Álgebra Linear

19/10/24

Lista 9

1. Seja B uma matriz 3×3 com autovalores 0, 1 e 2. Com essa informação, ache:

   (a) o posto de B;
   (b) o determinante de $B^T B$;
   (c) os autovalores de $B^T B$;
   (d) os autovalores de $(B^2 + I)^{-1}$.

a) Como 0 é autovalor, $N(B) \neq \{0\}$, logo, $\text{posto}(B) < 3$

Temos que MA de cada autovalor é 1, como $MG \leq MA$, MG de cada um também é 1, como $\dim E_0 = \dim N(B) = 1$, $\underline{\text{posto}(B) = 2}$

b) $\det B^T B = \det B^T \cdot \underbrace{\det B}_{0}$

$\Leftrightarrow \underline{\det B^T B = 0}$

c) Não podemos saber com certeza quais são, mas 0 é um deles

d) Vamos calcular os autovalores de $B^2 + I$ primeiro

Tomando o autovetor x; $Bx = x$, temos:

$(B^2 + I)x = \lambda x \Leftrightarrow B^2 x + x = \lambda x \Leftrightarrow x + x = \lambda x \Rightarrow \underline{\lambda = 2}$

Tomando o autovetor y; $By = 2y$, temos:

$(B^2 + I)y = \mu y \Leftrightarrow B^2 y + y = \mu y \Leftrightarrow 4y + y = \mu y \Leftrightarrow \underline{\mu = 5}$

Tomando os autovetores; $Bz = 0$

$(B^2 + I)z = \omega z \Leftrightarrow z = \omega z \Leftrightarrow \underline{\omega = 1}$

Sabemos que se $Bx = \lambda x$, se $\exists B^{-1}$, então $\frac{1}{\lambda} x = B^{-1} x$

logo os autovalores de $(B^2+I)^{-1}$ são:

$$1, \frac{1}{5}, \frac{1}{2}$$

2. Ache os autovalores das seguintes matrizes

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\rightarrow$ autovalores: $\underline{1, 4, 6}$

$\sum_{i=1}^{n} \lambda_i = 11 \quad \prod_{i=1}^{n} \lambda_i = \det A$

(b) $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ $\rightarrow Bx = \lambda x \Rightarrow \det(B - \lambda I) = 0$

$\rightarrow B - \lambda I = \begin{bmatrix} -\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & -\lambda \end{bmatrix} \Rightarrow \lambda^2(2-\lambda) - 3(2-\lambda)$

$(2-\lambda)(\lambda^2-3) \Rightarrow \underline{(2-\lambda)(\lambda+\sqrt{3})(\lambda-\sqrt{3})}$

Autovalores: $\underline{2, \sqrt{3}, -\sqrt{3}}$

(c) $C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

Como C não é invertível e $\dim N(C) = 2$, 0 é um autovalor e $\dim E_0 = 2$.

Como eu tenho 3 autovalores, e eu sei que $\sum_{i=1}^{n} \lambda_i = tr(C)$ e 2 são $0 \Rightarrow$ autovalores são:

$\underline{0, 6}$

3. Descreva todas as matrizes S que diagonalizam as matrizes A e A^{-1}:

$$A = \begin{bmatrix} 0 & 4 \\ 1 & 2 \end{bmatrix}.$$

$$\begin{bmatrix} -\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix} \Rightarrow \det(A-\lambda I) = \lambda^2 - 2\lambda - 4 \begin{cases} \to \lambda = 1+\sqrt{5} \\ \to \lambda = 1-\sqrt{5} \end{cases}$$

Achando os autovetores

$\lambda = 1+\sqrt{5}$

$$\begin{bmatrix} -1-\sqrt{5} & 4 \\ 1 & 1-\sqrt{5} \end{bmatrix}^{=B} \longrightarrow \begin{bmatrix} -1-\sqrt{5} & 4 \\ 0 & 0 \end{bmatrix} \div \frac{-1}{1+\sqrt{5}} = \begin{bmatrix} 1 & -4/_{1+\sqrt{5}} \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow N(B) = \text{span}\left\{\begin{bmatrix} 4/_{1+\sqrt{5}} \\ 1 \end{bmatrix}\right\}$$

$\lambda = 1-\sqrt{5}$

$$\begin{bmatrix} \sqrt{5}-1 & 4 \\ 1 & 1+\sqrt{5} \end{bmatrix}^{=C} \longrightarrow \begin{bmatrix} \sqrt{5}-1 & 4 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4/_{\sqrt{5}-1} \\ 0 & 0 \end{bmatrix}$$

$$\to N(C) = \text{span}\left\{\begin{bmatrix} \frac{-4}{\sqrt{5}-1} \\ 1 \end{bmatrix}\right\}$$

