university of groningen

Languages and Machines

L4: Finite State Machines (Part 2)

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Languages and Their Machines

Regular	$\leftrightarrow$	Finite State Machines (FSMs)
Context-free	$\leftrightarrow$	Pushdown Machines
Context-sensitive	$\leftrightarrow$	Linearly-bounded Machines
Decidable	$\leftrightarrow$	Always-terminating Turing Machines
Semi-decidable	$\leftrightarrow$	Turing Machines

Three Machines for Regular Languages

Regular Languages

- Built from $\emptyset$, $\{\epsilon\}$, and $\{a_i\}$ (for every $a_i \in \Sigma$) by applications of union, concatenation, and Kleene star operators

Finite State Machines (FSMs)

1. Deterministic FSMs (DFSMs)
2. Nondeterministic FSMs (NFSMs)
3. Nondeterministic FSMs with ϵ-transitions (NϵFSMs)

The Three Machines are Equivalent

Previous Lecture:

- Every DFSM can be regarded as an equivalent NFSM, and
- Every NFSM can be regarded as an equivalent NϵFSM.

Thus, DFSMs $\rightsquigarrow$ NϵFSMs.

Also:

- For every regexp there is an N(ϵ)FSM.

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Today:

NϵFSMs $\rightsquigarrow$ DFSMs.

- Every N(ϵ)FSM is equivalent to a DFSM (possibly larger).
- Given an N(ϵ)FSM M, we will determine a regexp for $L(M)$

Overview

From NFSMs to DFSMs

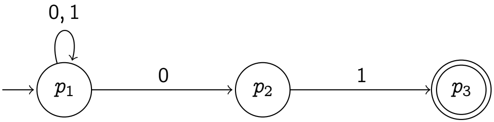
From NϵFSMs to DFSMs

The regular expression for a machine

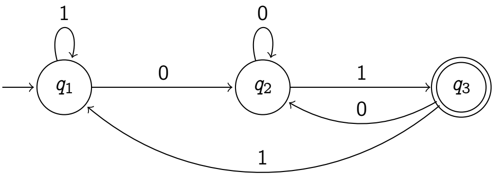
From NFSMs to DFSMs: Idea

Let $L = \{x \in \{0,1\}^* \mid x \text{ has suffix '01'}\}$.

An NFSM N that recognizes L:



We will see how to transform N into the equivalent DFSM D:



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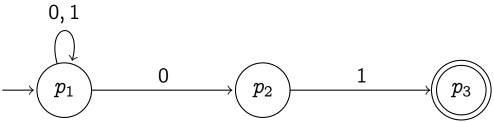

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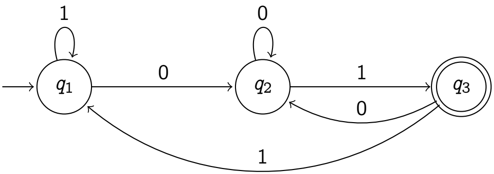

Here *equivalence* means $L(N) = L(D) = L$.

From NFSMs to DFSMs

Suppose given an NFSM $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$.
The DSFM D is defined as

$$D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$$

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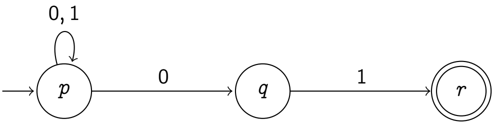
Intuition: For every $q \in S$, we check the states that N goes to from q on input a, and then take the union of all those states.

Example 1

An NFSM for $L = \{x \in \{0,1\}^* \mid x \text{ has suffix '01'}\}$:



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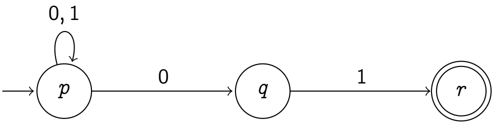

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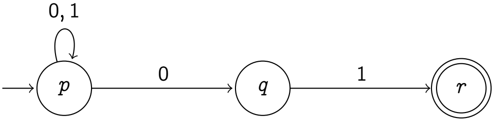

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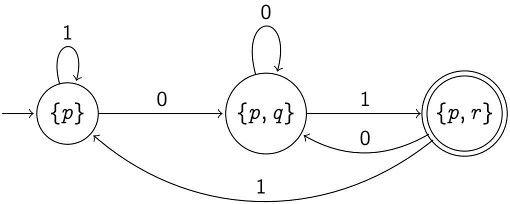
Let's transform it into the DFSM:



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From NFSMs to DFSMs

From NϵFSMs to DFSMs

The regular expression for a machine

The Subset Construction

Recall: For each $S \subseteq Q_N$ and for each a in Σ,

$$\delta_D(S, a) = \bigcup_{q \in S} \delta_N(q, a)$$

To compute $\delta_D(S, a)$, for every $q \in S$: we see what states $q_1, \ldots, q_k$ N goes to from q on input a, and take their union $\{q_1, \ldots, q_k\}$.

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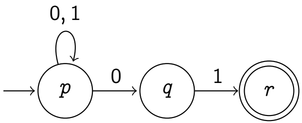

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δ_D	0	1
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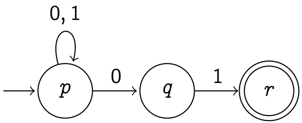

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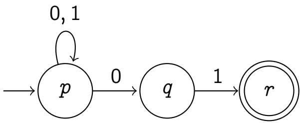

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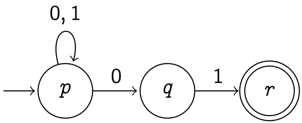

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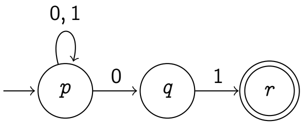

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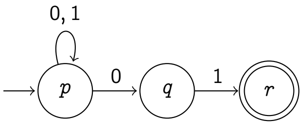

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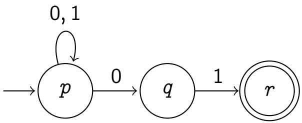

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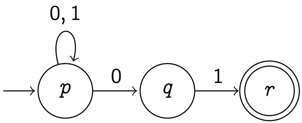

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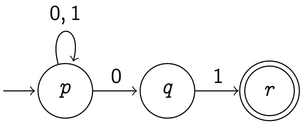

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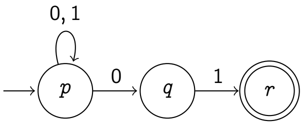

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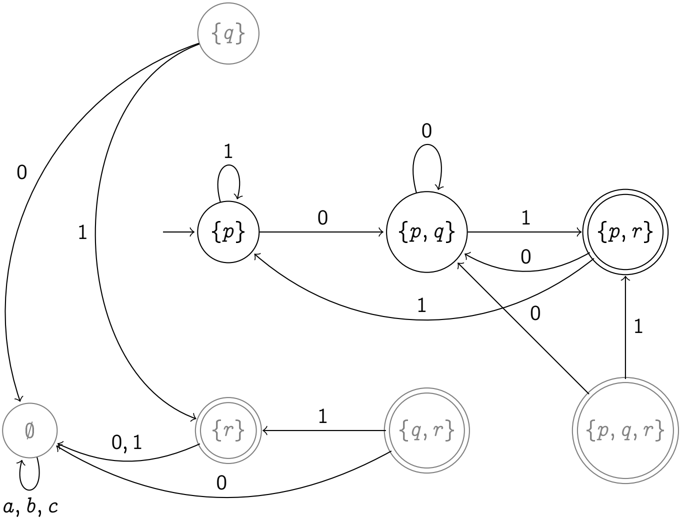
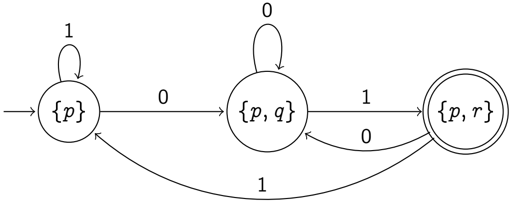
Not all States are Reachable

States in gray are inaccessible from the starting state $\{p\}$:



Not all States are Reachable

Hence, by erasing inaccessible states, we obtain:



Note: The resulting DFSM has the same number of states as the given NFSM (3) but has more transitions (6 vs 4).

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If the machine is an NϵFSM, then the general idea of the subset construction is as just discussed. We just need an additional notion.

Give a set of states S, its **ϵ-closure** is the set of states that can be reached from S via zero or more ϵ-transitions.

