# Models and Semantics of Computation

Jorge A. Pérez
Bernoulli Institute
University of Groningen

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## CCS: A Calculus of Communicating Systems (I)

- informal introduction
- syntax and operational semantics
- value-passing CCS

# Acknowledgment

This set of slides was originally produced by Jiri Srba, and makes part of the course material for the book

**Reactive Systems: Modelling, Specification and Verification**
by L. Aceto, A. Ingolfsdottir, K. G. Larsen and J. Srba
URL: `http://rsbook.cs.aau.dk`

I have adapted them slightly for the purposes of this course.

# Classical View

## Characterization of a Classical Program

Program transforms an input into an output.

- Denotational semantics:
  a meaning of a program is a partial function

$$states \hookrightarrow states$$

- Nontermination is bad!
- In case of termination, the result is unique.

Is this all we need?

# Reactive systems

What about:

- Operating systems?
- Communication protocols?
- Control programs?
- Mobile phones?
- Vending machines?

# Reactive systems

## Characterization of a Reactive System

Reactive System is a system that computes by reacting to stimuli from its environment.

Key Issues:

- communication and interaction
- parallelism

Nontermination is good!

The result (if any) does not have to be unique.

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# Analysis of Reactive Systems

## Questions

- How can we develop (design) a system that "works"?
- How do we analyze (verify) such a system?

## Fact of Life

Even short parallel programs may be hard to analyze.

# The Need for a Theory

## Conclusion

We need formal/systematic methods (tools), otherwise ...

- Intel's Pentium-II bug in floating-point division unit
- Ariane-5 crash due to a conversion of 64-bit real to 16-bit integer
- Mars Pathfinder
- ...

# Classical vs. Reactive Computing

| | Classical | Reactive/Parallel |
|---|---|---|
| interaction | no | yes |
| nontermination | undesirable | often desirable |
| unique result | yes | no |
| semantics | $states \hookrightarrow states$ | **?** |

# How to Model Reactive Systems

## Question

What is the most abstract view of a reactive system (process)?

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## Answer

A process performs an action and becomes another process.

# Labelled Transition System

## Definition

A labelled transition system (LTS) is a triple $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ where

- $Proc$ is a set of states (or processes),
- $Act$ is a set of labels (or actions), and
- for every $a \in Act$, $\xrightarrow{a} \subseteq Proc \times Proc$ is a binary relation on states called the transition relation.

We will use the infix notation $s \xrightarrow{a} s'$ meaning that $(s, s') \in \xrightarrow{a}$.

Sometimes we distinguish the initial (or start) state.

# Sequencing, Nondeterminism and Parallelism

LTS explicitly focuses on interaction.

LTS can also describe:

- sequencing $(a; b)$
- choice (nondeterminism) $(a + b)$
- limited notion of parallelism (by using interleaving) $(a \parallel b)$

# Binary Relations

## Definition

A binary relation $\mathcal{R}$ on a set $A$ is a subset of $A \times A$.

$$\mathcal{R} \subseteq A \times A$$

Sometimes we write $x \mathcal{R} y$ instead of $(x, y) \in \mathcal{R}$.

## Properties

- $\mathcal{R}$ is reflexive if $(x, x) \in \mathcal{R}$ for all $x \in A$
- $\mathcal{R}$ is symmetric if $(x, y) \in \mathcal{R}$ implies that $(y, x) \in \mathcal{R}$ for all $x, y \in A$
- $\mathcal{R}$ is transitive if $(x, y) \in \mathcal{R}$ and $(y, z) \in \mathcal{R}$ implies that $(x, z) \in \mathcal{R}$ for all $x, y, z \in A$

# Closures

Let $\mathcal{R}$, $\mathcal{R}'$ and $\mathcal{R}''$ be binary relations on a set $A$.

## Reflexive Closure

$\mathcal{R}'$ is the reflexive closure of $\mathcal{R}$ if and only if

1. $\mathcal{R} \subseteq \mathcal{R}'$,
2. $\mathcal{R}'$ is reflexive, and
3. $\mathcal{R}'$ is the smallest relation that satisfies the two conditions above, i.e., for any relation $\mathcal{R}''$:
   if $\mathcal{R} \subseteq \mathcal{R}''$ and $\mathcal{R}''$ is reflexive, then $\mathcal{R}' \subseteq \mathcal{R}''$.

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## Transitive Closure

$\mathcal{R}'$ is the transitive closure of $\mathcal{R}$ if and only if

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2. $\mathcal{R}'$ is transitive, and
3. $\mathcal{R}'$ is the smallest relation that satisfies the two conditions above, i.e., for any relation $\mathcal{R}''$:
   if $\mathcal{R} \subseteq \mathcal{R}''$ and $\mathcal{R}''$ is transitive, then $\mathcal{R}' \subseteq R''$.