Diagonalizando

$$A = \begin{bmatrix} \frac{4}{1+\sqrt{5}} & \frac{-4}{\sqrt{5}-1} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1+\sqrt{5} & 0 \\ 0 & 1-\sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ \frac{4}{\sqrt{5}-1} & \frac{4}{1+\sqrt{5}} \end{bmatrix} \cdot \frac{1}{2\sqrt{5}}$$

4. Ache Λ e S que diagonalizem A

$$A = \begin{bmatrix} 0.6 & 0.9 \\ 0.4 & 0.1 \end{bmatrix}.$$

Qual limite de Λ^k quando $k \to +\infty$? E o limite de A^k?

$$\begin{bmatrix} 0.6-\lambda & 0.9 \\ 0.4 & 0.1-\lambda \end{bmatrix} \Rightarrow (0.6-\lambda)(0.1-\lambda) - 0.36$$

$$0.06 - 0.6\lambda - 0.1\lambda + \lambda^2 - 0.36$$

$$\lambda^2 - 0.7\lambda - 0.3 = 0$$

$$\lambda = 1 \longrightarrow \lim_{k\to\infty} S\Lambda^k S^{-1} = S(\lim_{k\to\infty} \Lambda^k) S^{-1}$$

$$\lambda = -\frac{3}{10}$$

Calculando autovetores:

$\lambda = 1$

$$\begin{bmatrix} -0.4 & 0.9 \\ 0.4 & -0.9 \end{bmatrix} \longrightarrow \begin{bmatrix} -0.4 & 0.9 \\ 0 & 0 \end{bmatrix} \rightarrow N(A-I) = \text{span}\left\{\begin{bmatrix} 2.25 \\ 1 \end{bmatrix}\right\}$$

$\lambda = \frac{-3}{10}$

$$\begin{bmatrix} 0.9 & 0.9 \\ 0.4 & 0.4 \end{bmatrix} = N(A + \tfrac{3}{10}I) = \text{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$$

$$\text{Achando } S\Lambda S^{-1} \Rightarrow \underbrace{\begin{bmatrix} 1 & 2.25 \\ -1 & 1 \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} -0.3 & 0 \\ 0 & 1 \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} 1 & -2.25 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{3.25}}_{S^{-1}}$$

Aplicando

$$\lim_{k\to\infty} \Lambda^k = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \lim_{k\to\infty} A^k = \underbrace{\begin{bmatrix} 1 & 2.25 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}\begin{bmatrix} 1 & -2.25 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{3.25}$$

$$\frac{1}{3.25}\begin{bmatrix} 2.25 & 2.25 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2.25 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2.25 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{3.25}$$

5. Seja $Q(\theta)$ a matriz de rotação do ângulo θ em $\mathbb{R}^2$:

$$Q(\theta) = \begin{bmatrix} \cos\theta & -\text{sen}\theta \\ \text{sen}\theta & \cos\theta \end{bmatrix}.$$

Ache os autovalores e autovetores de $Q(\theta)$ (eles podem ser complexos).

$$Q - \lambda I = \begin{bmatrix} \cos - \lambda & -\text{sen} \\ \text{sen} & \cos - \lambda \end{bmatrix} \rightarrow \cancel{\cos^2} - 2\lambda\cos + \lambda^2 + \cancel{\text{sen}^2}$$

$$\lambda^2 - 2\lambda\cos\theta + 1$$

$$\Delta = 4\cos^2\theta - 4 \longrightarrow 4(\cos^2\theta - 1)$$

$$\lambda = \frac{\cancel{2}\cos\theta \pm \cancel{2}\sqrt{-\text{sen}^2\theta}}{\cancel{2}} \Rightarrow \underbrace{\lambda = \cos\theta + i|\text{sen}\theta|}$$

6. Suponha que A e B são duas matrizes $n \times n$ com os mesmo autovalores $\lambda_1, \ldots, \lambda_n$ e os mesmos autovetores $x_1, \ldots, x_n$. Suponha ainda que $x_1, \ldots, x_n$ são LI. Prove que $A = B$.

