

Three-Way Chess by Varga

Quick Start and Technical Documentation

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March 12, 2025

1 Introduction

Three-Way Chess is a three-player chess variant played on a board consisting of 96 hexagonal tiles. Besides White and Black, as in standard chess, there is a third player, Gray, represented by the green color in this documentation. The game is played with standard chess pieces; however, the movement rules are altered. White plays first, followed by Gray and then Black. The game continues in this order, with each player making one move per turn until the game ends. All basic chess rules apply, including check, castling, promotion, and en passant, with the following exceptions:

1. When a player is checkmated, their pieces are removed from the board, and the game continues with the remaining two players. If only one player remains, they win the game. A player is considered checkmated if it is **their turn**, their king is in check, and there is no legal move to escape check.
2. A player's king can be captured by another player. If this happens, their pieces are **instantly** removed from the board, and the game continues with the remaining two players. If only one player remains, they win the game.
3. There are 3 different types of bishops since they are placed on different colored tiles. See the section on movement rules for more information.

The board and the initial position of the pieces are shown below [1](#).

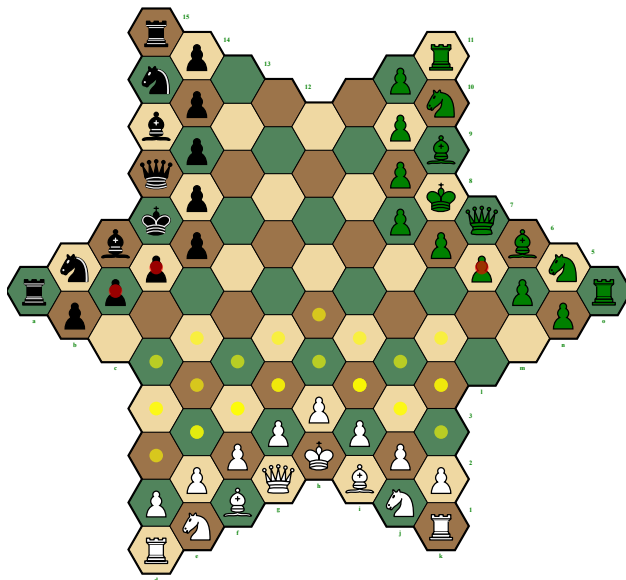


Figure 1: Starting Position

2 Movement Rules

In this section, we describe the movement rules for each piece. Since the tiles are hexagonal, the movement rules differ from those in standard chess. Each piece, except for pawns, has an additional degree of freedom in its movement.

Line movement is defined as moving in a straight line in any of the six directions. **Diagonal movement** is defined as jumping to the next same-colored hexagonal tile in any of the five possible directions. These diagonal moves can enable pieces to visually jump over pieces and in some cases even the border of the board.

2.1 Pawns

Pawns move and capture in the same way as in standard chess. They can be promoted to either a queen, rook, bishop, or knight when they reach a tile from which they cannot move further [2](#). Pawns can also capture en passant, allowing them to take an opponent's pawn that has moved two tiles forward on the previous move [3](#). En passant opportunities reset when the player who made the two-tile move makes another move.

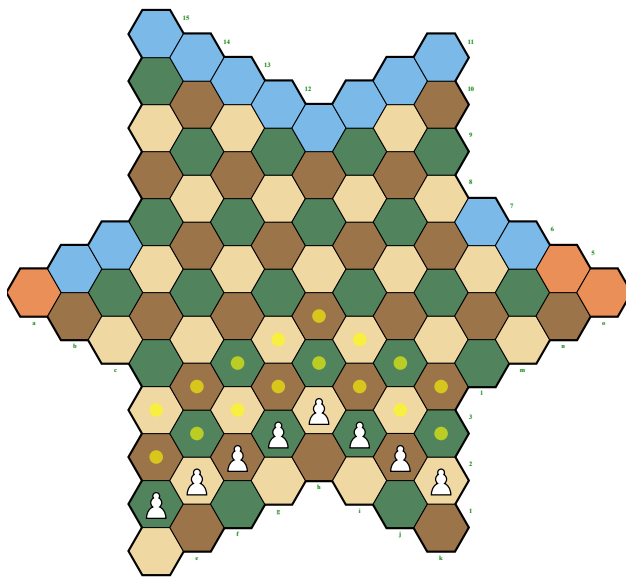


Figure 2: All possible pawn moves for White from the starting position. Highlighted tiles are promotion tiles. Orange tiles are, by definition, promotion tiles; however, they are not reachable in a normal game.

