Quantum Circuits as Novel Activation Functions in Cryptography

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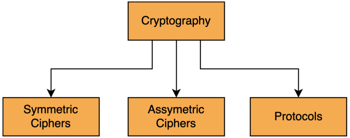
Conceptual framework

Definition 1 (Cryptography)

Literature shows different definitions, and some of these are [2]

- The discipline that embodies the principles, means, and methods for transforming data to encrypt semantic content, prevent unauthorized use or undetected modification .
- It is the science of encrypting information to conceal its content .



Conceptual framework

Definition 2 (Shannon Entropy)

It is given by

$$S = -\sum_{i=1}^{k} p_i \log_2 p_i \tag{1}$$

The entropy of uncertainty of a random variable X with probabilities $p_i, \ldots, p_n$.

Definition 3 (Von Neumann Entropy)

In the quantum information context,

$$H_V = -\sum_{i=1}^{n} \lambda_i \log_2 \lambda_i \tag{2}$$

Where λ_i are the eigenvalues of a density operator .

(some) Concepts

Definition 4 (Trapdoor function, trapdoor one-way function)

A function $f : \{0,1\}^* \to \{0,1\}^*$ is called **one-way** if the following two conditions hold . We additionally have the following two definitions,

1. A function that is simple to compute but difficult to invert without additional information is known as a **trapdoor function** .
2. A function that is easy to compute, but whose inverse is computationally infeasible to determine without specific privileged information.

Definition 5 (Protocol)

A **set of rules** used by two or more communicating entities that describe the message order and data structures for information exchanged between the entities is called **protocol** .

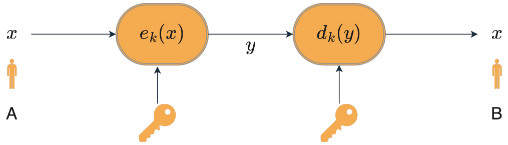
Definition 6 (One-Time-Pad protocol)

The protocol encrypts a message using a public channel and uses the XOR operation.

Implementation: QC and ML

We use B the text in binary is H$= 1001000_2 = 72_{10}$ and ciphertext in binary system is Z$= 1011010_2 = 90_{10}$. The subscripts refer to binary and decimal systems.
We notice that

$$\begin{aligned} B = DEC(C,K) &= DEC(ENC(B,K),K) \\ &= DEC(B \oplus K, K) \\ &= B \oplus K \oplus K \\ &= B \end{aligned} \tag{3}$$



Example

1. Encryption: To obtain ciphertext, $C = ENC(B, K) = B \oplus K$
2. Decryption: To retrieve the original text, $B = DEC(C, K) = C \oplus K$

Example

1. Encryption.

$$\begin{array}{rrcr} & 1001000 & \rightarrow & \mathrm{H} \\ \oplus & 0010010 & \rightarrow & 18 \\ \hline & 1011010 & \rightarrow & \mathrm{Z} \end{array}$$

2. Decryption

$$\begin{array}{rrcr} & 1011010 & \rightarrow & \mathrm{Z} \\ \oplus & 0010010 & \rightarrow & 18 \\ \hline & 1001000 & \rightarrow & \mathrm{H} \end{array}$$

Quantum Concepts

Definition 7 (Quantum key exchange (QKE))

It is the idea of leveraging quantum mechanics to improve classical protocols.

Definition 8 (BB84 protocol)

Let A and B, two individuals, exchange information using two points. Person A employs two distinct orthogonal bases to transmit the data.

Definition 9 (B92 protocol)

This protocol implements one non-orthogonal basis to send information.

Applications

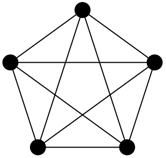
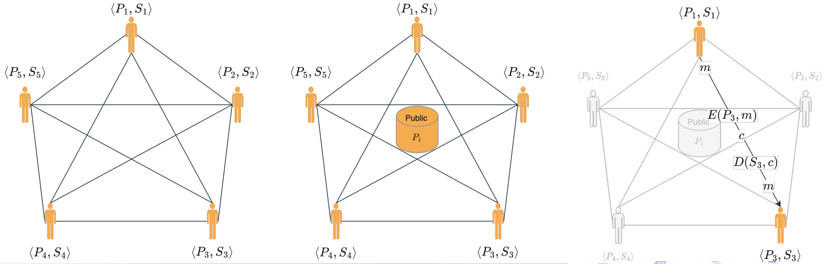


Figure: Graph for $n = 5$ and $k = 2$: This represents a network with $n = 5$ users, where $k = 2$ users are engaged in pairwise communication.

