

Languages and Machines

L12: Non Context-Free Grammars/Languages

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Last Lecture(s)



- Church-Turing's thesis
- A Universal Turing machine
- Undecidability results / problem reducibility
- Acceptance of the empty string (the blank tape problem)
- Incompleteness of arithmetic

Today: Non-CF grammars and Course evaluation.

Outline



Chomsky's Hiearchy

Rewriting Systems

Unrestricted Grammars

Context-Sensitive Grammars

From Lecture 2: Context-Free Grammars



A quadruple (V, Σ, P, S) where

- V is a set of variables or nonterminals
- $ightharpoonup \Sigma$ is an alphabet of **terminals**, disjoint from V
- ▶ P is a finite set of **production rules**, taken from $V \times (V \cup \Sigma)^*$. We write $A \to w$ instead of (A, w).
- $ightharpoonup S \in V$ is the start symbol.

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Notice:

▶ $A \to w$ involves rewriting: A **symbol** $A \in V$ is rewritten into a **string** in $w \in (V \cup \Sigma)^*$

Today:

lacktriangle Grammars that rewrite a **string** u into **another string** w

The Chomsky Hierarchy



Grammar	Language	Machine(s)
Type 0	R.e./Semi-decidable	TMs
?	Recursive / Decidable	Always-terminating TMs
Type 1	Context-sensitive	Linear-bounded automata
Type 2	Context-free	Pushdown automata
Type 3	Regular	N∈FSM / NFSM / DFSM

where

- Type 0 = Unrestricted
- Type 1 = Context-sensitive
- Type 2 = Context-free
- Type 3 = Regular

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Rewriting Systems



Let u, v, \ldots range over strings over an alphabet Σ .

- A rewriting rule is a pair (u, v), written $u \to v$
- A (string) rewriting system is a set of rewriting rules

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ightarrow v_1, \cdots, u_n
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Given a rewriting system P, we write $x \Rightarrow_P y$ if x = uvw, $(v, z) \in P$, and y = uzw.

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The Nullability Problem (NULL):

Given a string w and a rewriting system P, does $w \Rightarrow_{P}^{*} \epsilon$ hold?



• Key Idea: Given a simple TM $M = (Q, \Sigma, \Gamma, \delta, q_0)$, define a set of rewriting rules P such that:

M terminates on input w iff $[q_0w] \Rightarrow_P^* \epsilon$



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Parts I and II rely on different rewriting rules (see next slide).

A reduction of HALT into NULL:
 If NULL were decidable, then we could use the rewriting system P to solve HALT.



$$\underbrace{[q_0w] \Rightarrow_P^* [xEy]}_{\mathsf{Part \, I}} \text{ followed by } \underbrace{[xEy] \Rightarrow_P^* [xE] \Rightarrow_P^* [E] \Rightarrow_P \epsilon}_{\mathsf{Part \, II}}$$

Rewriting rules for Part I (assume $q, r \in Q$ and $X, Y, Z \in \Gamma$):

- 1. $[qX \rightarrow [BqX]]$
- 2. $q \rightarrow qB$ ("surround" the configuration with blank symbols)
- 3. ZqX o ZYr if $\delta(q,X) = (r,Y,R)$ (moving right)
- 4. $ZqX \rightarrow rZY$ if $\delta(q, X) = (r, Y, L)$ (moving left)
- 5. $ZqX \rightarrow ZEX$ if $\delta(q, X) = \bot$ (termination: add E)



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Rewriting rules for Part II:

- 6. $EX \rightarrow E$
- 7. XE] $\rightarrow E$]
- 8. $[E]
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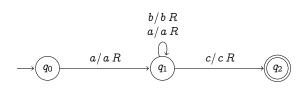
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Since $[xEy] \Rightarrow_P^* \epsilon$, we have that $[q_0w] \Rightarrow_P^* \epsilon$ if $w \in L(M)$. If $w \notin L(M)$, symbol E is never generated - ϵ is not reached.

Example

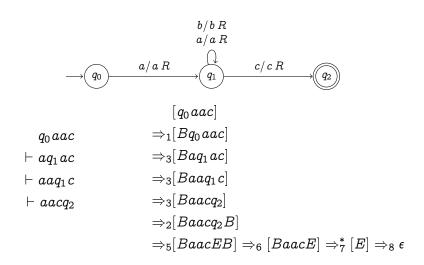




 q_0aac $\vdash aq_1ac$ $\vdash aaq_1c$ $\vdash aacq_2$

Example





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The most powerful grammars in Chomsky's hierarchy

- A grammar $G = (V, \Sigma, P, S)$ in which P is a **rewriting system**
- Derivations, sentential forms, sentences: as before



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- **Example**: Consider $V = \{S, A, C\}, \Sigma = \{a, b, c\}$ and

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We have $S \Rightarrow \epsilon$ and $S \Rightarrow a^{i+1}b^{i+1}c^{i+1}$ (not a CFL):

$$egin{aligned} S &\Rightarrow aAbc \ &\Rightarrow aaAbCbc \Rightarrow aaaAbCbCbc \Rightarrow \cdots \Rightarrow a^{i+1}A(bC)^ibc \ &\Rightarrow a^{i+1}(bC)^ibc \Rightarrow a^{i+1}b^{i+1}C^ic \Rightarrow a^{i+1}b^{i+1}c^{i+1} \end{aligned}$$