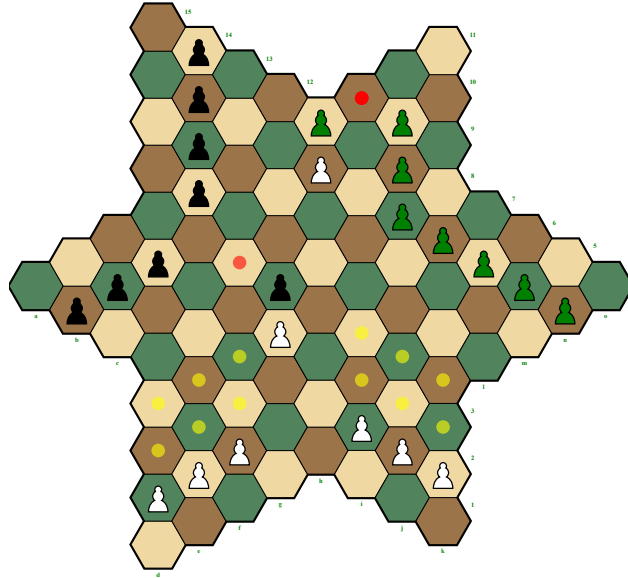
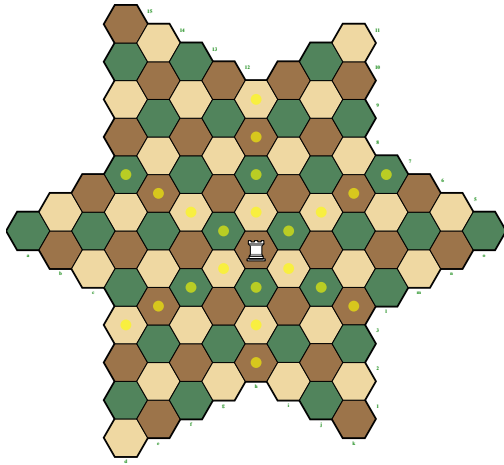


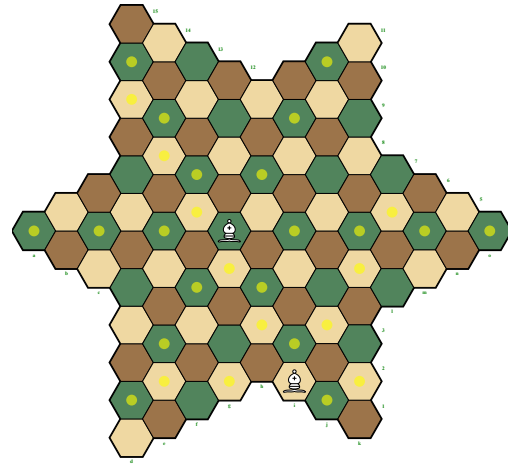
Figure 3: En passant example. In this theoretical example, both Black and Gray played their pawns two tiles forward in their respective previous moves. Notably, White's pawn on the *h*-file will also promote when capturing en passant.

2.2 Rooks, Bishops, and Queens

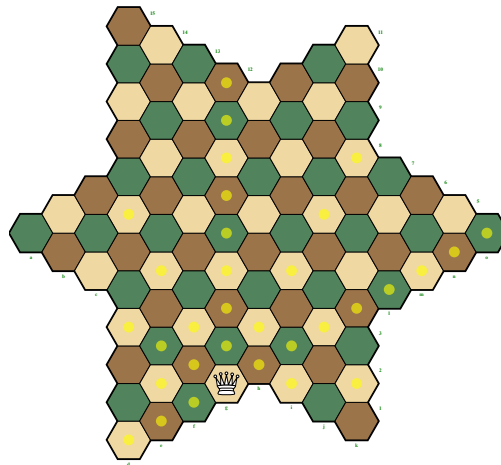
Rooks, bishops, and queens move in a similar way to standard chess, with the only difference being the additional degree of freedom mentioned before [4](#).



(a) Rook movement



(b) Bishop movement



(c) Queen movement

Figure 4: Movement patterns of the rook, bishop, and queen.

2.3 Knight movement

The knight has 12 possible moves from its starting position [5](#).

