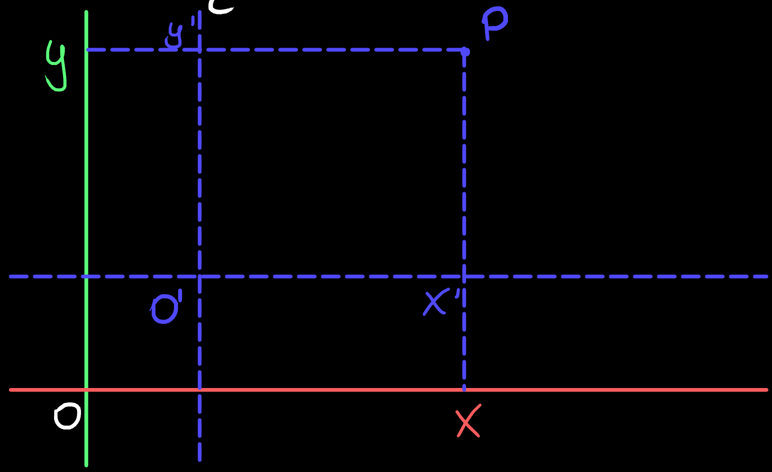
G.A

02/05/24

TRANSFORMAÇÃO DE COORDENADAS

① TRANSLAÇÃO



$O' = (a, b)$

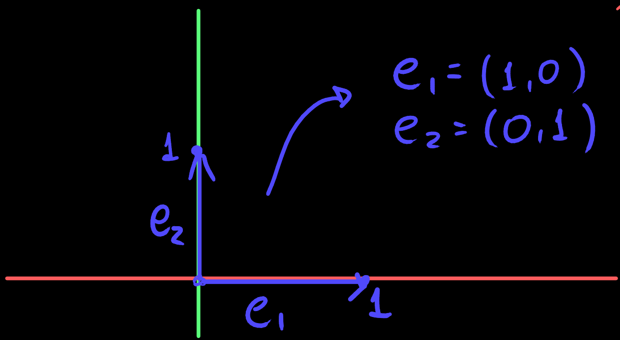
$$\begin{cases} y = y' + b \\ x = x' + a \end{cases}$$

$$\begin{cases} y' = y - b \\ x' = x - a \end{cases}$$

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \qquad y = \pm\sqrt{1 - \frac{(x-x_0)^2}{a^2}} + y_0$$

② ROTAÇÃO

- VETOR IDENTIDADE



$e_1 = (1,0)$

$e_2 = (0,1)$

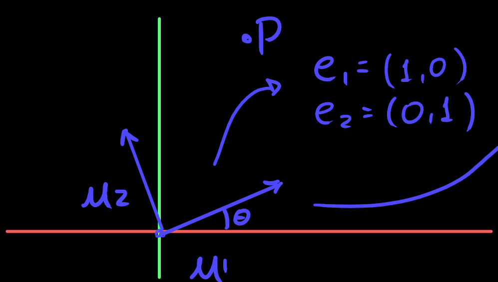
→ TODO VETOR DO PLANO PODE SER REPRESENTADO COMO UMA COMBINAÇÃO LINEAR DESSES VETORES.

$P = (x, y) \Rightarrow (x, 0) + (0, y)$

$\Rightarrow x(1,0) + y(0,1)$

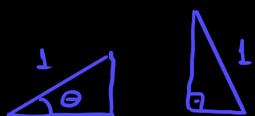
$\Rightarrow \underline{P = xe_1 + ye_2}$

VETOR IDENTIDADE DO EIXO ROTACIONADO



$e_1 = (1,0)$

$e_2 = (0,1)$



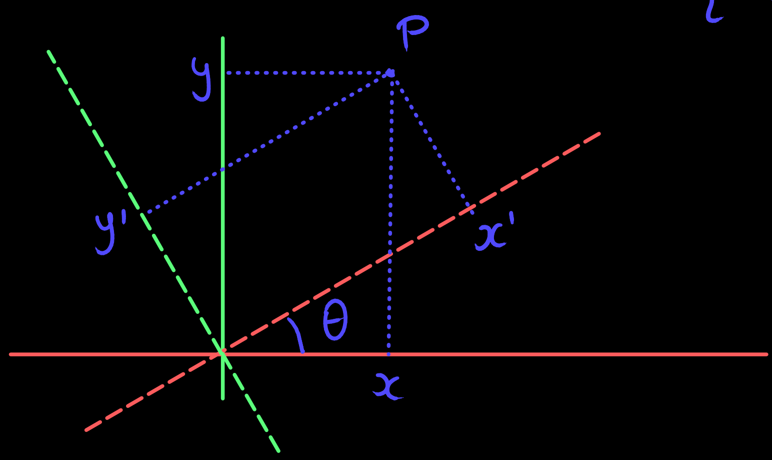
$u_1 = (\cos\theta, \operatorname{sen}\theta)$

