

# Dynamic Identification of flexible joint manipulators with an efficient closed loop output error method based on motor torque output data

M. Gautier, A. Jubien, A. Janot, P-Ph. Robet

**Abstract**— This paper deals with joint stiffness off-line identification with new closed loop output error method which minimizes the quadratic error between the actual motor force/torque and the simulated one. The measurement of the joint position and its derivatives are not necessary. This method called *DIDIM* (Direct and Inverse Dynamic Identification Models) was previously validated on rigid robots and is now extended to a flexible joint manipulator. *DIDIM* method for flexible joint manipulators is derived into a three-step procedure: first, a rigid low frequency dynamic model is identified with *DIDIM* method; second, approximate values of the inertia ratio and stiffness are identified using the total inertia and friction values of step 1 and classical non linear programming algorithm; third, all the dynamic parameters (inertia, friction, stiffness) of the flexible robot are more accurately identified all together, starting from the values identified in step 1 and 2 and using the *DIDIM* method. An experimental setup exhibits results and shows the effectiveness of our approach compared with a classical output error methods.

## I. INTRODUCTION

Accurate dynamic robots models are needed to control and simulate their motions with precision and reliability. Identification of rigid robots has been widely investigated in the last decades. The usual identification process is based on the Inverse Dynamic Model (*IDM*) and Least Squares (*LS*) estimation. This method, called *IDIM-LS* (Inverse Dynamic Identification Model with Least Squares), has been performed on several prototypes and industrial robots with accurate results [1].

Identification of flexibilities is more complex than identification of rigid body dynamics: only a subset of state variables is measured and one cannot use directly linear regressions [2]. This can be solved by adding sensors [3] and/or external excitations [4]. In [5], the authors identify joint stiffness of a 6 degrees of freedom (*dof*) robot with motion capture, which needs using an expensive motion capture system. In [6], the authors have rigorously compared three minimal identification models depending on data availability. All these techniques provide good results but they need either the flexibility measures variables or high-order derivatives of motor position (1 to 4) and motor force

(1 to 2). In [7], the authors use the System Identification Toolbox for Matlab to identify both joint and structural flexibilities of one axis of an industrial robot: inertia and stiffness parameters seem well identified. But, they do not discuss about the repartition of Coulomb friction and data filtering. In [8], a three mass flexible model of robot arm is identified but the methodology uses a nonlinear grey-box identification method and needs both motor torque and motor position.

Recently, a new identification process needing only actual force/torque data was validated on rigid robots [9]. This method called *DIDIM* (Direct and Inverse Dynamic Identification Models) is now extended to joint stiffness identification. A first attempt is presented in [10]: the non-linear *LS* problem is solved by using the Nelder-Mead simplex algorithm. Good results are obtained but the algorithm converges slowly and it is quite sensitive to initialization.

The success of *DIDIM* method needs to keep the closed-loop joint position bandwidth in the simulator close to the actual one at each step of the iterative procedure. In [11] and [12] a gains updating is present in the simulated robot in order to keep the bandwidth of the rigid controlled *dof* and to keep the natural frequency of the flexible *dof*, close to the actual ones, at each step of the *DIDIM* algorithm.

This paper presents a new Closed Loop Output Error (*CLOE*) based on *DIDIM* method for identifying dynamics parameters of a flexible joint robot. It proposes an improvement of *DIDIM* method to avoid the gains updating in the simulated robot to identify rigid and flexible robot. A three-step identification procedure is used. In the first step, the low frequency rigid dynamic model is identified with *DIDIM* method. In the second step the inertias ratio and stiffness of the flexible model are identified with the Nelder-Mead simplex algorithm, given the total mass and friction identified in step 1. In the third step the dynamic parameters of the flexible dynamic model (inertia, friction, stiffness) are identified all together with *DIDIM* method, starting from the values identified separately in step 1 and 2, to get the simulated positions, velocities and accelerations close to the actual one.

This paper is divided into six sections. Section 2 describes the experimental setup and its modeling. Section 3 presents the usual method for dynamic identification of rigid robots, based on *IDIM-LS* method. Section 4 presents the *DIDIM* method for rigid robots. Section 5 presents the *DIDIM* procedure for a flexible joint manipulator. Section 6 is devoted to the experimental identification of one prismatic flexible joint manipulator. The identified values are

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compared with those identified with *IDIM-LS* method and with classical Output Error (*OE*) methods.

## II. MODELING OF A FLEXIBLE JOINT ROBOT

### A. Experimental setup

The *EMPS* is a high-precision linear Electro-Mechanical Positioning System (see fig. 1 and fig. 2). It is a standard configuration of a drive system for prismatic joint of robots or machine tools. It is composed of a Maxon DC motor which drives a carriage by a Star high-precision low-friction ball screw. The carriage moves a load in translation. The motor rotor and the ball screw are connected by a flexible coupling. Two incremental encoders are present on the robot. The motor encoder measures the motor position and the load encoder measures the position of the ball screw. The motor position is controlled with a proportional-derivative PD controller. A payload (10(Kg)) can be added on the carriage.