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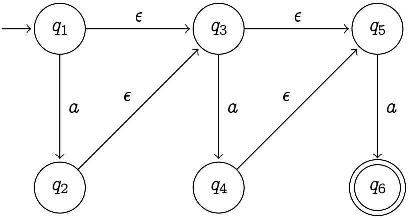

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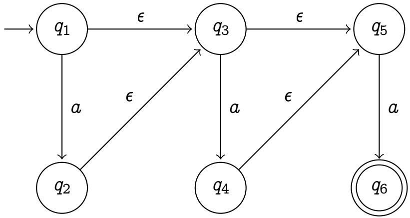


	ϵ-closure
$\{q_1\}$	$\{q_1, q_3, q_5\}$
$\{q_2\}$	$\{q_2, q_3, q_5\}$
$\{q_3\}$	$\{q_3, q_5\}$
$\{q_4\}$	$\{q_4, q_5\}$
$\{q_5\}$	$\{q_5\}$
$\{q_6\}$	$\{q_6\}$

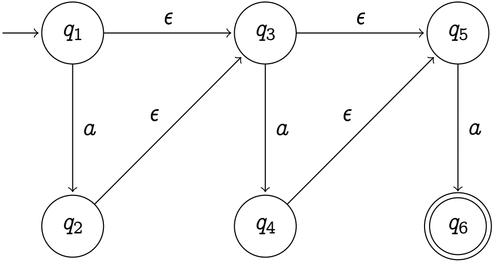
From NϵFSMs to DFSMs - Recipe

To transform an NϵFSM N into a DFSM D:

1. Compute the ϵ-closure of N's start state (as a singleton). The resulting set is D's start state.
2. For every state S of D (a set of states of N) and every $a \in \Sigma$, construct a new state: the set of states reachable from some $q \in S$ by an a-transition followed by zero or more ϵ-transitions. (That is, this step concerns computing $a\epsilon^*$.)
3. Recurse until no new states are created.

Example 2

Let's transform the NϵFSM



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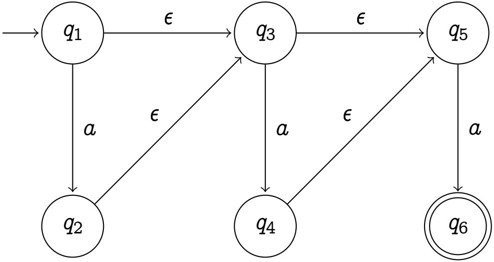
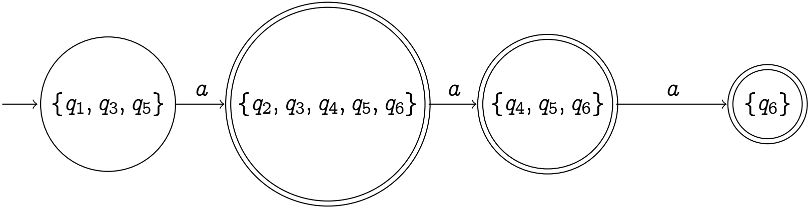
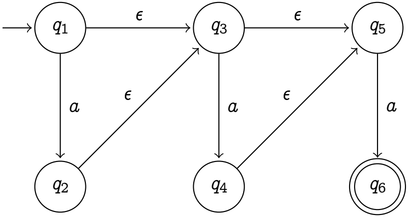


into the DFSM



The Two-Table Method (cf. Ex. 3.7)



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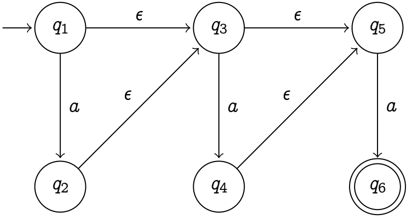
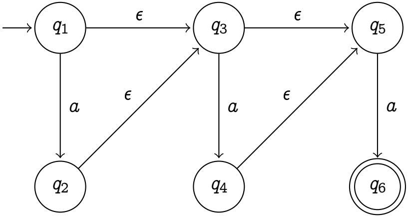


First table:

	a	ϵ^*
$\rightarrow \{q_1\}$	$\{q_2\}$	$\{q_1, q_3, q_5\}$
$\{q_2\}$	$\emptyset$	$\{q_2, q_3, q_5\}$
$\{q_3\}$	$\{q_4\}$	$\{q_3, q_5\}$
$\{q_4\}$	$\emptyset$	$\{q_4, q_5\}$
$\{q_5\}$	$\{q_6\}$	$\{q_5\}$
$*\{q_6\}$	$\emptyset$	$\{q_6\}$

