Are combination forecasts of S&P 500 volatility statistically superior?
Ralf Becker, Adam E. Clements (2008)

Jan Wälty

University of Zurich

jan.waelty@uzh.ch

December 15, 2023

Paper overview

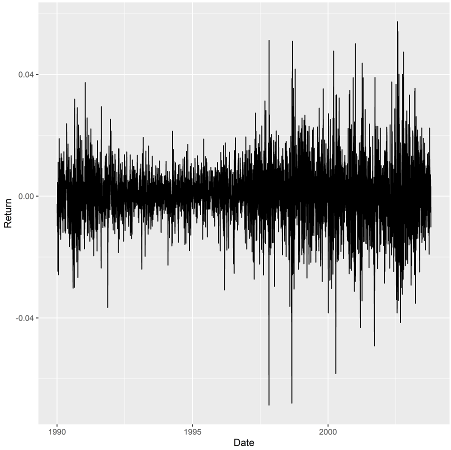


Figure: Daily returns S&P 500 (1990-2003)

- Volatility of high-frequency returns usually not constant over time
- Apply rolling-window procedure to test volatility models
  - Fit: 1000 data points
  - Forecast horizon: 22 days (equivalent to VIX prediction)
- Compare to implied volatility (VIX)
- Combine models to assess whether they perform better than single models

Narrow replication

- Model (de-meaned) returns Y_t as function of σ_t (conditional standard deviation) and ϵ_t (mean zero random process):

$$Y_t = \sigma_t \epsilon_t,\ \epsilon_t \sim N(0,1)$$
$$E_{t-1}[Y_t^2] = E_{t-1}[\sigma_t^2 \epsilon_t^2] = \sigma_t^2$$

- Perform rolling-window procedure and compare performance of models (subset of models used in paper)

ARCH(1)

$$\sigma_t^2 = \gamma + \alpha \cdot y_{t-1}^2$$

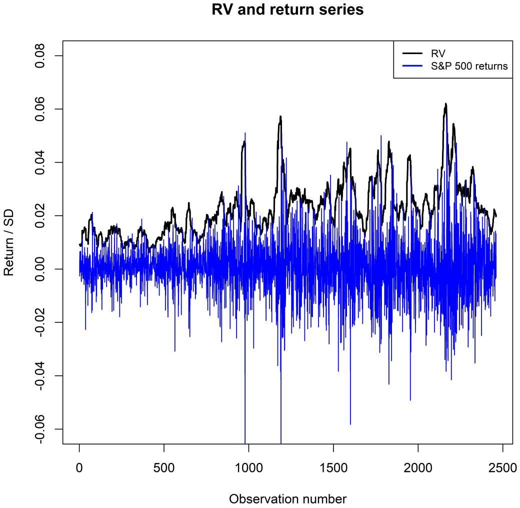
Garch(1,1)

$$\sigma_t^2 = \gamma + \alpha \cdot y_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$$

GJR-Garch(1,1)

$$\sigma_t^2 = \gamma + (\alpha + \delta 1_{(y_{t-1}<0)}) \cdot y_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$$

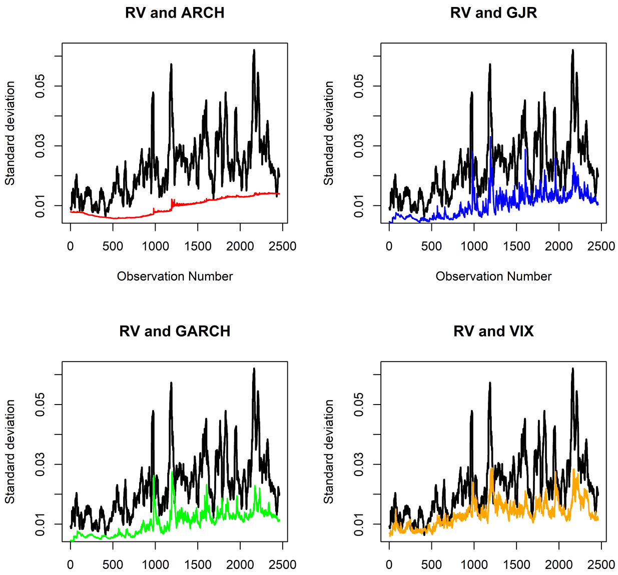
Results I



Model	MSE
ARCH(1,1)	0.618
GJR-Garch(1,1)	0.534
Garch(1,1)	0.532
VIX	0.356

Table: Model results (out-of-sample)

Results II


RV and ARCH
RV and GJR
RV and GARCH
RV and VIX
Standard deviation
0.01
0.03
0.05
0
500
1000
1500
2000
2500
Observation Number

Reflection and further steps

Reflection

- Authors do not provide data and code
- Models incorporating realized volatility cannot be estimated
- Chosen benchmark not optimal given its stochastic nature
- Results cannot directly be compared to paper (very sensitive to benchmark)

Further steps

- Choose a different benchmark (e.g. intraday low-to-high ratio)
- Extend models by adding more lags
- Various Garch extensions are possible
- Change distribution of ϵ_t (e.g. log-normal or other heavy-tail distribution)