Macroeconomics
TOPIC 10: The Long Run

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Long run: we will zoom out.

We won't focus on cycles and fluctuations as we did in the previous topics. We will look at the trend: economic growth in the long-run.

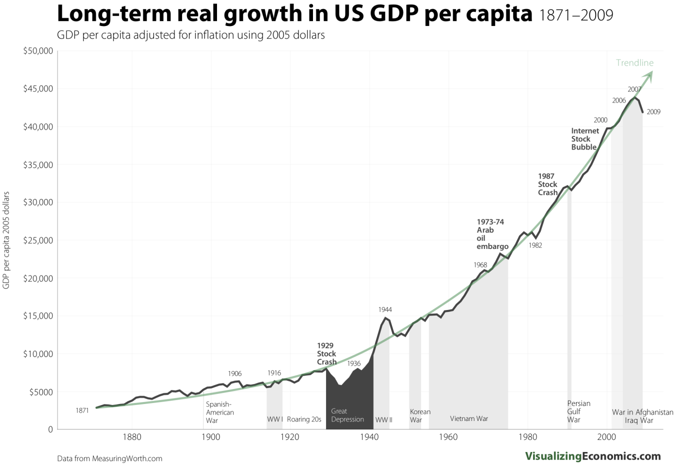


Figure: Long term real growth in US GDP per capita

We will make different assumptions compared to the previous topics:

- capital is no longer fixed
- the number of firms is no longer fixed
- population is no longer fixed

Road map:

- 1. Introduction
- 2. Aggregate production function
- 3. The Solow model

Road map:

- **1. Introduction**
- 2. Aggregate production function
- 3. The Solow model

1. Introduction

Growth entails the expansion of an economy's productive capacity. In this Topic we will discuss the basic concept of growth and how additions to capital, population, savings, and technology impact growth.

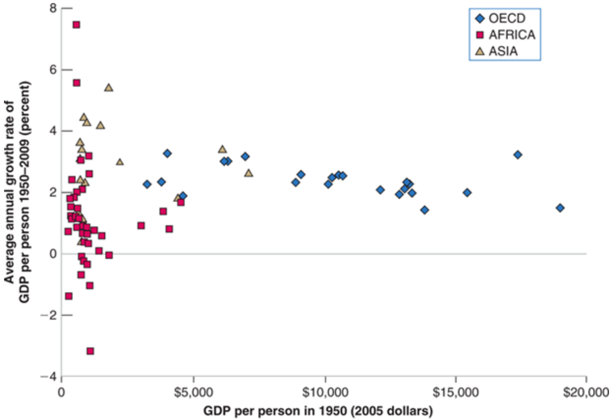
Generally, growth is measured by the growth rate of real GDP per capita.
[Remember: Why does growth matter? Topic 1: growth entails the expansion of an economy's productive capacity but also of its output, its income, and the living standards of its citizens.]

1. Introduction

This Topic will help us:

- understanding the determinants of GDP in the long-run (and therefore why some countries are rich and others poor)
- understanding why poor countries remain poor: no convergence (see next figure)
- design policies that help poor countries to grow
- learn how the growth rate is affected by shocks

1. Introduction



Source: See Table 10-1.

Growth rate of GDP per person since 1950 versus GDP per person in 1950. Did poor countries grow faster? No.

Road map:

- 1. Introduction
- **2. Aggregate production function**
- 3. The Solow model

2. Aggregate production function
   - 2.1. Introduction
   - 2.2. Returns
     - 2.2.1. Returns to scale
     - 2.2.2. Marginal returns
   - 2.3. Output per capita

2.1. Aggregate production function - Introduction

Because we are studying the long run evolution of output, we need to analyze the determinants of output:

- the **factors of production**
  - labor
  - capital
- the **state of technology**

For the moment, we will be agnostic on the state of technology. Later on, we will explicit and explain it.

2.1. Aggregate production function - Introduction

Aggregate production function: $Y = F(K, N)$

where K is capital (machines, plants) and N is labor (number of workers).

- Relation between output and inputs
- Level of output for given levels of inputs

2.2. Returns

2.2.1. Returns to scale

What happens when we double the amount of capital and the number of workers?

The new level of output might be:

- twice the initial level: constant returns to scale
- less than twice the initial level: decreasing returns to scale
- more than twice the initial level: increasing returns to scale

2.2.1. Returns to scale

More generally:

- $F(xK, xN) = xY$: constant returns to scale
- $F(xK, xN) < xY$: decreasing returns to scale
- $F(xK, xN) > xY$: increasing returns to scale

2.2.2. Marginal returns

What happens when we increase the amount of capital, keeping the number of workers constant?

- Additions of capital yield constant increases in output: constant (marginal) returns to capital
- Additions of capital yield progressively smaller, or diminishing, increases in output: decreasing or diminishing (marginal) returns to capital
- Additions of capital yield progressively larger increases in output: increasing (marginal) returns to capital

2.2.2. Marginal returns

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2.2.2. Marginal returns

What happens when we increase the number of workers, keeping the amount of capital constant?

- Additions of workers yield constant increases in output: constant (marginal) returns to labor
- Additions of workers yield progressively smaller, or diminishing, increases in output: decreasing or diminishing (marginal) returns to labor
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2.2.2. Marginal returns

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2.3. Output per worker

2.3. Output per worker

Let suppose that we have constant returns to scale:

$$xY = F(xK, xN)$$

If we set $x = 1/N$, we get:

$$\frac{Y}{N} = F(\frac{K}{N}, \frac{N}{N}) = F(\frac{K}{N}, 1)$$

where Y/N and K/N are output per worker and capital per worker.