The Two-Table Method (cf. Ex. 3.7)



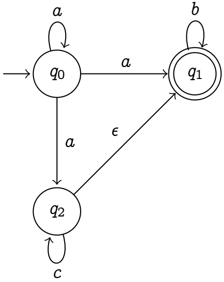
First table:

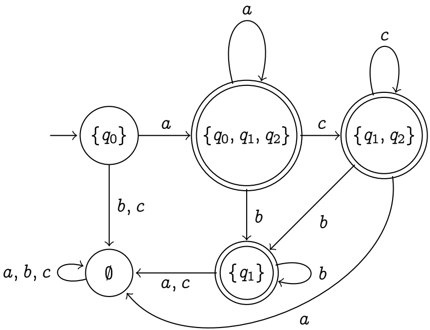
	a	ϵ^*
$\rightarrow \{q_1\}$	$\{q_2\}$	$\{q_1, q_3, q_5\}$
$\{q_2\}$	$\emptyset$	$\{q_2, q_3, q_5\}$
$\{q_3\}$	$\{q_4\}$	$\{q_3, q_5\}$
$\{q_4\}$	$\emptyset$	$\{q_4, q_5\}$
$\{q_5\}$	$\{q_6\}$	$\{q_5\}$
$*\{q_6\}$	$\emptyset$	$\{q_6\}$

Second table:

δ_D	$a\epsilon^*$
$\rightarrow \{q_1, q_3, q_5\}$	$\{q_2, q_3, q_5\} \cup \{q_4, q_5\} \cup \{q_6\} = \{q_2, q_3, q_4, q_5, q_6\}$
$*\{q_2, q_3, q_4, q_5, q_6\}$	$\emptyset \cup \{q_4, q_5\} \cup \emptyset \cup \{q_6\} \cup \emptyset = \{q_4, q_5, q_6\}$
$*\{q_4, q_5, q_6\}$	$\emptyset \cup \{q_6\} \cup \emptyset = \{q_6\}$
$*\{q_6\}$	$\emptyset$

Example 3

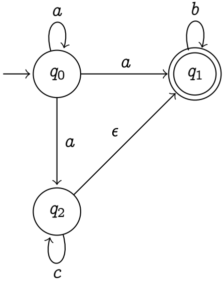


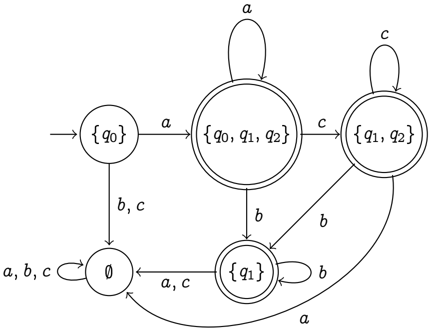


First table:

	ϵ^*	a	b	c
$\rightarrow \{q_0\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$	$\emptyset$	$\emptyset$
$*\{q_1\}$	$\{q_1\}$	$\emptyset$	$\{q_1\}$	$\emptyset$
$\{q_2\}$	$\{q_1, q_2\}$	$\emptyset$	$\emptyset$	$\{q_2\}$

Example 3

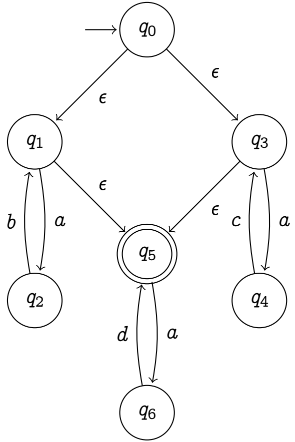




Second table:

δ_D	$a\epsilon^*$	$b\epsilon^*$	$c\epsilon^*$
$\rightarrow \{q_0\}$	$\{q_0, q_1, q_2\}$	$\emptyset$	$\emptyset$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1\}$	$\{q_1, q_2\}$
$*\{q_1\}$	$\emptyset$	$\{q_1\}$	$\emptyset$
$*\{q_1, q_2\}$	$\emptyset$	$\{q_1\}$	$\{q_1, q_2\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

