Supplementary material for: "Imitation: Theory and Experimental Evidence"

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Abstract

This supplement to Apesteguia, Huck, and Oechssler (2006) is organized as follows. The proofs to the propositions in the main text are collected in Appendix A. Appendix B contains a treatment of Selten and Ostmann's imitation equilibrium. The instructions for the experiments are shown in Appendix C, and Appendix D contains additional regression results.

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A Proofs

Notation: Let player $(i,j)_t$ be the player who has role $i \in \{X,Y,Z\}$ in group $j \in \{1,2,3\}$ at time t, and let $s_i^j(t)$ be that player's strategy in t. We refer to the set of individuals whose actions and payoffs can be observed by individual $(i,j)_t$, as $(i,j)_t$'s *reference group*, $R(i,j)_t$. Individual $(i,j)_t$'s *set of observed actions* includes all actions played by someone in his reference group and is denoted by $O(i,j)_t := \{s_h^k(t) | (h,k)_t \in R(i,j)_t\}$. Notice that $(i,j)_t \in R(i,j)_t$ and $s_i^j(t) \in O(i,j)_t$ in all our experimental treatments.

Proposition 1 *If agents follow either a WIBA ("weakly imitate the best average") or a WIBM ("weakly imitate the best max") rule and if the reference group is as in treatment GROUP, the Walrasian state ω^e is the unique stochastically stable state.*

Proof of Proposition 1. First notice that if agents observe only strategies played in the own group, the max and average evaluation rules coincide. By standard arguments (see e.g. Samuelson, 1994) only sets of states that are absorbing under the unperturbed ($\varepsilon = 0$) process can be stochastically stable. A straightforward generalization of Proposition 1 in Vega–Redondo (1997) shows that only uniform states can be absorbing (in all other states there is at least one agent who observes a strategy that fared better than his own), which is why we can restrict attention in the following to uniform states.[1] We will show that ω^e can be reached with one mutation from any other uniform state $\omega^s \neq \omega^e$. The proof is then completed by showing that it requires at least two mutations to leave the Walrasian state.

Consider any uniform state $\omega^s \neq \omega^e$ and suppose that some player $(i,j)_t$ switches to the Walrasian strategy e. As a consequence $(i,j)_t$ will have the highest payoff in group j which will be observed by the other group members. By property (v) all players who were in group j at time t will play e in $t+1$ with positive probability. Moreover, due to the random matching it is possible that the three players who were in group j at time t will be in three distinct groups in $t+1$. In that case, each of them will achieve the highest payoff in their respective group which will be observed by their group members who then can also switch to the Walrasian strategy e, such that ω^e is reached. (If there are more than three groups, it will simply take a few periods more to reach ω^e.) It remains to be shown that ω^e cannot be left with a single mutation. This is straightforward. In fact, it follows from exactly the same argument as in Vega–Redondo's result. If a player switches to some strategy $s \neq e$, he will have the lowest payoff in his group and will

[1] Notice that the random rematching of agents into groups is crucial here. If group compositions were fixed, different groups could, of course, use different strategies.

therefore not be imitated. Moreover, he observes his group members who still play e and earn more than himself. Thus, he will switch back eventually. ■

Proposition 2 *If agents follow a WIBA or a WIBM rule and if the reference group is as in treatment ROLE, the Cournot state ω^c is the unique stochastically stable state.*

Proof of Proposition 2. Although with reference groups as in treatment ROLE, the max and average evaluation rules do not coincide, we can use identical arguments for both rules to prove the claim. This is due to the fact, that we can establish the claim by restricting attention to one-shot mutations that do not induce different payoffs for any particular strategy an agent observes.

By a similar argument as above, only states in which all role players in a given role receive the same payoff can be candidates for stochastic stability. We will show that the Cournot state ω^c can be reached with a sequence of one–shot mutations from any other absorbing state. The proof will be completed by showing that it requires at least two mutations to leave ω^c. It is easy to see that every non–equilibrium state can be left with one mutation. One of the players who is currently not best replying, say (i, j), must simply switch to his best reply. This will increase (i, j)'s payoff which will also be observed by all other players in role i. Hence, in the next period all players in role i may have switched to their best replies against their opponents. Thus, for the first claim it remains to be shown that there exists for any state $\omega \neq \omega^c$ a sequence of (unilateral) best replies that leads into ω^c. This is easy to see by inspecting the payoff matrix, but follows more generally from the observation that the game has a potential (see Monderer and Shapley, 1996).

