ÁLGEBRA LINEAR

18/09/24

ORTOGONALIDADE NOS ESPAÇOS FUNDAMENTAIS

- $N(A) \perp C(A^T)$

DEM. 1

$x \in N(A) \Leftrightarrow Ax = 0$ →

$$A = \begin{bmatrix} - & a_1 & - \\ & \vdots & \\ - & a_i & - \\ & \vdots & \\ - & a_n & - \end{bmatrix} \cdot x = 0$$

a_i = LINHA i DE X, LOGO ← $a_i \cdot x = 0$

$a_i \in C(A^T) \Rightarrow x \perp C(A^T) \Rightarrow N(A) \perp C(A^T)$

DEM. 2

$w \in C(A^T) \Leftrightarrow y \in \mathbb{R}^m \rightarrow A^T y = w \quad (x \in N(A))$

$x^T A^T y = x^T w \Rightarrow (\underbrace{Ax}_{0})^T y = x^T w \Rightarrow \underline{0 = x^T w}$

- $C(A) \perp N(A^T)$

DEM

Resultado anterior com $A^T \Rightarrow N(A^T) \perp C((A^T)^T) = C(A)$