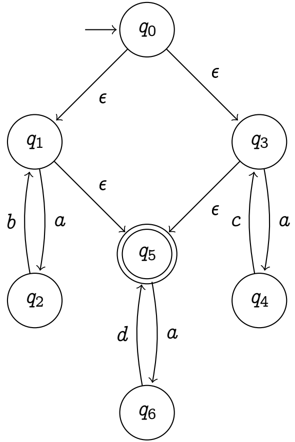
Example 4: A Single Table



The transition function δ_D (n stands for q_n):

δ_D	a	b	c	d
$\rightarrow *\{0,1,3,5\}$	$\{2,4,6\}$	$\emptyset$	$\emptyset$	$\emptyset$

Example 4: A Single Table



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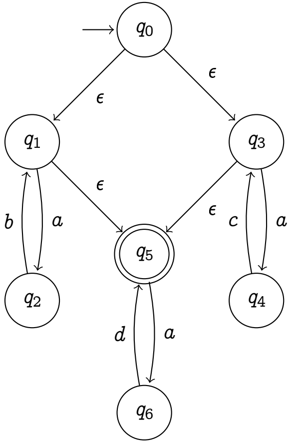

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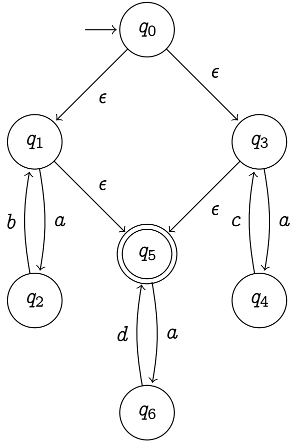

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$\{2,4,6\}$	$\emptyset$	$\{1,5\}$	$\{3,5\}$	$\{5\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$*\{5\}$	$\{6\}$	$\emptyset$	$\emptyset$	$\emptyset$

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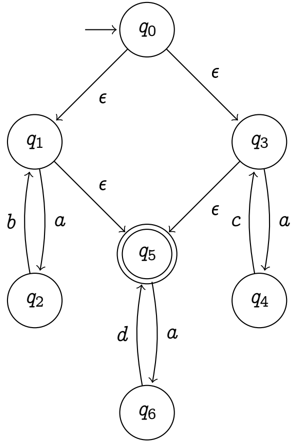

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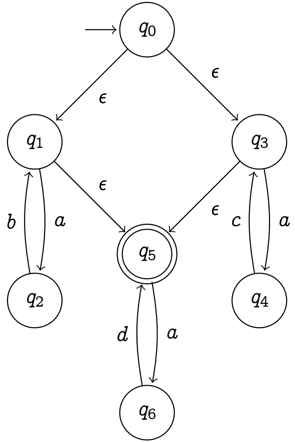

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$*\{5\}$	$\{6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$*\{1,5\}$	$\{2,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$*\{3,5\}$	$\{4,6\}$	$\emptyset$	$\emptyset$	$\emptyset$

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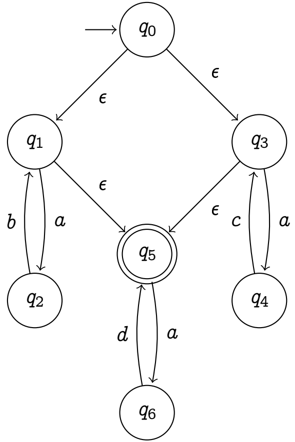

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$*\{1,5\}$	$\{2,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$*\{3,5\}$	$\{4,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\{6\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{5\}$

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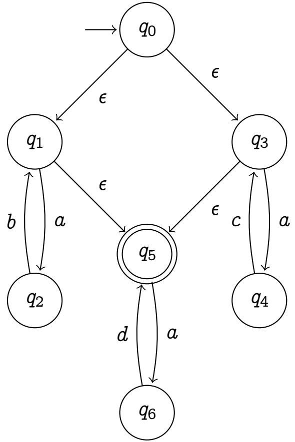

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$*\{1,5\}$	$\{2,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$*\{3,5\}$	$\{4,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\{6\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\{5\}$
$\{2,6\}$	$\emptyset$	$\{1,5\}$	$\emptyset$	$\{5\}$