Now, consider ω^c and see what happens when a single player (i, j) switches to some other strategy. As he moves away from his best reply, he will earn less than the other agents in the same role i. As he can observe these other agents, he will not be imitated and will eventually switch back. Thus, it is impossible to leave ω^c with one mutation which completes the proof.■

Proposition 3 *If agents follow a WIBA (WIBM) rule and if the reference group is as in treatment FULL, then the Cournot state ω^c (Walrasian state ω^e) is the unique stochastically stable state.*

Proof of Proposition 3. WIBA: Note again that only uniform states can be candidates for stochastic stability. We will show that it takes one mutation to reach ω^c from any other absorbing state while it takes two mutations to leave ω^c. Consider first a possible transition from ω^e to ω^c. With 1 mutation a transition to the state $\omega = (cee)(eee)(eee)$ is possible. The two e–players in group 1 observe two e–players (including themselves) that earn 400 and two others that earn 0, which is on average 200. But they also observe one c–player who gets 300. Thus, with positive probability in the next round all players in group 1 play c and one round later everyone plays c. We denote this possible transition in short as:

$$\omega^e \xrightarrow{1} (cee)(eee)(eee) \to (ccc)(eee)(eee) \to \omega^c,$$

where the number above the arrow denotes the required number of mutations.

Similarly, it can be checked that the following transitions from ω^x, $x = a, b, d$ to ω^c require one mutation only,

$$\omega^x \xrightarrow{1} (cxx)(xxx)(xxx) \to (cxx)(cxx)(cxx) \to \omega^c.$$

On the other hand, any transition from state ω^c to some other absorbing state ω^x, $x \neq c$, is impossible with one mutation as the process must return to ω^c

$$\omega^c \xrightarrow{1} (xcc)(ccc)(ccc) \to \omega^c.$$

Thus, ω^c is the unique stochastically stable state.

WIBM: We shall construct sequences that require just one mutation and lead from ω^a and ω^b to ω^e. Furthermore, we shall show that two simultaneous mutations are sufficient to go from ω^c and ω^d to ω^e. The proof is completed by showing that three simultaneous mutations are required to leave ω^e.

It is easy to see that the following transitions from $x = a, b$ to e require one mutation only,

$$\omega^x \xrightarrow{1} (exx)(xxx)(xxx) \to (exx)(exx)(exx) \to \omega^e.$$

Next, the following transitions from $x = c, d$ to e require two mutation only,

$$\omega^x \xrightarrow{2} (exa)(xxx)(xxx) \to (eee)(exx)(exx) \to \omega^e.$$

To complete the proof note that any sequence from ω^e with two or less arbitrary mutations x, y will lead back to ω^e.

$$\omega^e \xrightarrow{2} (xye)(eee)(eee) \to \omega^e$$

or

$$\omega^e \xrightarrow{2} (xee)(yee)(eee) \to \omega^e,$$

as the e–player in the group that contains the mutation(s) will always have a higher payoff than the mutation(s). ■

Remark The proof of Propositions 3 shows that specifics of the payoff function matter for the exact prediction under WIBA. A generalization for a larger class of payoff functions would predict outcomes ranging from the Cournot to some more competitive outcomes (without exactly specifying the boundary). On the other hand, the proofs of Propositions 3 for WIBM and of Propositions 1 and 2 do not make use of anything that is specific to our chosen Cournot–type payoff function and it is easy to see that they could be generalized to a large class of Cournot games in exactly the same form as above. The number of players and/or groups could also be varied without altering those results.

B Imitation Equilibrium

We shall briefly review the recently introduced notion of an *imitation equilibrium (IE)* (Selten and Ostmann, 2001), and derive its predictions for our treatments. Unlike the preceding models, imitation equilibrium is a static equilibrium notion. Following Selten and Ostmann (2001) we will say that player (i, j) has an *imitation opportunity* if there is an $s_h^k \neq s_i^j$, $s_h^k \in O(i, j)$, such that the payoffs of player (h, k) are the highest in $R(i, j)$ and there is no player in $R(i, j)$ playing s_i^j with payoffs as high as (h, k).[2] A *destination* is a state without imitation opportunities. An *imitation path* is a sequence of states where the transition from one element of the sequence to the next is defined by all players with imitation opportunities taking one of them. The imitation path continues as long as there are imitation opportunities.