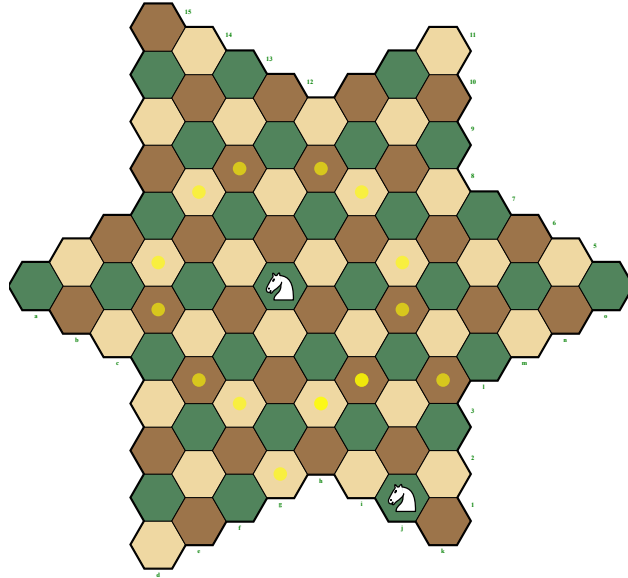


Figure 5: Knight movement

2.4 King movement

The king can move one tile in any direction (including diagonally), resulting in a total of eleven possible moves if no pieces are blocking the way.

As in standard chess, the king can castle - either kingside or queenside - provided that the respective rooks and the king have not moved, there are no pieces between the king and the rook, and the king is not in check [6](#).

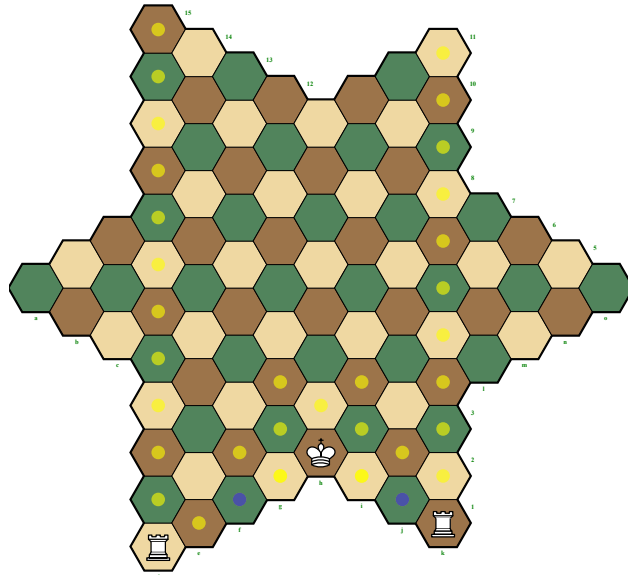


Figure 6: King movement, with blue circles representing possible castling moves.

2.5 In practice

Figure 7 shows an example position with all possible moves for Black player.

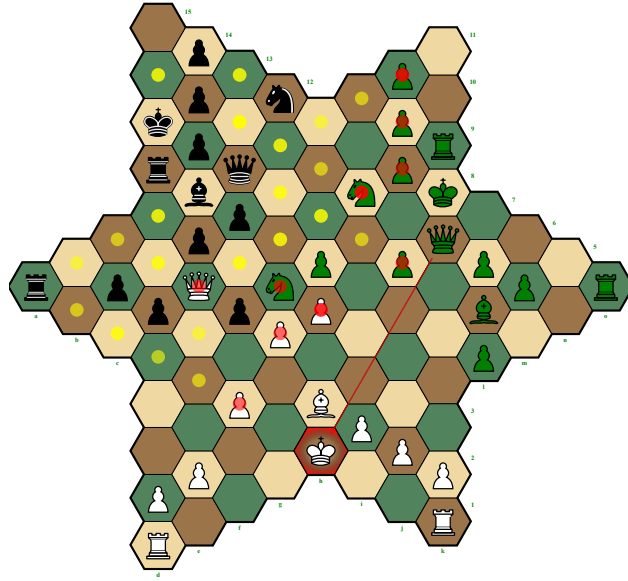


Figure 7: Example position with all possible moves for Black player.

3 Technical documentation

Internally, the board is represented using two coordinate systems.

1. **Flat array:** The board is stored as a one-dimensional array of 96 elements. Each element is a byte, where the first two bits represent the player's color, and the last three bits represent the piece type. This representation is used for storing the game state.
2. **Skewed cube coordinates:** This is a custom representation based on the cube-coordinate system used for hexagonal grids. Each hexagon is assigned three coordinates: q , r , and s . In standard cube coordinates, the equation $q + r + s = 0$ holds. However, in skewed cube coordinates, the equation is modified to $q + r + s = -1$. The s coordinate is redundant and can be derived from the other two. This representation is used for calculating possible moves for each piece and for displaying the board.

