THE RATIONALIZABILITY OF SURVEY RESPONSES

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Abstract. We propose the concept of survey rationalizability, that we base on classical item response theories in psychology. Survey rationalizability involves positioning survey questions on a common scale such that, in the case of attitude (respectively, aptitude) surveys, each respondent gives higher support to questions that are more aligned with her views (respectively, less difficult). We first demonstrate that ideas from standard revealed preference analysis can be used to characterize when deterministic responses on dichotomous surveys are rationalizable. We then show that these results readily extend to polytomous questions and stochastic responses. Furthermore, we investigate the identification of the models.

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1. Introduction

Surveys are becoming increasingly popular in economics as tools for gathering data on various economic variables, such as subjective expectations, happiness, contingent valuations, beliefs, and political attitudes. Yet, the rationality foundations and the resulting empirical content of survey responses are not sufficiently understood from an economic point of view. This paper takes a first step in this direction by formalizing classical psychological theories of item response and then studying their empirical content using concepts inspired by traditional revealed preference analysis. Ultimately, this type of exercise is crucial for better assessing the external validity of survey information and for improving the structure of surveys in terms of informativeness and accuracy, aspects that ultimately lead to better decision-making and policies.

We study the most commonly used types of surveys, distinguishing between questions aimed at measuring attitudes or aptitudes, questions allowing for yes/no responses or Likert-type responses, and theories of response that are deterministic or stochastic. In all cases, we demonstrate that the study of surveys can be entirely based on the revealed analysis of responses, i.e., we employ the observed responses of individuals to infer the structure of the survey and the processes by which individuals respond to the questions. Furthermore, we show that the analysis of the simplest case—dichotomous surveys with deterministic responses—forms the basis for the analysis of all other cases.

We start our analysis with attitude surveys to measure individuals' attitudes, opinions, beliefs, or preferences on a given topic. We use the classical treatment in psychology of item response theory with ideal points.[1] Accordingly, we model rationalizability as the possibility of positioning survey questions on a common scale and describing the behavior of each individual by means of a strictly quasi-concave utility such that questions associated with larger utility receive larger support. In the stylized case of dichotomous surveys and deterministic responses, rationalizability requires each individual to endorse questions with utility above a threshold level, and only those questions. Given the strict quasi-concavity of utilities, this must correspond to each individual endorsing questions located on an interval of the real line.[2]

[1]See Thurstone (1928) and especially the unfolding theory of Coombs (1964) for early developments, and see Tay and Ng (2018) and Van der Linden (2018) for more recent accounts.

[2]We note here that our formalization of survey rationalizability can be applied to a wide variety of economic settings that deviate from the traditional framework involving choices among alternatives. The common characteristic of these settings is the absence of rivalry among the alternatives. For instance, in the context of surveys, individuals express their degree of conformity with a statement

We show that the study of rationalizability can be built on the basis of an intuitive revelation principle. Consider an individual who endorses questions q_2 and q_3, but not question q_1. Given the interval nature of responses, the individual reveals that, within this triple, q_1 must occupy an extreme position within the triple. Following the logic of revealed preference analysis, a minimal rationalizability requirement should be that these direct revelations cannot be contradictory; if there are individuals that reveal two questions as extreme within a triple, it cannot be the case that another individual reveals the third question as also extreme in the triple.[3] For obvious reasons, we refer to this asymmetry type of property as the Weak Axiom of Revealed Extremeness (WARE). However, in our first result, Proposition 1, we show that the study of direct revelations is not sufficient for rationalizability. Fortunately, we are able to show how to incorporate indirect revelations, something that can be done in two ways. First, we expand survey data with the inclusion of particular fictitious responses in a way that applying WARE to the expanded survey becomes sufficient. Second, we parallel the standard revealed preference analysis by appropriately extending the revealed extremeness relation, and then imposing asymmetry on the extended relation, a property that we call the Strong Axiom of Revealed Extremeness (SARE). In both cases, we are minimally accounting for the exact indirect revelations contained in the data, leading to our characterization of survey rationalizability in the first part of Theorem 1. Moreover, this characterization also helps to show that testing for rationalizability is computationally tractable, as we show in the second part of the theorem.

We then show that our techniques for the dichotomous and deterministic case prove useful in the study of more general settings. In particular, we consider the cases of polytomous surveys, where questions can be answered using one of multiple levels of endorsement, and probabilistic survey data, which allows for belief expressions or noisy responses. After adjusting our notion of survey rationalizability to consider ordered collections of thresholds, one for each endorsement level, and random thresholds, respectively, Theorems 2 and 3 establish the corresponding characterizations of rationalizability. Both results follow from a simple transformation of these richer datasets

rather than choosing which of several statements they prefer. We discuss this point in greater detail in Section 3.1.

[3]Importantly, notice that we are required to work not with binary relations but with ternary relations, which substantially departs from the revealed preference literature. Additionally, another departure is that here we do not have menu variation, but individual variation.

into analytically equivalent dichotomous datasets, allowing us to use the techniques developed for the proof of Theorem 1.

We then shift our focus to studying the identification of these models. Since they are essentially ordinal, the key step in the analysis is determining the order of questions and the location of individuals' ideal points. When the survey data is rationalizable, the extremeness revelations are key to learning the order of questions. In Proposition 2, we show that a richness condition guarantees that this order is unique. Regarding the location of individuals, Proposition 3 demonstrates that it is possible to establish bounds for them, with precise information being conveyed by probabilistic surveys, whereas polytomous surveys provide more information than dichotomous surveys. Furthermore, in Proposition 4, we show that all locations can be cardinally and uniquely identified when a specific parametric model is considered.

We close the analysis of attitude surveys by studying the assumption on the unidimensional location of questions and individuals. We argue that this assumption is not only relevant because a large number of theoretical settings and empirical applications in psychology, sociology, political science, and economics adhere to it, but it is also critical. This is so because the multidimensional analysis can be addressed in a straightforward manner. Essentially, in a multidimensional model, survey rationalizability loses its empirical content; we show in Theorem 5 that every survey can be rationalized when already two dimensions are considered.

We then study the alternative popular case of surveys oriented toward the study of cumulative aptitudes, such as ability, health status, or the extent of a practice. A common approach in the psychology literature is to apply dominance item response theory, with the dichotomous and deterministic version known as a Guttman scale (see Guttman (1944)). According to this approach, the difficulty of questions and the aptitude levels of individuals can be represented on the real line, with individuals responding positively to questions that are below their aptitude levels. We show in Theorem 4 that the revealed analysis of Guttman scales is facilitated by their cumulative nature, reducing the problem to a more standard asymmetry check on a binary relation.

2. Related Literature

In economics, the empirical literature using surveys is large and rapidly growing. Here, our focus is on its theoretical and methodological counterpart. Bertrand and

Mullainathan (2001) propose an econometric-based framework that accounts for errors in responses, enabling a meaningful interpretation of individuals subject to cognitive biases. Prelec (2004) proposes a scoring method for the elicitation of truthful subjective judgments based on answers that are more common than collectively predicted. Falk, Neuber, and Strack (2021) develop an individual-response model based on imperfect self-knowledge, where individuals' responses depend on a combination of private signals and the population mean. Manski (2004) and Stantcheva (2022) discuss several methodological issues in designing and interpreting survey studies. Benjamin, Guzman, Fleurbaey, Heffetz, and Kimball (2023) propose a methodology to uncover the informational content of self-reported well-being surveys, considering various potential response biases. Liu and Netzer (2023) study the use of response time to enhance the informational content of happiness survey responses, aiming to overcome the identification problems established in Bond and Lang (2019). Our contribution to this literature is to offer a theoretical framework that studies the rationality of survey responses based on classical accounts in psychology. We also show how the ordered structure of questions and individual attitudes or aptitudes is revealed. Finally, our probabilistic models of survey responses enable systematic consideration of various types of errors.

Our results relate to various strands of literature across scientific disciplines, such as the use of sortability and seriation in archaeology, anthropology, biology, computer science, and psychology. The common connection among these fields is the so-called consecutive-ones property (c1p) of a 0-1 matrix. A matrix satisfies c1p if there exists a permutation of its columns such that the ones in every row are consecutive.[4] In essence, for the dichotomous and deterministic case, survey rationalizability can be expressed as a c1p problem, enabling the use of existing results.[5] Importantly, our analysis, inspired by classical revealed preference techniques, provides a conceptually simple and tractable characterization of survey rationalizability and, in doing so, of

[4]As an illustration of an application of c1p in a different field, consider the statistical archaeology problem discussed in Kendall (1969). The data consists of a set of graves and a set of objects that are either present or absent in these graves. C1p is used to determine whether graves and objects can be arranged on a time scale such that each grave contains only objects that were created and not yet obsolete during the relevant period. See Coombs and Smith (1973) and Hubert (1974) for connections between these techniques and survey responses, and Liiv (2010) for an overview of their applications across different disciplines.

[5]See the discussion following the proof of Theorem 1 for the use and connection of our results with classical c1p results, such as Tucker (1972).

c1p. Our proof demonstrates that c1p requires only a check of asymmetry in an appropriately defined ternary relation. Additionally, we show how our techniques can be extended to address more complex settings, such as those involving polytomous surveys or stochastic responses.

Our paper can also be related to the study of ternary relations. Huntington and Kline (1917) and Fishburn (1971) represent early treatments of abstract ternary relations, with the purpose of capturing the intermediateness notion on the real line. Most of our results are presented in terms of an extremeness relation, but they can be equivalently presented in terms of an intermediateness relation. Our results contribute to this literature both conceptually and technically. Conceptually, we apply ternary relations to a concrete economic problem and, instead of taking them as given, we derive them from data. Technically, we show which incomplete ternary relations can be extended into the ternary relation of a linear order. We achieve this by combining techniques from the c1p literature and the revealed preference literature.

3. Attitude surveys: the dichotomous and deterministic case

We begin our analysis with the study of attitude surveys, deferring the case of aptitude surveys to Section 7. To facilitate intuitive understanding, we start by examining the simplest case: dichotomous questions with deterministic responses. In Section 4, we extend the techniques developed here to analyze more general cases, including polytomous questions and stochastic responses.