An *imitation equilibrium* is a destination that satisfies that all imitation paths generated by any deviation of any one player return to the original state. Two classes of imitation paths generated by a deviation (henceforth called *deviation paths*) that return to the original state are distinguished.

(i) *Deviation paths with deviator involvement*: the deviator himself takes an imitation opportunity at least once and the deviation path returns to the original state.

(ii) *Deviation paths without deviator involvement*: the destination reached by a deviation path where the deviator never had an imitation opportunity gives lower payoffs to the deviator than those at the original state, making that the deviator returns to the original strategy. This creates an imitation path that returns to the original state.

The following proposition reveals remarkable similarities between Selten and Ostmann's imitation equilibrium and the dynamic class of WIBA rules.

Proposition 4 *Imitation equilibrium (IE) is characterized by the following.*

(a) In Treatment GROUP the Walrasian state ω^e is the unique IE.

[2] This requirement is analogous to the one in IBM.

(b) In Treatment ROLE the Cournot state ω^c is the unique IE.

(c) In Treatment FULL the Cournot state ω^c is the only uniform IE.

Proof (a) Only uniform states can be imitation equilibria, otherwise there would be an imitation opportunity. To see that ω^e is an imitation equilibrium note that if (i, j) deviates from ω^e will experience lower payoffs than any other player in group j; nobody follows and (i, j) returns to e. To see that any other uniform state is not an imitation equilibrium consider the deviation of (i, j) to the immediate higher production level. This creates an imitation opportunity to players in group j. By random matching this deviation spreads out the whole population, in which case a destination is reached. At the destination the payoffs of (i, j) are lower than at the original distribution. Player (i, j) returns to the original action. Now players in group j have higher payoffs than (i, j), do not imitate him, and (i, j) has an imitation opportunity to go back to the deviation strategy.

(b) If (i, j) deviates from ω^c, he will get lower payoffs than players in role i. Nobody follows the deviation, and (i, j) returns to c. This shows that ω^c is an imitation equilibrium. It is easy to show that any state other than ω^c where members of the same role play the same action, but where differences between roles are not excluded, is not an imitation equilibrium. Note then that there is a (i, j) that is not best-replying, then a deviation of (i, j) to his best-reply gives to him higher payoffs, creating an imitation opportunity to players in role i. At this destination (i, j) has higher payoffs than at the original state, and hence does not return to the original action. It remains to be shown that a state where at least one role whose members play different actions is not an imitation equilibrium. If in such a case, in any random matching any player has an imitation opportunity, then the assertion holds. Assume the opposite, then since there are not two different best-replies that give the same payoffs, at least one player is not best-replying, and hence the above argument shows that such a state is not an imitation equilibrium.

(c) To show in FULL that non-uniform states are not imitation equilibria is tedious, and hence we concentrate on uniform states. We first show that ω^c is an imitation equilibrium. At ω^c let (i, j) deviate to $s_i^j \neq c$. Then players in role i will have higher payoffs than (i, j) and players in group j will observe that those players in their respective role have higher payoffs than (i, j). Hence, nobody follows. Then, (i, j) observes that c gives higher payoffs to players in role i and hence returns to c.

To see that ω^d and ω^e are not IE, let $x \in \{d, e\}$ and consider a deviation from ω^x of any one player to action a. It is easy to see that this generates an infinite deviation path: $\omega^x \to (axx)(xxx)(xxx) \to (xxx)(axx)(axx) \to (axx)(xxx)(xxx) \to (xxx)(axx)(axx) \to \cdots$.

Now to show that ω^a and ω^b are not imitation equilibria it is enough to show that there exists a sequence of random matchings that makes that the imitation paths do not return to the original state. Let that if $x = a$, then $y = b$ and if $x = b$ if $y = a$ and $y = c$. Then, one can check that the following path can be generated: $\omega^x \to (yxx)(xxx)(xxx) \to (yyx)(yxy)(yxx) \to \omega^y \to (xyy)(yyy)(yyy) \to \omega^y$. ■

C Instructions

Welcome to our experiment! Please read these instructions carefully. Do not talk with the person sitting next to you and remain quiet during the entire experiment. If you have any questions please ask us. We will come to you.

During this experiment, which takes 60 rounds, you will be able to earn points in every round. The number of points you are able to earn depends on your actions and the actions of the other participants. The rules are very easy. At the end of the experiment the points will be converted to Euros at a rate of 3000:1.

