Engineering with Computers (2021) 37:2159–2172
https://doi.org/10.1007/s00366-020-00935-x

ORIGINAL ARTICLE

An approach for dynamic analysis of planar multibody systems with revolute clearance joints

Yu Chen[1,3] · Jun Feng[2,3] · Xu Peng[2] · Yu Sun[3] · Qiang He[3,4] · Chengtao Yu[1]

Received: 28 February 2019 / Accepted: 4 January 2020 / Published online: 14 January 2020

Abstract
Clearance is inevitable for manufacture and assembly in the revolute joints of multibody systems. Excessive value of joint clearance will lead to the poor dynamic performance of mechanism, namely noise, vibration and fatigue failure. This paper presents the development of a dynamic model for planar multibody systems with clearance joints. Based on the continuous contact law, the contact force model proposed by Lankarani and Nikravesh is employed to describe the contact–impact phenomenon between pin and hole. And, the incorporation of the friction influence on revolute joint clearance is conducted by a modified friction force model. According to the geometric relationship between contact elements, the kinematics of the multibody systems is mapped into the global coordinate systems. Additionally, an experimental test platform is set up whereby a slider–crank mechanism with two clearance joints is used as an example to demonstrate the correctness and effectiveness of the proposed approach. Finally, the effects of joint clearance on the dynamic characteristics of planar multibody systems are investigated.

Keywords Multibody · Systems · Joint clearance · Contact force · Dynamic characteristics

1 Introduction

In general, the revolute joint is assumed as an ideal pair joint without clearance and friction in the dynamic characteristics analysis of mechanism. However, the appearance of fabrication and design tolerances will cause the dynamic behavior changes, such as noise, vibration and fatigue failure. This is a typical problem in multibody systems with revolute clearance joints, which can be one of the most important applications. Then, one of this problem is how to define a suitable contact law for describing the contact–impact process [1–3]. The geometric parameter, material properties and energy loss should be considered in the contact force model. And, some contact parameters, namely the contact stiffness, damping coefficient and friction coefficient are evaluated by experiment and analysis. Moreover, the critical point is how to present the contact–impact events of revolute joint with clearances in the multibody systems during the motion of mechanism, which will effects the accuracy of dynamic analysis for the multibody systems [4–6]. Therefore, it is given that the revolute clearance joint plays a significant role in the design and analysis of multibody systems. Then, we should develop an appropriate approach for eliminating the undesirable influence.

During the past years, many researchers have paid attention to the modeling and numerical calculation of mechanism with revolute clearance joints, which mainly includes the modeling technique of contact force and the analytical method of multibody systems [7, 8]. In the first technology, Lankarani and Nikravesh [9] developed the continuous contact force model for describing the contact–

✉ Qiang He
heqiang@just.edu.cn

Yu Chen
chenyunjust@126.com

1 School of Mechanical Engineering, Jiangsu University of Technology, Changzhou 213001, China

2 National Key Laboratory of Transient Physics, Nanjing University of Science and Technology, Nanjing 210094, China

3 School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China

4 School of Mechanical Engineering, Jiangsu University of Science and Technology, Zhenjiang 212000, China

impact behavior of the contact elements, which was established by the Hertz contact law. This method considered the effects of energy loss on the contact–impact process and introduced the hysteresis damping function. With the actual joint taken into account, Bauchau and Rodriguez [10] presented a method for modeling the clearance joint, which included the clearance, lubrication and friction. The model was developed based on the energy preserving law and the effectiveness could be validated by the numerical example. Moreover, Pérez-González [11] built a modified elastic contact model based on the Hertz model and finite element method. This methodology could carry out the pressure distribution and contact area with accuracy and short computation times for interfaces of contact elements. The contact detection played a significant role in the simulation of multibody systems in the instant of contact–impact. Flores [12, 13] developed a comprehensive approach to calculate this point, which had the characteristics of automatically adjusting the time step with the variable time-step integration algorithms. The demonstration case provided a support discussion for the computational efficiency and availably to deal with the nonlinear contact of presented method. The issue that how to select a suitable model for a given contact problem was still discussed. On account of the LuGre friction law, the model for the stick–slip friction between the joint elements was extended by Muvengei [14], which could illustrate the mapping relationship between the friction force and slip velocity. And, the influence of stick–slip friction on the overall dynamic behavior of the multibody systems with different speeds was discussed. The contact–impact phenomenon appeared in the engineering problem, and the methods of impulse–momentum and compliance could provide sustainability for this issue. Bai and Zhao [15, 16] introduced a new contact force model of revolute clearance joint, which compared with the Lankarani–Nikravesh model and the improved elastic foundation model. The obtained results illustrated that the ratio of nonlinear stiffness coefficient and contact stiffness was improved by the modified elastic foundation model and damping theory. The nature of constitutive contact force law was employed as a description to illustrate the contact–impact events between contact elements. Wang and Li [17] performed an approach for the modeling of the revolute clearance joint between two rods. To investigate the nonlinearity produced by the contact–impact phenomenon, the dynamic characteristics analysis of the mechanism was conducted by the averaging methodology. And then, the influences of parameters of clearance size on the dynamic response of multibody systems were illustrated. Moreover, Qi [18] investigated the contact–impact phenomenon of the contact elements in the multibody systems of bearing. For the complex contact situations and massive description parameters, the stable contact situation was used to confirm the relationship between contact force and joint reaction force. And then, the contact force model could be established without the releasing the kinematic constraints of joints. Ma [19] studied a new method for describing the relationship between the contact elements of multibody systems, which was established based on the L–N contact force model and the elastic contact law. The evaluation of contact process between contact elements could be obtained by the discrete element theory and Gaussian quadrature. The application case was conducted to illustrate that the new method for modeling contact force was more suitable for discussing the situation with a smaller clearance size and a lower restitution coefficient. Furthermore, Tian [20, 21] developed a new dynamic spherical joint model for multibody systems in the unified global coordinate systems, which had the characteristics of elastic and lubrication. The proposed novel lubricant finite-difference grid rotation method could eliminate the problem of lubrication interface non-conformance between fluid and solid. This methodology could be extended to engineering application field. In addition, some researchers extended related experimental studies [22, 23]. Haines [24] performed an experimental study for investigating the dynamic response of revolute joints with varying degrees of clearance. In this study, the results revealed that the deflections associated with contact elasticity of bearing were found to be much greater and less linear than predicted. And then, Bouyer and Fillon [25] carried out an experimental analysis for the influence of hydrodynamic characteristics on the performance of revolute joint. The motion relationship of the contact elements in the different driving velocities could be illustrated by the testing results. To study the friction behavior better, Xu [26] focused on a new analysis method for evaluating the friction of a ball screw; the model was established based on the roll contact theory. A relative experiment was used as a validation to demonstrate the effectiveness of the model, which provided a new perspective for the modeling of friction force. Compared with an industrial case, Akhadkar [27] devoted to build a dynamic analysis model for illustrating the dynamic response of mulitibody systems with joint clearance. Based on the event-capturing time-stepping scheme with a second-order cone complementarity solver, this approach could be used to achieve the numerical. The validation experiment data revealed the effectiveness and accuracy of the proposed method.