# Labelled Transition Systems – Notation

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

- we extend $\xrightarrow{a}$ to the elements of $Act^*$
- $\longrightarrow = \bigcup_{a \in Act} \xrightarrow{a}$
- $\longrightarrow^*$ is the reflexive and transitive closure of $\longrightarrow$
- $s \xrightarrow{a}$ and $s \not\xrightarrow{a}$
- reachable states

# How to Describe LTS?

| | | |
|---|---|---|
| Syntax<br>unknown entity | $\longrightarrow$ | Semantics<br>known entity |
| programming language | $\longrightarrow$ | what (denotational) or<br>how (operational) it computes |
| ??? | $\longrightarrow$ | Labelled Transition Systems |

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| CCS | $\longrightarrow$ | Labelled Transition Systems |

# Calculus of Communicating Systems

## CCS

Process calculus called “Calculus of Communicating Systems”.

## Insight of Robin Milner (1989)

Concurrent (parallel) processes have an algebraic structure.

$$\boxed{P_1} \; op \; \boxed{P_2} \Rightarrow \boxed{P_1 \; op \; P_2}$$

# Process Calculus

## Basic Principle

1. Define a few atomic processes (modeling the simplest process behavior).
2. Define compositionally new operations (building more complex process behavior from simple ones).

## Example

1. atomic instruction: assignment (e.g. x:=2 and x:=x+2)
2. new operators:
   - sequential composition ($P_1; P_2$)
   - parallel composition ($P_1 \parallel P_2$)

   E.g. (x:=1 $\parallel$ x:=2); x:=x+2; (x:=x-1 $\parallel$ x:=x+5) is a process.

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# CCS Basics (Sequential Fragment)

- $Nil$ (or $\mathbf{0}$) process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\stackrel{\text{def}}{=}$)
- nondeterministic choice ($+$)

**This is Enough to Describe Sequential Processes**

Any finite LTS can be (up to isomorphism) described by using the operations above.

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# CCS Basics (Parallelism and Renaming)

- parallel composition ( $\parallel$ )
  (synchronous communication between two components $=$ handshake synchronization)
- restriction of a set of actions $(P \smallsetminus L)$
  alternative notation: $(\nu \tilde{a})P$
- relabelling $(P[f])$

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# Some Examples

Assigning names to processes (as in procedures) allows us to give recursive definitions of process behaviors.

Some examples:

- $Clock \stackrel{\text{def}}{=} tick.Clock$
- $CM \stackrel{\text{def}}{=} coin.\overline{coffee}.CM$
- $VM \stackrel{\text{def}}{=} coin.\overline{item}.VM$
- $CTM \stackrel{\text{def}}{=} coin.(\overline{coffee}.CTM + \overline{tea}.CTM)$
- $CS \stackrel{\text{def}}{=} \overline{pub}.\overline{coin}.coffee.CS$
- $SmUni \stackrel{\text{def}}{=} (CM \parallel CS) \setminus coin \setminus coffee$

# Some Examples, in CAAL

Small CCS processes can be simulated in CAAL:
`http://caal.cs.aau.dk`.

The syntax is very similar to CCS expressions:

```
Clock = tick.Clock;
CM = coin.'coffee.CM;
VM = coin.'item.VM;
CTM = coin.('coffee.CTM + 'tea.CTM);
CS = 'pub.'coin.coffee.CS;
SmUni = (CM | CS) \ {coin,coffee};
```

In CAAL you may "explore" process transitions.

# Defining CCS (channels, actions, process names)

Let

- $\mathcal{A}$ be a set of channel names (e.g. $tea$, $coffee$)
- $\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ be a set of labels where
  - $\overline{\mathcal{A}} = \{\overline{a} \mid a \in \mathcal{A}\}$
    ($\mathcal{A}$ are called names and $\overline{\mathcal{A}}$ are called co-names)
  - by convention $\overline{\overline{a}} = a$
- $Act = \mathcal{L} \cup \{\tau\}$ is the set of actions where
  - $\tau$ is the internal or silent action

  (e.g. $\tau$, $tea$, $\overline{coffee}$ are actions)
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# Definition of CCS (expressions)

| | | | |
|---|---|---|---|
| $P :=$ | $K$ | $\mid$ | process constants ($K \in \mathcal{K}$) |
| | $\alpha.P$ | $\mid$ | prefixing ($\alpha \in Act$) |
| | $\sum_{i \in I} P_i$ | $\mid$ | summation ($I$ is an arbitrary index set) |
| | $P_1 \parallel P_2$ | $\mid$ | parallel composition |
| | $P \setminus L$ | $\mid$ | restriction ($L \subseteq \mathcal{A}$) |
| | $P[f]$ | $\mid$ | relabelling ($f : Act \to Act$) such that |

- $f(\tau) = \tau$
- $f(\overline{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is called CCS process expressions (and denoted by $\mathcal{P}$).

## Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i \qquad Nil = 0 = \sum_{i \in \emptyset} P_i$$

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# Precedence

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1. restriction and relabelling (tightest binding)
2. action prefixing
3. parallel composition
4. summation

Example: $R + a.P \parallel b.Q \setminus L$ means $R + \big((a.P) \parallel (b.(Q \setminus L))\big)$.

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# Definition of CCS (defining equations)

## CCS program

A collection of defining equations of the form

$$K \stackrel{\mathrm{def}}{=} P$$

where $K \in \mathcal{K}$ is a process constant and $P \in \mathcal{P}$ is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g. $A \stackrel{\mathrm{def}}{=} \overline{a}.A \parallel A$.

# Semantics of CCS

Syntax
CCS
(collection of defining equations)

$\longrightarrow$

Semantics
LTS
(labelled transition systems)

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# Structural Operational Semantics for CCS

## Structural Operational Semantics (SOS) – Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules.

Given a collection of CCS defining equations, we define the following LTS $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$:

- $Proc = \mathcal{P}$ (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$ (the set of all CCS actions including $\tau$)
- transition relation is given by SOS rules of the form:

$$\text{RULE} \quad \frac{premises}{conclusion} \quad conditions$$

# SOS rules for CCS ($\alpha \in Act$, $a \in \mathcal{L}$)

ACT $\dfrac{}{\alpha.P \xrightarrow{\alpha} P}$

SUM$_j$ $\dfrac{P_j \xrightarrow{\alpha} P_j'}{\sum_{i \in I} P_i \xrightarrow{\alpha} P_j'}$ $j \in I$

COM1 $\dfrac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q}$

COM2 $\dfrac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'}$

COM3 $\dfrac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'}$

RES $\dfrac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L}$ $\alpha, \overline{\alpha} \notin L$

REL $\dfrac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$

CON $\dfrac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'}$ $K \stackrel{\text{def}}{=} P$

# Deriving Transitions in CCS

Let $A \stackrel{\mathrm{def}}{=} a.A$. Then

$$\big((A \parallel \overline{a}.Nil) \parallel b.Nil\big)[c/a] \xrightarrow{c} \big((A \parallel \overline{a}.Nil) \parallel b.Nil\big)[c/a].$$

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# LTS of the Process $a.Nil \parallel \overline{a}.Nil$

$$
\begin{tikzcd}
 & a.Nil \parallel \overline{a}.Nil \arrow[dl, "a"'] \arrow[dd, "\tau"] \arrow[dr, "\overline{a}"] & \\
Nil \parallel \overline{a}.Nil \arrow[dr, "\overline{a}"] & & a.Nil \parallel Nil \arrow[dl, "a"'] \\
 & Nil \parallel Nil &
\end{tikzcd}
$$

# Value Passing CCS

## Main Idea

Handshake synchronization is extended with the possibility to exchange integer values.

$$\overline{pay(6)}.Nil \parallel pay(x).\overline{save(x/2)}.Nil$$

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$$\downarrow \tau$$

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## Parametrized Process Constants

For example: $Bank(total) \stackrel{\text{def}}{=} save(x).Bank(total + x)$.

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$$\downarrow \tau$$

$$Nil \parallel Nil \parallel Bank(103)$$

## Parametrized Process Constants

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# Translation of Value Passing CCS to Standard CCS

## Value Passing CCS

$$C \stackrel{\text{def}}{=} in(x).C'(x)$$

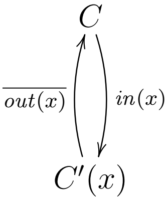
$$C'(x) \stackrel{\text{def}}{=} \overline{out(x)}.C$$

$\longrightarrow$

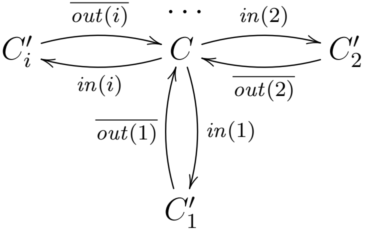
## Standard CCS

$$C \stackrel{\text{def}}{=} \sum_{i \in \mathbb{N}} in(i).C'_i$$

$$C'_i \stackrel{\text{def}}{=} \overline{out(i)}.C$$




symbolic LTS




infinite LTS

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## Fact

CCS can simulate a computation of any Turing machine.

## Remark

Hence CCS is as expressive as any other programming language but its use is to rather describe the behaviour of reactive systems than to perform specific calculations.

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