- We will swap points and
- edges



QC and ML

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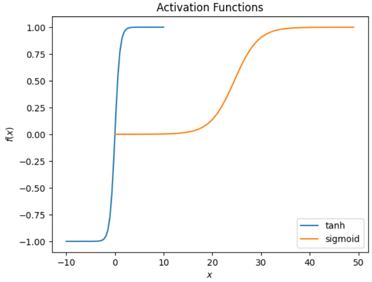
Definition 10 (Activation Functions)

Nonlinear functions that send a weighted sum of the inputs to a node.

Examples: Sigmoid, ReLU, softplus, $\tanh x$



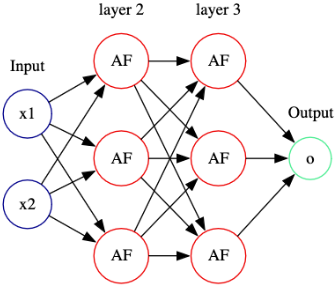


Figure: Examples of Activation functions and a fully connected network.

Neural Networks for Cryptography

Theorem 11 (Cybenko)

Let $f : \mathbb{R}^d \to \mathbb{R}$ be a function[1] *on a compact set $K \subset \mathbb{R}^d$. Then for any $\epsilon > 0$ there exists a neural network with a single hidden layer of the form*

$$\phi(x) = \sum_{i=1}^{N} \sum_{j=1}^{d} w_i^{(1)} \sigma(w_{ij}^{(0)} x_j + b_i^{(0)}) + b^{(1)},$$

$\theta = \{w_{ij}^{(0)}, w_i^{(1)}, b_i^{(0)}, b^{(1)}\}$, *where $\sigma : \mathbb{R} \to \mathbb{R}$ is an activation function*[2], *such that*

$$\sup_{x \in K} |f(x) - \phi(x)| < \epsilon.$$

The parameter N is known as the **width** [1].

[1] continuous

[2] non-polynomial non-linear

Examples: Activation Function

$$\sigma(x) = \mathrm{ReLU}(x) = \max(0, x)$$

$$\sigma(x) = \mathrm{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

$$\sigma(x) = \tanh x$$

$$\sigma(x) = \cos x, \sin x$$

Figure: Webbook: https://earlham-college.github.io/quaker-ece/

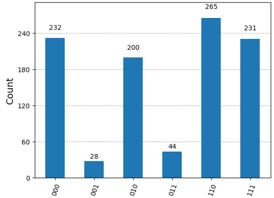


Figure: Addition=2 + 3 operation with QCircuits and simulator.

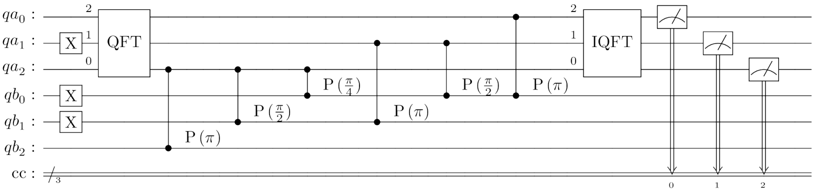


Figure: Partial implementation of AF

Conclusions and Discussion

- This paper explored fundamental concepts in cybersecurity/cryptography and their equivalents in the quantum domain.
- It also provided foundational information on key protocols in both classical and quantum cryptography.
- We replaced the activation function with a quantum circuit and analyzed potential options for security.

Future directions

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- Future work aims to expand on these fundamental concepts, incorporating emerging ideas from quantum computing, machine learning, and deep learning to contribute to developing next-generation cryptographic methods, particularly in the post-quantum cryptography era.
- Implement this and more experiments on a real quantum computer.

Explore and implement other options/circuits.

Figure: Webbook: https://github.com/jaorduz/QCandAI

Thank you!

References

Javier Orduz[1,2,3], Nagaraj[1]

[1] Jim Halverson. Tasi lectures on physics for machine learning, 2024.

[2] Javier Orduz. Mathematical foundations for Modern Cryptography in the Quantum Era. 2025. Accepted to be published soon.

Figure: https://jaorduz.github.io/files/2025/APSsldsA.pdf

Thanks to Earlham College for partially funding this project through the Stephen and Sylvia Tregidga Burges Endowed Fund for Student-Faculty Collaborative Research. JO acknowledges support from APS through award 9615914607. References will be included in my next paper: Mathematical Foundations for Modern Cryptography in the AI. Authored by JO.