Models and Semantics of Computation

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The π-calculus: A Calculus of Mobile Processes (I)

- informal introduction; contrast with CCS
- syntax and basic examples
- expressiveness (polyadic communication, recursion vs replication)

The π-calculus: A Calculus of Mobile Processes

Arguably, the paradigmatic calculus for concurrency

- Proposed by Milner, Parrow, and Walker in 1992.
  Developed significantly by Sangiorgi.

Interactive systems with dynamic connectivity (topology).
A dual role:

- A model of networked computation:
  Exchanged messages which contain links referring to communication channels themselves
- A basic model of computation:
  Interaction as the primitive notion of concurrent computing
  (Just as the λ-calculus for functional computing)

The π-calculus, in these lectures

- The theory of the π-calculus is richer than that of CCS.
  In some aspects, however, it is also more involved.
- We will overview this theory, contrasting it with CCS
- Hence, we present the π-calculus without going too much into technical details

Mobility as dynamic connectivity (1)

Towards the meaning of 'mobility':

- What kind of entity moves? In what space does it move?

Many possibilities—the two most relevant in this course are:

1. Processes move, in the virtual space of linked processes
2. Links move, in the virtual space of linked processes

Observe that

- A process' location is given by the links it has to other processes (think of your contacts in your mobile phone)
- Hence, the movement of a process can be represented by the movement of its links

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Mobility as dynamic connectivity (2)

1. Processes move, in the virtual space of linked processes
2. Links move, in the virtual space of linked processes

The π-calculus commits to mobility in the sense of (2)...

- Economy, flexibility, and simplicity (at least wrt CCS)

...while models of higher-order concurrency stick to (1):

- Inspired in the λ-calculus
- It might be difficult/inconvenient to "normalize" all concurrency phenomena in the sense of (2)

We will argue that (1) and (2) need not be mutually exclusive

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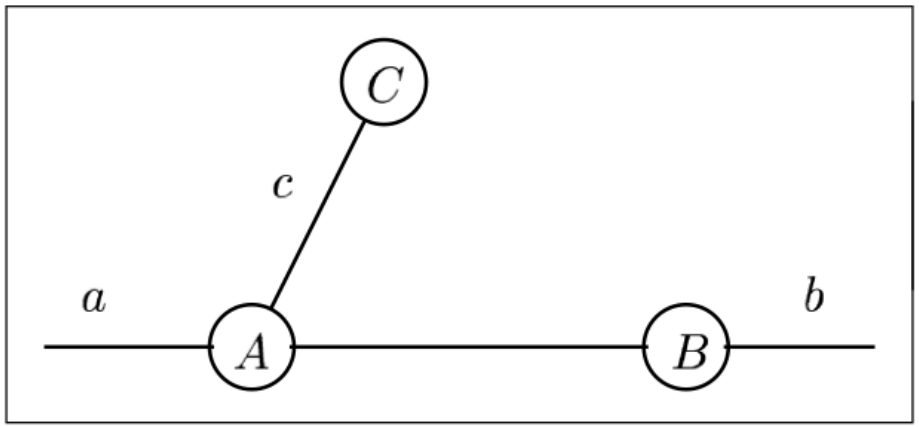
Dynamic connectivity in CCS is limited (1)

What's the main difference of the π-calculus wrt CCS?
The answer is **dynamic connectivity**.

Suppose a CCS process $S \stackrel{\text{def}}{=} (\nu x)(A \parallel C) \parallel B$.
Name a is free in A, while b is free in B. Graphically:



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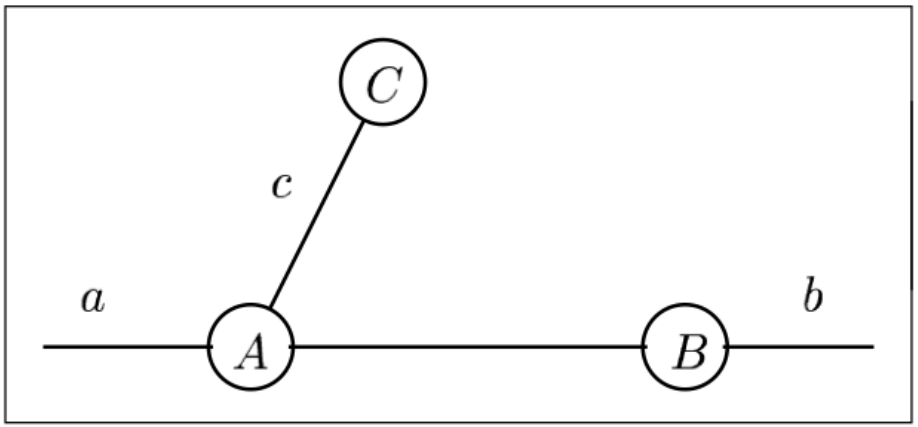