Fig. 1: EMPS prototype, flexible coupling and payload

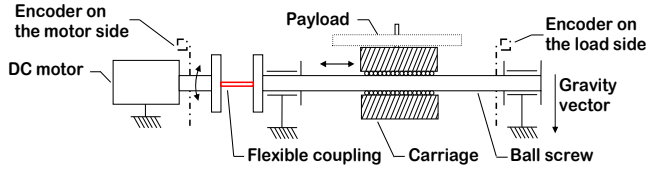


Fig. 2: EMPS Components

All variables and parameters are given in SI units on the load side. The motor force (unit on load side) is proportional to the motor torque (unit on motor side) according to the reduction ratio of the ball screw *red* (rd/m) :

$$\tau(N) = red \cdot \tau(N.m) \quad (1)$$

### A. Inverse dynamic model of the flexible joint robot

The mechanical system has  $n=2$  *dof*, one rigid *dof*  $q_1$  and one flexible *dof*  $q_2$  where:  $q_1$  (m/s) is the motor position,  $q_2$  (m/s) is the flexible *dof* position where  $q_{12} = q_1 + q_2$  is the load position. In our case the robot is not affected by gravity and is modeled with two inertias, one spring, and friction forces (see fig. 3). The link flexibility is not taken into account because it is insignificant compared to the joint flexibility.

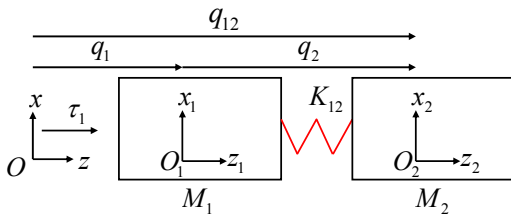


Fig. 3: EMPS modeling

The Inverse Dynamic Model (*IDM*) calculates the motor force according to the joint positions and their derivatives. Newton–Euler equations give the following *IDM* [13]:

$$\begin{aligned} \tau_{idm1} &= M_1 \ddot{q}_1 + F_{v1} \dot{q}_1 + F_{c1} \text{sign}(\dot{q}_1) - K_{12} q_2 \\ 0 &= M_2 \ddot{q}_{12} + F_{v2} \dot{q}_{12} + F_{c2} \text{sign}(\dot{q}_{12}) + K_{12} q_2 \end{aligned} \quad (2)$$

Where:  $\dot{q}_1$ ,  $\ddot{q}_1$ , are respectively the motor velocity and acceleration;  $\tau_{idm1}$  (N) is the motor force;  $\dot{q}_{12}$ ,  $\ddot{q}_{12}$  are respectively the load velocity and acceleration;  $\dot{q}_2$ ,  $\ddot{q}_2$  are respectively the flexible *dof* velocity and acceleration with,  $q_{12} = q_1 + q_2$ ,  $\dot{q}_{12} = \dot{q}_1 + \dot{q}_2$  and  $\ddot{q}_{12} = \ddot{q}_1 + \ddot{q}_2$ ;  $M_1$  (Kg) is the total motor side equivalent mass,  $F_{v1}$  (N/m/s) and  $F_{c1}$  (N) are respectively the viscous and Coulomb motor side friction parameters;  $M_2$  (Kg) is the total load side equivalent mass (screw, nut, carriage and load),  $F_{v2}$  (N/m/s) and  $F_{c2}$  (N) are respectively the viscous and Coulomb load side friction parameters;  $K_{12}$  (N/m) is the stiffness of the flexible joint ;  $\text{sign}(u)$  denotes the sign function.

The *IDM* can be written as follows:

$$\tau_{idm} = M(q) \ddot{q} + N(q, \dot{q}) + Kq \quad (3)$$

$$\begin{aligned} \text{With: } q &= \begin{pmatrix} q_1 \\ q_{12} \end{pmatrix}, \quad \dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_{12} \end{pmatrix}, \quad \ddot{q} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_{12} \end{pmatrix}, \quad \tau_{idm} = \begin{pmatrix} \tau_{idm1} \\ 0 \end{pmatrix} \\ M(q) &= \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \quad N(q, \dot{q}) = \begin{pmatrix} F_{v1} \dot{q}_1 + F_{c1} \text{sign}(\dot{q}_1) \\ F_{v2} \dot{q}_{12} + F_{c2} \text{sign}(\dot{q}_{12}) \end{pmatrix} \text{ and } K = \begin{pmatrix} K_{12} & -K_{12} \\ -K_{12} & K_{12} \end{pmatrix} \end{aligned}$$

The motor force  $\tau_{idm}$  can be written in a linear relation to the dynamic parameters as follows:

$$\tau_{idm} = IDM_{ST} \chi_{ST} \quad (4)$$

$$\begin{aligned} \text{With: } \chi_{ST} &= (M_1 \quad F_{v1} \quad F_{c1} \quad M_2 \quad F_{v2} \quad F_{c2} \quad K_{12})^T \\ IDM_{ST} &= \begin{pmatrix} \ddot{q}_1 & \dot{q}_1 & \text{sign}(\dot{q}_1) & 0 & 0 & 0 & -q_2 \\ 0 & 0 & 0 & \ddot{q}_{12} & \dot{q}_{12} & \text{sign}(\dot{q}_{12}) & q_2 \end{pmatrix} \end{aligned}$$

There are  $n_{ST} = 7$  parameters to be identified.

