## Models and Semantics of Computation

Jorge A. Pérez Bernoulli Institute University of Groningen

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#### CCS: A Calculus of Communicating Systems (I)

- informal introduction
- syntax and operational semantics
- value-passing CCS



## Acknowledgment

This set of slides was originally produced by Jiri Srba, and makes part of the course material for the book

Reactive Systems: Modelling, Specification and Verification by L. Aceto, A. Ingolfsdottir, K. G. Larsen and J. Srba URL: http://rsbook.cs.aau.dk

I have adapted them slightly for the purposes of this course.

### Classical View

## Characterization of a Classical Program

Program transforms an input into an output.

Denotational semantics:a meaning of a program is a partial function

$$states \hookrightarrow states$$

- ► Nontermination is bad!
- ▶ In case of termination, the result is unique.

Is this all we need?



## Reactive systems

#### What about:

- Operating systems?
- Communication protocols?
- Control programs?
- ► Mobile phones?
- Vending machines?

## Reactive systems

### Characterization of a Reactive System

Reactive System is a system that computes by reacting to stimuli from its environment.

#### Key Issues:

- communication and interaction
- parallelism

#### Nontermination is good!

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## Analysis of Reactive Systems

#### Questions

- ▶ How can we develop (design) a system that "works"?
- ► How do we analyze (verify) such a system?

#### Fact of Life

Even short parallel programs may be hard to analyze.

## The Need for a Theory

#### Conclusion

We need formal/systematic methods (tools), otherwise ...

- ▶ Intel's Pentium-II bug in floating-point division unit
- ▶ Ariane-5 crash due to a conversion of 64-bit real to 16-bit integer
- Mars Pathfinder
- **...**

# Classical vs. Reactive Computing

	Classical	Reactive/Parallel
interaction	no	yes
nontermination	undesirable	often desirable
unique result	yes	no
semantics	$states \hookrightarrow states$	?

## How to Model Reactive Systems

#### Question

What is the most abstract view of a reactive system (process)?

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#### Answer

A process performs an action and becomes another process.

## Labelled Transition System

#### Definition

A labelled transition system (LTS) is a triple  $(Proc, Act, \{\stackrel{a}{\longrightarrow} | a \in Act\})$  where

- ► *Proc* is a set of states (or processes),
- ightharpoonup Act is a set of labels (or actions), and
- ▶ for every  $a \in Act$ ,  $\stackrel{a}{\longrightarrow} \subseteq Proc \times Proc$  is a binary relation on states called the transition relation.

We will use the infix notation  $s \xrightarrow{a} s'$  meaning that  $(s, s') \in \xrightarrow{a}$ . Sometimes we distinguish the initial (or start) state.

## Sequencing, Nondeterminism and Parallelism

LTS explicitly focuses on interaction.

LTS can also describe:

- $\triangleright$  sequencing (a;b)
- ightharpoonup choice (nondeterminism) (a+b)
- ightharpoonup limited notion of parallelism (by using interleaving)  $(a \parallel b)$

## Binary Relations

#### Definition

A binary relation  $\mathcal{R}$  on a set A is a subset of  $A \times A$ .

$$\mathcal{R} \subseteq A \times A$$

Sometimes we write  $x \mathcal{R} y$  instead of  $(x, y) \in \mathcal{R}$ .

### **Properties**

- $ightharpoonup \mathcal{R}$  is reflexive if  $(x,x) \in \mathcal{R}$  for all  $x \in A$
- $ightharpoonup \mathcal{R}$  is symmetric if  $(x,y) \in \mathcal{R}$  implies that  $(y,x) \in \mathcal{R}$  for all  $x,y \in A$
- ▶  $\mathcal{R}$  is transitive if  $(x,y) \in \mathcal{R}$  and  $(y,z) \in \mathcal{R}$  implies that  $(x,z) \in \mathcal{R}$  for all  $x,y,z \in A$

### Closures

Let  $\mathcal{R}$ ,  $\mathcal{R}'$  and  $\mathcal{R}''$  be binary relations on a set A.

#### Reflexive Closure

 $\mathcal{R}'$  is the reflexive closure of  $\mathcal{R}$  if and only if

- 1.  $\mathcal{R} \subseteq \mathcal{R}'$ ,
- 2.  $\mathcal{R}'$  is reflexive, and
- 3.  $\mathcal{R}'$  is the smallest relation that satisfies the two conditions above, i.e., for any relation  $\mathcal{R}''$ : if  $\mathcal{R} \subseteq \mathcal{R}''$  and  $\mathcal{R}''$  is reflexive, then  $\mathcal{R}' \subseteq \mathcal{R}''$ .

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- 2.  $\mathcal{R}'$  is symmetric, and
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## Labelled Transition Systems – Notation

Let  $(Proc, Act, \{\stackrel{a}{\longrightarrow} | a \in Act\})$  be an LTS.