$u_2 = (-\operatorname{sen}\theta, \cos\theta)$

→ $P = (x', y')$

$$\boxed{\begin{aligned} P &= x'u_1 + y'u_2 \\ u_1 &= \cos\theta\, e_1 + \operatorname{sen}\theta\, e_2 \\ u_2 &= \cos\theta\, e_2 - \operatorname{sen}\theta\, e_1 \end{aligned}}$$

COORDENADAS NOVAS EM FUNÇÃO DAS ANTIGAS



$P = x'u_1 + y'u_2 \rightarrow P = x'(\cos\theta\, e_1 + \operatorname{sen}\theta\, e_2) + y'(-\operatorname{sen}\theta\, e_1 + \cos\theta\, e_2)$

$\rightarrow e_1(\underbrace{x'\cos\theta - y'\operatorname{sen}\theta}_{x}) + e_2(\underbrace{x'\operatorname{sen}\theta + y'\cos\theta}_{y})$

$$\begin{cases} x = x'\cos\theta - y'\operatorname{sen}\theta & (\cos\theta) \\ y = x'\operatorname{sen}\theta + y'\cos\theta & (\operatorname{sen}\theta) \end{cases} \Rightarrow \begin{cases} x\cos\theta = x'\cos^2\theta - \cancel{y'\operatorname{sen}\theta\cos\theta} \\ y\operatorname{sen}\theta = x'\operatorname{sen}^2\theta + \cancel{y'\cos\theta\operatorname{sen}\theta} \end{cases}$$

$$\boxed{\begin{gathered} x\cos\theta + y\operatorname{sen}\theta = x' \\ \text{ANALOGAMENTE} \\ -x\operatorname{sen}\theta + y\cos\theta = y' \end{gathered}}$$

$$\rightarrow \begin{cases} x' = x\cos\theta + y\operatorname{sen}\theta \\ y' = -x\operatorname{sen}\theta + y\cos\theta \end{cases}$$

- FÓRMULA GERAL DA EQUAÇÃO QUADRÁTICA

$$Ax^2 + Bxy + Cy^2 + \underbrace{Dx + Ey}_{\text{ELIMINA POR TRANSLAÇÃO}} + F = 0$$

$\rightarrow Ax^2 + Bxy + Cy^2 = K$

$A(x'\cos\theta - y'\operatorname{sen}\theta)^2 + B(x'\cos\theta - y'\operatorname{sen}\theta)(x'\operatorname{sen}\theta + y'\cos\theta) + C(x'\operatorname{sen}\theta + y'\cos\theta)^2 = K$

$Ax'^2\cos^2\theta - 2Ax'y'\operatorname{sen}\theta\cos\theta + Ay'^2\operatorname{sen}^2\theta +$
$Bx'^2\cos\theta\operatorname{sen}\theta + Bx'y'\cos^2\theta - Bx'y'\operatorname{sen}^2\theta - By'^2\operatorname{sen}\theta\cos\theta +$
$Cx'^2\operatorname{sen}^2\theta + 2Cx'y'\operatorname{sen}\theta\cos\theta + Cy'^2\cos^2\theta = K$

A NOVA EQUAÇÃO TEM A CARA:

$$A'x'^2 + B'x'y' + C'y'^2$$

$$B' = 0 \Rightarrow B(\underbrace{\cos^2\theta - \operatorname{sen}^2\theta}_{\cos 2\theta}) + (C-A)(\underbrace{2\operatorname{sen}\theta\cos\theta}_{\operatorname{sen} 2\theta}) = 0$$

$$\to B\cos 2\theta = (A-C)\operatorname{sen} 2\theta$$

$$\to \boxed{\operatorname{tg} 2\theta = \frac{B}{A-C}}$$

$$\cos 2\theta = \pm \frac{1}{\sqrt{1+\operatorname{tg}^2 2\theta}}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\operatorname{sen}\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$