It is important to note that many researchers mainly focus on the dynamic analysis of multibody systems with revolute clearance joints [28, 29]. Schwab [30] proposed a comparison study for the influences of flexible element on the dynamic characteristics of mechanism with clearance joint. The simulation models were established with rigid and elastic elements, respectively. The dynamic response of contact–impact process could be evaluated by this method. Then, Flores [31, 32] applied a computational method for dynamic simulation of the slider–crank mechanism with clearance joint. The application was employed as an example for illustrating the effect of clearance on the dynamic characteristics. And, the center trajectory of journal showed the different motion modes between journal and bearing. The effects of clearance joint on the dynamic characteristics of mechanism was conducted by Muvengei and Kihiu [33]. The slider–crank mechanism was employed as a demonstration case for studying the clearance size and driving speed influenced on the dynamic behavior of planar multibody systems. In addition, a new methodology for searching the trajectory optimization of a mechanical system with clearance joint was developed by Erkaya [34]. In this study, the adaptive network-based fuzzy inference system (ANFIS) was employed to represent the features of clearance joint. Although the clearance size was small, it was clear that the track was very sensitive to the joint clearance. With the uncertain clearance joint taken into account, the nonlinear dynamic behavior of a flexible mechanism was conducted by Tian [35, 36]. The kinetic model of revolute clearance joint was established by the absolute nodal coordinate formulation. And then, the effects of the LundGrenoble and modified Coulomb's friction model on the dynamic characteristics of mechanism were discussed in the work. With the effects of mixed lubrication taken into account, Zhao [37] presented a new approach for investigating the planar multibody systems with clearance joint. The dynamics equations were obtained by Lagrange's method and the dynamic analysis model of the mechanism systems was computed by finite element method. The numerical results of the mixed lubricated revolute joint may reduce the friction power and vibration phenomenon. The wear phenomenon always appeared in the mechanical systems with revolute joint, which might induce clearance and non-circularity. To obtain the dynamic analysis results of mechanism accurately, Xu and Han [38] made a general method for researching the dynamic characteristics of mechanism with clearance joint. Compared with traditional methods, the presented approach could be employed to discuss the influence of noncircular contact on the revolute joint. Meanwhile, Ma and Qian [39] represented a new approach for modeling and analysis of mechanical systems with multiple revolute clearance joints. The contact force model was acquired based on the Lankarani–Nikravesh model and elastic foundation model, and the LuGre friction model was applied in the friction force analysis. The numerical results illustrated that the strong dynamic interaction of contact elements appeared and the number of revolute clearance joint was considered in the dynamic analysis of mechanical systems. Tan [40] implemented a continuous analysis methodology of the multibody systems with revolute clearance joints. The modified contact force model and improved friction force model were introduced in Newton–Euler equations. The numerical results showed that the mutual coupling region was easily found because of the existing of two clearance joints. Based on the Nodal coordinate formulation and absolute nodal coordinate formulation, Li [41] investigated a new approach for modeling the rigid–flexible coupling dynamic model of mechanical systems. The Newmark-β method was employed for calculating the motion equations of multibody systems. Then, the influence of clearance number, clearance size and clearance stiffness on the mechanism was studied. Furthermore, the experimental investigations of multibody systems with clearance joints have attracted the attention of researchers [42, 43]. For the transmission systems with clearance, Crowther [44] developed the free vibration experiment for analysis of the influences of clearance on the dynamic response of mechanical systems. The results could be contributed to improve the investigation of dynamic characteristics for the mechanical systems with clearances. In general, compliant mechanism was employed in the engineering equipment, which had the flexible member between conventional rigid links. Flores and Koshy [45] proposed a computational and experimental study on the dynamic characteristics of mechanism with joint clearances. The experimental test provided the validation that the revolute clearance joint could play an important role on the dynamic characteristics of the mechanism with clearance joint. The main reason for wear between journal and bearing was the presence of clearance. For the problem of wear, Lai [46] implemented the calculate method for revolute joint wear of a multibody systems with a lower driving speed. The measurement of wear demonstrated the validation of the presented approach and the method might provide a higher prediction accuracy.

In contrast to the extensive investigations on the dynamic analysis of multibody systems with clearance joint, devoting to the industrial case investigation with the experimental validation is more limited. In the work, we developed a suitable method to indicate the dynamic behavior of multibody systems considering the effects of revolute joint with clearances. Based on the geometric relationship between pin and hole, the kinematics of the multibody systems is mapped into the global coordinate systems. And, the penetration value of impact effects on the extra displacements against the clearance size is modeled as the relative position in the revolute joint with clearance. Then, the mathematic model of multibody systems can be obtained conveniently as an accurate and complete description for the accuracy dynamic characteristics of the mechanism, from which not only the maximum elastic deformation and corresponding orientation of centers but also the coupling relationship of them can be derived. In addition, the accuracy analysis model of a planar slider–crank mechanism is established by the proposed methodology and the experiment is conducted as an example to demonstrate the correctness and effectiveness of the presented approach. In the end, the nonlinear dynamic behaviors of a slider–crank mechanism caused by clearance joint are studied. This paper is organized as follows: Sect. 2 represents the model of a planar multibody systems with joint clearance. A case of planar mechanical system and the experimental platform is introduced in Sect. 3. Section 4 shows the effect of joint clearance on the dynamic behavior of multibody systems. Finally, the conclusions are presented in the Sect. 5.

2 Modeling of a planar multibody systems with joint clearance

2.1 Relative motion between pin and hole

The revolute joint is widely employed in the planar multibody systems, which could provide a conversion function between translational motion and rotational motion. The relation motion analysis of revolute joint with clearance is displayed in this section, and the motion is divided into different phases [47]. In conformity with the former investigations, the clearance plays a crucial role in the dynamic analysis of multibody systems. As shown in Fig. 1, the mode of relation motion are free flight mode, continuous contact mode and impact penetration mode. In the global coordinate systems, R_b and R_j describe the radius of hole and pin, and the eccentricity (e) and clearance (c) are important parameters of revolute joint. The motion mode of revolute joint can be described by the difference between pin and hole, and the relative penetration (δ) is given by [48]:

$$\delta = e - c \tag{1}$$

When $\delta < 0$, the motion is free flight mode, and when $\delta = 0$, it is indicated as continuous contact mode. Besides, when $\delta > 0$, the motion is defined as impact penetration mode.

Figure 2 presents that body $k+1$ can rotate with body k with any point within the constraint circle, and the relative position of centers for pin and hole is indicated by eccentricity [49]. Then, Q represents any point on body $k+1$; the relative position of Q is defined as:

$$\begin{cases} X_Q = e\cos(\eta) + l_Q\cos(\theta) \\ Y_Q = e\sin(\eta) + l_Q\sin(\theta) \end{cases}, \tag{2}$$

where η denotes the corresponding orientation of centers, and $\eta \in [0, 2\pi)$. l_Q and θ are the length of point from center of pin and rotation angle, respectively.

As shown in Fig. 2b, the constraint area of point Q can be considered as the envelope with the size of eccentricity (e), and the length of rotation radius is l_Q. According to the envelope theory, it is clearly shown that the envelope is part of constraint area as the rotation angle increases from 0 to 2π. Based on the geometry dimensions, the influence of clearance on the motion of revolute joint can be indicated expediently [50, 51]. Then, the velocity of point Q is written by the time-derivative of position:

$$\begin{cases} \dot{X}_Q = -e\sin(\eta) - l_Q\sin(\theta) \\ \dot{Y}_Q = e\cos(\eta) + l_Q\cos(\theta) \end{cases}. \tag{3}$$

The Eq. 3 is also expressed as:

$$\begin{aligned} \begin{Bmatrix} \dot{X}_Q \\ \dot{Y}_Q \end{Bmatrix} &= [K_Q] \cdot \begin{Bmatrix} \dot{\eta} \\ \dot{\theta} \end{Bmatrix} \\ &= \begin{bmatrix} -e\sin(\eta) & -l_Q\sin(\theta) \\ e\cos(\eta) & l_Q\cos(\theta) \end{bmatrix} \cdot \begin{Bmatrix} \dot{\eta} \\ \dot{\theta} \end{Bmatrix}, \end{aligned} \tag{4}$$

where $[K_Q]$ denotes the matrix of velocity coefficients of pin.

The derivative of Eq. 4 gives the acceleration of point Q, which can be represented as:

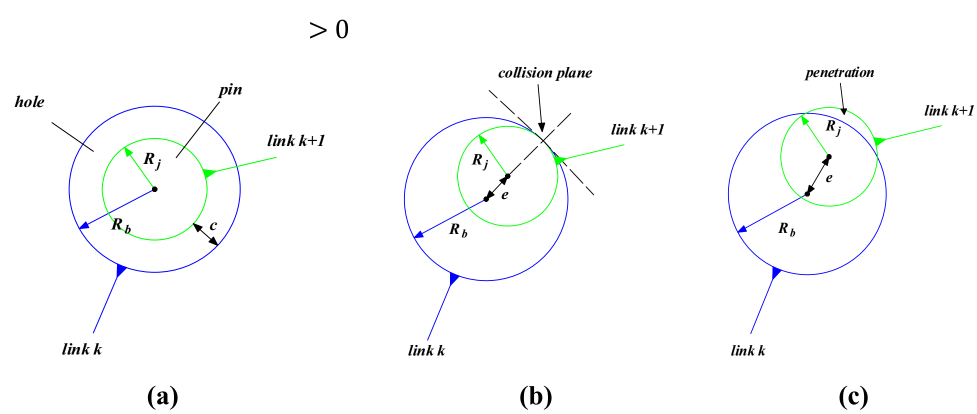


Fig. 1 The different phases of revolute joint with clearance: **a** free flight motion; **b** continuous contact motion; **c** Impact penetration motion