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Dynamic connectivity in CCS is limited (2)

Suppose a CCS process $S \stackrel{\text{def}}{=} (\nu x)(A \parallel C) \parallel B$.
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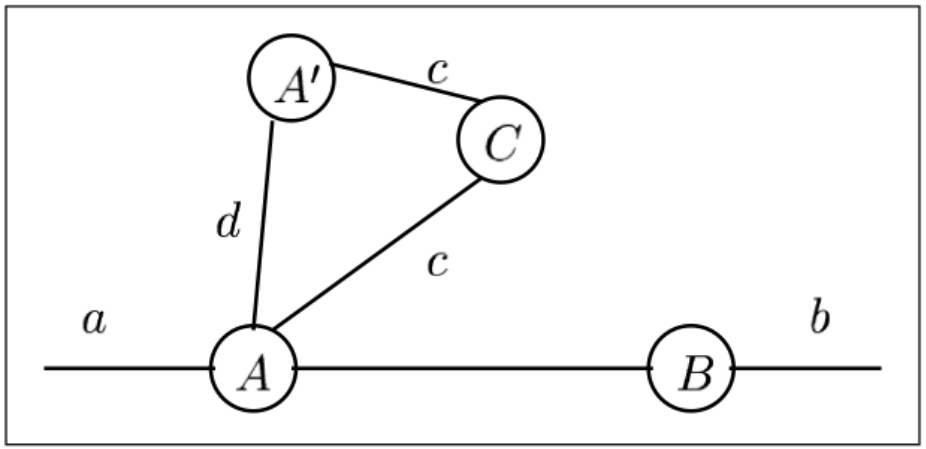
Suppose now that $A \stackrel{\text{def}}{=} a.(\nu d)(A \parallel A') + c.A''$. Graphically:



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Dynamic connectivity in CCS is limited (3)

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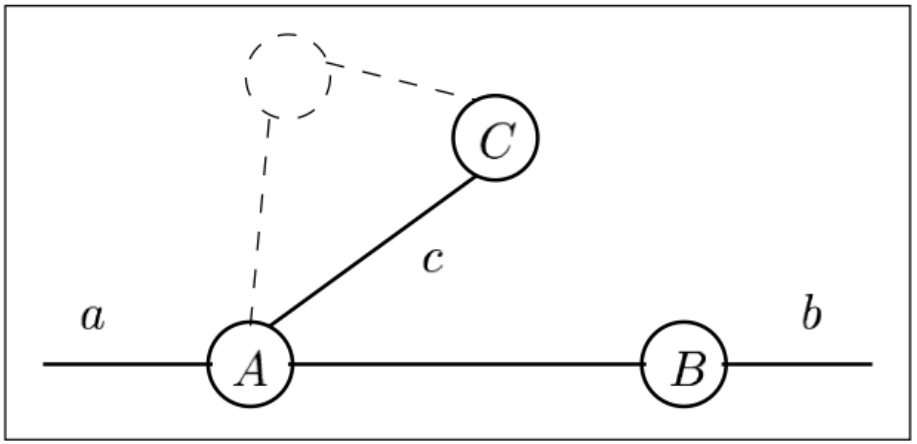
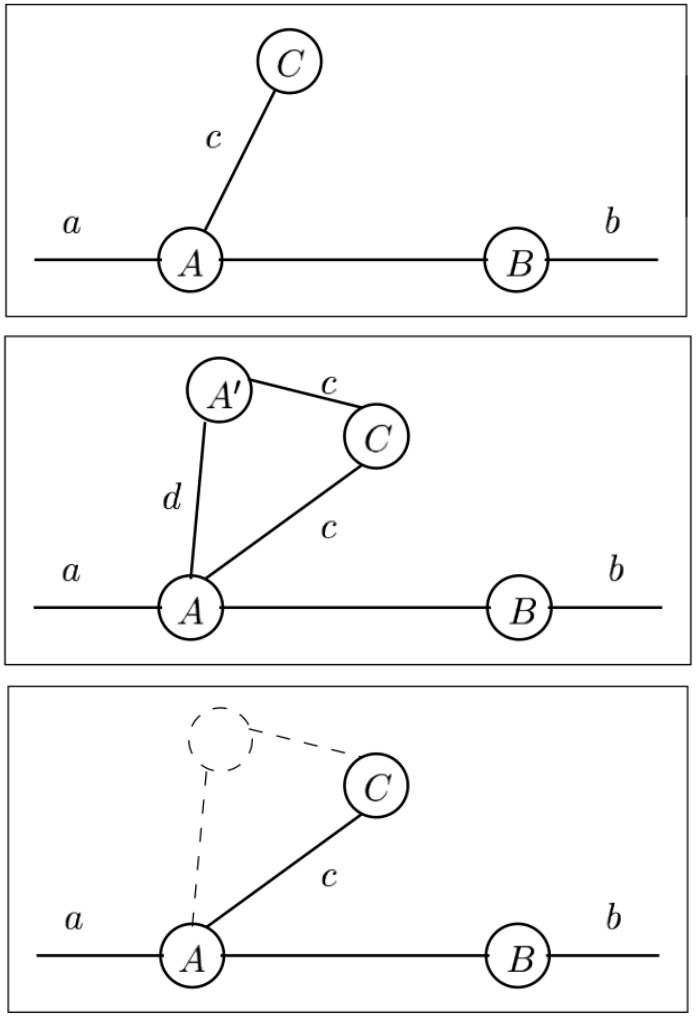
Finally, suppose that $A' = c.\mathbf{0}$. Process A' then dies. Graphically:



(3)

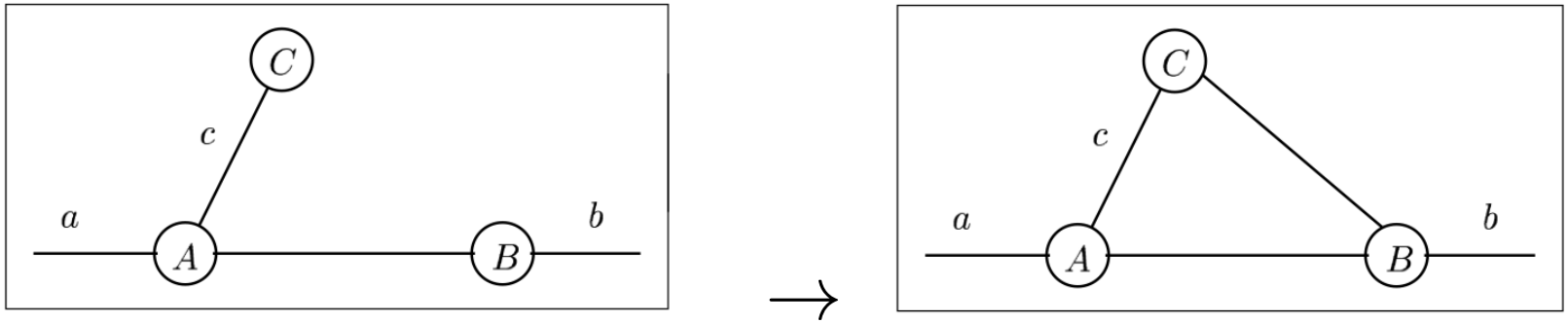
Dynamic connectivity in CCS is limited (4)

In CCS, links can proliferate and die:



Dynamic connectivity in CCS is limited (5)

However, new links between existing processes cannot be created.
The following transition is not possible in CCS:



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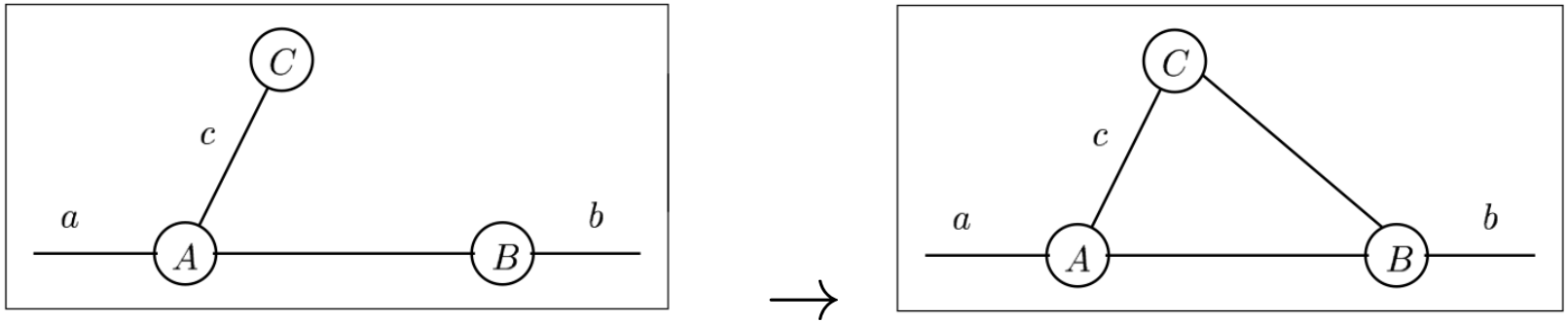

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The π-calculus goes beyond CCS by allowing dynamic communication topologies.

The π-calculus, more formally

We now formally introduce the π-calculus. Some highlights:

- The major novelty is communication of names
- Dynamic connectivity formalized as scope extrusion
- Two different semantics:
  - A reduction semantics, using a relation of structural congruence
  - A semantics based on labeled transitions (LTS)

  Roughly: a step in the reduction semantics, noted $P \longmapsto P'$, corresponds to a transition $P \xrightarrow{\tau} P'$ in the LTS (and vice versa).

The π-calculus, more formally

We use $x, y, z, \ldots$ to range over $\mathcal{N}$, an infinite set of names.

The action prefixes of the π-calculus generalize the actions of CCS:

$$\begin{array}{lcll} \alpha & ::= & \overline{x}\langle y \rangle & \text{send name } y \text{ along } x \\ & & x(y) & \text{receive a name along } x \\ & & \tau & \text{unobservable action} \end{array}$$

Brackets in $\overline{x}\langle y \rangle$ represent a tuple of values.

- Above, monadic communication: exactly one name is sent.
- In polyadic communication more than one value (a list of values) may be sent.

Process expressions of the π-calculus

P, Q	$::=$	$\mathbf{0}$	Inactive process
	$\mid$	$\alpha.P$	Prefix
	$\mid$	$P + P$	Sum
	$\mid$	$P \parallel Q$	Parallel composition
	$\mid$	$(\nu y)P$	Name creation/restriction
	$\mid$	$A\langle y_1, \ldots, y_n \rangle$	Identifier

We assume each identifier A is equipped with a recursive definition $A(x_1, \ldots, x_n) \stackrel{\text{def}}{=} P$, where $i \neq j$ implies $x_i \neq x_j$.

Free and bound names

- Restriction and input actions are name binders:
  In $(\nu y)\, P$ and $x(y).P$ name y is bound with scope P.
- In contrast, in $\overline{x}\langle y \rangle$ name y is free.

Structural Congruence: Intuitions

- The syntax of processes is too concrete: there are syntactically different terms that represent the "same behavior". Examples:

$$a(x).\overline{b}\langle x\rangle \quad \text{and} \quad a(y).\overline{b}\langle y\rangle$$
$$P \parallel Q \quad \text{and} \quad Q \parallel P$$

  [We often omit trailing $\mathbf{0}$s, and write $\overline{b}\langle y\rangle$ instead of $\overline{b}\langle y\rangle.\mathbf{0}$.]
- Structural congruence identifies processes which are "obviously the same" based on their syntactical structure
- In this sense, structural congruence will be stronger than any behavioral equivalence — that is, will equate less processes.

Structural Congruence

P and Q structurally congruent, written $P \equiv Q$, if we can transform one into the other by using the following equations:

1. α-conversion: change of bound names
2. Laws for parallel composition:

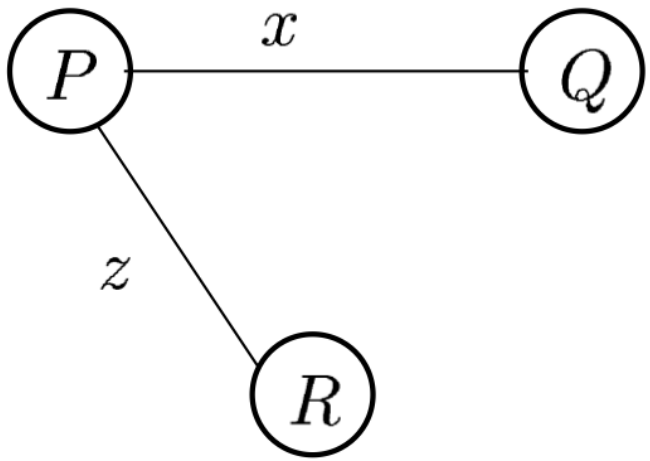
$$\begin{aligned} P \parallel \mathbf{0} &\equiv P \\ P \parallel Q &\equiv Q \parallel P \\ P \parallel (Q \parallel R) &\equiv (P \parallel Q) \parallel R \end{aligned}$$

3. Law for recursive definitions: $A\langle \tilde{y} \rangle \equiv P\{\tilde{y}/\tilde{x}\}$ if $A(\tilde{x}) \stackrel{\text{def}}{=} P$
4. Laws for restriction:

$$\begin{aligned} (\nu x)(P \parallel Q) &\equiv P \parallel (\nu x)Q \quad \text{if } x \notin \mathsf{fn}(P) \\ (\nu x)\mathbf{0} &\equiv \mathbf{0} \\ (\nu x)(P + Q) &\equiv P + (\nu x)Q \\ (\nu x)(\nu y)\, P &\equiv (\nu y)(\nu x)\, P \end{aligned}$$

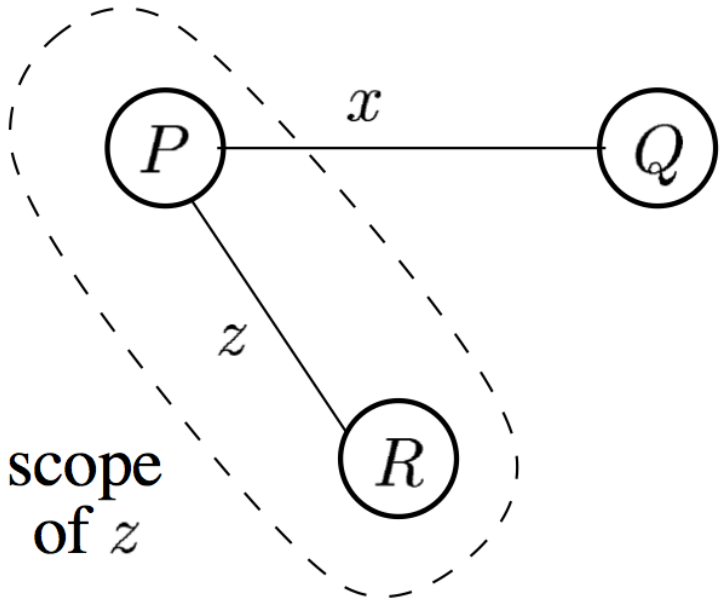
Scope Extrusion (1)

A process $P \parallel Q \parallel R$.
Name x is free in P and Q, while z is free in P and R:



Scope Extrusion (2)

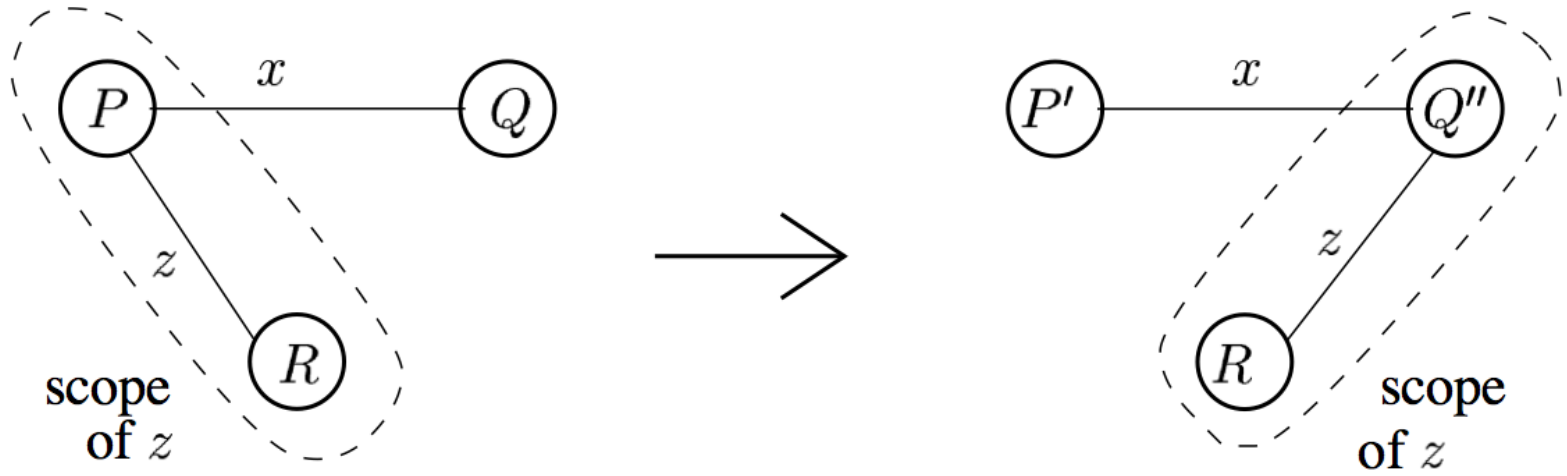
Suppose that z is restricted to P and R, while x is free in P and Q. That is, we have the process $(\nu z)(P \parallel R) \parallel Q$:



What happens if P wishes to send z to Q?

Scope Extrusion (3)

Suppose $P = \overline{x}\langle z \rangle.P'$, with $z \notin \mathrm{fn}(P')$.
Suppose also $Q = x(y).Q'$, with $z \notin \mathrm{fn}(Q')$.



where $Q'' = Q'\{z/y\}$. We have graphically described the reduction

$$(\nu z)(P \parallel R) \parallel Q \longmapsto P' \parallel (\nu z)(R \parallel Q'')$$

The above describes a movement of a way of accessing R (rather than a movement of R).

Initial Examples

We present some simple examples of scope extrusion.
For the moment, we use an informal account of reduction ($\longmapsto$), based on three postulates:

1. A law for inferring interactions:

$$a(x).P \parallel \overline{a}\langle b\rangle.Q \longmapsto P\{^{b}/_{x}\} \parallel Q$$

2. Restrictions respect silent transitions:

$$P \longmapsto Q \text{ implies } (\nu x)P \longmapsto (\nu x)Q$$

3. Structurally congruent processes should have the same behavior.

A Simple Example

We use str. congruence to infer an interaction for the process

$$a(x).\overline{c}\langle x\rangle \parallel (\nu b)\overline{a}\langle b\rangle$$

Since $b \notin \mathrm{fn}(a(x).\overline{c}\langle x\rangle)$, we have

$$a(x).\overline{c}\langle x\rangle \parallel (\nu b)\overline{a}\langle b\rangle \equiv (\nu b)(a(x).\overline{c}\langle x\rangle \parallel \overline{a}\langle b\rangle)$$

We can infer that

$$(\nu b)(a(x).\overline{c}\langle x\rangle \parallel \overline{a}\langle b\rangle) \longmapsto (\nu b)(\overline{c}\langle b\rangle \parallel \mathbf{0})$$

because $a(x).\overline{c}\langle x\rangle \parallel \overline{a}\langle b\rangle \longmapsto \overline{c}\langle b\rangle \parallel \mathbf{0}$ is a valid interaction.

Removing $\mathbf{0}$, in general we have, for any $b \notin fn(P)$:

$$a(x).P \parallel (\nu b)\overline{a}\langle b\rangle.Q \longmapsto (\nu b)(P\{^{b}/_{x}\} \parallel Q)$$

and the scope of b has moved from the right to the left.

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Another Example

$$P = (\nu z)((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \parallel x(u).\overline{u}\langle v\rangle \parallel \overline{x}\langle z\rangle)$$

Observe that $\mathsf{fn}(P) = \{x, v, y\}$ and $\mathsf{bn}(P) = \{z, w, u\}$.

There are two possibilities for reduction.

1. Interaction among the first and second components:

$$\begin{array}{rcl} P & \longmapsto & (\nu z)(\mathbf{0} \parallel \overline{u}\langle v\rangle\{y/u\} \parallel \overline{x}\langle z\rangle) \\ & = & (\nu z)(\mathbf{0} \parallel \overline{y}\langle v\rangle \parallel \overline{x}\langle z\rangle) = P_1 \end{array}$$

   Process $P\{y/u\}$ represents the process P in which the free occurrences of name u have been substituted with y.

