Álgebra Linear

18/10/24

Polinômio Característico

- Def: $p(\lambda) = (-1)^n \cdot \det(A - \lambda I) = (-1)^n (\lambda^n + c_{n-1}\lambda^{n-1} + \dots + c_0)$

  $\hookrightarrow \det(\lambda I - A)$

- Se λ_i são raízes de $p(\lambda)$, então $p(\lambda) = (-1)^n \prod_{i=1}^{n} (\lambda - \lambda_i)$

- Propriedades

  ▷ $\det A = \prod_{i=1}^{n} \lambda_i$

  DEM

  $p(0) = \det(A - I \cdot 0) = \det A$

  $p(0) = \prod_{i=1}^{n} \lambda_i \Rightarrow \det A = \prod_{i=1}^{n} \lambda_i$

  Além disso:

  $p(0) = (-1)^n \cdot c_0$

  $c_0 = (-1)^n \cdot \det A$

  ▷ $Tr(A) = \sum_{i=1}^{n} \lambda_i$

  DEM

  $p(\lambda) = (-1)^n \prod_{i=1}^{n} (\lambda - \lambda_i) \rightsquigarrow c_{n-1} = -\lambda_1 - \lambda_2 - \dots - \lambda_n = -\sum_{i=1}^{n} \lambda_i$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - a_{11} & -a_{12} & \cdots & \\ a_{21} & \lambda - a_{22} & & \\ \vdots & & \ddots & \\ \vdots & & & \ddots \end{vmatrix} = (\lambda - a_{11}) \underbrace{\begin{vmatrix} \lambda - a_{22} & & \vdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \lambda - a_{nn} \end{vmatrix}}_{\text{grau } n-1} + \underset{\substack{\downarrow \\ \text{grau} \\ n-2}}{q(\lambda)}$$

$$(\lambda - a_{11})(\lambda - a_{22}) \begin{vmatrix} \lambda - a_{33} & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \lambda - a_{nn} \end{vmatrix} + \bar{q}(\lambda)$$

$$\hookrightarrow \prod_{i=1}^{n} (\lambda - a_{ii}) + \bar{q}(\lambda) \Rightarrow \lambda^n - \left(\sum_{i=1}^{n} a_{ii}\right)\lambda^{n-1}$$