Let $\mathcal{Q} = \{1, \ldots, q, \ldots, Q\}$ be a set of questions related to a specific topic, such as capital punishment or immigration, on which individuals $\mathcal{N} = \{1, \ldots, n, \ldots, N\}$ are surveyed. We interpret question $q \in \mathcal{Q}$ as a request for the individuals to provide a graded evaluation or endorsement of the statement described in the question; given the present case of dichotomous questions, these are yes/no, agree/disagree or positive/negative evaluations. Accordingly, a dichotomous survey (D-survey) is a collection of the form $\{Q_n\}_{n \in \mathcal{N}}$, with $Q_n \subseteq \mathcal{Q}$ describing the set of questions that are endorsed by individual n. Without loss of generality, we assume that all response sets are different and that any two questions are different in the sense that the subset of individuals endorsing them is not the same. We study when a D-survey can be rationalized in the following sense.

D-survey rationalizability. We say that $\{Q_n\}_{n\in\mathcal{N}}$ is rationalizable whenever there exist $[\{U_n, \tau_n\}_{n\in\mathcal{N}}, \{\mu_q\}_{q\in\mathcal{Q}}]$, with $U_n : \mathbb{R} \to \mathbb{R}$ being strictly quasi-concave and $\tau_n, \mu_q \in \mathbb{R}$, such that for every $n \in \mathcal{N}$ and $q \in \mathcal{Q}$, it holds that $q \in Q_n$ if and only if $U_n(\mu_q) \geq \tau_n$.

The notion of rationalizability contains two major assumptions: (i) views are to be located on the real line, and (ii) utility functions are strictly quasi-concave. As for the first assumption, unidimensionality is a standard assumption in psychology, sociology, political science, and many economic settings of interest.[6] In any case, we extend our analysis to the multidimensional case in Section 6. Regarding the second assumption, once views are considered unidimensional, we believe that this is the most natural economic modeling of interest in the presence of trade-offs and it captures the notion of endorsement-by-proximity described in the psychology literature.[7] Note that since the model is essentially ordinal, all that is required is the assumption of single-peaked preferences over the ordered questions. We use strict quasi-concavity to capture this, as it simplifies the presentation of the more complex surveys studied later in the paper.

Before continuing, note that the interpretation of the utility function depends on the application at hand. Some surveys contain questions where the utility function can be interpreted in line with classical economic terms, even though the question may be hypothetical and utility may result from the hypothetical contemplation of the question. In these situations, the threshold level may represent some internal expectations. In other types of surveys, the utility function may be interpreted as the internal value derived from supporting a specific statement, with rejected statements being those that entail high costs of departing from personal beliefs or views. The specific interpretation is not crucial to our study; the representation we adopt helps to formalize the idea of survey rationalizability in a language common to economists.

3.1. **Examples of the setting.** Our formal setting uses the language of attitude surveys on a given topic, which represents its most direct application. These surveys are extensively used in economics to gather information on a variety of topics and, similarly,

[6] In many settings, the order of questions is not explicitly known, and we show how it can be learned from the response data. Section 3.1 provides several examples and, in most of them, the order of questions is not necessarily known.

[7] It can be shown that rationalizability can be defined equivalently as the existence of $[\{\mu_n, \epsilon_n\}_{n\in\mathcal{N}}, \{\mu_q\}_{q\in\mathcal{Q}}]$ with μ_n representing the location or ideal view of the individual, such that for every $n \in \mathcal{N}$ and $q \in \mathcal{Q}$, it is $q \in Q_n$ if and only if $d(\mu_n, \mu_q) \leq \epsilon_n$, where d is a distance function.

widely used in other disciplines, such as psychology, to measure personality traits or attitudes. Examples of their use can be seen in Andrich (1988), where individuals are asked to endorse or reject a number of statements on capital punishment and Grable and Lytton (1999), that surveys the risk attitude of individuals.[8] However, our setting is also applicable well beyond the survey interpretation. Here, we provide some economic examples that show the potential applicability of our results. The common characteristic of these examples is that there is little rivalry between the options, and the issue is whether individuals like/use/consider/accept each of the options.[9]

Activities. Behavioral traits, such as risk aversion, can be analyzed by gathering information on individual participation in various potentially risky activities, such as drug use or extreme sports. Similarly, altruism can be assessed by collecting data on participation in pro-social activities. This approach has a long tradition in psychology; an example is Fromme, Katz, and Rivet (1997), who designed and studied the cognitive appraisal of risky activities to assess risk preferences.[10] In this type of scenario, $\mathcal{N}$ is the set of individuals surveyed about their participation in a number of activities $\mathcal{Q}$.

Text analysis. Consider the particular use of language by various individuals or institutions on a given dimension, such as behavioral traits or political attitudes. The purpose is to order a set of linguistic expressions and, as a result, to categorize individuals by their language use. See Laver, Benoit, and Garry (2003) for an application to political texts to extract policy positions.[11] Using our terminology, Q_n represents the subset of linguistic expressions used by individual n.

[8] The former includes statements/questions such as: (i) capital punishment is justified because it does act as a deterrent to crime, (ii) I think capital punishment is necessary but I wish it were not or (iii) until we find a more civilized way to prevent crime we must have capital punishment. The latter includes questions such as: (i) it's hard for me to pass up on a bargain, (ii) it is more important to be protected from rising consumer prices (inflation) than maintaining the safety of your money from loss and theft, or (iii) how/are you comfortable investing in stocks or stock mutual funds? Note that these questions have not a clear ordering.

[9] We use the dichotomous case, but the more general cases with Likert scales and probabilistic responses also apply to most of them.

[10] The authors gathered information on activities such as substance use (e.g., alcohol and marijuana), interpersonal behaviors (e.g., going on a blind date), and sports (e.g., rock climbing). Again, note that these activities do not have a clear ex-ante ranking of riskiness.

[11] They consider words such as 'drugs,' 'markets,' 'nation,' 'security,' or 'representation,' and order them in the ideological spectrum. See also the recent use of large language models to position political texts in several unidimensional ideological and policy spaces in Le Mens and Gallego (2024).

Product Space and Recommendations. Many recommendation systems, such as those for movies or music bands, are based on arranging products and consumers within a characteristic space, with consumers favoring products closer to them.[12] In some cases, objects may be arranged on a single scale, and understanding this can be instrumental in improving recommendation systems. This approach is rooted in classical psychology literature, where tastes are investigated by observing the acceptability or rejection of stimuli that can be ordered. See Pfaffmann (1960) for an early treatment, and consider, as discussed there, products with different salt concentrations as an example of ordered stimuli. Following this line of thought, Q_n represents the set of products liked by individual n.

Wishlisting. The previous example involves recommendations to other users, but one may also consider recommendations to oneself in the form of a wishlist. In this context, the wishlist Q_n represents the set of items for which an individual expresses interest or a positive attitude. See Manzini, Mariotti, and Ülkü (2024) for a sequential model of individual "wishlisting."

Attention studies. Another case that deviates from the traditional question-response paradigm of surveys involves the use of more implicit expressions of interest. This is common in attentional studies, where a number of individuals $\mathcal{N}$ is offered a number of stimuli $\mathcal{Q}$, and the analyst infers the subset of these stimuli that attract the individual's attention. Often, objects can be ordered, and individuals pay attention to an interval of stimuli. A classical paper in this field is Thomas (1973), which studies children's interest in faces drawn with different degrees of detail.[13]

Approval voting. The idea of allowing individuals to pick any number of options has been suggested as a voting mechanism. See Brams and Fishburn (1978) for an early analysis of approval voting. Here, Q_n represents the subset of candidates approved by individual n. Interestingly, Laslier (2009) and Alós-Ferrer and Buckenmaier (2019)

[12]The classical Hotelling setting is also based on this simple idea to ultimately model choice behavior.

[13]Eye-tracking techniques are used in economics to gather information on attention; see Lahey and Oxley (2016) for a review. Polonio, Di Guida and Coricelli (2015) employ seriation techniques to understand the linkage between saccades and strategies of play within a game.

have shown that equilibrium behavior in approval voting games may have the upper-contour-set structure analyzed in this paper.[14]

Judgment aggregation. There is a set of individuals $\mathcal{N}$ making judgments on a set of logical propositions $\mathcal{Q}$. Dietrich and List (2010) study conditions that facilitate judgment aggregation; one of these conditions involves the case where propositions can be ordered on a line such that the accepted judgments of each individual form an interval.

Networks and homophily. Consider a set of individuals who have directed links to others. The question is whether the network can be explained by ordering individuals such that they form links with those most similar to them. For an application using Twitter data to gather political ideology, see Barberá (2014). We can understand this within our setting by considering $\mathcal{Q} = \mathcal{N}$.

Matching. Let $\mathcal{N}$ be a set of individuals and $\mathcal{Q}$ a set of objects (as in housing problems) or a set of different individuals (as in marriage problems). Individuals submit a set Q_n of acceptable elements. Typically, individuals have a common preference on $\mathcal{Q}$ determined, e.g., by quality. However, in some settings, individuals may have idiosyncratic single-peaked preferences on $\mathcal{Q}$, perhaps given by quasi-linear preferences on quality and price in the housing problem, or by affinity in the marriage problem (see Bade (2019)).

3.2. **Revealed information: extremeness.** We start by discussing how survey responses reveal the order of questions, which will soon be proven to be the critical feature for survey rationalizability. First, recall that the upper contour set of a strictly quasi-concave utility function must correspond to an interval on the real line. Then, consider individual n, and suppose we observe $Q_n \cap \{q_1, q_2, q_3\} = \{q_2, q_3\}$. In other words, individual n endorses questions q_2 and q_3 but not question q_1. Given the interval nature of responses, this observation reveals that q_1 occupies an extreme position within the triple. When this is the case, we write $q_1 \mid \{q_2, q_3\}$ and say that q_1 has been revealed as extreme in the triple $\{q_1, q_2, q_3\}$.

For rationalizability, the revealed information across individuals should not be contradictory. We can then consider the following asymmetry type of property.

[14]Núñez and Xefteris (2017) study approval voting under single-peaked preferences, which is related to our notion of endorsement-by-proximity. They show that every Nash-implementable welfare optimum can indeed be implemented by means of approval voting mechanisms.

Weak Axiom of Revealed Extremeness (WARE). We say that $\{Q_n\}_{n\in\mathcal{N}}$ satisfies WARE if, for every triple $\{q_1, q_2, q_3\}$ of questions, $q_1 \mid \{q_2, q_3\}$ and $q_2 \mid \{q_1, q_3\}$ imply that $q_3 \mid \{q_1, q_2\}$ cannot hold.