### B. Direct dynamic model of the flexible joint robot

From (3), the implicit form of the Direct Dynamic Model (*DDM*) of the flexible joint robot is given by:

$$M(q) \ddot{q} = \tau_{idm} - N(q, \dot{q}) - Kq \quad (5)$$

### C. Inverse dynamic model of the rigid robot

In case of low frequencies less than the first flexible mode, the robot is considered as a rigid structure,  $q_{12} = q_1$  in (2) this reduces to:

$$\tau_{idm1} = M_{tot} \ddot{q}_1 + F_{v_{tot}} \dot{q}_1 + F_{c_{tot}} \text{sign}(\dot{q}_1) \quad (6)$$

Where:  $M_{tot} = M_1 + M_2$  is the total inertia,  $F_{v_{tot}} = F_{v1} + F_{v2}$  and  $F_{c_{tot}} = F_{c1} + F_{c2}$  are the total viscous and Coulomb friction parameters respectively.

The motor force of the rigid robot  $\tau_{idm1}$  can be written in a linear relation to the dynamic parameters as follows:

$$\tau_{idm1} = IDM_{ST} \chi_{ST} \quad (7)$$

with:  $IDM_{ST} = (\ddot{q}_l \quad \dot{q}_l \quad \text{sign}(\dot{q}_l))$ ,  $\chi_{ST} = (M_{tot} \quad F_{v_{tot}} \quad F_{c_{tot}})^T$

There are  $n_{ST} = 3$  parameters to be identified.

Equation (6) can be written as follows:

$$\tau_{idm1} = M_r(q_l) \ddot{q}_l + N_r(q_l, \dot{q}_l) \quad (8)$$

With:  $M_r(q_l) = M_{tot}$  and  $N_r(q_l, \dot{q}_l) = F_{v_{tot}} \dot{q}_l + F_{c_{tot}} \text{sign}(\dot{q}_l)$ .

#### D. Direct dynamic model of the rigid robot

From (8), the DDM of the rigid robot is the following:

$$M_r(q_l) \ddot{q}_l = \tau_{idm1} - N_r(q_l, \dot{q}_l) \quad (9)$$

### III. IDIM-LS: INVERSE DYNAMIC IDENTIFICATION MODEL WITH LEAST SQUARES METHOD

Because of perturbations due to noise measurement and modeling errors, the actual force/torque  $\tau$  differs from  $\tau_{idm}$  by an error  $e$ , such that:

$$\tau = \tau_{idm} + e = IDM_{ST}(q, \dot{q}, \ddot{q}) \chi_{st} + e \quad (10)$$

The vector  $\hat{\chi}_{st}$  is the least squares (LS) solution of an over determined system built from the sampling of (10), while the robot is tracking exciting trajectories [14]:

$$Y = W_{st} \chi_{st} + \rho \quad (11)$$

Where:  $Y$  is the  $(rx1)$  measurement vector,  $W_{st}$  the  $(rxn_{st})$  observation matrix, and  $\rho$  is the  $(rx1)$  vector of errors. The number of rows is  $r = n * n_e$ , where the number of recorded samples is  $n_e$ . When  $W_{st}$  is not a full rank matrix, the LS solution is not unique. The system (11) is rewritten:

$$Y = W \chi + \rho \quad (12)$$

Where a subset  $W$  of  $b$  independent columns of  $W_{st}$  is calculated, which defines the vector  $\chi$  of  $b$  base parameters [15][16]. Standard deviations  $\sigma_{\hat{\chi}_i}$ , are estimated assuming that  $W$  is a deterministic matrix and  $\rho$ , is a zero-mean additive independent Gaussian noise, with a covariance matrix  $C_{\rho\rho} = E(\rho\rho^T) = \sigma_\rho^2 I_r$  [17].

Where  $E$  is the expectation operator and  $I_r$ , the  $(r \times r)$  identity matrix. An unbiased estimation of the standard deviation  $\sigma_\rho$  is the following:

$$\hat{\sigma}_\rho^2 = \|Y - W \hat{\chi}\|^2 / (r - b) \quad (13)$$

The covariance matrix of the estimation error is given by:

$$C_{\hat{\chi}\hat{\chi}} = E[(\chi - \hat{\chi})(\chi - \hat{\chi})^T] = \hat{\sigma}_\rho^2 (W^T W)^{-1} \quad (14)$$

The relative standard deviation  $\% \sigma_{\hat{\chi}_{i1}}$  is given by:

$$\% \sigma_{\hat{\chi}_{i1}} = 100 \sigma_{\hat{\chi}_{i1}} / |\hat{\chi}_{i1}|, \text{ for } |\hat{\chi}_{i1}| \neq 0 \quad (15)$$

Where  $\sigma_{\hat{\chi}_i}^2 = C_{\hat{\chi}\hat{\chi}}(i, i)$  is the  $i^{\text{th}}$  diagonal coefficient of  $C_{\hat{\chi}\hat{\chi}}$ . Calculating the LS solution of (12) from perturbed data in  $W$  and  $Y$  may lead to bias if  $W$  is correlated to  $\rho$ . Then, it is essential to filter data in  $Y$  and  $W$  before computing the LS solution. Velocities and accelerations are estimated by means of a band-pass filtering of the positions. To eliminate high frequency noises and torque ripples, a parallel decimation is performed on  $Y$  and on each column of  $W$ . More details about data filtering can be found in [17] and [18].