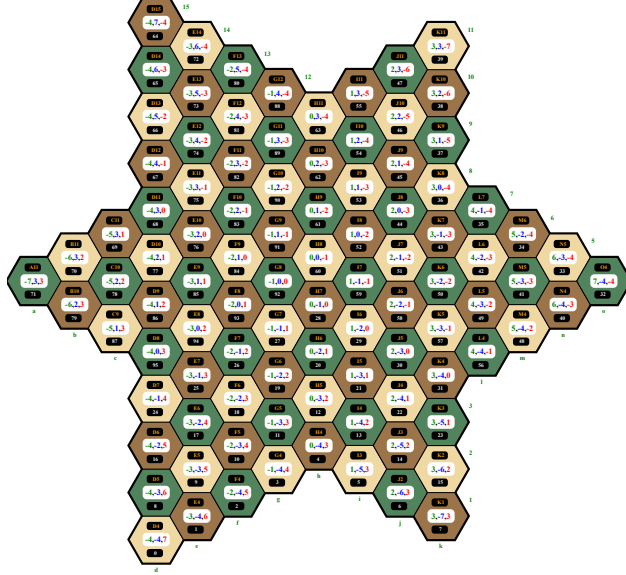


Figure 8: Cube coordinates

3.1 Creating the board

Using the skewed cube coordinates, we have a nice symmetrical equation to check if a hexagon is part of the board by imagining that the board is made out of two triangles.

$$(q \leq 3 \wedge q \leq 3 \wedge r \leq 3) \vee (q \geq -4 \wedge r \geq -4 \wedge q + r \geq -4) \quad (1)$$

And iteratively we can generate the board by:

```

1 hex_dict: dict[str, Hex] = {}
2
3 for q in range(-4, 8):
4     for r in range(-4, 4 - q):
5         s = -1 - q - r
6         hex_dict[f"{q}-{r}"] = Hex(q, r, s)
7
8 for q in range(-7, 4):
9     for r in range(-4 - q, 4):
10        s = -1 - q - r
11        hex_dict[f"{q}-{r}"] = Hex(q, r, s)

```

To draw each square then we can use the following:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \text{size} \cdot \begin{pmatrix} \frac{3}{2} & 0 \\ -\sqrt{3} & -\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} q \\ r \end{pmatrix} + \begin{pmatrix} \frac{\text{size}}{2} \\ -\frac{\sqrt{3} \cdot \text{size}}{2} \end{pmatrix}$$

where size refers to the size of the hexagon in the flat-top representation. Additional information can be found in the [Hexagonal Grids](#) article by Red Blob Games.

3.2 HexFEN

HexFEN is a compact ASCII notation for describing the game state, based on the Forsyth-Edwards Notation [9](#).

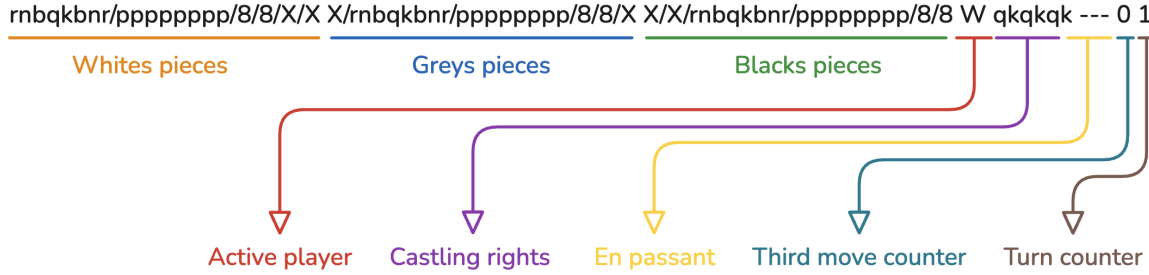


Figure 9: HexFEN example notation of the starting position with tagged pieces.

The eight whitespace-separated parts of the HexFEN string represent the following:

1. **The first three parts** represent each player's pieces, starting with White, then Gray, and finally Black. These strings are split by rows, with each row separated by a forward slash. Logically, it follows the flat array representation presented earlier. *X* represents an empty third of the board (32 unoccupied tiles).
2. **The active player** is represented by a single character: either W, G, or B.
3. **Castling availability** is represented by six characters: $((q|-)(k|-))^3$. Here, *q* represents queenside castling, *k* represents kingside castling, and *-* indicates no castling availability.
4. **The en passant target** is represented by three characters: $(-|(1-8))^3$. Digits represent the local column of the en passant target, while *-* indicates no en passant target.
5. **The third-move clock** is a number representing the count of third moves since the last capture or pawn move.
6. **The full-move number** is a number representing the count of full moves since the start of the game.