Having observed the revelation of $q_1 \mid \{q_2, q_3\}$ and $q_2 \mid \{q_1, q_3\}$, rationalization requires positioning q_1 and q_2 as extremes within the triple, which in turn necessitates that q_3 is not extreme. The following result shows that WARE is a necessary property for rationalizability, but not sufficient.

Proposition 1. *If $\{Q_n\}_{n\in\mathcal{N}}$ is rationalizable, then $\{Q_n\}_{n\in\mathcal{N}}$ satisfies WARE, but the converse is not necessarily true.*

Proof of Proposition 1: For the first part, suppose that $\{Q_n\}_{n\in\mathcal{N}}$ is rationalizable. Assume that $q_1 \mid \{q_2, q_3\}$ and $q_2 \mid \{q_1, q_3\}$ hold. From these extremeness revelations, it must be that $\{\mu_{q_1}, \mu_{q_2}\} = \{\min\{\mu_{q_1}, \mu_{q_2}, \mu_{q_3}\}, \max\{\mu_{q_1}, \mu_{q_2}, \mu_{q_3}\}\}$. Then, if we observe any (interval) response set containing both q_1 and q_2, it must also contain q_3, and WARE is satisfied.

For the second part, consider the following example of a D-survey: $\mathcal{N} = \{1, 2, 3\}$, $\mathcal{Q} = \{q_1, q_2, q_3, q_4\}$ and the response sets are $Q_1 = \{q_1, q_2\}$, $Q_2 = \{q_1, q_3\}$ and $Q_3 = \{q_1, q_4\}$. By construction, there are at most two extremeness revelations in every triple of alternatives, and WARE holds. However, the responses are not rationalizable; the information contained in Q_1 requires q_4 to be located as the maximum or minimum in the triple $\{q_1, q_2, q_4\}$. If it is a maximum (respectively, a minimum), the information contained in Q_2 requires q_4 to be also the maximum (respectively, the minimum), in the triple $\{q_1, q_3, q_4\}$ and, as a consequence, the maximum (respectively, the minimum) in the set $\{q_1, q_2, q_3, q_4\}$. However, the same argument can be applied to q_3 by considering response sets Q_1 and Q_3, and to q_2 by considering response sets Q_2 and Q_3. Since the three alternatives cannot be extreme in the set of four alternatives, a contradiction arises and the result follows. ■

3.3. **A characterization of rationalizability.** Proposition 1 demonstrates that the absence of direct contradictions required by WARE is insufficient for rationalizability. We now outline two equivalent approaches for appropriately expanding survey information and characterize survey rationalizability. The first approach involves augmenting the information revealed by the survey through the inclusion of fictitious responses. The second approach extends the extremeness relation to account for all indirect extremeness revelations. While these approaches turn out to be equivalent, they highlight different aspects of the problem, which can be instrumental for addressing various issues, as we demonstrate below in our study of the complexity of testing for rationalizability.

In order to discuss the first approach, we need to consider pairs of overlapping responses, namely:

$$(Q_i, Q_j) \text{ such that } Q_i \cap Q_j \neq \emptyset,\ Q_i \setminus Q_j \neq \emptyset \text{ and } \ Q_j \setminus Q_i \neq \emptyset.$$

Recall that, under rationalizability, response sets correspond to intervals of the real line. Hence, the union of two overlapping response sets also corresponds to a (larger) interval.[15] If this larger set of responses does not belong to the original set of responses, it brings new information that can be added to the original survey. We then consider this principle recursively until no more novel unions of overlapping responses can be added. Denote $\mathcal{N}_0 = \mathcal{N}$ and consider a sequence of D-surveys $\{Q_n\}_{n\in\mathcal{N}_m}$, $m = 0, 1, \ldots, M$, such that for every $m < M$, the $(m+1)$-th D-survey is equal to the m-th D-survey with the addition of a (novel) union of two overlapping sets in the m-th D-survey. Given finiteness, this sequence ends up in a D-survey that admits no further addition, $\{Q_n\}_{n\in\mathcal{N}_M}$, that we call the expanded D-survey of $\{Q_n\}_{n\in\mathcal{N}}$. Importantly, notice that this survey is closed up to unions of overlapping responses.[16]

As for the second approach, we now define the indirect extremeness revelations derived from the direct extremeness revelations contained in the original D-survey. Let T be a generic ternary relation on a finite set X, and define the following operation, that we call overlapping:[17]

$$xT\{y, z_1\} \text{ and } xT\{y, z_2\} \text{ implies } xT\{z_1, z_2\},$$

Given T, the overlapping closure of T, denoted $\widehat{T}$, is the minimal ternary relation containing T and being closed under the overlapping operation. We then use the overlapping closure of $|$ to define a stronger version of WARE.

[15]Notice that this statement does not necessarily apply if the responses are not overlapping: if two intervals are disjoint, the union may fail to be an interval. Also, notice that the intersection and difference of overlapping intervals must also be intervals, but our revelation technique does not require including this information.

[16]The order in which information is added is irrelevant and the expanded D-survey is unique.

[17]The property is a type of transitivity and we use the term overlapping to relate it to the notion of overlapping sets.

Strong Axiom of Revealed Extremeness (SARE). We say that $\{Q_n\}_{n\in\mathcal{N}}$ satisfies SARE if, for every triple $\{q_1, q_2, q_3\}$ of questions, $q_1 \widehat{|} \{q_2, q_3\}$ and $q_2 \widehat{|} \{q_1, q_3\}$ imply that $q_3 \mid \{q_1, q_2\}$ cannot hold.

We are now in a position to state the main theorem of this section.

Theorem 1. *The following statements are equivalent:*

(1) $\{Q_n\}_{n\in\mathcal{N}}$ *is rationalizable.*
(2) $\{Q_n\}_{n\in\mathcal{N}_M}$ *satisfies WARE.*
(3) $\{Q_n\}_{n\in\mathcal{N}}$ *satisfies SARE.*

Moreover, determining whether $\{Q_n\}_{n\in\mathcal{N}}$ *is rationalizable is polynomial with a time complexity of* $O(Q^4)$*.*

Proof of Theorem 1: We start by proving that (1) implies (2). Consider any appropriate sequence of D-surveys $\{Q_n\}_{n\in\mathcal{N}_m}$, $m = 0, 1, \ldots, M$ leading to the expanded survey. Given (1), $\{Q_n\}_{n\in\mathcal{N}_0}$ is rationalizable. We now claim that if $\{Q_n\}_{n\in\mathcal{N}_m}$ is rationalizable, $\{Q_n\}_{n\in\mathcal{N}_{m+1}}$ is also rationalizable. To see this, suppose that $\{Q_n\}_{n\in\mathcal{N}_m}$ is rationalizable with parameters $[\{U_n, \tau_n\}_{n\in\mathcal{N}_m}, \{\mu_q\}_{q\in\mathcal{Q}}]$. Let $Q_i \cup Q_j$ be the novel response set added to form $\{Q_n\}_{n\in\mathcal{N}_{m+1}}$. This requires Q_i, Q_j to belong to $\{Q_n\}_{n\in\mathcal{N}_m}$ and to be overlapping. To construct the new rationalization, define the location of questions and the individual parameters of any individual other than the novel one as in the rationalization of $\{Q_n\}_{n\in\mathcal{N}_m}$; we then simply need to define parameters for the novel individual and prove the rationalization of her response set $Q_i \cup Q_j$. To do so, define $a = \min\{\mu_q : q \in Q_i \cup Q_j\}$, $b = \max\{\mu_q : q \in Q_i \cup Q_j\}$ and define the utility of the novel individual to be any strictly increasing utility function up to $\frac{a+b}{2}$ and strictly decreasing after this point, with the utility at a and b being equal to the threshold level. Given the original rationalizability and the overlapping nature of Q_i and Q_j, the interval $[a, b]$ contains exactly questions $Q_i \cup Q_j$, and rationalizability of this response set follows. Given finiteness, this inductive argument guarantees that the expanded D-survey is rationalizable and, by Proposition 1, it must satisfy WARE.

We now prove that (2) implies (1). We use the characterization of c1p of Theorem 6 in Tucker (1972), via the absence of asteroidal triples. In order to be able to use it, we first need to bring the concepts used there to our setting. Formally, c1p requires the existence of a linear order $\succ$ over $\mathcal{Q}$ such that every response set Q_n is formed by consecutive questions in $\succ$. An asteroidal triple is a tripe of questions $\{q_1, q_2, q_3\}$ such that for every pair of questions $\{q_i, q_j\}$ in this triple, there exists a sequence of individuals $n_{ij}^1, \ldots, n_{ij}^K$ such that: (i) $q_i \in Q_{n_{ij}^1}$ and $q_j \in Q_{n_{ij}^K}$, (ii) for every $k = 1, \ldots, K-1$, $Q_{n_{ij}^k} \cap Q_{n_{ij}^{k+1}} \neq \emptyset$, and (iii) the remaining alternative in the triple does not belong to the response set of any individual in this sequence.

We now show that if the expanded D-survey satisfies WARE, the original survey does not contain asteroidal triples. Suppose by contradiction that $\{q_1, q_2, q_3\}$ is an asteroidal triple. Consider first the response sets of n_{ij}^1 and n_{ij}^2. Given (ii), it must be either that one of these sets contains the other, or that they overlap. In the former case, the larger of these two response sets, denoted $\bar{Q}_{ij}^2$, belongs to the expanded D-survey (as it belongs to the survey), contains q_i but not q_o, and intersects $Q_{n_{ij}^3}$. In the latter case, since the expanded D-survey is closed under overlapping unions, $\bar{Q}_{ij}^2 = Q_{n_{ij}^1} \cup Q_{n_{ij}^2}$ must belong to the expanded D-survey, contains q_i but not q_o, and intersects $Q_{n_{ij}^3}$. Consider the response sets $\bar{Q}_{ij}^2$ and $Q_{n_{ij}^3}$. Again, it must be either the case that one of them contains the other or they overlap. With the same reasoning, we can identify $\bar{Q}_{ij}^3$ in the expanded D-survey, containing q_i but not q_o, and intersecting $Q_{n_{ij}^4}$. Proceed recursively until obtaining the response set $\bar{Q}_{n_{ij}^K}$. This set contains q_i and q_j, but not q_o. Since this argument applies to any combination in the triple, we have identified three (fictitious) individuals in the expanded D-survey violating WARE, a contradiction. Now, having proved that the original survey does not contain asteroidal triples, the application of Theorem 6 in Tucker (1972) guarantees that the original survey must satisfy c1p.