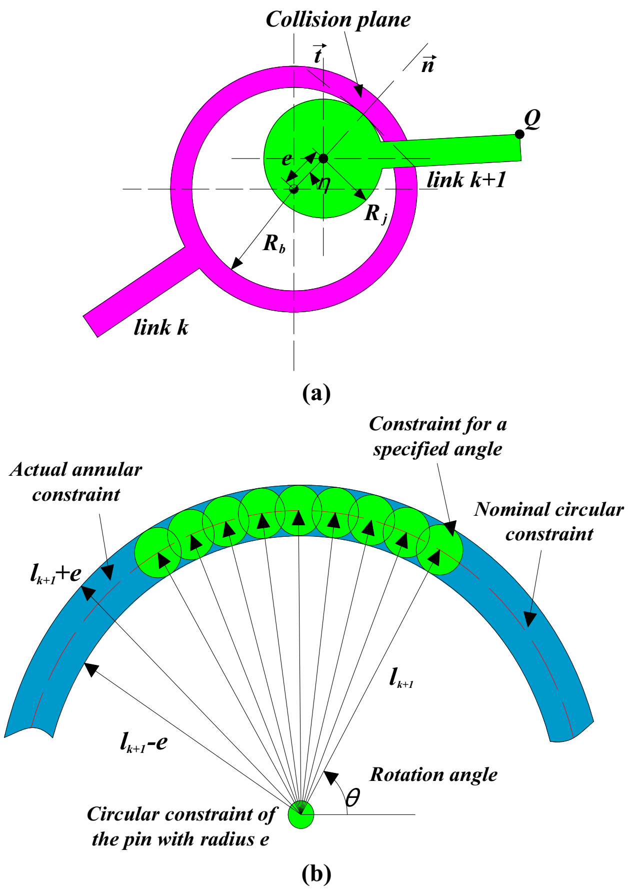


Fig. 2 Kinematic modeling for the revolute joint with clearance: **a** revolute joint with clearance; **b** motion constraint

$$\begin{Bmatrix} \ddot{X}_Q \\ \ddot{Y}_Q \end{Bmatrix} = [K_Q] \cdot \begin{Bmatrix} \ddot{\eta} \\ \ddot{\theta} \end{Bmatrix} + \dot{\eta} \cdot [L_{\eta Q}] \cdot \begin{bmatrix} \dot{\eta} \\ \dot{\theta} \end{bmatrix} + \dot{\theta} \cdot [L_{\theta Q}] \cdot \begin{Bmatrix} \dot{\eta} \\ \dot{\theta} \end{Bmatrix}, \tag{5}$$

where $[L_{\eta Q}]$ and $[L_{\theta Q}]$ represent the partial derivative matrices of velocity coefficient for pin, which can be described as:

$$[L_{\eta Q}] = \frac{\partial}{\partial \eta} \cdot [K_Q] = \begin{bmatrix} -e\cos(\eta) & 0 \\ -e\sin(\eta) & 0 \end{bmatrix}, \tag{6}$$

$$[L_{\theta Q}] = \frac{\partial}{\partial \theta} \cdot [K_Q] = \begin{bmatrix} 0 & -l_Q\cos(\theta) \\ 0 & -l_Q\sin(\theta) \end{bmatrix}. \tag{7}$$

2.2 Mathematic model of planar mechanism with joint clearance

The purposes of kinematic analysis for planar mechanism are to provide theoretical basis for the investigation of dynamic behavior, and the joint clearance should be considered during the analysis of dynamic response, vibration and noise characteristics of mechanism. Then, the slider–crank mechanism has the characteristics of two revolute joints with clearances, as shown in Fig. 3. In the kinematic analysis of these multibody systems, it is necessary to ensure the position of mass center for link. And, the velocity and acceleration of mass center can be obtained by the time-derivatives of position [52, 53]. According to Fig. 3, the positions of mass centers can be written as:

$$\begin{Bmatrix} X_{m1} \\ Y_{m1} \end{Bmatrix} = \begin{bmatrix} l_{Om_1}\cos(\theta_1) \\ l_{Om_1}\sin(\theta_1) \end{bmatrix}, \tag{8}$$

$$\begin{Bmatrix} X_{m2} \\ Y_{m2} \end{Bmatrix} = \begin{bmatrix} l_{OA}\cos(\theta_1) \\ l_{OA}\sin(\theta_1) \end{bmatrix} + \begin{bmatrix} e_A\cos(\eta_2) \\ e_A\sin(\eta_2) \end{bmatrix} + \begin{bmatrix} l_{A_1m_2}\cos(\theta_2) \\ l_{A_1m_2}\sin(\theta_2) \end{bmatrix}, \tag{9}$$

$$\begin{Bmatrix} X_{m3} \\ Y_{m3} \end{Bmatrix} = \begin{bmatrix} l_{OA}\cos(\theta_1) \\ l_{OA}\sin(\theta_1) \end{bmatrix} + \begin{bmatrix} e_A\cos(\eta_2) \\ e_A\sin(\eta_2) \end{bmatrix} + \begin{bmatrix} l_{A_1B_1}\cos(\theta_2) \\ l_{A_1B_1}\sin(\theta_2) \end{bmatrix} + \begin{bmatrix} e_B\cos(\eta_3) \\ e_B\sin(\eta_3) \end{bmatrix}, \tag{10}$$

where l_i and e_i represent the length and eccentricity, respectively.

The velocity and acceleration of mass center can be obtained from the derivative of position equations:

$$\begin{Bmatrix} \dot{x}_{m_i} \\ \dot{y}_{m_i} \end{Bmatrix} = \begin{Bmatrix} \sum_{p=1}^{3} \dot{\theta}_p \frac{\partial x_i}{\partial \theta_p} + \sum_{q=2}^{3} \dot{\eta}_q \frac{\partial x_i}{\partial \eta_q} \\ \sum_{p=1}^{3} \dot{\theta}_p \frac{\partial y_i}{\partial \theta_p} + \sum_{q=2}^{3} \dot{\eta}_q \frac{\partial y_i}{\partial \eta_q} \end{Bmatrix}, \tag{11}$$

$$\begin{Bmatrix} \ddot{x}_{m_i} \\ \ddot{y}_{m_i} \end{Bmatrix} = \begin{Bmatrix} \sum_{p=1}^{3} \ddot{\theta}_p \frac{\partial x_i}{\partial \theta_p} + \sum_{p=1}^{3} \dot{\theta}_p^2 \frac{\partial^2 x_i}{\partial \theta_p^2} + \sum_{q=2}^{3} \ddot{\eta}_q \frac{\partial x_i}{\partial \eta_q} + \sum_{q=2}^{3} \dot{\eta}_q^2 \frac{\partial^2 x_i}{\partial \eta_q^2} \\ \sum_{p=1}^{3} \ddot{\theta}_p \frac{\partial y_i}{\partial \theta_p} + \sum_{p=1}^{3} \dot{\theta}_p^2 \frac{\partial^2 y_i}{\partial \theta_p^2} + \sum_{q=2}^{3} \ddot{\eta}_q \frac{\partial y_i}{\partial \eta_q} + \sum_{q=2}^{3} \dot{\eta}_q^2 \frac{\partial^2 y_i}{\partial \eta_q^2} \end{Bmatrix}, \tag{12}$$

where $\dot{x}$ and $\ddot{x}$ denote the velocity and acceleration values of corresponding parameters (i = 1, 2, 3).

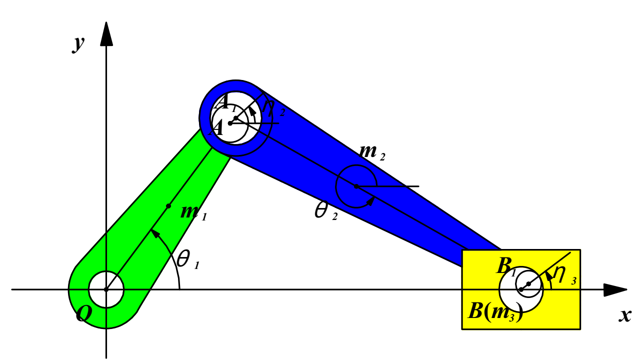


Fig. 3 The planar slider–crank mechanism with joint clearance

2.3 Numerical models for contact forces

It is known that an appropriate contact force model during the contact–impact event plays an important role in the dynamic analysis of planar mechanism with joint clearance, which could provide the improvements of evaluation. On the one hand, the impact velocity, material properties and geometry characteristics should be considered in the constitutive law of contact force model [54]. On the other hand, the integration of the motion equations of multibody systems is done in the numerical calculation process of contact force. Considering the effect of energy loss, the Lankarani–Nikravesh model is widely employed in the dynamic characteristics analysis of planar mechanism with joint clearance. The normal contact force is separated into the elastic and dissipative components. Moreover, different contact force models could be compared by Flores [55], and the results reveal that the Lankarani–Nikravesh model is more effective to depict the dynamic behavior of the contact–impact process [56]. Then, the Lankarani–Nikravesh model is applied to describe the contact–impact of joint clearance, which is written as:

$$F_N = K\delta^n\left[1 + \frac{3\left(1 - c_e^2\right)\dot{\delta}}{4\dot{\delta}^{(-)}}\right], \tag{13}$$

where n represents the exponent of metallic contacts, the value of n is 1.5. $\dot{\delta}^{(-)}$ and c_e are the initial impact velocity and restitution coefficient, respectively. The maximum value of δ is 0.01 mm. The generalized parameter K can be defined based on the material properties and radius of contact element, which is expressed as:

$$K = \frac{4}{3\pi(\sigma_1 + \sigma_2)}\left[\frac{R_b R_j}{R_b + R_j}\right]^{\frac{1}{2}}, \tag{14}$$

where R_b is the radius of bearing and R_j denotes the radius of journal, respectively. The material parameter σ_i is given by:

$$\sigma_i = \frac{1 - v_i^2}{\pi E_i}\ (i = 1, 2), \tag{15}$$

where v_i is the Poisson's ratio and E_i is the Young's modulus of each sphere.

