Models and Semantics of Computation

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CCS: A Calculus of Communicating Systems (II)

- value passing CCS
- translations vs encodings
- strong and weak bisimilarity: definition, games, properties

Acknowledgment

This set of slides was originally produced by Jiri Srba, and makes part of the course material for the book

Reactive Systems: Modelling, Specification and Verification
by L. Aceto, A. Ingolfsdottir, K. G. Larsen and J. Srba
URL: http://rsbook.cs.aau.dk

I have adapted them slightly for the purposes of this course.

Value Passing CCS

Main Idea

Handshake synchronization is extended with the possibility to exchange integer values.

$$\overline{pay(6)}.Nil \parallel pay(x).\overline{save(x/2)}.Nil$$

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$$Nil \parallel Nil \parallel Bank(103)$$

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Translation of Value Passing CCS to Standard CCS

Value Passing CCS (CCS^v)

$$C \stackrel{\text{def}}{=} in(x).C'(x)$$

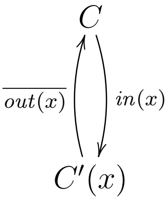
$$C'(x) \stackrel{\text{def}}{=} \overline{out(x)}.C$$

$$\longrightarrow$$

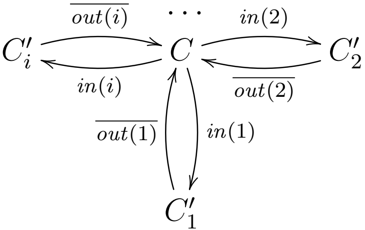
Standard CCS (CCS)

$$C \stackrel{\text{def}}{=} \sum_{i \in \mathbb{N}} in(i).C'_i$$

$$C'_i \stackrel{\text{def}}{=} \overline{out(i)}.C$$



symbolic LTS



infinite LTS

Language Translations and Encodings

- We would like to express the previous intuitions about the translation in a formal manner.
- We will rely on the concepts of translations and encodings. In a way, an encoding is a "compiler" between two different process calculi

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Later on the course we will be more precise about translations.]

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Language Translations and Encodings

Assume a source language $\mathcal{P}_{\mathsf{s}}$, and a target language $\mathcal{P}_{\mathsf{t}}$

A translation is a function $[\![\cdot]\!] : \mathcal{P}_{\mathsf{s}} \to \mathcal{P}_{\mathsf{t}}$

To be meaningful, translations should satisfy some criteria

- syntactic criteria ensuring, e.g., that the translation enjoys some compositionality principles
  Ex: homormorphism wrt parallel, i.e., $[\![P \parallel Q]\!] = [\![P]\!] \parallel [\![Q]\!]$
- semantic criteria ensuring, e.g., that the behavior of the translated target term is related to the behavior of the corresponding source term
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Encoding

A translation enjoying some syntactic and semantic criteria

Translation of Value Passing CCS to Standard CCS

Let V be a set of values.

Intuitively, each label l in CCS^v is a set $\{l_v \mid v \in V\}$ in CCS.

The translation $[\![\cdot]\!]_{\mathcal{V}} : \mathrm{CCS}^v \to \mathrm{CCS}$ is defined inductively as:

$$\begin{array}{rcl}
[\![a(x).P]\!]_{\mathcal{V}} & = & \sum_{v \in V} a_v.[\![P\{v/x\}]\!]_{\mathcal{V}} \\
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[\![A\langle e_1, \ldots, e_n \rangle]\!]_{\mathcal{V}} & = & A_{e_1, \ldots, e_n}
\end{array}$$

Also, the definition $A(\tilde{x}) \stackrel{\text{def}}{=} P$, with $\tilde{x} = x_1, \ldots, x_n$, is translated into a set of definitions $\{A_{\tilde{v}} \stackrel{\text{def}}{=} [\![P\{\tilde{v}/\tilde{x}\}]\!]_{\mathcal{V}} \mid \tilde{v} \in V^n\}$

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- The encoding shows that CCS^v can be safely used in examples; CCS is enough for theoretical purposes
- The encoding is possible because we have (guarded) choice on possibly infinite sums
- Recall the translations for parallel composition and sum:

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CCS is Turing Complete

Another expressiveness result:

Fact

CCS can simulate a computation of any Turing machine.

Question: how would you show this?

Remark

Hence CCS is as expressive as any other programming language but its use is to rather describe the behaviour of reactive systems than to perform specific calculations.

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Behavioural Equivalence

Implementation

$$CM \stackrel{\text{def}}{=} coin.\overline{coffee}.CM$$

$$CS \stackrel{\text{def}}{=} \overline{pub}.\overline{coin}.coffee.CS$$

$$Uni \stackrel{\text{def}}{=} (\nu\, coin, coffee)(CM \parallel CS)$$

Specification

$$Spec \stackrel{\text{def}}{=} \overline{pub}.Spec$$

Question

Are the processes Uni and $Spec$ behaviorally equivalent?

$$Uni \equiv Spec$$

where $\equiv$ is a binary relation on processes.