We now show that given that the original survey satisfies c1p, it is rationalizable. Given the finiteness of $\mathcal{Q}$, $\succ$ is representable and, hence, there exists a collection of real values $\{\mu_q\}_{q \in \mathcal{Q}}$ such that for every pair of questions $q, q' \in \mathcal{Q}$, it is $\mu_q > \mu_{q'} \Leftrightarrow q \succ q'$. For every $n \in \mathcal{N}$ such that $|Q_n| \geq 1$, consider any utility function $U_n(x)$ that is strictly increasing whenever $x \leq \frac{\min_{q \in Q_n} \mu_q + \max_{q \in Q_n} \mu_q}{2}$, and strictly decreasing otherwise. Moreover, we can select U_n to satisfy $U_n(\min_{q \in Q_n} \mu_q) = U_n(\max_{q \in Q_n} \mu_q) = 0$. This function is strictly quasi-concave. Set $\tau_n = 0$. Since Q_n is formed by consecutive questions in $\succ$, it is immediate that $Q_n = \{q : U_n(\mu_q) \geq \tau_n = 0\}$, proving rationalization. For every $n \in \mathcal{N}$ with $|Q_n| = 0$, we can rationalize responses by considering U_n to be strictly negative and strictly quasi-concave, and setting $\tau_n = 0$.

We now prove that the extremeness relation directly revealed by the expanded D-survey coincides with the overlapping closure of the extremeness relation of the original D-survey, implying that (3) is equivalent to (2). Consider again any appropriate sequence of D-surveys $\{Q_n\}_{n \in \mathcal{N}_m}$, $m = 0, 1, \ldots, M$, leading to the expanded D-survey. Denote by $|_m$ the extremeness revelations of survey $\{Q_n\}_{n \in \mathcal{N}_m}$. We need to show that

$|_M = \widehat{|}$. Consider two successive D-surveys $\{Q_n\}_{n\in\mathcal{N}_m}$ and $\{Q_n\}_{n\in\mathcal{N}_{m+1}}$ in the sequence, and assume that $Q_i \cup Q_j$, with $Q_i, Q_j \in \{Q_n\}_{n\in\mathcal{N}_m}$, is the novel response in $\{Q_n\}_{n\in\mathcal{N}_{m+1}}$. We claim that $|_{m+1}$ is composed by $|_m$ plus every overlapping $q\ |_m\ \{q', q_1\}, q\ |_m\ \{q', q_2\}$ in which $q \notin Q_i \cup Q_j$, $q_1 \in Q_i \setminus Q_j$ and $q_2 \in Q_j \setminus Q_i$. First, it is obvious that any such overlapping will be revealed by $Q_i \cup Q_j$ and, hence, present in $|_{m+1}$. Second, consider any revelation $q\ |_{m+1}\ \{q', q''\}$ that was not present in $|_m$. It must be the case that this revelation is generated by response set $Q_i \cup Q_j$, and for it to be novel, it must be $q \notin Q_i \cup Q_j$, $q' \in Q_i \setminus Q_j$ and $q'' \in Q_j \setminus Q_i$. But in such a case, since Q_i and Q_j are overlapping sets, there must exist $q^* \in Q_i \cap Q_j$, and the D-survey $\{Q_n\}_{n\in\mathcal{N}_m}$ must reveal $q\ |_m\ \{q^*, q'\}$ (via Q_i) and $q\ |_m\ \{q^*, q''\}$ (via Q_j), and hence the novel revelation can be seen as an overlapping of two original revelations. Using this recursively, we have shown that the extremeness revelations of the expanded D-survey can be seen as overlappings of the original extremeness relation and hence $| \equiv |_0 \subseteq |_M \subseteq \widehat{|}$. We now claim that $|_M$ is closed under overlappings. To see this, suppose that we observe $q\ |_M\ \{q^*, q'\}$ and $q\ |_M\ \{q^*, q''\}$. Then, we know that there exists Q_i, Q_j in the expanded D-survey such that $q^* \in Q_i \cap Q_j$ and $q' \in Q_i$ and $q'' \in Q_j$. If $q'' \in Q_i$ or $q' \in Q_j$, we can use the corresponding set to claim that $q\ |_M\ \{q', q''\}$. If none is true, then Q_i and Q_j are overlapping sets and, since the expanded D-survey is closed under overlapping unions, $Q_i \cup Q_j$ belongs to the expanded D-survey, and we can use it to claim that $q\ |_M\ \{q', q''\}$. Since the overlapping closure is the minimal ternary relation containing $|$ and closed under overlapping, the following must hold $\widehat{|} \subseteq |_M$.

Finally, in order to establish the computational complexity of survey rationalizability, consider the graph formed by $Q + Q(Q-1)$ nodes corresponding to all questions and pairs of questions in the D-survey. Connect every question q to every pair of questions $\{q', q''\}$, and associate to their edge a weight of 1 when $q \mid \{q', q''\}$, and a weight of ∞ otherwise. We denote these weights by $[q, \{q', q''\}, 0]$. We then use the Floyd-Warshall algorithm over this (survey) graph, in order to identify the shortest path from q to $\{q', q''\}$, possibly using other nodes as intermediate. Denoting by $[q, \{q', q''\}, l]$ the weight of the shortest path from q to $\{q', q''\}$, possibly involving the use of intermediate questions $\{1, \ldots, l\}$ via overlapping, it is immediate that these values satisfy

$$[q, \{q', q''\}, l] = \min\{[q, \{q', q''\}, l-1], [q, \{q', l\}, l-1] + [q, \{q'', l\}, l-1]\}.$$

We can iterate this process until l reaches Q. Note that edges with a finite value at the end of the process must correspond to the overlapping closure of $|$. Using the analogous reasoning to the standard result in which the Floyd-Warshall algorithm computes the transitive closure of a binary relation in polynomial time $\mathcal{O}(Q^3)$, it follows that: (i) computing the overlapping closure of the extremeness ternary relation can be done in polynomial time, and (ii) given the Q^2 order of the nodes of the survey graph (instead of Q in the transitive closure) this computation has polynomial order $\mathcal{O}(Q^4)$. This concludes the proof of the theorem. ■

Theorem 1 demonstrates that survey rationalizability can be understood through a familiar economic approach: (i) include all indirect revelations derivable from the data, and (ii) require these revelations to be asymmetric. Notably, these revelations pertain not to choice exercises but to attitude survey responses. They are unrelated to the preference ordering of a decision-maker and instead concern the positioning of questions/individuals on the corresponding attitude scale. Crucially, these revelations involve a ternary relation, not a binary one.

Statements (2) and (3) of Theorem 1 describe two conceptual views of indirect revelations. Statement (2) involves expanding the survey with fictitious responses and considering the direct revelations of the expanded survey. Instead, statement (3) extends the original survey's direct revelations into an indirect (overlapping closure) relation. The proof of their equivalence uses a recursive argument, showing that: (i) at each step, the fictitious response added contains only revelations that could be indirectly derived, and (ii) every possible indirect revelation is contained in a fictitious response added at some step in the sequence.

The equivalence of (2) and (3) with statement (1) is established as follows. First, survey rationalizability can be expressed in terms of the ordinal consecutive-ones property (c1p).[18] In essence, for a given order of questions $\succ$ where c1p holds, one can construct a corresponding strictly quasi-concave utility function and threshold for each agent. This utility function is strictly increasing up to the midpoint of the interval defined by the minimal and maximal positively responded questions, with zero utility (equal to the threshold) at those boundary questions. The consecutive nature of responses given by c1p ensures this construction is valid for rationalizability.

In order to show that our properties guarantee c1p, we use Theorem 6 in Tucker (1972), that is based upon the absence of "asteroidal triples," a concept introduced by Lekkerkerker and Boland (1962) in their study of interval graphs. In essence, the

[18] See the proof for the formal definitions of c1p, and that of asteroidal triple below, within our context.

asymmetry condition contained in WARE corresponds to the simplest possible asteroidal triple, and we need to prove that the asymmetry of indirect revelations rules out any asteroidal triple. This result follows from a recursive argument, with the indirect revelation approach facilitating the analysis.

Finally, given the equivalence between (1) and (3), the complexity of checking for rationalizability reduces to the complexity of checking for the asymmetry of overlapping closure of the extremeness relation of the original D-survey. In the proof we show that this can be done using a version of the classical Floyd-Warshall algorithm that is known to be polynomial in time.

4. Attitude surveys: polytomous and stochastic cases

In this section, we show how to exploit the structure of Theorem 1 when dealing with two related, but informationally richer, problems. The first one covers the Likert scale case in which individuals are allowed to use more than two labels in declaring their endorsement. The second analyzes the case of dichotomous surveys in which responses may be probabilistic.

4.1. **Polytomous surveys.** Given question $q \in \mathcal{Q}$, individual $n \in \mathcal{N}$ is now allowed to express strength of endorsement by using a label from $\mathcal{L} = \{0, 1, \ldots, L\}$, where $L > 0$. We write $\mathcal{L}^* = \mathcal{L} \setminus \{0\}$. Higher labels in the collection express stronger endorsement.[19] A polytomous survey or, simply, a P-survey is a map $P : \mathcal{N} \times \mathcal{Q} \to \mathcal{L}$, where $P(n, q) = l$ refers to the case where individual n assigns label l to question q. We consider the following notion of rationalizability, which reduces to the notion used for D-surveys when $L = 1$.

P-survey rationalizability. We say that P is rationalizable whenever there exist $[\{U_n, \{\tau_n^l\}_{l \in \mathcal{L}^*}\}_{n \in \mathcal{N}}, \{\mu_q\}_{q \in \mathcal{Q}}]$, with $U_n : \mathbb{R} \to \mathbb{R}$ being strictly quasi-concave, $\tau_n^1 < \cdots < \tau_n^L$, and $\mu_q \in \mathbb{R}$ such that, for every $n \in \mathcal{N}$, $q \in \mathcal{Q}$ and $l \in \mathcal{L}^*$, $P(n, q) = L$ if $U_n(\mu_q) \geq \tau_n^L$, $P(n, q) = l$, $0 < l < L$, if $\tau_n^{l+1} > U_n(\mu_q) \geq \tau_n^l$ and $P(n, q) = 0$ otherwise.

In words, the individual now uses different thresholds to determine her degree of endorsement. These thresholds naturally impose stronger requirements on stronger expressions of endorsement.