It is important to note that the Coulomb's friction law is the most fundamental and simplest model of friction on the dry contacting surfaces. The Coulomb's friction law can be independent of relative tangential velocity. Due to the presence of friction phenomenon, the different friction regimes appear on the surfaces of contacted bodies, such as sliding and sticking [57, 58]. Then, to avoid numerical difficulties, a modified Coulomb's friction law can be employed:

$$F_T = -c_f c_d F_N \frac{v_T}{||v_T||}, \tag{16}$$

where c_f and v_T are the friction coefficient and relative tangential velocity, respectively. c_d represents the dynamic correction coefficient, which is defined as:

$$c_d = \begin{cases} 0 & \text{if } ||v_T|| \le v_0 \\ \dfrac{v_T - v_0}{v_1 - v_0} & \text{if } v_0 \le ||v_T|| \le v_1 \\ 1 & \text{if } ||v_T|| \ge v_1 \end{cases}, \tag{17}$$

where v_0 and v_1 represent the tolerances for the tangential velocity of surfaces in contact. And, the dynamic correction factor c_d denotes that the friction force changes direction in the presence of almost null values of the tangential velocity, which is perceived by the integration algorithm as a dynamic response with high-frequency contents, forcing it to reduce the time-step size.

2.4 Multibody systems dynamic model

In the Cartesian coordinates, the position and orientation of a body reference frame can be defined [59, 60]. And, the position and orientation of rigid body i can be written as:

$$\mathbf{q}_i^* = \left[\mathbf{r}_i^T \mathbf{p}_i^T\right]^T, \tag{18}$$

where $\boldsymbol{r}_i = [xyz]^T$ denotes the translation coordinate. $\mathbf{p}_i = [e_0 e_1 e_2 e_3]^T$ is the rotational coordinates, which contains Euler parameters. The angular velocity $\boldsymbol{\omega}_i'$ and angular acceleration $\dot{\omega}_i'$ are employed instead of time-derivatives of the Euler parameters in the numerical calculation process. Then, the velocity and acceleration of body i are expressed by:

$$\dot{\mathbf{q}}_i = \left[\dot{\mathbf{r}}_i^T \boldsymbol{\omega}_i'^T\right]^T, \tag{19}$$

$$\ddot{\mathbf{q}}_i = \left[\ddot{\mathbf{r}}_i^T \dot{\boldsymbol{\omega}}_i'^T\right]^T. \tag{20}$$

For a constrained multibody system, the algebraic kinematic independent holonomic constraints are evaluated as:

$$\boldsymbol{\Phi}(q, t) = 0. \tag{21}$$

With the constraint equations and differential equations of motion taken into account, the unique solution can be obtained in the dynamic analysis, for a proper set of initial conditions. Then, based on the second time derivative of the constraint Eq. (21) and Lagrange multipliers technique, the motion equation of multibody systems can be written as:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \gamma \end{Bmatrix}, \tag{22}$$

where $\mathbf{M}$ denotes the global mass matrix and $\mathbf{g}$ is the force vector that contains the external and Coriolis forces acting on the bodies of this systems. $\boldsymbol{\lambda}$ and γ are vector of Lagrange multipliers and vector of quadratic velocity terms, that is,

$$\gamma = -\left(\boldsymbol{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}\right)_{\mathbf{q}}\dot{\mathbf{q}} - \boldsymbol{\Phi}_{tt} - 2\boldsymbol{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}}. \tag{23}$$

Based on the Baumgarte stabilization technique, Eq. 22 is rewritten as:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \gamma - 2\alpha\dot{\phi} - \beta^2\boldsymbol{\Phi} \end{Bmatrix}, \tag{24}$$

where α and β denote positive constants that represent the feedback control parameters for the velocity and position constraint violations. The interested reader can refer to Flores [61] for further details on the formulation used and to Baumgarte [62] for details on the stabilization procedures.

3 Application case: a planar slider–crank mechanism

3.1 Description of the slider–crank mechanism

The planar slider–crank mechanism is chosen as an example to illustrate the effects of joint clearance on the dynamic characteristics of multibody systems. The similar examples have been employed by researchers, which could provide references for this study. This mechanism, as shown in Fig. 4, is the transmission mechanism of the high-speed press systems, and this mechanical systems can be defined as a planar slider–crank mechanism. The lengths of crank l_1 and connecting rod l_2 are 0.025 m and 1.02 m. And, the mass of crank m_1, connecting rod m_2 and slider m_3 are given as 453.87 kg, 2000.55 kg and 21,594.71 kg, respectively. Material properties consists of the Young's modulus $E = 210$ GPa, density $\rho = 7960\,\mathrm{kg/m^3}$, the Poisson's ratio $v = 0.3$, the restitution coefficient of $c_e = 0.95$, and friction coefficient $c_f = 0.01$. The clearance exists for revolute joint between the crank and connecting rod, and other one is defined between connecting rod and slider. Then, the clearance value of revolute joint is chosen to $c = 0.1$ mm and the maximum rotational speed of crank is given by $\omega = 200$ rpm. Based on the real engineering data, the parameter values of numerical simulation can be defined.

3.2 Experimental set-up

To investigate the dynamic characteristics of a slider–crank mechanism with revolute clearance joints and validate the availability of proposed numerical simulation method, the experimental platform is designed and constructed [63]. The experimental platform consists of a slider–crank mechanism and measurement systems, as shown in Fig. 5. Meanwhile, the experimental platform allows that the slider–crank mechanism operates with the variable rotational speed and the measurement systems is equipped with a variety of wireless sensor, which can provide the safely testing and eliminate the restriction of testing conditions, such as temperature, distance and noise. To improve the precision of experimental, a locking device is installed in this mechanism, which assures the clearance size of journal and bearing. Furthermore, the measurement systems consist of data acquisition systems and data analysis systems. In the data acquisition systems, the acceleration sensor is connected with the data receiver, and the testing data are transmitted to PC. Then, the transformation, filtering, de-noising and extraction of testing data are conducted in the BeeTech software platform, which is installed into PC. It is worth mentioning that the acceleration sensor should be calibrated before the experimental.