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Goals

What should a reasonable behavioral equivalence satisfy?

- abstract from states (consider only the behavior – actions)
- abstract from nondeterminism
- abstract from internal behavior

What else?

- reflexivity $P \equiv P$ for any process P
- transitivity $Spec_0 \equiv Spec_1 \equiv Spec_2 \equiv \cdots \equiv Impl$ gives that
$$Spec_0 \equiv Impl$$
- symmetry $P \equiv Q$ iff $Q \equiv P$
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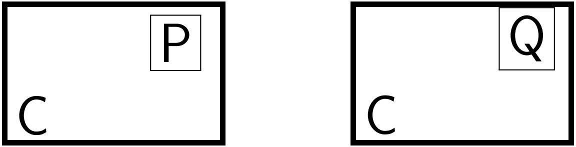

$C(P)$ $\qquad\qquad$ $C(Q)$

- We would like "equal" processes P and Q to "behave the same" under any context $C(\cdot)$.
- A context is a process with a hole. When the hole is filled in with a process P, we obtain another process (noted $C(P)$ or $C[P]$).

Congruence Property

$$P \equiv Q \text{ implies } C(P) \equiv C(Q)$$

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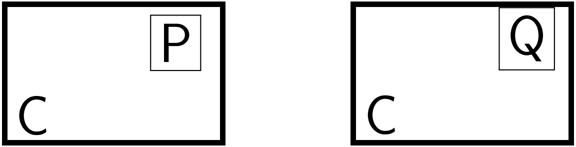

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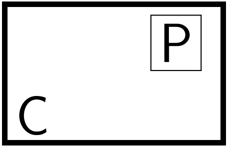

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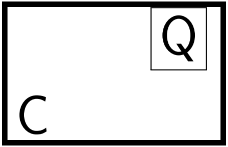

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Ideally

- Ideally, we would like to identify two processes unless there is some sequence of 'interactions' than an 'observer' may have with them leading to different 'outcomes'
- There are many different choices — many notions of behavioral equivalences.

We will explore two of them:

- Trace Equivalence
- Strong Bisimilarity

Trace Equivalence

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Trace Set for $s \in Proc$

$$Traces(s) = \{w \in Act^* \mid \exists s' \in Proc.\ s \xrightarrow{w} s'\}$$

Let $s \in Proc$ and $t \in Proc$.

Trace Equivalence

We say that s and t are trace equivalent ($s \equiv_t t$) if and only if

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Trace Equivalence: Example

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$$

We have $s \equiv_t t$ because

$$Traces(s) = Traces(t) = \{ab, ac\}$$

Black-Box Experiments

Main Idea

Two processes are behaviorally equivalent if and only if an external observer cannot tell them apart.

Strong Bisimilarity

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Strong Bisimulation

A binary relation $\mathcal{R} \subseteq Proc \times Proc$ is a strong bisimulation iff whenever $(s,t) \in \mathcal{R}$ then for each $a \in Act$:

- if $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some t' such that $(s',t') \in \mathcal{R}$
- if $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ for some s' such that $(s',t') \in \mathcal{R}$.

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Processes $p_1, p_2 \in Proc$ are strongly bisimilar $(p_1 \sim p_2)$ if and only if there exists a strong bisimulation $\mathcal{R}$ such that $(p_1, p_2) \in \mathcal{R}$.

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Basic Properties of Strong Bisimilarity

Theorem

$\sim$ is an equivalence (reflexive, symmetric and transitive)

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$s \sim t$ if and only if for each $a \in Act$:

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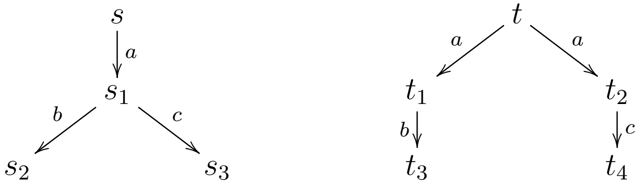
How to Show Nonbisimilarity?



