance, small instances of problem or special cases of problem can be tractable. An alternative to exact inference is approximate inference. The course book mentions the Monte Carlo algorithm ,Markov chain inference and Gibbs sampling.

Part IV: More extensions

After modelling H.S we got the following network. Note we have H.S in such a way that since here is incompetent we expect him in most cases to screw up, that is even if he takes action to prevent meltdown he will actually most of the time make things worse.

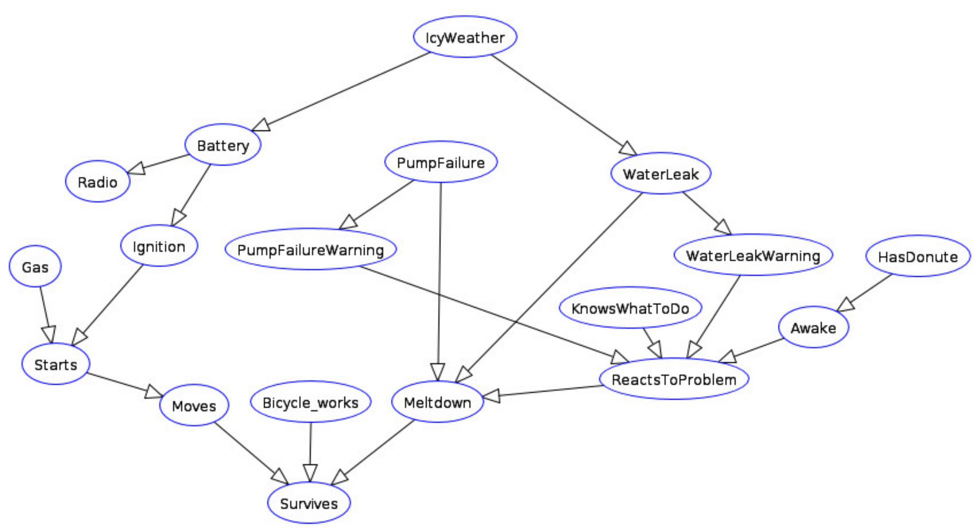

Figure 3: power plant with further extension of H.S

Q: ***The owner had an idea that instead of employing a safety person, to replace the pump with a better one. Is it possible, in your model, to compensate for the lack of Mr H.S.'s expertise with a better pump?***

A: Yes, in our model you could you simply change the probability of pumpfailure and to a certain degree make up for H.S incompetence.

Q: ***Mr H.S. fell asleep on one of the plant's couches. When he wakes up he hears someone scream: "There is one or more warning signals beeping in your control room!". Mr H.S. realizes that he does not have time to fix the error before it is to late (we can assume that he wasn't in the control room at all). What is the chance of survival for Mr H.S. if he has a car with the same properties as the owner? (notice that this question involves a disjunction which can not be answered by querying the network as is) Clarification: Maybe something could be added to or modified in the network.***

A: We add the node signal as bellow. H.s chances of survival are $\langle 0.995, 0.005\rangle$ which quite good even (risk of meltdown is still high).

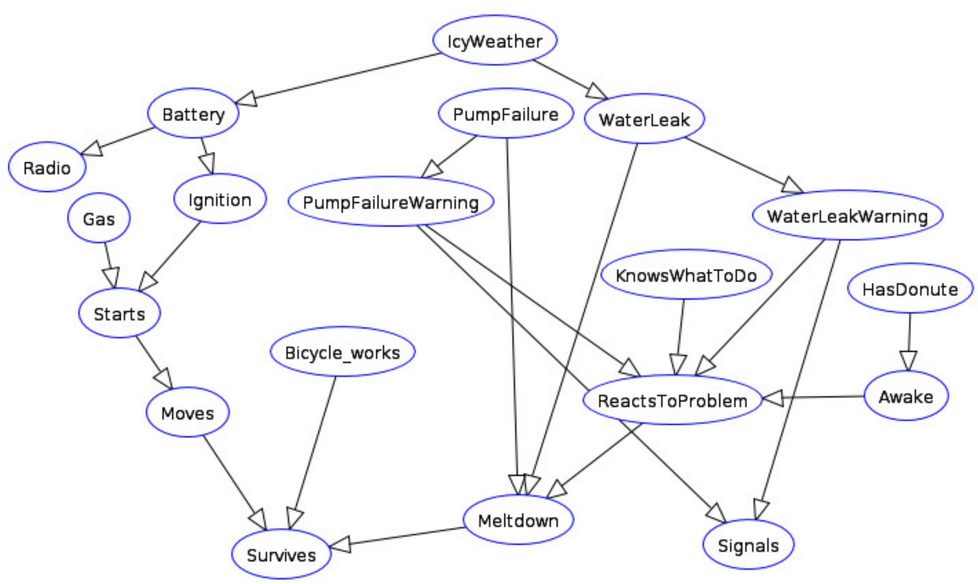

Figure 4: power plant model with node signal

Q: ***What unrealistic assumptions do you make when creating a Bayesian Network model of a person?***

A: Humans are complex beings it's unrealistic to try to model every aspect a human. When modelling something in a bayesian network we need to make a lot of simplifications. One human can differ to another a simple thing as age can make a huge difference in many other aspects of a human.