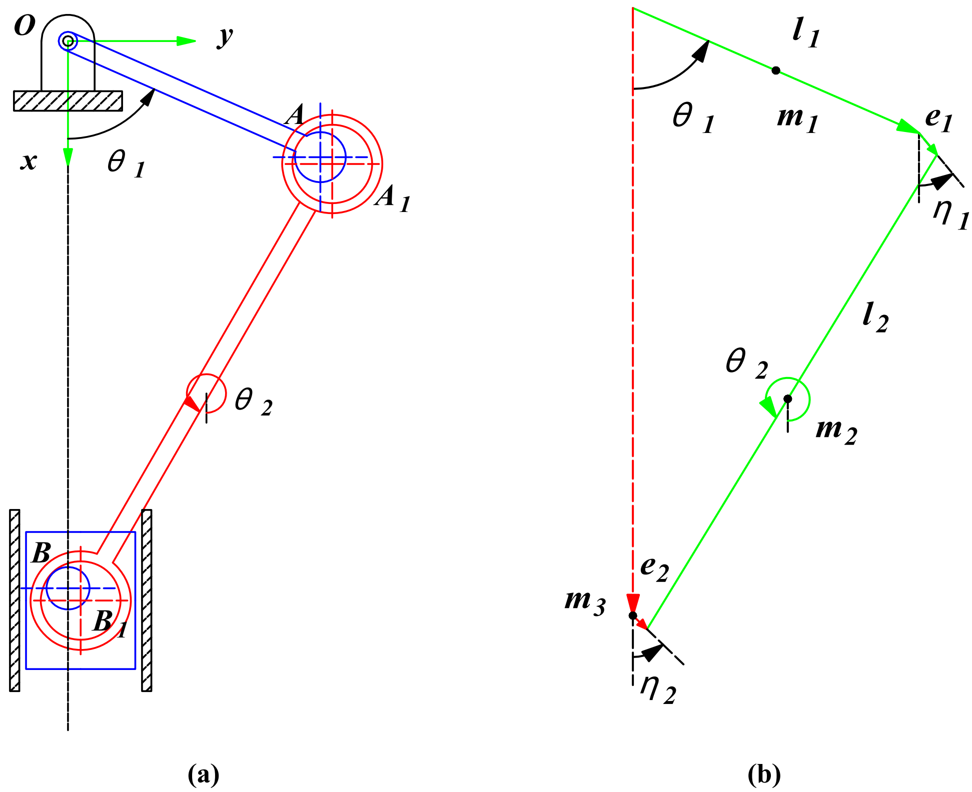
4 Results and discussion

In this section, the dynamic behavior of a mechanism with revolute clearance joints is studied based on the proposed methodology. In the first case, the planar slider–crank mechanism with clearance joints is employed as an application to demonstrate the effectiveness of the presented calculation method in the work. Meanwhile, the effects of driving velocities on the dynamic characteristics of the slider–crank mechanism are conducted in this case. In addition, the numerical calculation of other case could illustrate that how the clearance size affects the dynamic response of these multibody systems. And, the crank, connecting rod and slider are defined as rigid bodies.

4.1 Simulation and validate with different crank driving velocities

The purpose of this investigation is to illustrate the relationship between driving velocities and dynamic response of the slider–crank mechanism. The initial condition considers that the centers of journal and bearing are assumed to coincident, the initial position and angular velocities of centers are defined as zero. The driving velocities are 50 rpm, 100 rpm, 150 rpm and 200 rpm, respectively. It is

Fig. 4 The exaggerated model of slider–crank mechanism with clearance joints: **a** schematic representation; **b** vector representation



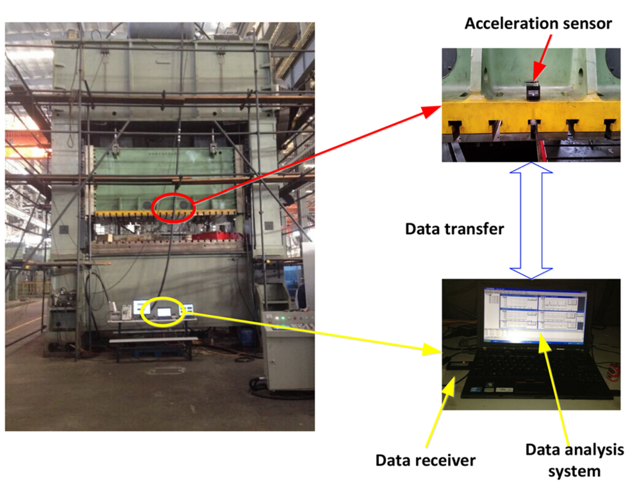


Fig. 5 The representation of experimental platform

important to note that the results of numerical calculation for slider–crank mechanism can be obtained after the steady state has been reached.

Figure 6 represents the motion trajectories of slider as the change of driving velocities. As shown in Fig. 6a, the displacements of slider keep a similar path with different driving velocities, which are relatively smooth. It is shown that when the increase of rotational speeds, the phenomenon of fluctuation appears in the position curve of slider. Although the value of fluctuation is smaller, the results also could reveal the influences of revolute joint clearance on the stability of this mechanism. As the performance index, the position of slider can be used to evaluate the machining accuracy of high-speed press systems.

Figure 7a displays the change trend of velocities curve for slider, which shows that the maximum value of velocity increases obviously with the growth of driving velocities. However, the difference of results for joint with clearance and ideal joint increases slowly with the increase of driving velocities. Then, the value of velocity plays an important role on the dynamic analysis of mechanical systems, which could describe the relationship between stability and efficiency of slider–crank mechanism. Furthermore, Fig. 7b shows the effects of revolute joint clearance on the velocities of slider, which cause a wave shape for the velocity versus rotational angle. Then, this phenomenon reveals that the journal could freely move inside the bearing boundaries at a constant velocity. The fluctuation phenomenon can be illustrated the existence of revolute joint clearance.

Figure 8 shows a comparison of the slider acceleration and experimental data with the different driving velocities, combined with the presented methodology. The effects of joint clearance on the acceleration of slider are exhibited in the numerical results. The difference of acceleration fluctuation between actual results and ideal state is smaller at a

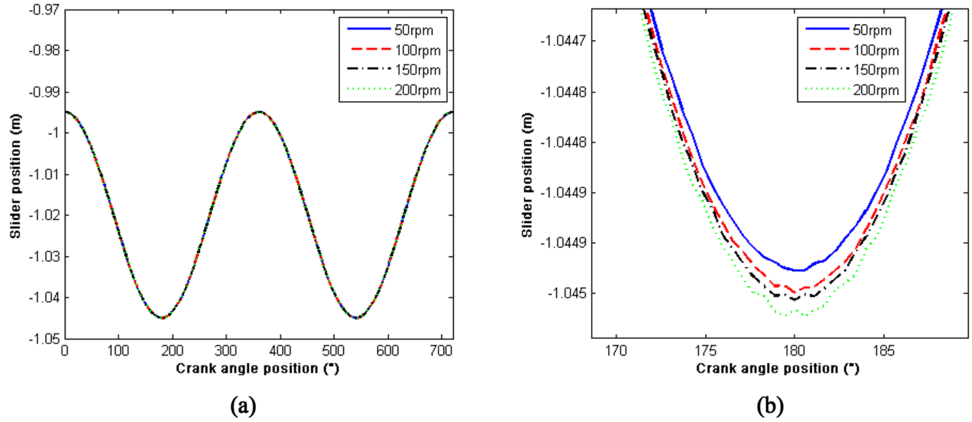


Fig. 6 Displacement of slider with different driving velocities: **a** the displacement representation, **b** the enlarged representation

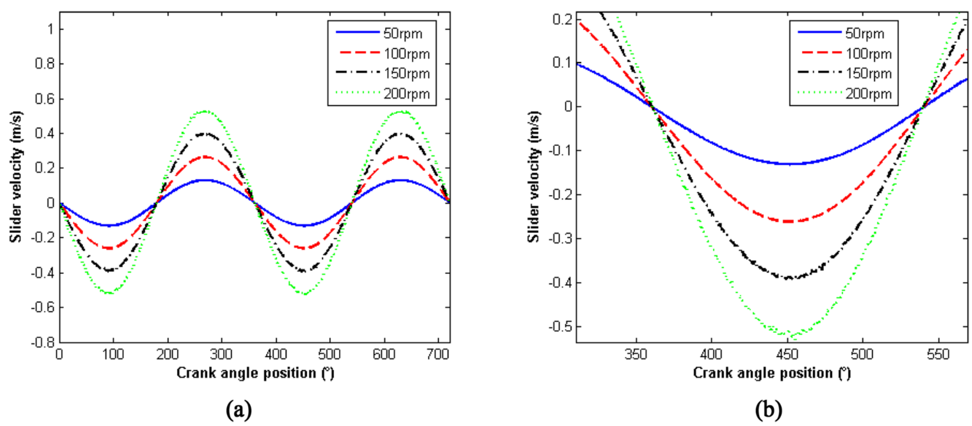


Fig. 7 Velocity of slider with different driving velocities: **a** the velocity representation, **b** the enlarged representation

lower rotational speed, which could conclude that the acceleration of slider is not greatly affected by clearance joints. However, the values of fluctuation increase obviously as the driving speed exceeds 100 rpm. The account of this phenomenon can be explained that the increase of driving velocity improves the contact velocity and contact–impact force, and a larger fluctuation appears in this condition. Furthermore, the numerical simulation results based on the proposed method agree quite well with experimental data with the different rotational speeds, which illustrates that the proposed method can be used as a reasonable approach to describe dynamic behavior of multibody systems with revolute clearance joints.