- ightharpoonup we extend  $\stackrel{a}{\longrightarrow}$  to the elements of  $Act^*$
- $\blacktriangleright \longrightarrow = \bigcup_{a \in Act} \xrightarrow{a}$
- ightharpoonup is the reflexive and transitive closure of  $\longrightarrow$
- $ightharpoonup s \stackrel{a}{\longrightarrow} and s \stackrel{a}{\longrightarrow}$
- reachable states

### How to Describe LTS?

Syntax unknown entity

programming language

777

 $\begin{tabular}{ll} \longrightarrow & Semantics \\ & known \ entity \\ \end{tabular}$ 

what (denotational) or how (operational) it computes

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Labelled Transition Systems

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CCS

→ Semantics known entity

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Labelled Transition Systems

## Calculus of Communicating Systems

#### **CCS**

Process calculus called "Calculus of Communicating Systems".

## Insight of Robin Milner (1989)

Concurrent (parallel) processes have an algebraic structure.

$$P_1 \quad op \quad P_2 \Rightarrow P_1 \quad op \quad P_2$$

### **Process Calculus**

## Basic Principle

- 1. Define a few atomic processes (modeling the simplest process behavior).
- Define compositionally new operations (building more complex process behavior from simple ones).

## Example

- 1. atomic instruction: assignment (e.g. x:=2 and x:=x+2)
- new operators
  - sequential composition  $(P_1; P_2)$
  - ▶ parallel composition  $(P_1 \parallel P_2)$
  - E.g.  $(x:=1 \parallel x:=2)$ ; x:=x+2;  $(x:=x-1 \parallel x:=x+5)$  is a process.

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## CCS Basics (Sequential Fragment)

- ► Nil (or 0) process (the only atomic process)
- ightharpoonup action prefixing (a.P)
- ightharpoonup names and recursive definitions  $(\stackrel{
  m def}{=})$
- ▶ nondeterministic choice (+)

## This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.

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# CCS Basics (Parallelism and Renaming)

- parallel composition ( || ) (synchronous communication between two components = handshake synchronization)
- restriction of a set of actions  $(P \setminus L)$  alternative notation:  $(\nu \tilde{a})P$
- ightharpoonup relabelling (P[f])

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## Some Examples

Assigning names to processes (as in procedures) allows us to give recursive definitions of process behaviors.

#### Some examples:

- $ightharpoonup Clock \stackrel{\text{def}}{=} tick.Clock$
- $ightharpoonup CM \stackrel{\text{def}}{=} coin.\overline{coffee}.CM$
- $VM \stackrel{\text{def}}{=} coin.\overline{item}.VM$
- $CS \stackrel{\text{def}}{=} \overline{pub}.\overline{coin}.coffee.CS$
- $ightharpoonup SmUni \stackrel{\text{def}}{=} (CM \parallel CS) \setminus coin \setminus coffee$

## Some Examples, in CAAL

```
Small CCS processes can be simulated in CAAL: http://caal.cs.aau.dk.
```

The syntax is very similar to CCS expressions:

```
Clock = tick.Clock;
CM = coin.'coffee.CM;
VM = coin.'item.VM;
CTM = coin.('coffee.CTM + 'tea.CTM);
CS = 'pub.'coin.coffee.CS;
SmUni = (CM | CS) \ {coin,coffee};
```

In CAAL you may "explore" process transitions.

- $\triangleright$  A be a set of channel names (e.g. tea, coffee)
- $ightharpoonup \mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$  be a set of labels where
  - $\overline{\mathcal{A}} = \{ \overline{a} \mid a \in \mathcal{A} \}$ ( $\mathcal{A}$  are called names and  $\overline{\mathcal{A}}$  are called co-names
  - by convention  $\overline{\overline{a}} = a$
- $\blacktriangleright \ Act = \mathcal{L} \cup \{\tau\}$  is the set of actions where
  - au is the internal or silent action g. au, tea,  $\overline{coffee}$  are actions)
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# Definition of CCS (expressions)

process constants  $(K \in \mathcal{K})$ prefixing  $(\alpha \in Act)$ summation (I is an arbitrary index set)parallel composition restriction  $(L \subseteq \mathcal{A})$ relabelling  $(f : Act \to Act)$  such that

- $\blacktriangleright f(\tau) = \tau$
- $f(\overline{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is called CCS process expressions (and denoted by  $\mathcal{P}$ ).

#### Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i$$

$$Nil = 0 = \sum_{i \in \emptyset} P_i$$

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$$P := K \qquad | \qquad \qquad \\ \alpha.P \qquad | \qquad \qquad \\ \sum_{i \in I} P_i \quad | \qquad \qquad \\ P_1 \parallel P_2 \quad | \qquad \qquad \\ P \smallsetminus L \quad | \qquad \qquad \\ P[f] \qquad | \qquad \qquad \\ \end{pmatrix}$$

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### Precedence

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- 1. restriction and relabelling (tightest binding)
- 2. action prefixing
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- 4. summation

Example:  $R + a.P \parallel b.Q \setminus L$  means  $R + ((a.P) \parallel (b.(Q \setminus L)))$ .