## IV. DIDIM METHOD FOR RIGID ROBOTS

### A. Theoretical approach

In this section, the method is briefly recalled, a complete presentation can be found in [9]. DIDIM is a CLOE method requiring only force/torque data. The output  $y = \tau$ , is the actual joint force/torque  $\tau$ , and the simulated output  $y_s = \tau_{idm}$ , is the simulated joint force/torque. The signal  $q_{ddm}(\chi, t)$  is the result of the integration of the linear implicit differential equation (5) for flexible robot and (9) for rigid robot. In both case the robot is simulated on closed loop with the same control law between the simulated robot and the actual one. The optimal solution  $\hat{\chi}$  minimizes the quadratic criterion:

$$J(\chi) = \|Y - Y_s\|^2 \quad (16)$$

Where  $Y(\tau)$  and  $Y_s(\tau_{idm})$  are vectors obtained by filtering and down sampling the vectors of samples of the actual force/torque  $\tau$ , and of the simulated force/torque  $\tau_{idm}$ , respectively. This non-linear LS problem is solved by the Gauss-Newton regression. It is based on a Taylor series expansion of  $y_s$ , at a current estimate  $\hat{\chi}^k$ . Because of the same closed loop control for the actual and for the simulated robot, the simulated position, velocity and acceleration have little dependence on  $\chi$  such as:

$$(q_{ddm}(\hat{\chi}^k), \dot{q}_{ddm}(\hat{\chi}^k), \ddot{q}_{ddm}(\hat{\chi}^k)) \approx (q, \dot{q}, \ddot{q}) \quad (17)$$

Then the jacobian matrix can be approximated by:

$$\left( \frac{\partial(y_s)}{\partial \chi} \right)_{\hat{\chi}^k} \approx IDM(q_{ddm}(\hat{\chi}^k), \dot{q}_{ddm}(\hat{\chi}^k), \ddot{q}_{ddm}(\hat{\chi}^k)) \approx IDM(q, \dot{q}, \ddot{q}) \quad (18)$$

Taking the approximation (18) of the jacobian matrix into the Taylor series expansion, it becomes:

$$y = \tau = IDM(q_{ddm}(\hat{\chi}^k), \dot{q}_{ddm}(\hat{\chi}^k), \ddot{q}_{ddm}(\hat{\chi}^k)) \chi^{k+1} + (o + e) \quad (19)$$

This is the Inverse Dynamic Identification Model, where  $(q, \dot{q}, \ddot{q})$  are estimated with  $(q_{ddm}, \dot{q}_{ddm}, \ddot{q}_{ddm})$  and  $(o + e)$  is an error. An over-determined linear system is obtained after a sampling and a parallel decimation of (18):

$$Y(\tau) = W_\delta(q_{ddm}, \dot{q}_{ddm}, \ddot{q}_{ddm}, \hat{\chi}^k) \chi + \rho \quad (20)$$

The *LS* solution of (20) calculates  $\hat{\chi}^{k+1}$ , at iteration  $k+1$ . This process is iterated until:

$$(\|\rho_{k+1}\| - \|\rho_k\|) / \|\rho_k\| \leq \text{tol}_l \quad (21)$$

Where  $\text{tol}_l$  is a value ideally chosen to be a small number to get fast convergence with good accuracy.

*DIDIM* method avoids tuning the band pass filter in the *IDIM* method by using the integration of the *DDM* in a closed-loop simulation. It combines the *IDM* and the *DDM* and validates, in the same identification procedure, both models for computed force control and for simulation. The robot controller must be known but this is not a real problem because working on identification for simulation or control of the robot requires a minimal knowledge about it.

### B. Initialization $\chi^0$

The success of *DIDIM* method is based on the approximations given by (17) and (18), this needs to keep the closed-loop joint position bandwidth in the simulator close to the actual one at each step of the iterative procedure. Because the joint position bandwidth mainly depends on the drive chain inertia [9],  $\chi^0$  is taken equal to 0 except for the *a priori* values of the drive chain inertia. In practice, the algorithm has a good convergence if *a priori* value is equal to the rotor inertia of the motor, which is given by the manufacturer. This initialization avoids the gains updating performed in [9], [11] and [12].

## V. DIDIM METHOD FOR FLEXIBLE JOINT ROBOTS

### A. Introduction

Difficulties arise with flexible systems because the flexible *dof*,  $q_2$  is not actuated and not controlled. The choice of *a priori* inertia and stiffness values for the initialization of the algorithm is less robust than rigid robot. If *a priori* values are too far from actual values (greater than 10%), the approximations given by (17) and (18) are not longer valid and the algorithm may diverge.

So before applying *DIDIM* method on the flexible robot, it is necessary to have good initialization values for inertia and stiffness. These values are identified with a classical non linear programming algorithm before identifying all the dynamic parameters thanks to the dynamic parameters of the rigid model. The complete identification procedure is described in the following subsections.

### B. Step 1: identification of the rigid model

*DIDIM* is applied to find the dynamic parameters of rigid model of the robot. Since this model is valid at low frequencies, the cut-off frequency of the decimate filter must be fixed to a low value (less than 10Hz in most cases). The optimal parameters minimize the quadratic criterion (16). We get the simulated states by simulating (9). The algorithm is initialized with:

$${}^0M_{tot} = {}^{ap}M, {}^0F_{v_{tot}} = {}^0F_{c_{tot}} = 0 \quad (22)$$

The *a priori* value  ${}^{ap}M$  can be a *CAD* value or can be given by the robot manufacturer or the rotor inertia of the motor.

