# G.A — ELIPSE

## PROPRIEDADES DA ELIPSE

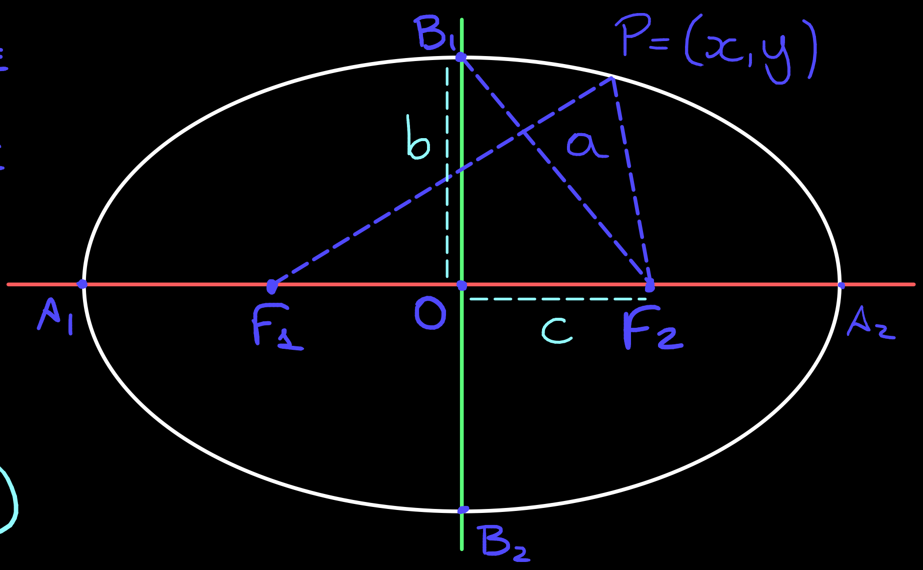
$F_1$ e $F_2$: Foco da elipse

O: Centro da elipse

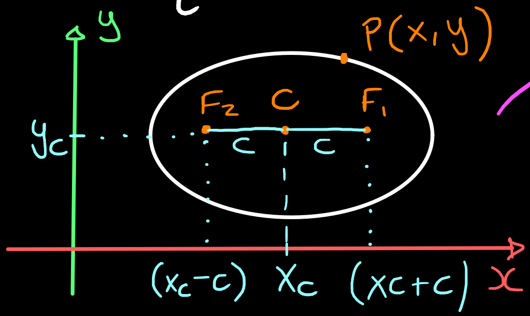
$\overline{A_1A_2}$: $2a$ (Eixo maior)

$\overline{B_1B_2}$: $2b$ (Eixo menor)

$e = \frac{c}{a}$ (Excentricidade)




## EQUAÇÃO REDUZIDA




$|\vec{F_1P}|+|\vec{F_2P}|=2a$

$\vec{F_1P}=(x-x_c+c,\ y-y_c)$

$\vec{F_2P}=(x-x_c-c,\ y-y_c)$

$x-x_c=s$

$y-y_c=t$

$$\sqrt{(\underbrace{x-x_c}_{s}+c)^2+(\underbrace{y-y_c}_{t})^2}+\sqrt{(\underbrace{x-x_c}_{s}-c)^2+(\underbrace{y-y_c}_{t})^2}=2a$$

$$\left(\sqrt{(s+c)^2+t^2}\right)^2=\left(2a-\sqrt{(s-c)^2+t^2}\right)^2 \Rightarrow (s+c)^2=4a^2-4a\sqrt{(s+c)^2+t^2}+(s-c)^2$$

$$\cancel{s^2}+\cancel{2sc}+\cancel{c^2}=4a^2-4a\sqrt{(s-c)^2+t^2}+\cancel{s^2}-\cancel{2sc}+\cancel{c^2} \Rightarrow \left(a\sqrt{(s+c)^2+t^2}\right)^2=\left(a^2-s\cdot c\right)^2$$

$$a^2\left((\underbrace{s-c^2}_{s^2-2sc+c^2})+t^2\right)=a^4-\cancel{2a^2sc}+s^2c^2=a^2s^2-\cancel{2a^2sc}+a^2c+at^2$$

$$a^2s^2-c^2s^2+a^2t^2=a^4-a^2c^2 \Rightarrow s^2(\overbrace{a^2-c^2}^{b^2})+a^2t^2=a^2(\overbrace{a^2-c^2}^{b^2})$$

$$\frac{s^2\cancel{b^2}}{a^2\cancel{b^2}}+\frac{\cancel{a^2}t^2}{\cancel{a^2}b^2}=\frac{\cancel{a^2b^2}}{a^2b^2} \rightarrow \boxed{\frac{(x-x_c)^2}{a^2}+\frac{(y-y_c)^2}{b^2}=1}$$