Se A e B tem autovetores independentes, entao ambos são diagonalizáveis:

$$A = \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix}^{-1}$$

$$B = \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix}^{-1}$$

É óbvio ver que ambas são inversíveis, e que

$$A^{-1} \cdot B = I \Rightarrow \boxed{B = A}$$

7. Seja $Q(\theta)$ como na Questão 5. Diagonalize $Q(\theta)$ e mostre que

$$Q(\theta)^n = Q(n\theta).$$

Autovalores de Q:

$\lambda_1 = \cos\theta + i|\text{sen}\,\theta|$

$\lambda_2 = \cos\theta - i|\text{sen}\,\theta|$

Achando os autovetores de Q

$Q - \lambda_1 I$

$$\begin{bmatrix} -i|\text{sen}\theta| & -\text{sen}\theta \\ \text{sen}\theta & -i|\text{sen}\theta| \end{bmatrix} \rightarrow \begin{bmatrix} i \\ 1 \end{bmatrix} \in N(Q - \lambda_1 I)$$

$$\begin{bmatrix} i|\text{sen}\theta| & -\text{sen}\theta \\ \text{sen}\theta & i|\text{sen}\theta| \end{bmatrix} \rightarrow \begin{bmatrix} i \\ -1 \end{bmatrix} \in N(Q - \lambda_2 I)$$

$$Q = \begin{bmatrix} i & i \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\theta - i|\operatorname{sen}\theta| & 0 \\ 0 & \cos\theta + i|\operatorname{sen}\theta| \end{bmatrix} \begin{bmatrix} -i/2 & 1/2 \\ -i/2 & -1/2 \end{bmatrix}$$

$\rightarrow$ Já sabemos que $(\cos\theta + i\operatorname{sen}\theta)^n = \cos(n\theta) + i\operatorname{sen}(n\theta)$

$$\Rightarrow Q^n = \begin{bmatrix} i & i \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}^n \begin{bmatrix} -i/2 & 1/2 \\ -i/2 & -1/2 \end{bmatrix}$$

$$\lambda_1^n = \cos(n\theta) - i|\operatorname{sen}(n\theta)|$$

$$\lambda_2^n = \cos(n\theta) + i|\operatorname{sen}(n\theta)|$$

8. Suponha que G_{k+2} é a média dos dois números anteriores G_{k+1} e G_k. Ache a matriz A que faz com que

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}.$$

(a) Ache os autovalores e autovetores de A;

(b) Ache o limite de A^n quando $n \to +\infty$;

(c) Mostre que G_n converge para $2/3$ quando $G_0 = 0$ e $G_1 = 1$.

(8) $G_{k+2} = \dfrac{G_{k+1} + G_k}{2}$

a)

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 1/2-\lambda & 1/2 \\ 1 & -\lambda \end{bmatrix}\right) = 0 \Rightarrow -\lambda(1/2-\lambda) - 1/2 = 0$$

$$\lambda^2 - \lambda/2 - 1/2 = 0$$

$$\Delta = \frac{1}{4} + \cancel{4}^{2} \cdot 1/2$$

$$\Delta = 1/4 + 2 = 9/4 \rightarrow \lambda = \frac{1/2 \pm 3/2}{2} \begin{cases} \lambda_1 = 1 \\ \lambda_2 = -\frac{1}{2} \end{cases}$$

$$N\left(\begin{bmatrix} -1/2 & 1/2 \\ 1 & -1 \end{bmatrix}\right) = \operatorname{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\} \rightarrow \text{autovetores}$$

$$N\left(\begin{bmatrix} 1 & 1/2 \\ 1 & 1/2 \end{bmatrix}\right) = \operatorname{span}\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$$

$$N\left(\begin{bmatrix} 1 & 1/2 \\ 1 & 1/2 \end{bmatrix}\right) = \operatorname{span}\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$$

b) Diagonalizando:

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \rightarrow \det = -3 \rightarrow S^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}$$

$$\lim_{n\to\infty} A^n = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \cancel{-1/2}_{0} \end{bmatrix}^n \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}}$$

c) Se $G_0 = 0$ e $G_1 = 1$

$$\begin{bmatrix} G_n \\ G_{n-1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad n \to \infty$$

$$\begin{bmatrix} G_n \\ G_{n-1} \end{bmatrix} = \begin{bmatrix} \cancel{2/3} & \cancel{1/3} \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\boxed{G_n = \frac{2}{3}}$$

9. Ache a solução do sistema de EDOs usando o método de diagonalização:

$$\begin{cases} u_1'(t) = 8u_1(t) + 3u_2(t), \\ u_2'(t) = 2u_1(t) + 7u_2(t), \end{cases}$$

onde $u(0) = (5, 10)$.