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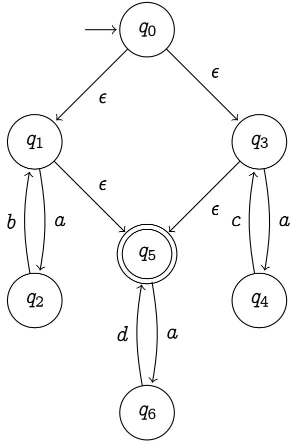

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$*\{1,5\}$	$\{2,6\}$	$\emptyset$	$\emptyset$	$\emptyset$
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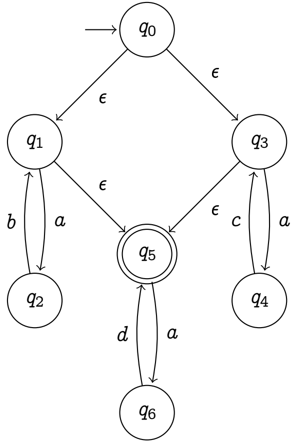

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Up to here

- Every DFSM can be regarded as an equivalent NϵFSM
- Every NϵFSM can be transformed into an equivalent DFSM

$\Rightarrow$ The languages accepted by DFSMs or NϵFSMs are equal
This is the class of the languages accepted by FSMs

Outline

From NFSMs to DFSMs

From NϵFSMs to DFSMs

The regular expression for a machine

A regular expression for a machine

Machines in **normal form**:

- the start state q_0 has no incoming arrows
- q_f is the only accepting state, and has no outgoing arrows.

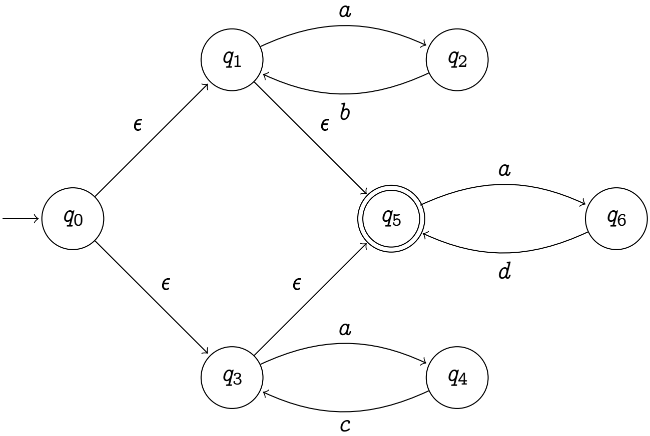
States different from q_0 and q_f are called **internal nodes.**

Intuition: The state diagram of the machine seen as a **directed graph**, with edges labelled by regular expressions.

- Initially, an edge's label is the finite set of the symbols at the edge.
- One by one, we eliminate the internal nodes of the graph.
- When all internal nodes are eliminated, there is only one remaining edge, namely from $q_0 \to q_f$.
- The label of the last edge is the resulting regular expression.

Example

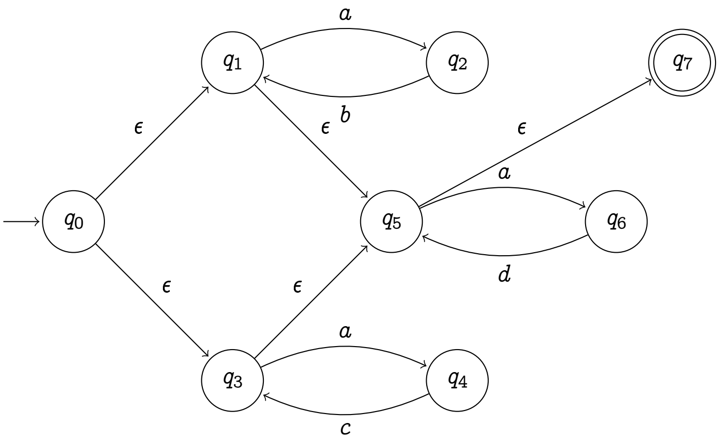
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}.$$



We first bring the machine to normal form.