2. Interaction among the second and third components:

$$\begin{array}{rcl} P & \longmapsto & (\nu z)((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \parallel \overline{u}\langle v\rangle\{z/u\} \parallel \mathbf{0}) \\ & = & (\nu z)((\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) \parallel \overline{z}\langle v\rangle \parallel \mathbf{0}) = P_2 \end{array}$$

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Try it yourself

- Three agents: a printing server, a client, and a printer. The client wishes to print a document d.
- The client and the server share a public name b.
- The server and the printer share a name a.
- However, the client doesn't share names with the printer, so it cannot contact it.
- The document d cannot be transmitted along public names.

Try it yourself

The client, the server, and the printer in three different processes:

$$
\begin{aligned}
C = \;& (\nu r)\overline{b}\langle r\rangle.r(y). \\
& \quad (\nu s)\overline{y}\langle s\rangle.s(c).\overline{s}\langle d\rangle.\mathbf{0} \\
S = \;& b(x).\overline{x}\langle a\rangle.\mathbf{0} \\
P = \;& a(u).\overline{u}\langle c\rangle.u(w).\overline{print}\langle w\rangle.\mathbf{0} \\
N = \;& C \parallel S \parallel P
\end{aligned}
$$

- The client uses private names (r, s) to send requests: first to the server and then to the printer
- For simplicity, the server ends after receiving a client's request. By design of C, name a is exchanged on a private name (r).
- The printer sends a confirmation c after getting a request on a. All exchanges are along s, private to C and P.
- The entire network is the composition of the three processes

Try it yourself

The client, the server, and the printer in three different processes:

$$
\begin{aligned}
C = \;& (\nu r)\overline{b}\langle r\rangle.r(y). \\
& \quad (\nu s)\overline{y}\langle s\rangle.s(c).\overline{s}\langle d\rangle.\mathbf{0} \\
S = \;& b(x).\overline{x}\langle a\rangle.\mathbf{0} \\
P = \;& a(u).\overline{u}\langle c\rangle.u(w).\overline{print}\langle w\rangle.\mathbf{0} \\
N = \;& C \parallel S \parallel P
\end{aligned}
$$

- The client uses private names (r, s) to send requests: first to the server and then to the printer
- For simplicity, the server ends after receiving a client's request. By design of C, name a is exchanged on a private name (r).
- The printer sends a confirmation c after getting a request on a. All exchanges are along s, private to C and P.
- The entire network is the composition of the three processes

Basic Expressiveness of the π-calculus

We examine the basic expressiveness of the π-calculus. We show:

- How to represent polyadicity using monadic name passing
- The relationship between recursive definitions and replication

We appeal to encodings from one language to another to justify our claims.

Polyadic Communication

- For any $n \leq 0$, we would like to have prefixes

$$x(y_1, \cdots, y_n).P \quad \text{and} \quad \overline{x}\langle z_1, \cdots, z_n\rangle.Q \quad (\text{all } y_i \text{ are distinct})$$

- The naive encoding (ne) sends one name at the time:

$$\begin{array}{rcl} [\![x(y_1, \cdots y_n).P]\!]_{ne} & = & x(y_1).\cdots x(y_n).P \\ [\![\overline{x}\langle z_1, \cdots, z_n\rangle.Q]\!]_{ne} & = & \overline{x}\langle z_1\rangle.\cdots .\overline{x}\langle z_n\rangle.Q \end{array}$$

- Problem: this encoding is sensible to undesirable interferences. Take the process

$$S = x(y_1 y_2).P \parallel \overline{x}\langle z_1 z_2\rangle.Q \parallel \overline{x}\langle v_1 v_2\rangle.R$$

Using the encoding we get:

$$[\![S]\!]_{ne} = x(y_1).x(y_2).P \parallel \overline{x}\langle z_1\rangle.\overline{x}\langle z_2\rangle.Q \parallel \overline{x}\langle v_1\rangle.\overline{x}\langle v_2\rangle.R$$

$[\![S]\!]_{ne}$ features unintended behavior: 'mix-ups' in sent values.