### C. Step 2: Estimation of the mass ratio and of the flexible mode, with given total mass and friction

In order to estimate  $M_1$  and  $K_{12}$ , for the given total mass  $M_{tot}$  and friction  $F_{v_{tot}}, F_{c_{tot}}$ , identified in step 1, two parameters are introduced which have a better physical meaning:

$${}^a f_{n\_flex} = \sqrt{{}^a K_{12} / {}^a M_2} / (2\pi) \quad (23)$$

$${}^a \lambda = {}^a M_1 / ({}^a M_1 + {}^a M_2) = {}^a M_1 / {}^a M_{tot} \quad (24)$$

Where  $f_{n\_flex}$  is the natural frequency when the rigid *dof* is blocked ( $q_1 = 0$ ) and  $\lambda$  is the ratio between the motor side drive chain inertia and the total drive chain inertia;  ${}^a M_1$  (Kg) is the actual total motor side equivalent mass;  ${}^a M_2$  (Kg) is the actual total load side equivalent mass and  $K_{12}$  (N/m) is the actual stiffness of the flexible joint.

In this step, only the values  $\lambda$  and  $f_{n\_flex}$  are estimated with a non linear programming algorithm. The optimal parameters  $\hat{\lambda}$  and  $\hat{f}_{n\_flex}$  minimize the following criterion:

$$(\hat{f}_{n\_flex}, \hat{\lambda}) = \arg \min_{f_{n\_flex}, \lambda} \|Y - Y_s\|^2 \quad (25)$$

Where  $Y$  and  $Y_s$  are vectors obtained by filtering and down sampling the vectors of samples of the actual force/torque, and of the simulated force/torque, respectively. The positions, velocities and accelerations are computed by numerical integration of (5). Only inertias and stiffness vary in the simulated robot according to  $\lambda$  and  $f_{n\_flex}$ :

$${}^k M_1 = {}^k \hat{\lambda} \hat{M}_{tot}, {}^k M_2 = (1 - {}^k \hat{\lambda}) \hat{M}_{tot} \quad (26)$$

$${}^k K_{12} = 4\pi^2 (1 - {}^k \hat{\lambda})^k \hat{f}_{n\_flex}^2 \hat{M}_{tot}$$

The other parameters are frozen such as:

$${}^k F_{v1} = \hat{F}_{v_{tot}}, {}^k F_{v2} = 0, {}^k F_{c1} = \hat{F}_{c_{tot}}, {}^k F_{c2} = 0, \text{ for any } k \quad (27)$$

The coefficients  $\hat{M}_{tot}$ ,  $\hat{F}_{v_{tot}}$  and  $\hat{F}_{c_{tot}}$  are the values of  $M_{tot}$ ,  $F_{v_{tot}}$  and  $F_{c_{tot}}$  respectively, identified in step 1, section 5.2. Only 2 parameters are estimated over the 7 flexible model parameters. That reduces significantly the computational time in the non linear programming procedure. The non linear programming algorithm is the Nelder-Mead simplex algorithm (the "fminsearch" MATLAB function). The initial conditions of algorithm must cover usual ranges of the flexible joint natural modes and of the ratios between the motor side drive chain inertia and the total drive chain inertia of robots. They can be chosen in these intervals according to results of section 6.4:

$${}^0 f_{n\_flex} \in [10; 80] \text{ Hz}, {}^0 \lambda \in [0.1; 0.9] \quad (28)$$

The iterative procedure stops when the relative decreasing of the criterion (21) is less than a value ideally chosen to be a small number to get fast convergence with good accuracy.

#### D. Step 3: global identification of the flexible model

*DIDIM* is applied to find the dynamic parameters of the flexible robot. The optimal parameters minimize the quadratic criterion (16). We get the simulated states by simulating (5). The algorithm is initialized with:

$$\begin{aligned} {}^0M_1 &= \hat{\lambda} \hat{M}_{tot}, \quad {}^0M_2 = (1 - \hat{\lambda}) \hat{M}_{tot}, \\ {}^0K_{12} &= 4\pi^2 (1 - \hat{\lambda}) \hat{f}_{n\_flex} \hat{M}_{tot} \end{aligned} \quad (29)$$

Where  $\hat{\lambda}$  and  $\hat{f}_{n\_flex}$  are the identified values with the procedure described in section V.C. The other parameters are initialized with zero:

$${}^0F_{v1} = {}^0F_{v2} = {}^0F_{c1} = {}^0F_{c2} = 0 \quad (30)$$

We recall the identification procedure in figure 4.

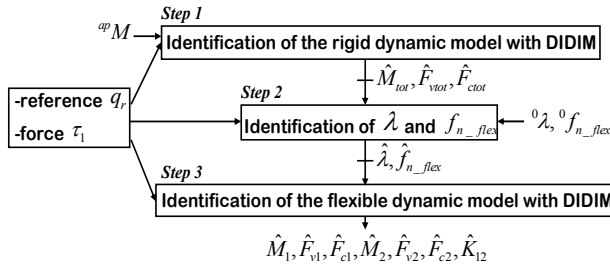


Fig. 4: Identification procedure

The initial conditions in step 3, calculated thanks to step 1 and step 2, are accurate enough to validate the approximation given by (17) and (18).