Always 9 of the present participants will be evenly divided into three roles. There are the roles X, Y, Z, taken in always by 3 participants. The computer randomly allocates the roles at the beginning of the experiment. You will keep your role for the course of the entire experiment.

In every round every X-participant will be randomly matched by the computer with one Y- and one Z-participant. After this, you will have to choose one of five different actions, actions A, B, C, D, and E. We are not going to tell you, how your payoff is calculated, but in every round your payoff depends uniquely on your own decision and the decision of the two participants you are matched with. The rule underlying the calculation of the payoff is the same in all 60 rounds.

After every round you get to know how many points you earned with your action and your cumulative points.

In addition, you will receive the following information:

[In ROLE and FULL] You get to know which actions the other two participants who have the same role as you (and who were matched with different participants) have chosen, and how many points each of them earned.

[In GROUP and FULL] You get to know which actions the other two participants you were matched with have chosen, and how many points each of them earned.

[In FULL] Furthermore you get to know how many points all 9 participants (in all the 3 roles) on average earned in this round.

Those are all the rules. Should you have any questions, please ask now. Otherwise have fun in the next 60 rounds.

C.1 Instructions for GROUP-π and ROLE-π

Welcome to our experiment! Please read these instructions carefully. Do not talk with the person sitting next to you and remain quiet during the entire experiment. If you have any questions please ask us. We will come to you.

During this experiment, which takes 60 rounds, you will be able to earn points in every round. The number of points you are able to earn depends on your actions and the actions of the other participants. The rules are very easy. At the end of the experiment the points will be converted to Euros at a rate of 3000:1.

Always 9 of the present participants will be evenly divided into three roles. There are the roles X, Y, Z, taken in always by 3 participants. The computer randomly allocates the roles at the beginning of the experiment. You will keep your role for the course of the entire experiment.

In each round every X-participant will be randomly matched by the computer with one Y- and one Z-participant. Each participant plays the role of a firm, and you have to decide which quantity you want to supply to the market. Each firm can choose among five quantities a, b, c, d, and e, ordered by size, with a being the lowest and e being the highest quantity. In every round your payoff depends uniquely on your own decision and the decision of the two participants you are matched with. On the next page you find a payoff table, which shows for each combination of your quantity and the quantities of the others how much you will earn.[3] The rule determining payoffs will be the same for all 60 periods.

After every round you get to know how many points you earned with your action and your cumulative points.

In addition, you will receive the following information:

[In ROLE-π] You get to know which actions the other two participants who have the same role as you (and who were matched with different participants) have chosen, and how many points each of them earned.

[In GROUP-π] You get to know which actions the other two participants you were matched with have chosen, and how many points each of them earned.

Those are all the rules. Should you have any questions, please ask now. Otherwise have fun in the next 60 rounds.

[3] A payoff table analogous to Table 1 in the main text was provided. The given table included the following example: ad means that one of the participants you are matched with chose a and the other chose d. The order in which quantities are listed does not matter. For example, if you choose d and the other two choose ad, then your payoff is 1008.

D Regressions

In this appendix we show all regression results for models (1) and (2). Tables 1, 2, and 3 show the results for what makes subjects switch to another strategy (model 1). Table 1 contains the estimations for treatment GROUP, Table 2 for ROLE, and Table 3 for FULL. The first two columns in each table show the results from the linear random effects model, also shown in the main body of the paper. The third and fourth columns show results obtained from a linear model with subject-specific fixed effects and the fifth and sixth column show estimates from a random effects probit model (marginal effects at population means).

Tables 4, 5, and 6 show the results for what makes subjects follow IBM (model 2). Table 4 contains the estimations for treatment GROUP, Table 5 for ROLE, and Table 6 for FULL. Again, the first two columns in each table show the results from the linear random effects model, also shown in the main body of the paper. The third and fourth columns show results obtained from a linear model with subject-specific fixed effects and the fifth and sixth column show estimates from a random effects probit model (marginal effects at population means).

Table 1: Estimating the likelihood that subjects change their actions in treatment ROLE.

ROLE	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	886***	997***	879***	979***	—	—
	(42.6)	(40.4)	(35.6)	(36.8)		
own payoff	−.316***	−.289***	−.311***	−.284***	−.369***	−.325***
	(.033)	(.033)	(.033)	(.033)	(.04)	(.04)
payoff diff.	.098***	.100***	.105***	.108***	.129***	.133***
	(.035)	(.034)	(.035)	(.034)	(.04)	(.04)
relative	—	−387***	—	−353***	—	−413***
propensity		(37.5)		(38.0)		(45.9)
R^2	.075	.131	.075	.129	—	—
# of obs.	3186	3186	3186	3186	3186	3186

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level.