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Polyadic Communication

- A correct encoding first establishes a fresh link in which values are sent, one by one
- More precisely, we define $[\![\cdot]\!]_{\mathsf{pc}} : \mathrm{poly}\pi \to \mathrm{mona}\pi$:

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\begin{array}{rcl}
[\![x(y_1 \cdots y_n).P]\!]_{\mathsf{pc}} & = & x(w).w(y_1).\cdots .w(y_n).[\![P]\!]_{\mathsf{pc}} \\
[\![\overline{x}\langle z_1 \cdots z_n\rangle.Q]\!]_{\mathsf{pc}} & = & (\nu w)(\overline{x}\langle w\rangle.\overline{w}\langle z_1\rangle.\cdots .\overline{x}\langle z_n\rangle.[\![Q]\!]_{\mathsf{pc}})
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  (the other operators are translated homomorphically)
- n-adic communication thus requires $n+1$ reductions.
- Question: would the above strategy work if the fresh name is created by the receiver?
- We say that polyadic communication is representable in the monadic π-calculus. The latter can be regarded as basic.

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Recursive Definitions and Replication

We have seen the π-calculus with recursive definitions of the form

$$A(\tilde{x}) \stackrel{\text{def}}{=} Q_A$$

where

- A is a process identifier
- Q_A is a process expression which may contain calls of A and other parametric processes.

Another form of introducing infinite behavior is replication.
The replication of process P, written $!P$, is given by the definition

$$!P \stackrel{\text{def}}{=} P \parallel !P$$

That is, $!P$ represents an unbounded number of copies of P:

$$!P \equiv P \parallel !P \equiv P \parallel P \parallel !P \equiv P \parallel P \parallel P \parallel !P \cdots$$

Recursive definitions and replication can be mutually represented!

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Encoding Replication into Recursive Definitions

- From the previous description, it is clear that replication is a particular case of recursive definition.
- So the encoding formalizes a simple idea: each $!P$ is translated into a process A_P, recursively defined as

$$A_P(\tilde{x}) \stackrel{\text{def}}{=} P \parallel A_P\langle\tilde{x}\rangle$$

which can provide an unbounded number of copies of P.

- More formally: let $\pi_!$ and π_{rd} denote the set of π-calculus expressions with replication and recursive definitions, resp. Let $[\![\cdot]\!]_{\text{rep}} : \pi_! \to \pi_{\text{rd}}$ be the encoding defined as

$$[\![!P]\!]_{\text{rep}} \quad = \quad A\langle\tilde{x}\rangle \quad \text{where } A(\tilde{x}) \stackrel{\text{def}}{=} P \parallel A\langle\tilde{x}\rangle \text{ with } \tilde{x} \subseteq \text{fn}(P)$$

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Encoding Recursive Definitions into Replication

The encoding in the other direction is a bit more interesting.

Intuitively, for each definition $A(\tilde{x}) \stackrel{\text{def}}{=} Q_A$ we use

- message passing to model the passing of parameters
- replication to model the multiple activations
- restriction to model the scope of the definition

Encoding Recursive Definitions into Replication

In two steps, we set up a "server" for each recursive definition.

First, we define the encoding $[\![\cdot]\!]_{\mathsf{rd}_0} : \pi_{\mathsf{rd}} \to \pi_!$:

$$\begin{array}{rcl} [\![A_i(\tilde{x}_i) \stackrel{\text{def}}{=} Q_i]\!]_{\mathsf{rd}_0} & = & !a_i(\tilde{x}_i).[\![Q_i]\!]_{\mathsf{rd}_0} \\ [\![A\langle\tilde{z}\rangle]\!]_{\mathsf{rd}_0} & = & \overline{a_i}\langle\tilde{z}\rangle \end{array}$$

(the other constructs are translated homomorphically)

Then, we treat processes which contain identifiers.

Let P be a process with $A_1(\tilde{x_1}) \stackrel{\text{def}}{=} Q_1, \ldots, A_k(\tilde{x_k}) \stackrel{\text{def}}{=} Q_k$ as the sets of definitions of its process identifiers. We have:

$$[\![P]\!]_{\mathsf{rd}} \quad = \quad (\nu a_1, \ldots, a_k)([\![P]\!]_{\mathsf{rd}_0} \parallel \prod_{i \in \{1,\ldots,k\}} [\![A_i(\tilde{x}_i) \stackrel{\text{def}}{=} Q_i]\!]_{\mathsf{rd}_0})$$

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where $a_1, \ldots, a_k \notin \mathsf{fn}(P)$.

Recursive Definitions and Replication

- The two encodings demonstrate that infinite behavior in the π-calculus is a natural concept
- We can choose the form of infinite behavior that fits better in our models
- Interestingly, a similar result does not hold for CCS