## VI. EXPERIMENTAL VALIDATION

#### A. Data acquisition and exciting trajectories

Motor and load positions are measured by means of high precision encoders working in quadrature count mode (accuracy of 12500 counts per revolution). The sample acquisition frequency for joint position and current reference (drive force) is 1 KHz. We calculate the motor force using the relation:

$$\tau_1 = {}^{ap}g_\tau v_\tau \quad (31)$$

where  $v_\tau$  is the current reference of the amplifier current loop, and  ${}^{ap}g_\tau$  is *a priori* value of the gain of the joint drive chain, which is taken as a constant value in the frequency range of the robot (less than 30Hz) because of the large bandwidth of the current loop (700 Hz). The motor position is PD controlled with a closed loop bandwidth tuned at 20Hz. Exciting reference trajectories are a trapezoidal velocity signal coupled with a chirp signal. Trapezoidal velocity excites inertia and friction parameters while chirp signal excites the flexibility.

#### B. Identification of the flexible model with *IDIM-LS* method

*IDIM-LS* method is performed with the *IDM* defined in (4). It needs the motor force  $\tau_1$ , the motor position  $q_1$  and the load position  $q_{12}$ . The cut-off frequency of the Butterworth filter and decimate filter are fixed at 100Hz and 60Hz respectively. Two identification tests are performed: one without payload on the load side and another one with payload on the load side (10(Kg)). Because all data are measured with high accuracy, the results given in Table 3 and Table 4 are our references for the study of other identification methods. If identification methods are accurate enough, variation  $\Delta M_2$  close to 10(Kg) must be observed on  $M_2$  whereas insignificant variations must be observed on the others.

#### C. Identification of the rigid dynamic model

The algorithm is initialized with  ${}^0M_{tot} = {}^{ap}M = 78(\text{Kg})$ ,  ${}^0F_{v_{tot}} = {}^0F_{c_{tot}} = 0$  and the cut-off frequency of the decimate filter is fixed at 5Hz.

The algorithm converges in only 2 steps and 4 seconds on a dual-core CPU 2Ghz clock in both cases (with and without payload). The "rigid" *DIDIM* identified values are given in Table 1.

TABLE 1: DIDIM IDENTIFIED VALUES WITH THE RIGID MODEL

Parameter	Without payload			With payload		
	$\hat{\chi}^0$	$\hat{\chi}^2$	$\% \sigma_{\hat{\chi}^2}$	$\hat{\chi}^0$	$\hat{\chi}^2$	$\% \sigma_{\hat{\chi}^2}$
$M_{tot}$ (Kg)	78	<b>107</b>	0.3	78	<b>117</b>	0.3
$F_{v_{tot}}$ (N/m/s)	0	<b>216</b>	1.4	0	<b>220</b>	1.4
$F_{c_{tot}}$ (N)	0	<b>18.2</b>	1.4	0	<b>17.6</b>	1.7
$\ Y - W.X\  / \ Y\ $	5.06%			5.47%		

The identified values  $\hat{M}_{tot}$ ,  $\hat{F}_{v_{tot}}$ , and  $\hat{F}_{c_{tot}}$  are used in the following step. The algorithm converges if  ${}^{ap}M$  is chosen between 25 and 1000(kg), which shows its robustness.

#### D. Identification of $\lambda$ and $f_{n\_flex}$

The cut-off frequency of the decimate filter is fixed at 60Hz. The results are given in Table 2. The values and according to iteration k are given in.

TABLE 2: IDENTIFIED VALUES

	Without payload		With payload	
	$k = 0$	$k = 16$	$k = 0$	$k = 17$
$\lambda$	0.5	<b>0.62</b>	0.5	<b>0.57</b>
$f_{n\_flex}$	40	<b>23.85</b>	40	<b>20.89</b>
$\ Y - Y_k\  / \ Y\ $	8.80%		10.8%	

Fig. 5:  $\lambda$  and  $f_{n\_flex}$  values according to iteration k

The algorithm converges in less than 20 iterations. In this case, it is robust with respect to initial values because  ${}^0\lambda$  can be chosen between 0.25 and 0.95 and  ${}^0f_{n\_flex}$  can be

chosen between 15(Hz) and 50(Hz). The variations of  $\lambda$  and  $f_{n\_flex}$  according to iteration  $k$  are given in Fig. 5.

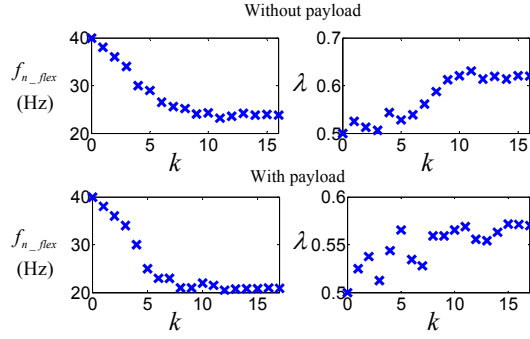


Fig. 5:  $\lambda$  and  $f_{n\_flex}$  values according to iteration  $k$

### E. Global Identification of the flexible model

The cut-off frequency of the decimate filter is fixed at 60Hz. We get the simulated states by simulating (5). Initial values of inertia and stiffness are calculated with (29). *DIDIM* method converges after only 3 iterations and takes 6 seconds only in both cases. The results are given in Table 3 and Table 4 and are very close to those identified with *IDIM-LS* method. Adding inertia, viscous and Coulomb friction parameters gives the “rigid” values. The payload is well identified. *DIDIM* method provides excellent results although only one variable measurement is used. However, we cannot separate motor friction from load friction. The estimated motor force follows closely the actual one as illustrated fig. 6.