We now show that the rationalizability of a P-survey can be obtained building upon Theorem 1, by studying the rationalizability of an associated D-survey. Formally, given

[19] For instance, strongly disagree, disagree, neutral, agree, strongly agree.

the response map P, define the D-survey

$$\{Q^P_{(n,l)}\}_{(n,l)\in\mathcal{N}\times\mathcal{L}^*} \text{ where } Q^P_{(n,l)} = \{q \in \mathcal{Q} : P(n,q) \geq l\}.$$

That is, the vector of responses of individual $n \in \mathcal{N}$ is used to construct $L-1$ response sets. Each of these response sets contains cumulative information, e.g., response set $Q^P_{(n,l)}$ contains all the questions endorsed by individual n with intensity l or above. Note that, formally, $\{Q^P_{(n,l)}\}_{(n,l)\in\mathcal{N}\times\mathcal{L}^*}$ is a D-survey where every (n,l) can be understood as an individual. It is immediate that $\{Q^P_{(n,l)}\}_{(n,l)\in\mathcal{N}\times\mathcal{L}^*}$ provides the same information as P does. We need to define the corresponding expanded D-survey, that follows the same logic than above, and that we denote by $\{Q^P_n\}_{n\in\mathcal{N}_M}$. Theorem 2 shows that rationalizability of P is equivalent to this expanded D-survey satisfying WARE.[20]

Theorem 2. *P is rationalizable if and only if $\{Q^P_n\}_{n\in\mathcal{N}_M}$ satisfies WARE.*

Proof of Theorem 2: To see the only if part, suppose that P is rationalizable. Then, it must be so for some collection of parameters $[\{U_n, \{\tau^l_n\}_{l\in\mathcal{L}^*}\}_{n\in\mathcal{N}}, \{\mu_q\}_{q\in\mathcal{Q}}]$. Consider the D-survey $\{Q^P_{(n,l)}\}_{(n,l)\in\mathcal{N}\times\mathcal{L}^*}$. By associating to response set $Q^P_{(n,l)}$ the utility function U_n and the threshold τ^l_n, this D-survey must be rationalizable. We can then use the only if part of Theorem 1 and the expanded D-survey $\{Q^P_n\}_{n\in\mathcal{N}_M}$ must satisfy WARE.

Suppose now that $\{Q^P_n\}_{n\in\mathcal{N}_M}$ satisfies WARE. By using the if part of Theorem 1, the D-survey $\{Q^P_{(n,l)}\}_{(n,l)\in\mathcal{N}\times\mathcal{L}^*}$ must satisfy c1p. We now show it is rationalizable. Let $\succ$ be a linear order on $\mathcal{Q}$ that guarantees so and consider any set of values $\{\mu_q\}_{q\in\mathcal{Q}}$ such that $\mu_q > \mu_{q'} \Leftrightarrow q \succ q'$. Given individual $n \in \mathcal{N}$, let $l_n > 0$ be the highest label used by this individual. If this label is used to describe endorsement for at least two questions, define $\mu_n = \frac{\min_{q\in Q^P_{(n,l_n)}} \mu_q + \max_{q\in Q^P_{(n,l_n)}} \mu_q}{2}$ and set $U_n(\mu_n) = l_n$ and $U_n(\min_{q\in Q^P_{(n,l_n)}} \mu_q) = U_n(\max_{q\in Q^P_{(n,l_n)}} \mu_q) = l_n - 1$. All values in the interval $[\min_{q\in Q^P_{(n,l_n)}}, \max_{q\in Q^P_{(n,l_n)}}]$ are set by considering the piecewise linear function determined by these three values. If the label was used for only one question, set μ_n equal to the μ-value of that question, and $U_n(\mu_n) = l_n - 1$. After this, consider recursively the rest of the labels, starting with $l_n - 1$ down to label 1. For any label l, determine the questions $\min_{q\in Q^P_{(n,l)}} \mu_q$ and $\max_{q\in Q^P_{(n,l)}} \mu_q$. If any of them is different to those previously considered (i.e., whenever these questions have label l exactly), extend the piecewise linear function U_n by forcing it to adopt value $l-1$ in the location of these new questions. After label 1, extend

[20] For brevity, this and the following result present the characterization based solely on expanded surveys. Nevertheless, the conclusions about overlapping closure and associated complexity remain equally applicable.

the function beyond its extremes with any strictly monotone linear functions. Set thresholds $\tau_n^l = l - 1$. The function U_n is strictly quasi-concave, and since $Q^P_{(n,l)}$ is formed by a set of consecutive questions in $\succ$, it is immediate that questions in $Q^P_{(n,l)}$, and only these questions, have utility above threshold τ_n^l. Rationalization follows. ■

4.2. **Stochastic responses.** We now study the case where survey responses may be stochastic. This can be understood in terms of the variation of responses of the individuals of a given subgroup, like those given by different age or gender groups, or the case in which individual responses express a value in some continuum, such as probabilistic beliefs or expectations, or intensity of support. Stochastic models of responses are also instrumental in dealing with actual data, since they are a way to model and account for noise resulting from, e.g., mistakes due to fatigue or inattention.

Formally, a dichotomous survey with stochastic responses, or S-survey, is a map $S : \mathcal{N} \times \mathcal{Q} \to [0,1]$ where $S(n,q)$ describes the probability with which individual $n \in \mathcal{N}$ endorses question $q \in \mathcal{Q}$. Consider the following definition of rationalizability.

S-survey rationalizability. We say that S is rationalizable whenever there exist $[\{U_n, F_n\}_{n\in\mathcal{N}}, \{\mu_q\}_{q\in\mathcal{Q}}]$, with $U_n : \mathbb{R} \to \mathbb{R}$ being strictly quasi-concave, $F_n : \mathbb{R} \to [0,1]$ being a cumulative distribution function (CDF) governing the realization of thresholds τ_n, and $\mu_q \in \mathbb{R}$ such that, for every $n \in \mathcal{N}$ and $q \in \mathcal{Q}$, $S(n,q) = F_n(U_n(\mu_q))$.

A question is endorsed whenever the utility of the view represented by this question is above a threshold but now, these thresholds are realized according to a distribution F_n. Hence, endorsement happens probabilistically, and the probability of individual n endorsing question q corresponds to the mass of thresholds below $U_n(\mu_q)$, i.e., $F_n(U_n(\mu_q))$. This is a convenient account of stochastic responses that aligns well with the models studied in this paper and can easily accommodate mistakes at the time of responding. Consider, e.g., an individual who with a large probability considers her true threshold, and with small probability trembles and considers a smaller or a larger threshold. This leads the individual to occasionally endorse questions with an associated utility below her true threshold and to reject questions that are above it.

We now argue that we can rely again on the techniques developed for the analysis of D-surveys. For every $n \in \mathcal{N}$, consider the (at most) Q response sets defined by $Q^S_{(n,q)} = \{q' \in \mathcal{Q} : S(n,q') \geq S(n,q)\}$, and the corresponding D-survey $\{Q^S_{(n,q)}\}_{(n,q)\in\mathcal{N}\times\mathcal{Q}}$. That is, the information of individual $n \in \mathcal{N}$ appears at most Q times, describing response sets that contain the questions chosen with at least as much probability as a given question. Denote the corresponding expanded D-survey by $\{Q_n^S\}_{n\in\mathcal{N}_M}$. Theorem 3 shows that rationalizability of S is equivalent to this expanded D-survey satisfying WARE.

Theorem 3. *S is rationalizable if and only if $\{Q_n^S\}_{n\in\mathcal{N}_M}$ satisfies WARE.*

Proof of Theorem 3: The proof is analogous to that of Theorem 2, so we only mention the most significant differences. Notice that from S we can construct a set of at most $Q \times N$ labels, and define the P-survey with the response map P having the property that, for every $n, n' \in \mathcal{N}$ and $q, q' \in \mathcal{Q}$, $P(n, q) \geq P(n', q')$ if and only if $S(n, q) \geq S(n', q')$. This allows us to construct the values μ_q and the functions U_n exactly as in Theorem 2. For every τ_n such that there exists $q \in \mathcal{Q}$ with $U_n(\mu_q) = \tau_n$, set $F_n(\tau_n) = S(n, q)$. Now, for every individual $n \in \mathcal{N}$, a larger threshold corresponds to a question with a larger utility and given the construction and the assumption that $P(n, q) \geq P(n, q')$ if and only if $S(n, q) \geq S(n, q')$, a larger utility corresponds to a larger probability of endorsement. We can then extend these values to a CDF F_n over the reals, and rationalization follows. ■

5. ATTITUDE SURVEYS: IDENTIFICATION

In this section, we discuss the identification of the models studied. In order to do so, we impose a richness condition, according to which for every triple of consecutive questions $\mu_{q_1} > \mu_{q_2} > \mu_{q_3}$, there are two individuals overlapping on this triple, i.e., $Q_{n_1} \cap \{q_1, q_2, q_3\} = \{q_1, q_2\}$ and $Q_{n_2} \cap \{q_1, q_2, q_3\} = \{q_2, q_3\}$.[21] To present results across different types of surveys, we make the following assumptions: for every individual $n \in \mathcal{N}$: (i) $\emptyset \neq Q_n \neq \mathcal{Q}$, and (ii) $S(n, q) = S(n, q') \Rightarrow P(n, q) = P(n, q') \Rightarrow [q \in Q_n$ if and only $q' \in Q_n]$. That is, (i) implies that we eliminate from the analysis the trivial individuals that endorse everything or nothing, and (ii) connects the response behavior in S-surveys, P-surveys and D-surveys.

As discussed above, the models are essentially ordinal, so we need to study the identification of the linear order $\succ$ corresponding to $\{\mu_q\}_{q\in Q}$. Later, we analyze what we can learn about the location of individuals. Finally, we show that a parametric version of the model allows for cardinal identification results.

[21]Notice that this requirement can be achieved with $Q - 1$ individuals in the expanded survey. If every triple involves different pairs of individuals, then $2(Q - 1)$ individuals would be needed.