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 means  $R + ((a.P) \parallel (b.(Q \setminus L)))$ .

## Definition of CCS (defining equations)

### CCS program

A collection of defining equations of the form

$$K \stackrel{\text{def}}{=} P$$

where  $K \in \mathcal{K}$  is a process constant and  $P \in \mathcal{P}$  is a CCS process expression.

- Only one defining equation per process constant.
- ▶ Recursion is allowed: e.g.  $A \stackrel{\text{def}}{=} \overline{a}.A \parallel A$ .

### Semantics of CCS

HOW?

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### Semantics of CCS

Syntax
CCS
(collection of defining equations)

SemanticsLTS(labelled transition systems)

HOW?

## Structural Operational Semantics for CCS

### Structural Operational Semantics (SOS) - Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules.

Given a collection of CCS defining equations, we define the following LTS  $(Proc, Act, \{\stackrel{a}{\longrightarrow} | a \in Act\})$ :

- $ightharpoonup Proc = \mathcal{P}$  (the set of all CCS process expressions)
- ►  $Act = \mathcal{L} \cup \{\tau\}$  (the set of all CCS actions including  $\tau$ )
- transition relation is given by SOS rules of the form:

RULE 
$$\frac{premises}{conclusion}$$
 conditions



## SOS rules for CCS ( $\alpha \in Act$ , $a \in \mathcal{L}$ )

$$\mathsf{ACT} \ \, \frac{}{\alpha.P \stackrel{\alpha}{\longrightarrow} P}$$

$$SUM_j \quad \frac{P_j \stackrel{\alpha}{\longrightarrow} P'_j}{\sum_{i \in I} P_i \stackrel{\alpha}{\longrightarrow} P'_j} \quad j \in I$$

$$\mathsf{COM1} \ \ \frac{P \overset{\alpha}{\longrightarrow} P'}{P \, \| \, Q \overset{\alpha}{\longrightarrow} P' \, \| \, Q}$$

$$\mathsf{COM2} \ \frac{Q \overset{\alpha}{\longrightarrow} Q'}{P \parallel Q \overset{\alpha}{\longrightarrow} P \parallel Q'}$$

COM3 
$$\xrightarrow{P \xrightarrow{a} P'} Q \xrightarrow{\overline{a}} Q'$$
 $P \parallel Q \xrightarrow{\tau} P' \parallel Q'$ 

RES 
$$\xrightarrow{P \xrightarrow{\alpha} P'} P \times L \xrightarrow{\alpha} P' \times L$$
  $\alpha, \overline{\alpha} \notin L$ 

$$\mathsf{REL} \ \, \frac{P \overset{\alpha}{\longrightarrow} P'}{P[f] \overset{f(\alpha)}{\longrightarrow} P'[f]}$$

$$\mathsf{CON} \ \ \frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \ \ K \stackrel{\mathrm{def}}{=} P$$

Let  $A \stackrel{\text{def}}{=} a.A$ . Then

$$\left((A \parallel \overline{a}.Nil) \parallel b.Nil\right)[c/a] \stackrel{c}{\longrightarrow} \left((A \parallel \overline{a}.Nil) \parallel b.Nil\right)[c/a].$$

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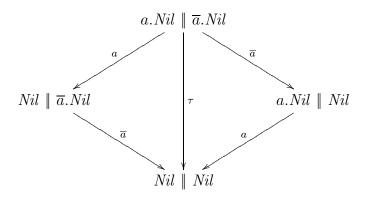
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## LTS of the Process $a.Nil \parallel \overline{a}.Nil$



#### Main Idea

Handshake synchronization is extended with the possibility to exchange integer values.

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#### Parametrized Process Constants

For example:  $Bank(total) \stackrel{\text{def}}{=} save(x).Bank(total + x).$ 

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$$\downarrow \tau$$

$$Nil \parallel Nil \parallel Bank(103)$$

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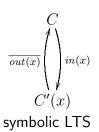
# Translation of Value Passing CCS to Standard CCS

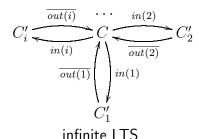
### Value Passing CCS

$$C \stackrel{\text{def}}{=} in(x).C'(x)$$
$$C'(x) \stackrel{\text{def}}{=} \overline{out(x)}.C$$

### Standard CCS

$$\longrightarrow C \stackrel{\text{def}}{=} \sum_{i \in \mathbb{N}} in(i).C'_i$$
$$C'_i \stackrel{\text{def}}{=} \overline{out(i)}.C$$





## CCS Has Full Turing Power

#### **Fact**

CCS can simulate a computation of any Turing machine.

#### Remark

Hence CCS is as expressive as any other programming language but its use is to rather describe the behaviour of reactive systems than to perform specific calculations.

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