Grammar G with alphabet $\{a, b, [,]\}$ and productions:

$$S
ightarrow aT[a] \mid bT[b] \mid [\] \ T[
ightarrow aT[A \mid bT[B \mid [\ Aa
ightarrow aA \ Ab
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ightarrow aB \ Bb
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 $L(G) = \{u[u] \mid u \in \{a, b\}^*\}.$ For instance:

$$S \Rightarrow a \underline{T[a]}$$
 $\Rightarrow a \underline{a} \underline{T[a]}$
 $\Rightarrow a \underline{a} \underline{T[a\underline{A]}}$
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Theorem 7.2: Simulate a Type 0 grammar G using a three-tape non-deterministic TM M such that L(M) = L(G)

- T1: the input string x
- T2: rules of G (e.g. u o v encoded as u # v)
- T3: the derivations of G



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Hence, if G is Type 0 then L(G) is recursively enumerable



Conversely, Type 0 grammars can simulate TMs (Theorem 7.3):

- Give a grammar G such that L(G) is the reversal of L(M) This can be done in two phases (Lemma 7.2):
 - 1. $S \Rightarrow^* w^R[q_0w]$, for some $w \in \Sigma^*$
 - 2. $w^R[q_0w] \Rightarrow^* w^R$, if $w \in L(M)$.
- Given M, construct a machine M' that
 - Reverts the input string $\boldsymbol{\mathit{w}}$
 - Applies M to w^R , so that $w_i \in L(M') \iff w_i^R \in L(M)$
 - Since $L(M') = \{w \mid w^R \in L(M)\}$, use Lemma 7.2 to obtain a G such that

$$L(G)=\{w_i^R\mid w_i\in L(M')\}=L(M)$$

Alternatively... (1/2)



- Let L be a r.e. language, recognized by $M=(Q,\Sigma,\Gamma,\delta,q_0,F)$.
- $G = (V, \Sigma, P, S)$ is designed to simulate the computations of M
- The effect of transition $\delta(q_i,x)=(q_j,y,R)$ on a configuration encoded as uq_ixvB is the derivation

$$u q_i x v B \Rightarrow u q_i y v B$$

- Deriving a terminal string in *G* in three phases:
 - a) Generate a string $u[q_0Bu]$, with $u \in \Sigma^*$
 - b) Simulate M on the string $u[q_0Bu]$
 - c) Remove the simulation substring
- Let $\Sigma=\{a_1,\ldots,a_n\}$ and $V=\{S,\,T,E_R,E_L,[,],A_1,\ldots,A_n\}\cup Q$

The rules (next slide) ensure that a derivation that begins by generating $u [q_0 B u]$ terminates with u whenever $u \in L(M)$; otherwise, the derivation doesn't produce a terminal

Alternatively... (2/2)



The first four rules are similar to the example given before:

- 1. $S
 ightarrow a_i \ T \left[\ a_i \ \right] \mid \left[\ q_0 \ B \ \right]$ for $1 \leq i \leq n$
- 2. $A_i a_j \rightarrow a_j A_i$ for $1 \leq i, j \leq n$
- 3. A_i] $\rightarrow a_i$] for $1 \leq i \leq n$
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The following rules follow δ :

- 5. $q_i \ x \ y o z \ q_j \ y$ whenever $\delta(q_i, x) = (q_j, z, R)$ and $y \in \Gamma$
- 6. $q_i x] \rightarrow z q_i B]$ whenever $\delta(q_i, x) = (q_i, z, R)$
- 7. $y \ q_i \ x o q_j \ y \ z$ whenever $\delta(q_i,x) = (q_j,z,L)$ and $y \in \Gamma$

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- 7. $y \ q_i \ x o q_j \ y \ z$ whenever $\delta(q_i,x) = (q_j,z,L)$ and $y \in \Gamma$

If an accepting state is reached, erase the string within brackets:

- 8. $q_i \ x o E_R$ whenever $\delta(q_i, x)$ is undefined and $q_i \in F$
- 9. $E_R x \to E_R$ for $x \in \Gamma$
- 10. E_R] $o E_L$ for $x \in \Gamma$
- 11. $x E_L \rightarrow E_L$ for $x \in \Gamma$
- 12. $[E_L o \epsilon]$

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Context-Sensitive (Type 1) Grammars



A Type 0 grammar is context sensitive if every u o v satisfies

- S doesn't occur in v
- If $u \neq S$ then $0 < |u| \leq |v|$

Thus, the length of the derived string remains the same or increases with each rule application (a monotonicity property).

We can see that every context-free language is context-sensitive:

- A context-free grammar is context-sensitive iff it is essentially non-contracting.
- Every context-free grammar is equivalent with an essentially non-contracting context-free grammar

Context-sensitive languages are accepted by always-terminating TMs. Hence, they are recursive.

Context-Sensitive (Type 1) Grammars



Let $V = \{S, A, C\}$, $\Sigma = \{a, b, c\}$ and the unrestricted grammar that generates the language $\{a^i b^i c^i | i > 0\}$:

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An equivalent context-sensitive grammar:

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Context-Sensitive (Type 1) Grammars



- A linear-bounded automaton is a non-deterministic, single-tape TM whose transitions never replace a blank symbol *B*.
- ► That is, the input string determines the length of the available tape. This effectively decreases the expressivity of TMs.
- ▶ *L* is accepted by a linear-bounded automaton iff *L* is context-sensitive.
- Simulating the derivations of a context-sensitive grammar requires some effort.

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Course Evaluation



- Lectures:
 Useful, understandable, too fast, too slow ...?
- Content:
 Too much, too little, useful for your (professional) life, ...?
- Tutorials: Helpful, interesting, ...?
- Homeworks: Easy, difficult, ...?
- Material (Reader and slides): Clear, complete, helpful, ...?

Feel free to send me an email with your constructive criticism!



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