5.1. **Identification of questions.** We start by showing that $\succ$ is fully identified. The reason for this is that, under the richness assumption, the extremeness revelations can be shown to be complete. Having the two extreme questions for every triple, the linear order is fully identified (up to inversion).[22]

Proposition 2. *Under the Richness assumption, if a survey is rationalizable, it is so for a unique linear order $\succ$ (up to inversion). This linear order is fully described by the extremeness revelations.*

Proof of Proposition 2: Let the survey be rationalizable. Suppose without loss of generality that it is so for the linear order $Q \succ Q-1 \succ \cdots \succ 1$. We now claim that for every triple of different questions in this sequence satisfying $i > j > k$, the expanded D-survey must reveal $i \mid \{j,k\}$ and $k \mid \{i,j\}$, i.e., that i,k are extreme in the triple (or, in a terminology that simplifies the exposition, that j must be the intermediate question in the triple). We prove this claim recursively, over the difference $i-k$. The result is trivially true for $i-k=2$ because in this case the triple must be formed by three consecutive questions and the richness condition guarantees that two individuals exist revealing the intermediateness of j. Suppose now that the result is true up to some value $i-k=t\geq 2$ and consider a triple of questions such that $i-k=t+1$. Consider question $j^* \neq j$ such that $i > j^* > k$, which must exist because $i-k=t\geq 2$. Let us assume that $j^* > j$ (the other case is dual and omitted). Given the recursive step, we know that $i \mid \{j,j^*\}$ and $j \mid \{i,j^*\}$, i.e., j^* has been revealed intermediate in $\{i,j^*,j\}$. We also know that $k \mid \{j,j^*\}$ and $j^* \mid \{j,k\}$, i.e., j has been revealed intermediate in $\{j^*,j,k\}$. Therefore, there exist four individuals $m_1,m_2,m_3,m_4 \in \mathcal{N}_M$ such that $Q_{m_1} \cap \{i,j^*,j\} = \{i,j^*\}$, $Q_{m_2} \cap \{i,j^*,j\} = \{j^*,j\}$, $Q_{m_3} \cap \{j^*,j,k\} = \{j^*,j\}$, and $Q_{m_4} \cap \{j^*,j,k\} = \{j,k\}$. Suppose first that $i \in Q_{m_3}$. Then, by rationalizability, since $j^* \notin Q_{m_4}$, it must be that $i \notin Q_{m_4}$ and, as a result, individuals m_3 and m_4 reveal that $k \mid \{i,j\}$ and $i \mid \{j,k\}$, as desired. Suppose now that $i \notin Q_{m_3}$. In this case, notice that by rationalizability, $k \notin Q_{m_1}$ (because it does not contain $j > k$) and, by assumption, $k \notin Q_{m_3}$. Since Q_{m_1} and Q_{m_3} are overlapping, the response $Q_{m_1} \cup Q_{m_3}$ belongs to the expanded D-survey and $[Q_{m_1} \cup Q_{m_3}] \cap \{i,j,k\} = \{i,j\}$. Then, responses $[Q_{m_1} \cup Q_{m_3}]$ and Q_{m_4} reveal that $k \mid \{i,j\}$ and $i \mid \{j,k\}$, as desired. All intermediate revelations of $\succ$ have been obtained through the extremeness revelations. Finally, to

[22]Since this result holds for all types of surveys, we do not specify the type in the formulation of the result. Also, we simplify the exposition of the proof by showing the key steps using only the D-survey notation.

show uniqueness, consider any other order $\succ'$ different than $\succ$ and its inverse. Hence, there must be three questions $i > j > k$ where the intermediate one is different to j. Without loss of generality, let i be the intermediate question in $\succ'$. But then, we know that there is a response in the expanded D-survey such that $Q_m \cap \{i,j,k\} = \{j,k\}$, and this is not rationalizable for $\succ'$. If the expanded D-survey is not rationalizable with $\succ'$, the proof of Theorem 1 guarantees that the original survey cannot be rationalizable with $\succ'$, concluding the proof. ■

Proposition 2 shows that, under the richness condition, the order of questions is identified by the extremeness revelations. Notice that this idea can also be used even in the absence of the richness condition. In essence, partial identification of the order of questions is obtained: any linear order with an extremeness relation containing the one revealed by the expanded D-survey is plausible.

5.2. **Identification of individuals.** We now discuss what can be potentially learnt about a specific individual $n \in \mathcal{N}$. As previously discussed, the model is basically ordinal. Accordingly, here we care about the identification of the location of the ideal view of the individual with respect to $\succ$.[23] We show that S-surveys are more informative in this respect than P-surveys, which in turn are more informative than D-surveys. For any individual $n \in \mathcal{N}$, denote by $\underline{q}_n^\alpha$ and $\overline{q}_n^\alpha$, with $\alpha \in \{\text{D,P,S}\}$, the minimum and maximum questions (according to μ_q or $\succ$) among: (i) the endorsed ones when $\alpha = \text{D}$, (ii) those with the highest strength of endorsement when $\alpha = \text{P}$, and (iii) those with the highest probability of endorsement when $\alpha = \text{S}$. Then, denote by $\alpha(n)$ the open interval of $\succ$ defined by the question immediately below $\underline{q}_n^\alpha$ and the one immediately above $\overline{q}_n^\alpha$ in $\succ$.[24] Importantly, in S-surveys whenever the CDF over the threshold is atomless, these two questions must either coincide or be consecutive, implying that the identification is almost complete.[25]

[23]Note that an individual with strictly increasing (decreasing) utility, and hence with no peak on the reals, is indistinguishable in ordinal terms with respect to an individual with a peak above the maximum (below the minimum) of all questions according to $\succ$.

[24]If one of these two questions does not exist, the position of the ideal view can be seen as unbounded in that specific direction.

[25]This follows immediately from the fact that, given strict quasi-concavity and the atomless of the CDF, only the closest question at each side of the ideal point may come with the highest endorsement probability (up to copies of these questions).

Proposition 3. *Under the Richness assumption, the ideal view of individual $n \in \mathcal{N}$ belongs to $S(n) \subseteq P(n) \subseteq D(n)$, and no further identification is possible.*

Proof of Proposition 3: Let the survey be rationalizable and $\succ$ be the unique linear order, up to inversion, guaranteeing so. Consider $n \in \mathcal{N}$ and an α-survey. First, we claim that the ideal view must belong to the interval $\alpha(n)$. Suppose by way of contradiction that this is not the case. We analyze the case in which the ideal view occupies a position equal or above the question immediately above $\overline{q}_n^\alpha$, and omit the other, dual case. In this case, notice that the question $\overline{q}_n^\alpha$ achieves a higher utility than the question above, contradicting the quasi-concavity of U_n. Second, we claim that $S(n) \subseteq P(n) \subseteq D(n)$. To see this, consider first $q \in S(n)$. By construction, q achieves the highest endorsement probability in $\mathcal{Q}$ and, given our basic assumptions, there exists at least one question q' with strictly less endorsement probability. Question q must belong to the equivalence class of questions with the highest label in the P-survey, and this level must be different than zero. It then follows that $S(n) \subseteq P(n)$. Consider now a question $q \in P(n)$. By construction, q achieves the highest label in $\mathcal{Q}$ and, given our basic assumptions, this label is strictly above zero. Question q must belong to the response set in the D-survey and $P(n) \subseteq D(n)$. Third, we claim that no further identification is possible. We analyze the case of D-surveys, with the other two being analogous. Consider the set Q_n. Modify the selection of the ideal point in the proof of Theorem 1 to occupy any position in $D(n)$. If this position is between $\overline{q}_n^\alpha$ and $\underline{q}_n^\alpha$, set the utility function to be piecewise linear, with value 1 for the ideal view and 0 for these two questions. Otherwise, it must be either above $\overline{q}_n^\alpha$ or below $\underline{q}_n^\alpha$. Consider the first case (the second being omitted as it is analogous). Then, set the utility function to be piecewise linear, with value 1 for the ideal view, 0 for $\underline{q}_n^\alpha$, and -1 for the question immediately above $\underline{q}_n^\alpha$ (if no question exists there, any strictly decreasing function works). This alternative representation also provides rationalizability and completes the proof. ■

5.3. **Exponential responses.** In our non-parametric models, the identification of questions and ideal points is ordinal. We now introduce a stochastic parametric model that illustrates how cardinal identification of the location of questions and ideal points, $\{\mu_q\}_{q\in\mathcal{Q}}$ and $\{\mu_n\}_{n\in\mathcal{N}}$ can be achieved. The model postulates that the individual endorsement probability is of the exponential form on the distance between the locations of the question and the ideal point of the individual, $S(n,q) = e^{-\frac{|\mu_n - \mu_q|}{\sigma_n}}$.[26] Intuitively, the probability of endorsing a question is 1 when its location coincides with that of the ideal point, decays with the distance between them, and is zero in the limit. It is modulated by a single parameter $\sigma_n > 0$ with a natural interpretation. Smaller values of σ_n accelerate the decay, representing individuals with a less tendency to endorse views away from their ideal point. Notice that the exponential endorsement probability could be alternatively presented via our stochastic framework by way of a strictly quasi-concave utility and a random threshold.[27]

We start by normalizing the location of any two questions, that we denote by 0_q and 1_q, to $\mu_{0_q} = 0$ and $\mu_{1_q} = 1$, respectively.[28]

Proposition 4. *Let $Q \geq 4$. Under the Richness assumption, in an S-survey with exponential responses, $\{\mu_q\}_{q \in \mathcal{Q} \setminus \{0_q, 1_q\}}$ and $\{\mu_n\}_{n \in \mathcal{N}}$ are cardinally identified.*

Proof of Proposition 4: Consider an S-survey with exponential responses. By Proposition 2, we know that $\succ$ is fully identified, and we set it to be $1_q \succ 0_q$. Given the atomless nature of the exponential model, for every $n \in \mathcal{N}$, the interval $S(n)$ contains either one or two questions. As in the proof of Proposition 3, denote these two questions by $\overline{q}_n^S$ and $\underline{q}_n^S$, and it must be either $\overline{q}_n^S \succ \underline{q}_n^S$ or $\overline{q}_n^S = \underline{q}_n^S \equiv q_n^S$. We now construct a map $h_n : \mathcal{Q} \to \{0, 1\}$ for each individual such that $h_n(q) = 1$ if and only if $\mu_q \geq \mu_n$ and $h_n(q) = 0$ otherwise. To do so, we consider two cases. If $|S(n)| = 2$, the exponential model of responses guarantees that μ_n must occupy a position strictly between these two questions, and we can assign $h_n(q) = 1$ to $\overline{q}_n^S$ and to every question above this one, according to $\succ$, and zero otherwise. If $|S(n)| = 1$, the map can be defined by $h_n(q) = 1$ whenever $q \succ q_n^S$ and $h_n(q) = 0$ whenever $q_n^S \succ q$. However, the relative location of q_n^S is yet undetermined. To do so, we consider another individual n' for which $h_{n'}(q_n^S)$ is known, and two questions q', q'' for which $h_n(q'), h_n(q''), h_{n'}(q'), h_{n'}(q'')$ are all known. It is immediate to see that the exponential model requires that

$$\frac{(-1)^{h_n(q_n^S)} \log S(n, q_n^S) - (-1)^{h_n(q'')} \log S(n, q'')}{(-1)^{h_n(q')} \log S(n, q') - (-1)^{h_n(q'')} \log S(n, q'')} = \frac{\mu_{q_n^S} - \mu_{q''}}{\mu_{q'} - \mu_{q''}} =$$

[26]The literature in psychology offers various probabilistic models of responses based on the notion of endorsement-by-proximity. See, e.g., Davison (1977). The present model is a convenient version of Andrich (1988) and Hoijtink (1990).