4.2 Clearance size affected analysis

This section investigates how the joint clearance size affects the dynamic characteristics of a planar–crank mechanism, i.e., the rotational speed of crank is defined as a constant value equal to 100 rpm during the simulation. The simulations of this mechanism are performed with different clearance sizes, e.g., 0.1 mm, 0.2 mm, 0.5 mm and 1 mm. Then, the numerical results are presented against the values obtained for ideal joint, and the results are painted for the two full rotational periods.

The displacement value of the crank–slider mechanism with the clearance joints in the different working conditions is shown in Fig. 9. When the contact–impact phenomenon occurs during the motion process of these mechanical systems, the fluctuation peak value tends to be comparable to what is observed in an ideal joint. These peaks are associated with the contact–impact forces developed during the contact between the journal and bearing. As the clearance size increases, the influences of clearance on the displacement of the slider–crank mechanism are severed. A larger clearance size will cause a higher amplitude of fluctuation, which is consistent with the conclusion of other published paper. The results reveal that the existence of revolute joint clearance has a crucial influence on the displacement of slider, which is taken into account during the process of analysis and design for a real mechanism.

The appearance of joint clearance will cause the sudden changes of velocity curves for the crank–slider mechanism. Figure 10 shows the variations of the slider velocity for these mechanical systems with four different clearance sizes. The amplitudes of slider velocity at the moments of

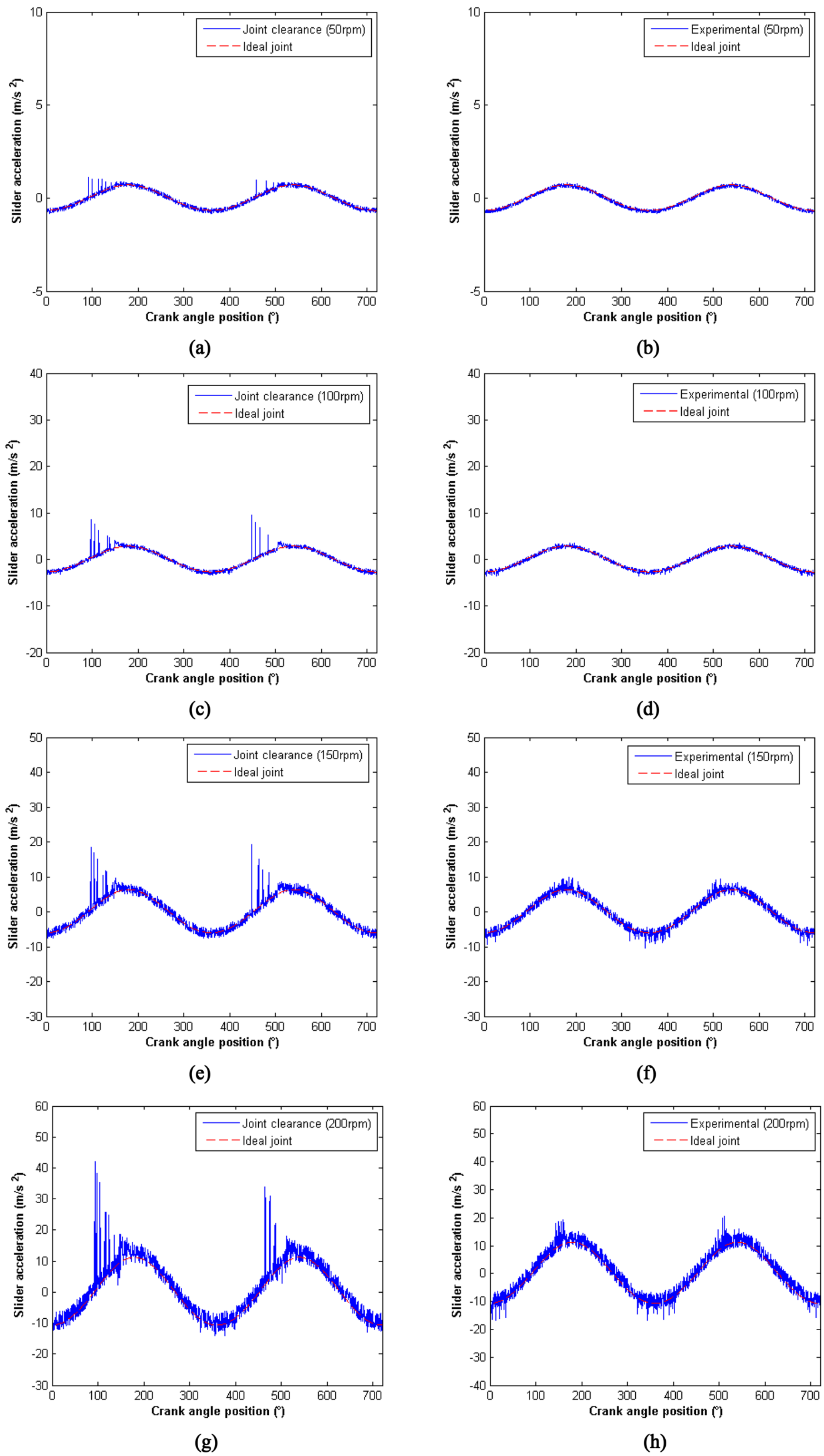


Fig. 8 Simulation and experimental of slider acceleration with different driving velocities: **a**, **b** 50 rpm; **c**, **d** 100 rpm; **e**, **f** 150 rpm; **g**, **h** 200 rpm. **a**, **c**, **e**, **g** are simulation results; **b**, **d**, **f**, **h** are experimental data

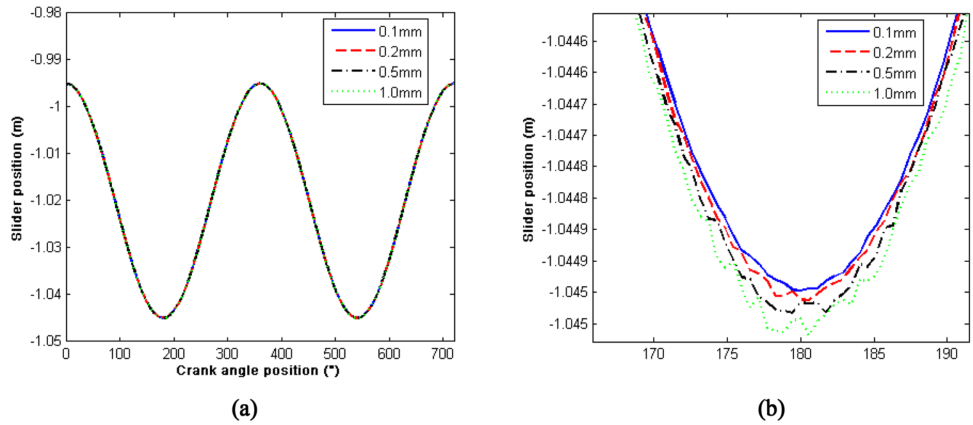


Fig. 9 Displacement of slider with different clearance sizes: **a** the displacement representation, **b** the enlarged representation

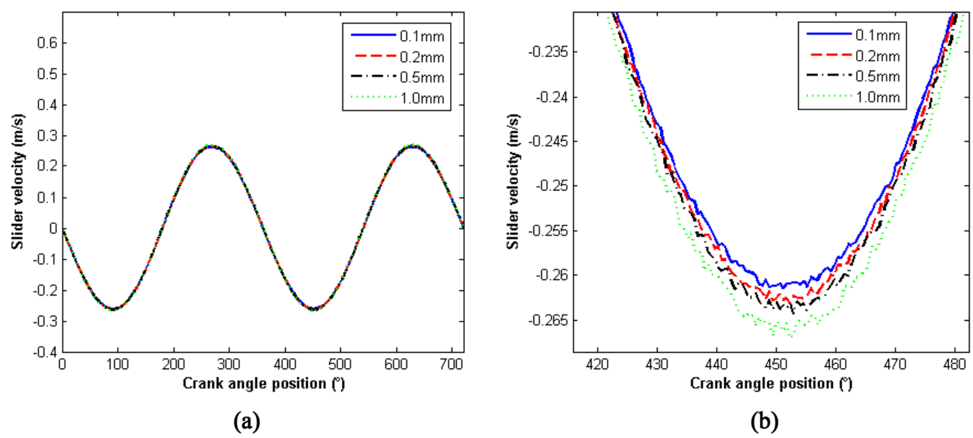


Fig. 10 Velocity of slider with different clearance sizes: **a** the velocity representation, **b** the enlarged representation

separation and collision of joint clearance element are displayed in the simulation results. The velocity fluctuation amplitude caused by contact–impact represents greater variation. By comparison, the bigger the clearance size, the higher the fluctuation amplitude caused by contact–impact force. The similar phenomenon is appeared in the profile of slider displacement displayed by Fig. 9. This is an accurate dynamic response of continuous contact for the revolute joint with clearances in the mechanical systems. The result could describe the interesting phenomenon, which illustrates the influence of clearance size on the stationarity of mechanical systems.