# EQUAÇÃO GERAL

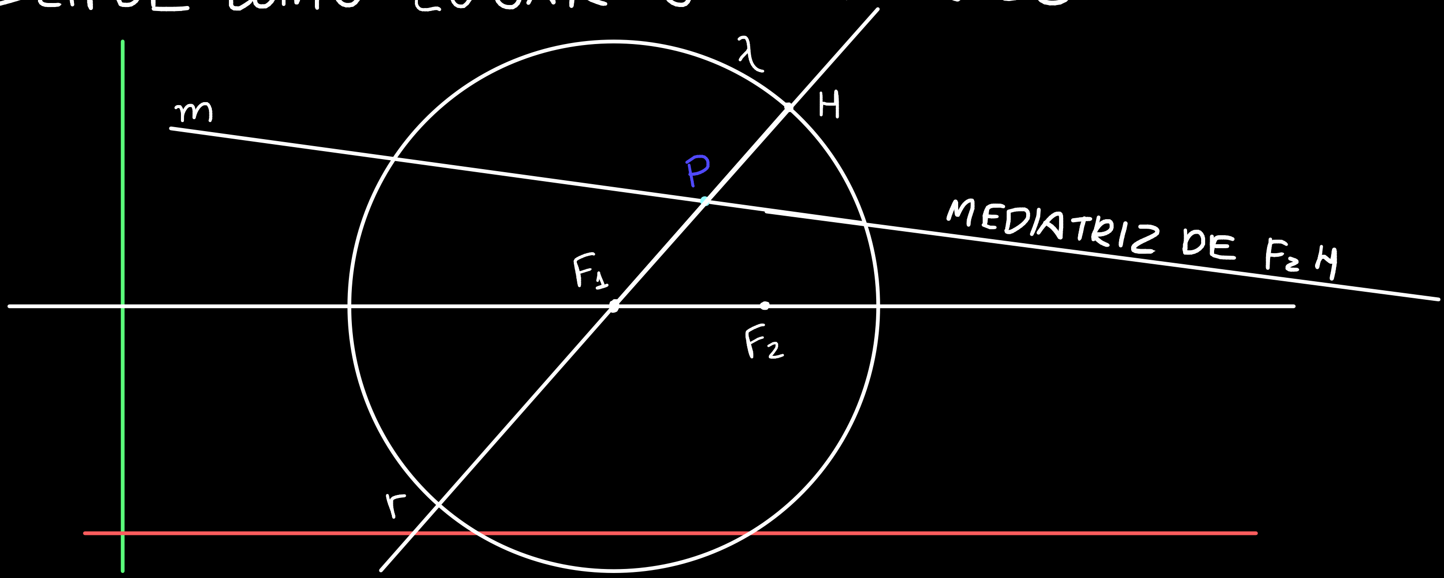
$$\frac{(x-x_c)^2}{a^2}+\frac{(y-y_c)^2}{b^2}=1 \Rightarrow \frac{x^2-2xx_c+x_c^2}{a^2}+\frac{y^2-2yy_c+y_c^2}{b^2}=1$$

$$0=b^2x^2-2xx_cb^2+b^2x_c^2+a^2y^2+2yy_ca^2+y_c^2a^2-a^2b^2$$

# ÁREA DA ELIPSE

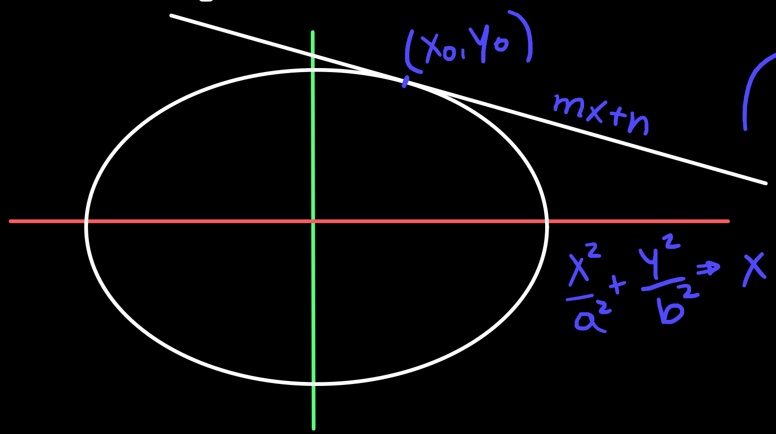
$$A_E=\pi\cdot a\cdot b$$

# ELIPSE COMO LUGAR GEOMÉTRICO




A ELIPSE É O LUGAR GEOMÉTRICO DOS PONTOS $P$ T.Q.
$P \in r \cap m \wedge r \cap \lambda = \{F_1, P\}$

# EQUAÇÃO DA RETA TANGENTE À ELIPSE




$$m(x-x_0)=y-y_0$$

$$m=f'$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} \Rightarrow x^2b^2+y^2a^2-a^2b^2=0$$

$$0=2xb^2+2yy'a^2$$

$$y'=-\frac{xb^2}{a^2y}$$

$$y - y_0 = \frac{-xb^2}{a^2y}\cdot(x - x_0) \Rightarrow a^2y^2 - a^2yy_0 = -x^2b^2 + xx_0b^2$$

$$\frac{a^2y^2 + x^2b^2 = xx_0b^2 + a^2yy_0}{a^2b^2}$$

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} = \frac{x_0}{a^2}x + \frac{y_0}{b^2}y$$

$$\boxed{1 = \frac{x_0}{a^2}\cdot x + \frac{y_0}{b^2}\cdot y}$$