[27]It suffices to set $U_n(x) = e^{-\frac{|\mu_n - x|}{\sigma_n}}$ and F_n the CDF of the uniform distribution in $[0, 1]$.

[28]Recall that the inverse of a linear order is equivalent to the linear order for our purposes, so the order of these two alternatives is without loss of generality.

$$= \frac{(-1)^{h_{n'}(q_n^S)} \log S(n', q_n^S) - (-1)^{h_{n'}(q'')} \log S(n', q'')}{(-1)^{h_{n'}(q')} \log S(n', q') - (-1)^{h_{n'}(q'')} \log S(n', q'')}.$$

Hence, $h_n(q_n^S)$ can be determined.[29] Given the complete map h_n, notice now that for every q other than 0_q and 1_q it is

$$\frac{(-1)^{h_n(q)} \log S(n, q) - (-1)^{h_n(0_q)} \log S(n, 0_q)}{(-1)^{h_n(1_q)} \log S(n, 1_q) - (-1)^{h_n(0_q)} \log S(n, 0_q)} = \mu_q,$$

$$\frac{-(-1)^{h_n(0_q)} \log S(n, 0_q)}{(-1)^{h_n(1_q)} \log S(n, 1_q) - (-1)^{h_n(0_q)} \log S(n, 0_q)} = \mu_n.$$

All locations are fully determined, concluding the proof. ■

6. ATTITUDE SURVEYS: MULTIDIMENSIONALITY

We now allow for the possibility that questions and individual ideal points belong to a multidimensional space. Given the result below, it suffices to consider the bidimensional case in which $U_n : \mathbb{R}^2 \to \mathbb{R}$ and $\mu_q \in \mathbb{R}^2$. The following result shows that the multidimensional case has no empirical power.

Proposition 5. *In the bidimensional case, every $\{Q_n\}_{n\in\mathcal{N}}$ is rationalizable.*

Proof of Proposition 5: Consider the collection of D-survey responses $\{Q_n\}_{n\in\mathcal{N}}$. We construct a rationalization of these responses. Locate all questions in different positions of the circumference of radius one centered at the origin, in any way that is desired. Consider Q_n. If $|Q_n| = 0$, rationalization is trivial by considering Euclidean preferences centered in the origin, with utility 0 at the origin and $\tau_n = 0$. If $|Q_n| = 1$, rationalization is trivial, by considering Euclidean preferences centered in the only question endorsed, with utility 0 at that point, and $\tau_n = 0$. If $|Q_n| \geq 3$, consider the convex hull formed by (the location) of all questions that belong to Q_n; we denote this set by $CH(Q_n)$. Given that all questions in Q are in different points of the unit circumference, it is evident that $q \in CH(Q_n) \cap Q$ if and only if $q \in Q_n$; every question in $Q \setminus Q_n$ lies outside this convex hull. We construct the utility function U_n as follows. Select any point in the interior of $CH(Q_n)$ and denote it by μ_n. Then,

- $U_n(\mu_n) = 0$.
- For every y in the frontier of $CH(Q_n)$, set $U_n(y_1, y_2) = -1$.

[29] Notice that in the extreme case in which $S(n, q_n^S) = 1$, it must be $\mu_n = \mu_{q_n^S}$ and $h_n(q_n^S)$ can be freely assigned.

- For every x outside $CH(Q_n)$, let y be the unique point in the frontier of $CH(Q_n)$ such that $y = \lambda x + (1 - \lambda)\mu_n$. We can then set $U_n(x) = \frac{-1}{\lambda}$.
- For every x in the interior of $CH(Q_n)$, let y be the unique point in the frontier of $CH(Q_n)$ such that $x = \lambda y + (1 - \lambda)\mu_n$. We can then set $U_n(x) = -\lambda$.

By construction, all indifference curves have the same shape (expansions or contractions of the polygon created by the set of positive responses of the individual), with utility normalized to 0 and -1 for the ideal point and the polygon indifference curve. This is obviously a strictly quasi-concave utility function. By setting $\tau_n = -1$, this rationalizes the responses of the individual.

If $|Q_n| = 2$, select a third point in the circumference that does not correspond to the location of any question, and consider the triangle formed by these three points. Then, proceed to construct the utility function and the threshold as per the above reasoning. This again rationalizes the set of responses. □

The intuition behind the result is that, unlike the unidimensional case where only two extreme questions can exist, two-dimensional representations permit an infinite number of extreme questions. Consequently, this imposes no constraints on the responses. Furthermore, the same result holds even if the rationalization concept is strengthened to require strictly convex contour sets.[30] Overall, any non-parametric analysis involving more than one dimension becomes trivial.

This result is unsurprising given the existing literature on convex representations of preference profiles. Bogomolnaia and Laslier (2007) demonstrate in their Theorem 16 that every preference profile admits a convex representation in two dimensions. In the current survey setting, we work with a single upper contour set per individual rather than the entire preference profile. As a result, our Proposition 5 can be viewed as a corollary to Theorem 16 of Bogomolnaia and Laslier (2007).[31]

[30]Although the indifference curves used in our construction are polygons formed by a finite number of points on the unit circumference, we can create strictly convex supersets of these polygons that intersect the unit circumference at exactly the same points.

[31]Notice though that the critical unidimensional cases are fundamentally different. Unlike other settings where information on the preference profile is assumed, such as Kalandrakis (2010) or Ballester and Haeringer (2011), the study of survey responses involves only very coarse information about individuals—specifically, a single upper contour set per individual.

Naturally, one might consider multi-dimensional analysis under versions of the model where utility functions are parametrically constrained. In such cases, not every collection of responses can be rationalized, with, e.g., Euclidean preferences. For example, consider a scenario with 4 individuals and 2^4 alternatives, where each alternative is endorsed by a unique subset of individuals. For these responses to be rationalizable with Euclidean preferences, there would need to exist four circles in the plane forming a Venn diagram. However, this is impossible due to Euler's relation, which governs the number of faces, edges, and vertices in a planar graph (see, e.g., Ruskey, Savage, and Wagon (2006)). This observation highlights an interesting direction for future research: exploring parametric versions of our otherwise nonparametric framework.

7. Aptitude surveys

We now briefly study the rationalizability of surveys measuring aptitudes, rather than attitudes. These are surveys with the aim of studying the absolute, cumulative, extent of a given variable, as it is often the case in the study of ability (say, the mathematical skills of pupils), health status (say, the extent of mobility for the aged), social functionality (say, the social skills of psychiatric patients), or the extent of a practice (say, the level of religiosity of citizens). Here we focus on the dichotomous and deterministic case, which is usually known as Guttman-scale and, accordingly, we refer to it as Guttman-rationalizability.[32]

Guttman-rationalizability. We say that $\{Q_n\}_{n\in\mathcal{N}}$ is Guttman-rationalizable whenever there exist $[\{\mu_n\}_{n\in\mathcal{N}}, \{\mu_q\}_{q\in\mathcal{Q}}]$ with $\mu_n, \mu_q \in \mathbb{R}$, such that for every $n \in \mathcal{N}$ and $q \in \mathcal{Q}$, it is $q \in Q_n$ if and only if $\mu_n \geq \mu_q$.

In Guttman-rationalizability, a question (or problem, or task) q receives a positive response whenever its location falls below the location of the individual, $\mu_q \leq \mu_n$. Notice that Guttman-rationalizability is a particular case of D-survey rationalizability since we could alternatively rationalize responses by constructing a strictly quasi-concave utility function U_n and a threshold τ_n such that the individual endorses every question located below the ideal point, and only these.[33] Hence, the properties in Theorem 1

[32] The extensions to non-dichotomous and probabilistic responses follow analogously to the cases studied above, and hence for the sake of space, we avoid them here.

[33] For instance, consider any utility function such that $\lim_{x\to-\infty} U_n(x) = 1$, increasing up to $U_n(\mu_n) > 1$, and decreasing afterwards with $U_n(\mu_n + \epsilon) < 1$ for any sufficiently small $\epsilon > 0$ such

are necessary for Guttman-rationalizability, but not sufficient. We now provide a necessary and sufficient condition. The cumulative nature of Guttman-rationalizability facilitates the survey revelation exercise; it allows us to analyze the responses by way of the transitive closure of a binary relation, instead of the ternary relation we adopted above.

For concreteness, from now on we simply use the language of an aptitude test where questions differ on complexity. Consider a pair of questions $q_1, q_2 \in \mathcal{Q}$ and an individual $n \in \mathcal{N}$ such that $Q_n \cap \{q_1, q_2\} = q_2$. It is evident that the aptitude of individual n separates the complexity of both questions, where the complexity of q_1 must be above that of q_2. We then say that q_1 has been revealed to be more complex than question q_2, and write $q_1 \succ_{mc} q_2$. We then use the standard transitive closure of $\succ_{mc}$, that we denote by $\widehat{\succ}_{mc}$, to define the following property.

Strong Axiom of Revealed Complexity (SARC). We say that $\{Q_n\}_{n\in\mathcal{N}}$ satisfies SARC if, for every pair $\{q_1, q_2\}$ of questions, $q_1 \widehat{\succ}_{mc} q_2$ implies that $q_2 \succ_{mc} q_1$ cannot hold.