Table 2: Estimating the likelihood that subjects change their actions in treatment GROUP.

GROUP	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	579***	730***	581***	709***	—	—
	(26.9)	(26.4)	(17.8)	(22.5)		
own payoff	−.197***	−.164***	−.195***	−.165***	−.225***	−.185***
	(.024)	(.024)	(.024)	(.024)	(.03)	(.03)
payoff diff.	.476***	.454***	.448***	.429***	.552***	.538***
	(.043)	(.043)	(.044)	(.043)	(.05)	(.05)
relative	—	−418***	—	−355***	—	−457***
propensity		(37.6)		(39.3)		(46.1)
R^2	.077	.146	.077	.145	—	—
# of obs.	3186	3186	3186	3186	3186	3186

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level.

Table 3: Estimating the likelihood that subjects change their actions in treatment FULL

FULL	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	611***	756***	613***	736***	—	—
	(44.1)	(37.3)	(31.2)	(32.9)		
own payoff	−.121***	−.077***	−.123***	−.089***	−.148***	−.104***
	(.029)	(.029)	(.029)	(.029)	(.04)	(.04)
payoff diff.	.211***	.208***	.208***	.204***	.275***	.277***
	(.032)	(.031)	(.032)	(.031)	(.04)	(.04)
relative	—	−467***	—	−389***	—	−516***
propensity		(36.5)		(38.1)		(50.1)
R^2	.042	.174	.042	.166	—	—
# of obs.	3186	3186	3186	3186	3186	3186

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level, ** denotes significance at the 5% level.

Table 4: Estimating the likelihood that subjects follow IBM in treatment ROLE.

ROLE	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	127***	146***	122***	148***	—	—
	(41.2)	(41.9)	(39.7)	(40.8)		
own payoff	−.001	.004	−.002	−.009	−.015	−.011
	(.038)	(.038)	(.038)	(.038)	(.04)	(.04)
payoff diff.	.248***	.246***	.249***	.248***	.234***	.233***
	(.038)	(.038)	(.038)	(.038)	(.04)	(.04)
relative	—	−90.1*	—	−126***	—	−113**
propensity		(47.7)		(48.6)		(50.7)
R^2	.038	.038	.038	.037	—	—
# of obs.	2079	2079	2079	2079	2079	2079

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level, ** denotes significance at the 5% level, * denotes significance at the 10% level.

Table 5: Estimating the likelihood that subjects follow IBM in treatment GROUP.

GROUP	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	145***	113***	116***	111***	—	—
	(22.5)	(25.4)	(21.2)	(24.9)		
own payoff	−.043	−.058*	−.013	−.015	−.069*	−.077**
	(.030)	(.030)	(.031)	(.030)	(.04)	(.04)
payoff diff.	.551***	.586***	.577***	.582***	.546***	.565***
	(.045)	(.047)	(.045)	(.047)	(.05)	(.05)
relative	—	131***	—	20.5	—	98.5*
propensity		(49.1)		(56.5)		(55.3)
R^2	.080	.087	.078	.080	—	—
# of obs.	1644	1644	1644	1644	1644	1644

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level ** denotes significance at the 5% level. * denotes significance at the 10% level.

Table 6: Estimating the likelihood that subjects follow IBM in treatment FULL

FULL	linear, random effects		linear, fixed effects		probit, random effects marginal effects only	
constant	164***	166***	154***	159***	—	—
	(43.6)	(45.3)	(40.5)	(45.3)		
own payoff	.056	.056	.059	.059	−.057	−.057
	(.038)	(.039)	(.038)	(.039)	(.04)	(.04)
payoff diff.	.156***	.156***	.152***	.151***	.165***	.164***
	(.040)	(.040)	(.040)	(.040)	(.04)	(.04)
relative	—	−13.4	—	−22.1	—	−15.2
propensity		(61.6)		(62.4)		(65.2)
R^2	.009	.009	.009	.009	—	—
# of obs.	1920	1920	1920	1920	3186	3186

Note: All coefficients and standard errors multiplied by 10^3. Standard errors in parentheses. ***denotes significance at the 1% level.

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