⑨ $\begin{cases} u_1'(t) = 8u_1(t) + 3u_2(t) \\ u_2'(t) = 2u_1(t) + 7u_2(t) \end{cases}$ $\quad u(0) = [5, 10]$

$$u'(t) = \overset{A}{\begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad u_1(t) = C_1 e^{\lambda t}$$

$$\det\left(\begin{bmatrix} 8-\lambda & 3 \\ 2 & 7-\lambda \end{bmatrix}\right) = (8-\lambda)(7-\lambda) - 6$$

$$\lambda^2 - 15\lambda + 56 - 6$$

$$\lambda^2 - 15\lambda + 50 = 0$$

$\Delta = 225 - 200$

$\Delta = 25$

$$\lambda = \frac{15 \pm 5}{2} \quad \lambda_1 = 10, \quad \lambda_2 = 5$$

$$N(A - 10I) = N\left(\begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}\right) \to \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$$

$$N(A - 5I) = N\left(\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\right) \to \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Diagonalizando:

$$S = \begin{bmatrix} 3/2 & 1 \\ 1 & -1 \end{bmatrix} \to \det = -3/2 - 1 = -5/2 \to \begin{bmatrix} \frac{2}{5} & 2/5 \\ \frac{2}{5} & -3/5 \end{bmatrix}$$

$$A = \begin{bmatrix} 3/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 2/5 & 2/5 \\ 2/5 & -3/5 \end{bmatrix}$$

$$v(0) = S^{-1} u(0)$$

$$\begin{bmatrix} v_1(0) \\ v_2(0) \end{bmatrix} = \begin{bmatrix} 2/5 & 2/5 \\ 2/5 & -3/5 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} v_1(0) \\ v_2(0) \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$v_1(t) = v(0) \cdot e^{\lambda_1 t} \Rightarrow v_1(t) = 6e^{10t}$$

$$v_2(t) = -4e^{5t}$$

$$u(t) = S v(t)$$

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 3/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 6e^{10t} \\ -4e^{5t} \end{bmatrix} \Rightarrow \begin{array}{l} u_1(t) = 9e^{10t} - 4e^{5t} \\ \underline{u_2(t) = 6e^{10t} + 4e^{5t}} \end{array}$$

10. Seja $\mathcal{F}(\mathbb{R};\mathbb{R})$ o espaço vetorial das funções reais de uma variável real. Considere em $\mathcal{F}(\mathbb{R};\mathbb{R})$ o subespaço

$$S := \text{Span}\left\{e^{2x}\,\text{sen}\,x,\, e^{2x}\cos x,\, e^{2x}\right\}.$$

e o operador linear $D : S \to S$ definido por $D(f) = f'$. Considere, ainda, as funções $f_1(x) = e^{2x}\,\text{sen}\,x$, $f_2(x) = e^{2x}\cos x$ e $f_3(x) = e^{2x}$ em $\mathcal{F}(\mathbb{R};\mathbb{R})$. Determine:

(a) a matriz de D em relação à base $\mathcal{B} = \{f_1, f_2, f_3\}$. Lembre-se de que, dada a base $\mathcal{B}$, podemos enxergar os elementos de como vetores em $\mathbb{R}^3$. Por exemplo:

$$(1,2,3)_{\mathcal{B}} = f_1 + 2f_2 + 3f_3.$$

(b) os autovalores de D e as funções de S que são autovetores de D.

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$S := \text{span}\{e^{2x}\text{sen}x, e^{2x}\cos x, e^{2x}\}$ $\quad D: S \to S \mid D(f) = f'$

$f_1(x) = e^{2x}\text{sen}x \quad f_2(x) = e^{2x}\cos x \quad f_3(x) = e^{2x}, \quad \beta = \{f_1, f_2, f_3\}$

a) $\begin{bmatrix} f_1' \\ f_2' \\ f_3' \end{bmatrix} = D \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$

$f_1'(x) = 2e^{2x}\text{sen}x + e^{2x}\cos x$

$f_2'(x) = 2e^{2x}\cos x - e^{2x}\text{sen}x$

$f_3'(x) = 2e^{2x}$

$\Rightarrow f_1' = 2f_1 + f_2$

$f_2' = 2f_2 - f_1$

$f_3' = 2f_3$

$D = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

b)

$D - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 0 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} \to (2-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = (2-\lambda)\left[(2-\lambda)^2 + 1\right]$

$\lambda_1 = 2$ é raiz $\to (\underbrace{2-\lambda}_{=i})^2 + 1 = 0 \quad\quad \lambda_1 = 2 \quad \lambda_2 = 2-i \quad \lambda_3 = 2+i$

$2-\lambda = i \quad\quad 2-\lambda = -i$

$\lambda_2 = 2-i \quad\quad \lambda = 2+i$

$D - 2I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \to \underline{e^{2x}}$

$D - (2-i)I = \begin{bmatrix} i & 1 & 0 \\ -1 & i & 0 \\ 0 & 0 & i \end{bmatrix} \to \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} \to \underline{i(e^{2x}\text{sen}x) + e^{2x}\cos x}$

$D - (2+i)I = \begin{bmatrix} -i & 1 & 0 \\ -1 & -i & 0 \\ 0 & 0 & -i \end{bmatrix} \to \begin{bmatrix} -i \\ 1 \\ 0 \end{bmatrix} \to \underline{e^{2x}\cos x - ie^{2x}\text{sen}x}$