To prove that $s \not\sim t$:

- Enumerate all binary relations and show that none of them at the same time contains (s, t) and is a strong bisimulation. (Expensive: $2^{|Proc|^2}$ relations.)
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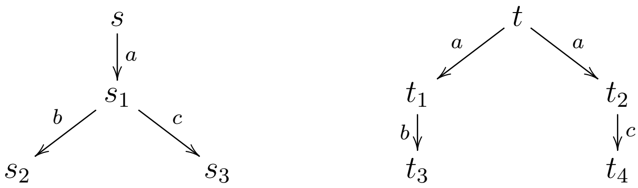

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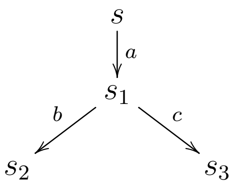

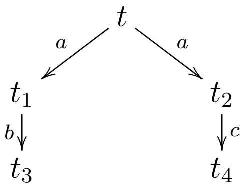

To prove that $s \not\sim t$:

- Enumerate all binary relations and show that none of them at the same time contains (s, t) and is a strong bisimulation. (Expensive: $2^{|Proc|^2}$ relations.)
- Make certain observations which will enable to disqualify many bisimulation candidates in one step.
- Use game characterization of strong bisimilarity.

Strong Bisimulation Game

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS and $s, t \in Proc$.

We define a two-player game of an 'attacker' and a 'defender' starting from s and t.

- The game is played in rounds and configurations of the game are pairs of states from $Proc \times Proc$.
- In every round exactly one configuration is called current. Initially the configuration (s, t) is the current one.

Intuition

The defender wants the show that s and t are strongly bisimilar while the attacker aims to prove the opposite.

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Rules of the Bisimulation Games

Game Rules

In each round the players change the current configuration as follows:

1. the attacker chooses one of the processes in the current configuration and makes an $\xrightarrow{a}$-move for some $a \in Act$, and
2. the defender must respond by making an $\xrightarrow{a}$-move in the other process under the same action a.

The newly reached pair of processes becomes the current configuration. The game then continues by another round.

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- If one player cannot move, the other player wins.
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Game Characterization of Strong Bisimilarity

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- States s and t are not strongly bisimilar if and only if the attacker has a universal winning strategy starting from the configuration (s,t).

Remark

Bisimulation game can be used to prove both bisimilarity and nonbisimilarity of two processes. It very often provides elegant arguments for the negative case.

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Strong Bisimilarity is a Congruence for CCS Operations

Theorem

Let P and Q be CCS processes such that $P \sim Q$. Then

- $\alpha.P \sim \alpha.Q$ *for each action* $\alpha \in Act$
- $P + R \sim Q + R$ *and* $R + P \sim R + Q$ *for each CCS process* R
- $P \mid R \sim Q \mid R$ *and* $R \mid P \sim R \mid Q$ *for each CCS process* R
- $(\nu \mathsf{a})\, P \sim (\nu \mathsf{a})\, Q$ *for any* a.

Other Properties of Strong Bisimilarity

Following Properties Hold for any CCS Processes P, Q and R

- $P + Q \sim Q + P$
- $P \parallel Q \sim Q \parallel P$
- $P + Nil \sim P$
- $P \parallel Nil \sim P$
- $(P + Q) + R \sim P + (Q + R)$
- $(P \parallel Q) \parallel R \sim P \parallel (Q \parallel R)$

Example – Buffer

Buffer of Capacity 1

$$B_0^1 \stackrel{\text{def}}{=} in.B_1^1$$
$$B_1^1 \stackrel{\text{def}}{=} \overline{out}.B_0^1$$

Buffer of Capacity n

$$B_0^n \stackrel{\text{def}}{=} in.B_1^n$$
$$B_i^n \stackrel{\text{def}}{=} in.B_{i+1}^n + \overline{out}.B_{i-1}^n \quad \text{for } 0 < i < n$$
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Example: $B_0^2 \sim B_0^1 \parallel B_0^1$

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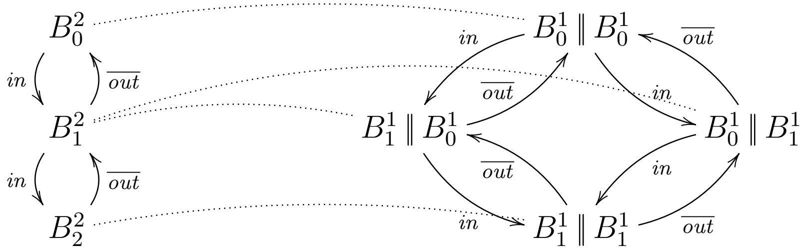

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Theorem

For all natural numbers n: $$B_0^n \sim \underbrace{B_0^1 \parallel B_0^1 \parallel \cdots \parallel B_0^1}_{n \text{ times}}$$

Proof.

The co-inductive proof method: to prove bisimilar processes, show an appropriate strong bisimulation that contains them. Construct the following binary relation where $i_1, i_2, \ldots, i_n \in \{0, 1\}$.

$$\mathcal{R} = \{\big(B_i^n,\ B_{i_1}^1 \parallel B_{i_2}^1 \parallel \cdots \parallel B_{i_n}^1\big) \mid \sum_{j=1}^{n} i_j = i\}$$

- $\big(B_0^n,\ B_0^1 \parallel B_0^1 \parallel \cdots \parallel B_0^1\big) \in R$
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Strong Bisimilarity – Summary

Properties of $\sim$

- an equivalence relation
- the largest strong bisimulation
- a congruence
- enough to prove some natural rules like
  - $P \parallel Q \sim Q \parallel P$
  - $P \parallel Nil \sim P$
  - $(P \parallel Q) \parallel R \sim Q \parallel (P \parallel R)$
  - $\ldots$

Question

Should we look any further???