Example

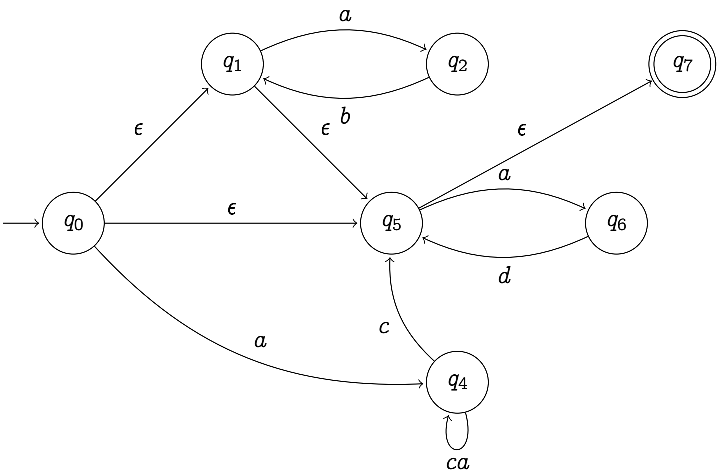
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}.$$



We added state q_7. Next we will remove q_3.

Example

$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$

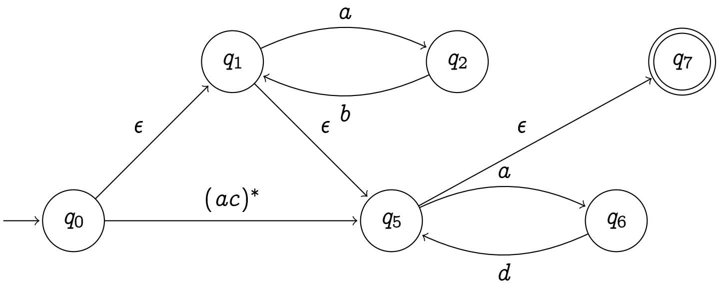


We removed q_3. Note: $(ac)^n = a(ca)^{n-1}c$

Next we will remove q_4.

Example

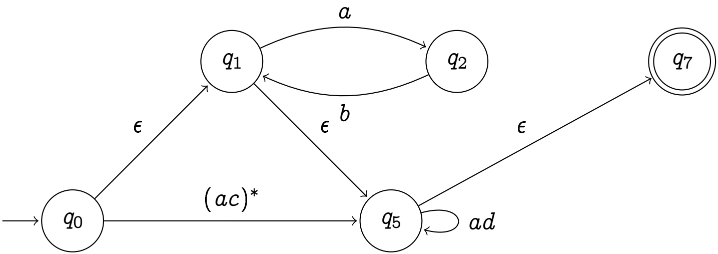
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$



We removed q_4. Next we will remove q_6.

Example

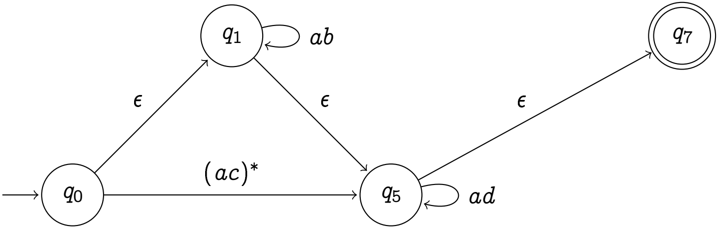
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$



We removed q_6. Next we will remove q_2.

Example

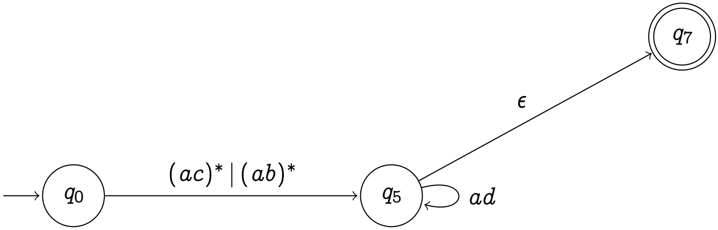
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$



We removed q_2. Next we will remove q_1.

Example

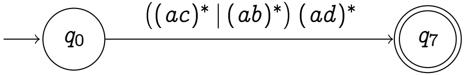
$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$



We removed q_1. Finally, we will remove q_5.

Example

$$W = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$



We are done!

Taking Stock

We may then conclude:

- A language is regular iff it is described by a regular expression
- A language is regular iff it is described by a regular grammar
- A language is regular iff it is described by a DFSM
- A language is regular iff it is described by a N(ϵ)FSM

Next Lecture

- Closure properties of regular languages
- The pumping lemma for regular languages