Figure 11 draws the evolution of acceleration of slider obtained in the simulations, and the results are given with the different clearance sizes in the steady state. Due to the contact–impact phenomenon between journal and bearing, it is clear that the acceleration is more sensitive to the clearance size, and it also indicates that the existence of joint clearance may cause impact dynamic load. As shown, there are peak values of acceleration appearing during the mechanism motion. The extreme value of acceleration variation is originated from the higher values of contact–impact force. As an example, when the clearance size is defined to 0.1 mm, the maximum acceleration is only 9.511 m/s^2. As the clearance size reaches 1 mm, the value rises to 17.47 m/s^2. Especially, the mechanism with a bigger clearance has a larger vibration relative to a smaller clearance in the same working condition. In addition, the results reveal that the presented method is very effective to conduct the dynamic analysis of clearance characteristics.

5 Conclusions

This paper proposes a general methodology for modeling and evaluating the dynamic characteristics of planar multibody systems considering the revolute clearance joints, and performs the relevant experimental verification. Theoretical analysis and numerical calculation are conducted and the conclusions can be summarized as follows:

1. A method for nonlinear dynamic analysis of a planar multibody systems with revolute clearance joints is presented. Considering the effects of energy loss, the contact force model is built based on the continuous contact law. Moreover, the motion equations of mechanism are established with some nonlinear characteristics, namely geometric relationship between

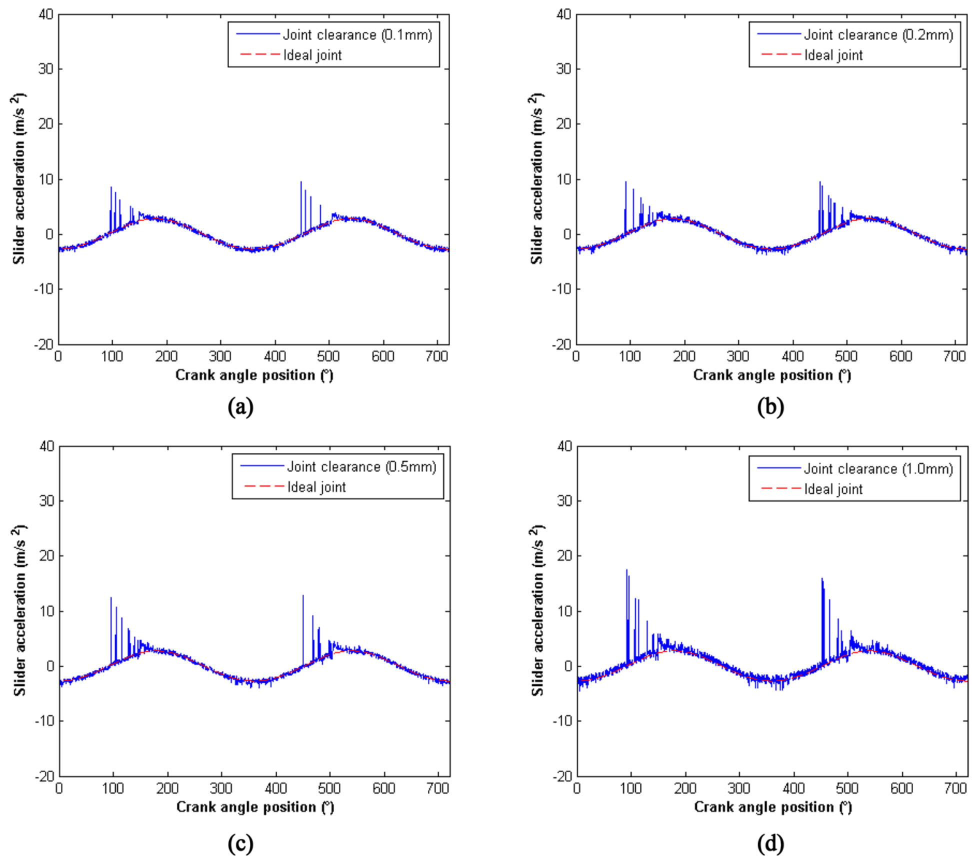


Fig. 11 Acceleration of slider with different clearance sizes: **a** 0.1 mm; **b** 0.2 mm; **c** 0.5 mm; **d** 1.0 mm

contact elements, material properties and contact–impact characteristics. Then, the presented approach could describe the nonlinear problems of multibody systems with clearance joints accurately.

2. Taking a slider–crank mechanism with revolute clearance joints as an example, a comparative investigation is implemented to investigate the effects of clearance joint on the dynamic behavior of a planar mechanism. For this objective, the experimental test platform is set up and the accelerations of slider are measured. Meanwhile, the numerical simulations of this planar slider–crank mechanism are performed with the same operational condition. The compared results present that the proposed approach is quite easy and effective to implement in a computational program.
3. To achieve better design criteria, the influences of clearance characteristics on the nonlinear dynamic behavior of the slider–crank mechanism are investigated, including driving velocity and clearance size. It is expressly found that the dynamic response of these multibody systems is sensitive to the changes of operation condition. The presented approach could provide the prediction and evaluation into in the actual design of equipment. In the future, the measurement of lubrication performance of journal bearings is developed and the lubrication characteristics will be introduced in the model of multibody system.

Acknowledgments This work is supported by the "National Natural Science Foundation of China (Nos. 51605208 and 11902161)", "Natural Science Foundation of the Jiangsu Higher Education Institutions of China (No. 19KJB460001)" and "The Talent Introduction Foundation of Jiangsu University of Technology (No. KYY18016)".

References

1. Ahmad A, Andersson K, Sellgren U (2014) A stiffness modeling methodology for simulation-driven design of haptic devices. Eng Comput 30(1):125–141
2. Bellary SAI, Samad A, Couckuyt I (2016) Acomparative study of kriging variants for the optimization of a turbomachinery system. Eng Comput 32(1):49–59
3. Wang Q, Tian Q, Hu H (2014) Dynamic simulation of frictional contacts of thin beams during large overall motions via absolute nodal coordinate formulation. Nonlinear Dyn 77:1411–1425
4. Erkaya S, Dogan S (2015) A comparative analysis of joint clearance effects on articulated and partly compliant mechanisms. Nonlinear Dyn 81:323–341
5. Kortelainen J, Mikkola A (2015) Semantic restrictions and rules in applications of multibody dynamics. Eng Comput 31(1):85–97
6. Wang Q, Tian Q, Hu H (2016) Dynamic simulation of frictional multi-zone contacts of thin beams. Nonlinear Dyn 83:1919–1937