TABLE 3: *IDIM-LS* AND *DIDIM* IDENTIFIED VALUES WITHOUT PAYLOAD

Method	<i>IDIM-LS</i>		<i>DIDIM</i>		
Parameter	$\hat{\chi}$	$\% \sigma_{\hat{\chi}}$	$\hat{\chi}^0$	$\hat{\chi}^3$	$\% \sigma_{\hat{\chi}^3}$
$M_I$ (Kg)	<b>68.0</b>	0.3	66.3	<b>65.3</b>	0.3
$F_{vI}$ (N/m/s)	<b>104</b>	1.3	0	<b>215</b>	0.7
$F_{cI}$ (N)	<b>10.0</b>	0.8	0	<b>17.6</b>	0.8
$M_2$ (Kg)	<b>38.0</b>	0.7	40.7	<b>37.4</b>	0.5
$F_{v2}$ (N/m/s)	<b>96.5</b>	1.4	0	-	-
$F_{c2}$ (N)	<b>9.5</b>	1.3	0	-	-
$K_{I2}$ (N/m)	<b>8.8 10<sup>5</sup></b>	0.7	9.1310 <sup>5</sup>	<b>8.4 10<sup>5</sup></b>	0.5
$M_I + M_2$	106		103		
$F_{vI} + F_{v2}$	211		215		
$F_{cI} + F_{c2}$	19.5		17.6		
$\ Y - W \cdot X\  / \ Y\ $	6.81%		8.33%		

TABLE 4: *IDIM-LS* AND *DIDIM* IDENTIFIED VALUES WITH PAYLOAD

Method	<i>IDIM-LS</i>		<i>DIDIM</i>		
Parameter	$\hat{\chi}$	$\% \sigma_{\hat{\chi}}$	$\hat{\chi}^0$	$\hat{\chi}^3$	$\% \sigma_{\hat{\chi}^3}$
$M_I$ (Kg)	<b>69</b>	0.3	66.7	<b>67</b>	0.4
$F_{vI}$ (N/m/s)	<b>110</b>	1.6	0	<b>217</b>	0.8
$F_{cI}$ (N)	<b>10.5</b>	1.3	0	<b>17.7</b>	1.1
$M_2$ (Kg)	<b>46.5</b>	0.7	50.3	<b>46.2</b>	0.6
$F_{v2}$ (N/m/s)	<b>98.2</b>	1.2	0	-	-
$F_{c2}$ (N)	<b>8.68</b>	1.5	0	-	-
$K_{I2}$ (N/m)	<b>8.6 10<sup>5</sup></b>	0.6	8.6710 <sup>5</sup>	<b>8.1 10<sup>5</sup></b>	0.6
$M_I + M_2$	115		113		
$F_{vI} + F_{v2}$	208		217		
$F_{cI} + F_{c2}$	19.1		17.7		
$\Delta M_2$	8.5		8.8		
$\ Y - W \cdot X\  / \ Y\ $	7.34%		8.52%		

### A. Comparison with other OE methods

In this section, the non-linear *LS* problem is solved by using the Nelder – Mead simplex algorithm. This method is compared with the new *DIDIM* method. Details about the procedure can be found in [10]. The optimal values are estimated with “*fminsearch*” MATLAB function. Two criteria are evaluated, the first one depends on the motor position and the second one depends on the motor force:

$$J(\chi)_{q_I} = \arg \min_{\chi} \|q_{Iddm} - q_I\|^2 \quad (32)$$

$$J(\chi)_{\tau_I} = \arg \min_{\chi} \|\tau_{Iddm} - \tau_I\|^2$$

Where:  $q_{Iddm}$  is the simulated motor position;  $q_I$  is the actual motor position.  $\tau_{Iddm}$  is the simulated motor force and  $\tau_I$  is the actual motor force. The cut-off frequency is fixed at 60(Hz) on the decimate filter. The algorithm is initialized with the pseudo regular initialization described in [10]:

$${}^0M_I = {}^0M_2 = \hat{M}_{tot}/2, {}^0F_{vI} = {}^0F_{v2} = \hat{F}_{v_{tot}}/2, {}^0F_{cI} = {}^0F_{c2} = \hat{F}_{c_{tot}}/2, \quad (33)$$

$${}^0K_{I2} = 4\pi^2 {}^0f_{n\_nat}^2 {}^0M_{eq} = \pi^2 {}^0f_{n\_nat}^2 \hat{M}_{tot}$$

Where  ${}^0f_{n\_nat}$  is the initialization of the natural frequency of system. The value of  ${}^0f_{n\_nat}$  is set to 30(Hz).

The relative errors between the Nelder-Mead identified values  $\hat{\chi}^{NM}$  and the *DIDIM* identified values  $\hat{\chi}^{DIDIM}$  are given in and are computed by the following formula:

$$\%e(\hat{\chi}_j) = 100(\hat{\chi}_j^{NM} - \hat{\chi}_j^{DIDIM}) / \hat{\chi}_j^{DIDIM} \quad (34)$$

The results are given in and in Table 5 and Table 6.