Theorem 4. *$\{Q_n\}_{n\in\mathcal{N}}$ is Guttman-rationalizable if and only $\{Q_n\}_{n\in\mathcal{N}}$ satisfies SARC.*

Proof of Theorem 4: Suppose first that $\{Q_n\}_{n\in\mathcal{N}}$ is Guttman-rationalizable. Consider two questions q, q' such that $q \widehat{\succ}_{mc} q'$. There must exist a sequence of questions such that $q_t \succ_{mc} q_{t-1}$ for every $t = 1, \ldots, T$, with $q_0 = q$ and $q_T = q'$. The latter implies that there must be a sequence of individuals n_t, $t = 1, \ldots, T$, such that $\mu_{q_T} > \mu_{n_T} \geq \mu_{q_{T-1}} > \mu_{n_{T-1}} \geq \mu_{q_{T-2}} > \cdots \geq \mu_{q_1} > \mu_{n_1} \geq \mu_{q_0}$ and hence, $\mu_{q_T} > \mu_{q_0}$. Then, it is evident that $q_0 \succ_{mc} q_T$ cannot hold, and SARC follows.

Suppose now that SARC holds, i.e., the transitive closure of $\succ_{mc}$ is asymmetric. Given the finiteness of $\mathcal{Q}$, we can find values $\{\mu_q\}_{q\in\mathcal{Q}}$ such that $q \succ_{mc} q'$ implies $\mu_q > \mu_{q'}$. If an individual responds negatively to all questions set $\mu_n < \min_{q\in\mathcal{Q}} \mu_q$; otherwise, set $\mu_n = \max_{q\in Q_n} \mu_q$. We claim that this constitutes a valid Guttman-rationalization. Suppose, by way of contradiction, that this is not the case. Notice that the construction guarantees that every question such that $\mu_q > \mu_n$ is negatively responded. The contradiction must involve an individual $n \in \mathcal{N}$ and a question $q \in \mathcal{Q}$ such that $\mu_n \geq \mu_q$ but $q \notin Q_n$. However, our construction guarantees that n responds positively to at least one question, and the definition of μ_n guarantees that there is a

that there is no question in the interval $(\mu_n, \mu_n + \epsilon)$, and $\tau_n = 1$. This is strictly quasi-concave and leads to positive answers until the ideal point of the individual, and negative ones afterwards.

question q' with $\mu_{q'} > \mu_q$ and $q' \in Q_n$. This would make n to reveal $q \succ_{mc} q'$, which leads to $\mu_q > \mu_{q'}$, a contradiction, concludig the proof. ■

8. Discussion

This paper represents a first attempt in the study of the rationality foundations of survey responses, setting the basis for the theoretical and empirical treatment of other important issues. For example, another direction of interest represents the combination of surveys with more standard economic data to enhance the information revelation process, in line with the recent suggestions of Caplin (2021), Gerasimou (2021) and Almås, Attanasio, and Jervis (2023).

Also, our framework remains valid in the presence of non-truthful responses and, indeed, it provides grounds to empirically study the nature of the potentially existing deviations. Notice first that a respondent trying to appear, say, more moderate would merely present a set of responses with the form of an interval, but shifted towards the moderate part of the scale. Hence, our techniques can still be used in the presence of non-truthful responses, providing correct information on the order of questions, and on the self-presentation of the individuals (that could deviate from her internal views due to conformism, shame, etc). As it is common in the survey literature, we could then compare survey responses run under different conditions (e.g., different degrees of confidentiality or anonymity, different informational structures on the social or specific support that questions may have, or different incentives, etc), or use implicit information (such as response times or attention) to identify better individuals' actual views. The ordered nature of questions obtained in the analysis would facilitate this. Moreover, we can shed light on which variables of the model, e.g., the threshold level or the individual utilities, are affected and how. Alternatively, another interesting approach would entail comparing survey responses with relevant choice data (e.g., risky activities versus lottery choices).

References

[1] Almås, I., Attanasio, O., and Jervis, P. (2023). Economics and Measurement: New measures to model decision making (No. w30839). National Bureau of Economic Research.

[2] Alós-Ferrer, C. and Buckenmaier, J. (2019). Strongly sincere best responses under approval voting and arbitrary preferences. *Games and Economic Behavior*, 117, 388-401.

[3] Andrich, D. (1988). The application of an unfolding model of the PIRT type to the measurement of attitude. *Applied Psychological Measurement*, 12(1), 33-51.

[4] Bade, S. (2019). Matching with single-peaked preferences. *Journal of Economic Theory*, 180, 81-99.

[5] Ballester, M. A., and Haeringer, G. (2011). A characterization of the single-peaked domain. *Social Choice and Welfare*, 36(2), 305-322.

[6] Barberá, P. (2014). Birds of the same feather tweet together: Bayesian ideal point estimation using Twitter data. *Political Analysis*, 23(1), 76-91.

[7] Benjamin, D. J., Guzman, J. D., Fleurbaey, M., Heffetz, O., and Kimball, M. (2023). What do happiness data mean? Theory and survey evidence. *Journal of the European Economic Association*, 21(6), 2377-2412.

[8] Bertrand, M. and Mullainathan, S. (2001). Do people mean what they say? Implications for subjective survey data. *American Economic Review*, 91(2), 67-72.

[9] Bogomolnaia, A., and Laslier, J. F. (2007). Euclidean preferences. *Journal of Mathematical Economics*, 43(2), 87-98.

[10] Bond, T. N., and Lang, K. (2019). The sad truth about happiness scales. *Journal of Political Economy*, 127(4), 1629-1640.

[11] Brams, S. J., and Fishburn, P. (1978). Approval Voting. *American Political Science Review*, 72(3), 831-847.

[12] Caplin, A. (2021). Economic Data Engineering (No. w29378). National Bureau of Economic Research.

[13] Coombs, C. H. (1964). *A theory of data*. Wiley.

[14] Coombs, C. H. and K. Smith (1973). On the Detection of Structure in Attitudes and Developmental Processes. *Psychological Review*, 80(5), 337-351.

[15] Davison, M. L. (1977). On a metric, unidimensional unfolding model for attitudinal and developmental data. *Psychometrika*, 42(4), 523-548.

[16] Dietrich, F., and List, C. (2010). Majority voting on restricted domains. *Journal of Economic Theory*, 145(2), 512-543.

[17] Falk, A., Neuber, T., and Strack, P. (2021). Limited self-knowledge and survey response behavior. Mimeo.

[18] Fishburn, P. C. (1971). Betweenness, orders and interval graphs. *Journal of Pure and Applied Algebra*, 1(2), 159-178.

[19] Fromme, K., Katz, E. C., and Rivet, K. (1997). Outcome expectancies and risk-taking behavior. *Cognitive Therapy and Research*, 21(4), 421-442.

[20] Gerasimou, G. (2021). Simple preference intensity comparisons. *Journal of Economic Theory*, 192, 105199.

[21] Grable, J., and Lytton, R. H. (1999). Financial risk tolerance revisited: the development of a risk assessment instrument. *Financial Services Review*, 8(3), 163-181.

[22] Guttman, L. (1944). A basis for scaling qualitative data. *American Sociological Review*, 9(2), 139-150.

[23] Hoijtink, H. (1990). A latent trait model for dichotomous choice data. *Psychometrika*, 55, 641-656.

[24] Huntington, E. V. and Kline, J. R. (1917). Sets of independent postulates for betweenness. *Transactions of the American Mathematical Society*, 18(3), 301-325.

[25] Hubert, L. (1974). Problems of seriation using a subject by item response matrix. *Psychological Bulletin*, 81(12), 976.

[26] Kalandrakis, T. (2010). Rationalizable voting. *Theoretical Economics*, 5(1), 93-125.

[27] Kendall, D. G. (1969). Some problems and methods in statistical archaeology. *World Archaeology*, 1(1), 68-76.

[28] Lahey, J. N., and Oxley, D. (2016). The power of eye tracking in economics experiments. *American Economic Review P&P*, 106(5), 309-313.

[29] Laslier, J. F. (2009). The leader rule: A model of strategic approval voting in a large electorate. *Journal of Theoretical Politics*, 21(1), 113-136.

[30] Laver, M., Benoit, K., and Garry, J. (2003). Extracting policy positions from political texts using words as data. *American Political Science Review*, 97(2), 311-331.

[31] Lekkeikerker, C., and Boland, J. (1962). Representation of a finite graph by a set of intervals on the real line. *Fundamenta Mathematicae*, 51(1), 45-64.

[32] Le Mens, G., and Gallego, A. (2024). Scaling political texts with large language models: Asking a chatbot might be all you need. Mimeo.

[33] Liu, S., and Netzer, N. (2023). Happy times: measuring happiness using response times. *American Economic Review*, 113(12), 3289-3322.

[34] Liiv, I. (2010). Seriation and matrix reordering methods: An historical overview. *Statistical Analysis and Data Mining: The ASA Data Science Journal*, 3(2), 70-91.

[35] Manski, C. F. (2004). Measuring expectations. *Econometrica*, 72(5), 1329-1376.

[36] Manzini, P., Mariotti, M., and Ülkü, L. (2024). A model of approval with an application to list design. *Journal of Economic Theory*, 217, 105821.

[37] Núñez, M. and Xefteris, D. (2017). Implementation via approval mechanisms. *Journal of Economic Theory*, 170, 169-181.

[38] Pfaffmann, C. (1960). The pleasures of sensation. *Psychological Review*, 67(4), 253.

[39] Polonio, L., Di Guida, S. and Coricelli, G. (2015). Strategic sophistication and attention in games: An eye-tracking study. *Games and Economic Behavior*, 94, 80-96.

[40] Prelec, D. (2004). A Bayesian truth serum for subjective data. *Science*, 306(5695), 462-466.

[41] Ruskey, F., Savage, C. D., and Wagon, S. (2006). The search for simple symmetric Venn diagrams. *Notices of the AMS*, 53(11), 1304-1311.

[42] Stantcheva, S. (2022). How to run surveys: A guide to creating your own identifying variation and revealing the invisible. *Annual Review of Economics*, 15.

[43] Tay, L. and Ng, V. (2018). Ideal point modeling of non-cognitive constructs: Review and recommendations for research. *Frontiers in Psychology*, 2423.

[44] Thomas, H. (1973). Unfolding the baby's mind: The infant's selection of visual stimuli. *Psychological Review*, 80(6), 468-488.

[45] Thurstone, L. L. (1928). Attitudes can be measured. *American Journal of Sociology*, 33(4), 529-554.

[46] Tucker, A. (1972). A structure theorem for the consecutive 1's property. *Journal of Combinatorial Theory*, Series B, 12(2), 153-162.

[47] Van der Linden, W. J. (2018). *Handbook of item response theory: Three volume set*. CRC Press.