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Problems with Internal Actions

Question

Does $a.\tau.Nil \sim a.Nil$ hold? NO!

Problem

Strong bisimilarity does not abstract away from τ actions.

Example: SmUni $\not\sim$ Spec

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\text{SmUni} \arrow[d, "\overline{pub}"] & \not\sim & \text{Spec} \arrow[loop below, "\overline{pub}"] \\
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Weak Transition Relation

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS such that $\tau \in Act$.

Definition of Weak Transition Relation

Below, $\circ$ stands for function composition.

$$\stackrel{a}{\Longrightarrow} = \begin{cases} (\xrightarrow{\tau})^* \circ \xrightarrow{a} \circ (\xrightarrow{\tau})^* & \text{if } a \neq \tau \\ (\xrightarrow{\tau})^* & \text{if } a = \tau \end{cases}$$

What does $s \stackrel{a}{\Longrightarrow} t$ informally mean?

- If $a \neq \tau$ then $s \stackrel{a}{\Longrightarrow} t$ means that
  from s we can get to t by doing zero or more τ actions, followed by the action a, followed by zero or more τ actions.
- If $a = \tau$ then $s \stackrel{\tau}{\Longrightarrow} t$ means that
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Weak Bisimilarity

Let $(Proc, Act, \{\xrightarrow{a} | \; a \in Act\})$ be an LTS such that $\tau \in Act$.

Weak Bisimulation

A binary relation $\mathcal{R} \subseteq Proc \times Proc$ is a weak bisimulation iff whenever $(s, t) \in \mathcal{R}$ then for each $a \in Act$ (including τ):

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Two processes $p_1, p_2 \in Proc$ are weakly bisimilar ($p_1 \approx p_2$) if and only if there exists a weak bisimulation $\mathcal{R}$ such that $(p_1, p_2) \in \mathcal{R}$.

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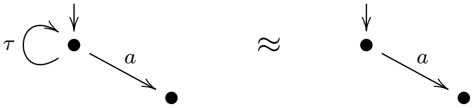
Weak Bisimilarity – Properties

Properties of $\approx$

- an equivalence relation
- the largest weak bisimulation
- validates lots of natural laws, e.g.
  - $a.\tau.P \approx a.P$
  - $P + \tau.P \approx \tau.P$
  - $a.(P + \tau.Q) \approx a.(P + \tau.Q) + a.Q$
  - $P + Q \approx Q + P \qquad P \parallel Q \approx Q \parallel P \qquad P + Nil \approx P \qquad \ldots$
- strong bisimilarity is included in weak bisimilarity ($\sim \subseteq \approx$)
- abstracts from τ loops



Is Weak Bisimilarity a Congruence for CCS?

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- $\alpha.P \approx \alpha.Q$ *for each action* $\alpha \in Act$
- $P \mid R \approx Q \mid R$ *and* $R \mid P \approx R \mid Q$ *for each CCS process* R
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What about choice?

$\tau.a.Nil \approx a.Nil$ but $\tau.a.Nil{+}b.Nil \not\approx a.Nil{+}b.Nil$

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Case Study: Communication Protocol

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\mathsf{Sending} &\stackrel{\text{def}}{=} \overline{\mathsf{send}}.\mathsf{Wait} \\
\mathsf{Wait} &\stackrel{\text{def}}{=} \mathsf{ack}.\mathsf{Send} + \mathsf{error}.\mathsf{Sending}
\end{aligned}
\qquad
\begin{aligned}
\mathsf{Rec} &\stackrel{\text{def}}{=} \mathsf{trans}.\mathsf{Del} \\
\mathsf{Del} &\stackrel{\text{def}}{=} \overline{\mathsf{del}}.\mathsf{Ack} \\
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Verification Question

$$\mathsf{Impl} \stackrel{\text{def}}{=} (\nu\, \mathsf{send}, \mathsf{trans}, \mathsf{ack}, \mathsf{error})(\mathsf{Send} \parallel \mathsf{Med} \parallel \mathsf{Rec})$$

$$\mathsf{Spec} \stackrel{\text{def}}{=} \mathsf{acc}.\overline{\mathsf{del}}.\mathsf{Spec}$$

Question

$$\mathsf{Impl} \stackrel{?}{\approx} \mathsf{Spec}$$

Check it by yourself in CAAL!

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