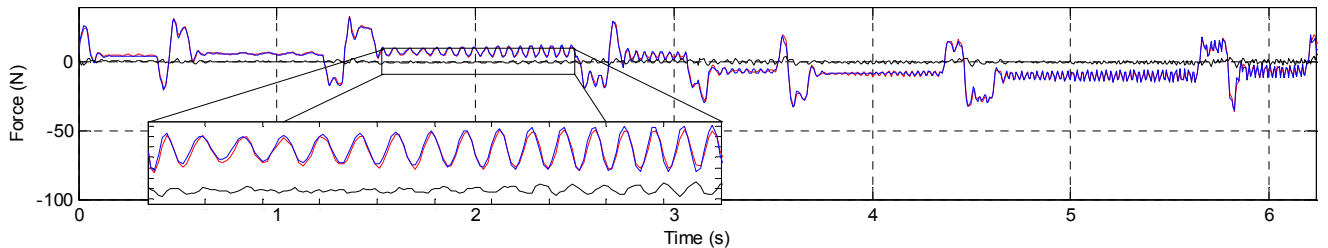


Fig. 6: Cross test validation with *DIDIM* method without payload. Red: measurement, Blue: Simulated force, Black: error

The Nelder – Mead simplex algorithm converges after a minimum of 172 iterations (276 simulations of *MDD*) and 9 minutes. The computation time is 9 times slower than *DIDIM* method (64 seconds). The results are close to those given by *DIDIM* except for frictions parameters for the two criterias. However, the algorithm is not as robust as *DIDIM* with respect to the initial values. *DIDIM* is a dramatic improvement of the usual *OE* method.

TABLE 5: NELDER-MEAD IDENTIFIED VALUES WITH  $J(\chi)q_i$

Parameter	Without payload		With payload	
	$\hat{\chi}^{179}$	$\%e(\chi_j)$	$\hat{\chi}^{185}$	$\%e(\chi_j)$
$M_I$ (Kg)	<b>64.1</b>	-1.8%	<b>65.9</b>	-1.6%
$F_{v1}$ (N/m/s)	<b>126</b>	-	<b>117</b>	-
$F_{c1}$ (N)	<b>13.2</b>	-	<b>10.2</b>	-
$M_2$ (Kg)	<b>40.1</b>	+7.2%	<b>48.3</b>	+4.5%
$F_{v2}$ (N/m/s)	<b>84.6</b>	-	<b>113</b>	-
$F_{c2}$ (N)	<b>5.2</b>	-	<b>8.0</b>	-
$K_{12}$ ( $10^5$ N/m)	<b>8.7</b>	+3.6%	<b>8.1</b>	0%
$M_I + M_2$	<b>104.2</b>	+1.2%	<b>114.2</b>	+1.1%
$F_{v1} + F_{v2}$	<b>211</b>	-1.9%	<b>230</b>	+6.0%
$F_{c1} + F_{c2}$	<b>18.4</b>	+4.5%	<b>18.2</b>	+2.8%
$\Delta M_2$	-	-	<b>8.2</b>	-6.8%

TABLE 6: NELDER-MEAD IDENTIFIED VALUES WITH  $J(\chi)\tau_i$

Parameter	Without payload		With payload	
	$\hat{\chi}^{172}$	$\%e(\chi_j)$	$\hat{\chi}^{188}$	$\%e(\chi_j)$
$M_I$ (Kg)	<b>67.1</b>	+2.8%	<b>68.1</b>	+1.6%
$F_{v1}$ (N/m/s)	<b>76.5</b>	-	<b>128</b>	-
$F_{c1}$ (N)	<b>12.1</b>	-	<b>8.4</b>	-
$M_2$ (Kg)	<b>37.8</b>	+1.1%	<b>45.7</b>	-1.1%
$F_{v2}$ (N/m/s)	<b>135</b>	-	<b>80</b>	-
$F_{c2}$ (N)	<b>6.40</b>	-	<b>10</b>	-
$K_{12}$ ( $10^5$ N/m)	<b>7.681</b>	-8.6%	<b>8.2</b>	+1.2%
$M_I + M_2$	<b>105</b>	+1.9%	<b>113.8</b>	+0.7%
$F_{v1} + F_{v2}$	<b>212</b>	-1.4%	<b>208</b>	-4.1%
$F_{c1} + F_{c2}$	<b>18.5</b>	+5%	<b>18.4</b>	+4%
$\Delta M_2$	-	-	<b>7.9</b>	-10%
$\ \tau_{Idkm} - \tau_I\  / \ \tau_I\ $	7.38%	-	8.21%	-

## VI. CONCLUSION

This paper has presented a new *CLOE* method to identify the dynamic parameters of joint flexible manipulator. The method is carried out with three steps, based on *DIDIM* method. Furthermore, only actual force/torque data are used. To highlight the effectiveness of our approach, results obtained with *DIDIM* method were compared with those obtained with *IDIM-LS* and *OE* methods. The experimental results show that results given by *DIDIM* method are comparable with *IDIM-LS* results which need three signals measurements (actual force/torque data, motor and load positions). *DIDIM*

method converges 9 times faster than other *OE* method and avoids tuning the bandpass filter in the *IDIM-LS* method by using the integration of the *DDM*. Future works concern the use of *DIDIM* method to identify a multi *dof* robot with both joint flexibilities and gravity effect.

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