7. Zheng EL, Zhou XL (2014) Modeling and simulation of flexible slider-crank mechanism with clearance for closed high speed press system. Mech Mach Theory 74:10–30
8. Naeeni IP, Keshavarzi A, Fattahi I (2018) Parametric study on the geometric and kinetic aspects of the slider-crank mechanism. Iran J Sci Technol Trans Mech Eng 77:405–417
9. Lankarani HM, Nikravesh PE (1994) Continuous contact force models for impact analysis in multibody systems. Nonlinear Dyn 5:193–207
10. Bauchau OA, Rodriguez J (2002) Modeling of joints with clearance in flexible multi-body systems. Int J Solids Struct 39(1):41–63
11. Pérez-González A, Fenollosa-Esteve C, Sancho-Bru JL (2008) A modified elastic foundation contact model for application in 3D models of the prosthetic knee. Med Eng Phys 30(3):387–398
12. Flores P, Ambrósio J (2010) On the contact detection for contact-impact analysis in multibody systems. Multibody Syst Dyn 24(1):103–122
13. Machado M, Flores P, Ambrósio J, Completo A (2011) Influence of the contact model on the dynamic response of the human knee joint. Proc Inst Mech Eng Part K J Multi-body Dyn 225(4):344–358
14. Muvengei O, Kihiu J, Ikua B (2012) Dynamic analysis of planar multi-body systems with LuGre friction at differently located revolute clearance joints. Multibody Syst Dyn 28(4):369–393
15. Bai ZF, Zhao Y (2012) Dynamic behaviour analysis of planar mechanical systems with clearance in revolute joints using a new hybrid contact force model. Int J Mech Sci 54(1):190–205
16. Bai ZF, Zhao Y (2013) A hybrid contact force model of revolute joint with clearance for planar. Int J Nonlinear Mech 48:15–36
17. Wang Y, Li FM (2015) Nonlinear dynamics modeling and analysis of two rods connected by a joint with clearance. Appl Math Model 39:2518–2527
18. Qi ZH, Wang G, Zhang ZG (2015) Contact analysis of deep groove ball bearings in multibody systems. Multibody Syst Dyn 33(2):115–141
19. Ma J, Qian LF, Chen GS (2015) Dynamic analysis of mechanical systems with planar revolute joints with clearance. Mech Mach Theory 94:148–164
20. Tian Q, Xiao Q, Sun Y, Hu HY, Flores P (2015) Coupling dynamics of a geared multibody system supported by elastohydrodynamic lubricated cylindrical joints. Multibody Syst Dyn 33:259–284
21. Tian Q, Lou J, Mikkola A (2017) A new elastohydrodynamic lubricated spherical joint model for rigid-flexible multibody dynamics. Mech Mach Theory 107:210–228
22. Shi X, Polycarpou AA (2005) Measurement and modeling of normal contact stiffness and contact damping at the meso scale. J Vibr Acoust 127:52–60
23. Akhadkar N, Acary V, Brogliato B (2017) 3D revolute joint with clearance in multibody systems. In: 7th International workshop on computational kinematics. Springer, France
24. Haines RS (1985) An experimental investigation into the dynamic behaviour of revolute joints with varying degrees of clearance. Mech Mach Theory 20(3):221–231
25. Bouyer J, Fillon M (2002) An experimental analysis of misalignment effects on hydrodynamic plain journal bearing performances. J Tribol 124:313–319
26. Xu NN, Tang WC, Chen YJ (2015) Modeling analysis and experimental study for the friction of a ball screw. Mech Mach Theory 87:57–69
27. Akhadkar N, Acary V, Brogliato B (2017) Multibody systems with 3D revolute joints with clearances: an industrial case study with an experimental validation. Multibody Syst Dyn 2:1–34
28. Jayaprakash G, Thilak M, Sivakumar K (2014) Optimal tolerance design for mechanical assembly considering thermal impact. Int J Adv Manuf Technol 73:859–873
29. Bai ZF, Zhao Y, Chen J (2013) Dynamics analysis of planar mechanical system considering revolute clearance joint wear. Tribol Int 64:85–95
30. Schwab AL, Meijaard JP, Meijers P (2002) A comparison of revolute joint clearance model in the dynamic analysis of rigid and elastic mechanical systems. Mech Mach Theory 37(9):895–913
31. Flores P, Ambrósio J (2004) Revolute joints with clearance in multibody systems. Comput Struct 82(17–19):1359–1369
32. Flores P, Ambrósio JAC, Claro JCP, Lankarani HM (2006) Influence of the contact-impact force model on the dynamic response of multibody systems. Proc Inst Mech Eng Part K J Multi-body Dyn 220(1):21–34
33. Muvengei O, Kihiu J, Ikua B (2012) Numerical study of parametric effects on the dynamic response of planar multi-body systems with differently located frictionless revolute clearance joints. Mech Mach Theory 53:30–49
34. Erkaya S (2013) Trajectory optimization of a walking mechanism having revolute joints with clearance using ANFIS approach. Nonlinear Dyn 71:75–91
35. Tian Q, Sun Y, Liu C, Hu HY, Flores P (2013) Elastohydrodynamic lubricated cylindrical joints for rigid–flexible multibody dynamics. Comput Struct 114:106–120
36. Wang Z, Tian Q, Hu HY, Flores P (2016) Nonlinear dynamics and chaotic control of planar flexible multibody systems with interval clearance size joint. Nonlinear Dyn 86(3):1571–1597
37. Zhao B, Zhou K, Xie YB (2016) A new numerical method for planar multibody system with mixed lubricated revolute joint. Int J Mech Sci 113:105–119
38. Xu LX, Han YC (2016) A method for contact analysis of revolute joints with noncircular clearance in a planar multibody system. Proc Inst Mech Eng Part K J Multi-body Dyn 230(4):589–605
39. Ma J, Qian LF (2017) Modeling and simulation of planar multibody systems considering multiple revolute clearance joints. Nonlinear Dyn 90:1907–1940
40. Tan HY, Hu YJ, Li L (2017) A continuous analysis method of planar rigid-body mechanical systems with two revolute clearance joints. Multibody Syst Dyn 40:347–373
41. Li YY, Wang ZL, Wang C, Huang WH (2018) Planar rigid-flexible coupling spacecraft modeling and control considering solar array deployment and joint clearance. Acta Astronaut 142:138–151
42. Erkaya S, Uzmay I (2010) Experimental investigation of joint clearance effects on the dynamics of a slider-crank mechanism. Multibody Syst Dyn 24(1):81–102
43. Flores P, Koshy CS, Lankarani HM (2011) Numerical and experimental investigation on multibody systems with revolute clearance joints. Nonlinear Dyn 65(4):383–398
44. Crowther AR, Singh R, Zhang N, Chapman C (2007) Impulsive response of an automatic transmission system with multiple clearances: formulation, simulation and experiment. J Sound Vib 306(3):444–466
45. Koshy CS, Flores P, Lankarani HM (2013) Study of the effect of contact force model on the dynamic response of mechanical systems with dry clearance joints: computational and experimental approaches. Nonlinear Dyn 73:325–338
46. Lai X, He H, Lai Q, Wang C, Yang J, Zhang Y (2017) Computational prediction and experimental validation of revolute joint clearance wear in the low-velocity planar mechanism. Mech Syst Signal Process 85:963–976
47. Machado M, Moreira P, Flores P, Lankarani HM (2012) Compliant contact force models in multibody dynamics: evolution of the Hertz contact theory. Mech Mach Theory 53:99–121

48. Zheng EL, Jia F, Sha HW, Shi JF (2012) Non-circular belt transmission design of mechanical press. Mech Mach Theory 57:126–138
49. Daniel GB, Cavalca KL (2011) Analysis of the dynamics of a slider-crank mechanism with hydrodynamic lubrication in the connecting rod-slider joint. Mech Mach Theory 46:1434–1452
50. Xiao MH, Geng GS, Li GH, Li H (2017) Analysis on dynamic precision reliability of high-speed precision press based on Monte Carlo method. Nonlinear Dyn 90:2979–2988
51. Chen GL, Wang H, Lin ZQ (2013) A unified approach to the accuracy analysis of planar parallel manipulators both with input uncertainties and joint clearance. Mech Mach Theory 64:1–17
52. Reis VL, Daniel GB, Cavalca KL (2014) Dynamic analysis of a lubricated planar slider-crank mechanism considering friction and Hertz contact effects. Mech Mach Theory 74:257–273
53. Gummer A, Sauer B (2014) Modeling planar slider-crank mechanisms with clearance joints in RecurDyn. Multibody Syst Dyn 31:127–145
54. Erkaya S, Dogan S, Ulus S (2015) Effects of joint clearance on the dynamics of a partly compliant mechanism: numerical and experimental studies. Mech Mach Theory 88:125–140
55. Flores P, Machado M, Silva MT, Martins JM (2011) On the continuous contact force models for soft materials in multibody dynamics. Multibody Syst Dyn 25(3):357–375
56. Flores P, Lankarani HM (2012) Dynamic response of multibody systems with multiple clearance joints. J Comput Nonlinear Dyn 7(3):031003–031013
57. Zheng EL, Jia F, Zhou SH (2014) Thermal modeling and characteristics analysis of the high speed press system. Int J Mach Tool Manuf 85:87–99
58. Zhong GY, Wang CQ, Yang SF, Zheng EL (2015) Position geometric error modeling, identification and compensation for large 5-axis machining center prototype. Int J Mach Tool Manuf 89:142–150
59. Li YY, Wang C, Huang WH (2019) Dynamics analysis of planar rigid-flexible coupling deployable solar array system with multiple revolute clearance joints. Mech Syst Signal Process 117:188–209
60. Zheng EL, Zhu R, Zhu SH, Lu XJ (2016) A study on dynamics of flexible multi-link mechanism including joints with clearance and lubrication for ultra-precision presses. Nonlinear Dyn 83:137–159
61. Alves J, Peixinho N, Silva MTD, Flores P (2015) A comparative study on the viscoelastic constitutive laws for frictionless contact interfaces in multibody dynamics. Mech Mach Theory 85:172–188
62. Baumgarte J (1972) Stabilization of constraints and integrals of motion in dynamical systems. Comput Methods Appl Mech Eng 1:1–16
63. Erkaya S (2012) Prediction of vibration characteristics of a planar mechanism having imperfect joints using neural network. J Mech Sci Technol 26:1419–1430