If both K and N double, capital per worker does not change. Then the right hand side of the equation does not change.
We can check that the left hand side does not change neither: because of CRS, Y doubles as well, so that output per worker does not change.

Moreover, the number of workers does not affect the relation between output per worker and capital per worker.

2.3. Output per worker

$$\frac{Y}{N} = F(\frac{K}{N}, 1)$$

Keeping N constant, an increase in capital increases output.
Therefore, an increase in capital per worker leads to an increase of output per worker.

Decreasing marginal returns to capital:
As capital increases, the associated increase in output is smaller and smaller.

2.3. Output per worker

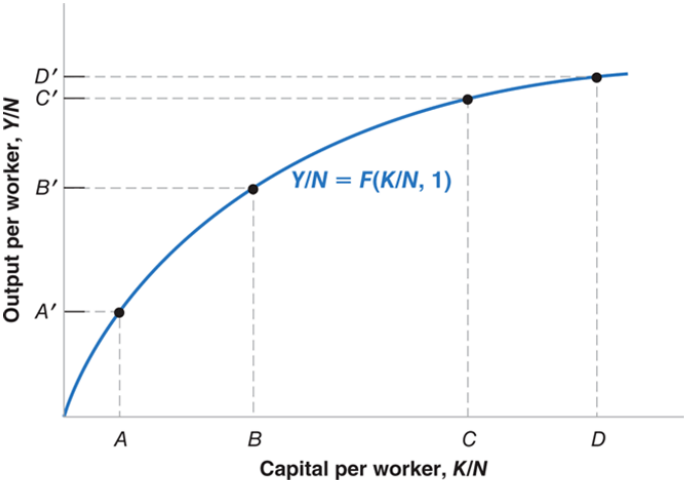


Figure: Output per worker as a function of capital per worker.

The slope of the production function shows how much extra output a worker produces when given an extra unit of capital.

This amount is called the **marginal product of capital**.

Decreasing marginal returns to capital:

- When capital per worker increases from A to B, output per worker increases from A' to B': large increase.
- When capital per worker increases from C to D, output per worker increases from C' to D': small increase.

$\Rightarrow$ the production function is increasing with a decreasing slope: the production function is **concave**. The production function exhibits diminishing marginal product of capital.

2.3. Output per worker

When capital per worker is low, the average worker has only a little capital to work with, so an extra unit of capital is very useful and produces a lot of additional output.

When capital per worker is high, the average worker has a lot of capital, so an extra unit increases production only slightly.

2.3. Output per worker

$$\frac{Y}{N} = F(\frac{K}{N}, 1)$$

When there is a change in the amount of capital per worker, we move along the production function.

When there is a change in the production function, keeping constant K/N, the production function shifts:

- When the state of technology increases, output increases, for a given value of capital per worker. The production function shifts upwards.

2.3. Output per worker

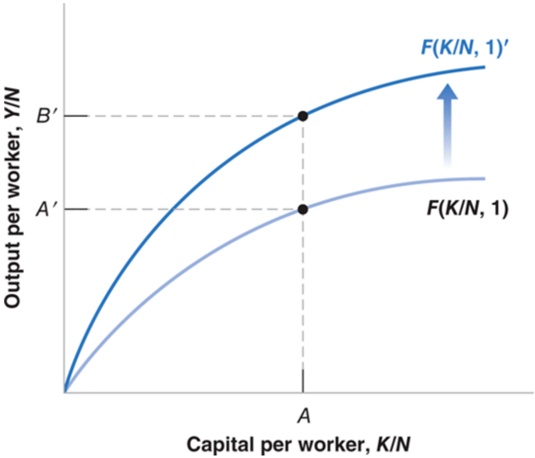


Figure: Improvement in the state of technology.

Road map:

- 1. Introduction
- 2. Aggregate production function
- **3. The Solow model**

3. The Solow model

- 3.1. Output and capital
- 3.2. Savings
- 3.3. Population growth
- 3.4. Technological progress

3.1. The Solow model - Output and Capital

How does capital determine output?

Aggregate production function: $\frac{Y}{N} = F(\frac{K}{N}, 1)$

We often use lower-case letters to denote quantities per worker:

$$y = \frac{Y}{N}$$

$$k = \frac{K}{N}$$

$$y = \frac{Y}{N} = F(\frac{K}{N}, 1) = f(k)$$

3.1. The Solow model - Output and Capital

How does output determine capital?

3.1. The Solow model - Output and Capital

Starting point: Equilibrium on the goods market

Lets denote private savings by $S^{private}$ and public savings by S^{public}:

- $S^{private} = Y - T - C$: the part of disposable income that is not consumed
- $S^{public} = T - G$: the part of government revenues that is not spent

Lets rewrite the equilibrium equation in the goods market:
$Y = C + I + G$
$Y - T = C + I + G - T$
$I = S^{private} + S^{public}$

In a closed economy, the equilibrium in the goods market requires that investment equals total saving.

3.1. The Solow model - Output and Capital

For simplicity, 2 assumptions:

- no government purchases, no taxes
  - $S^{public} = 0$
  - $S^{private}$ is total savings S
- people save a fraction s of their income and consume a fraction $1 - s$ of their income

3.1. The Solow model - Output and Capital

Therefore: $I = S = sY$ or $i = sy = sf(k)$ (in per worker terms)

Because $I = S$, s is also the fraction of output devoted to investment.

$F(\frac{K}{N}, 1)$ determines how much output is produced, and the saving rate s determines the allocation of this output between consumption and investment.

3.1. The Solow model - Output and Capital

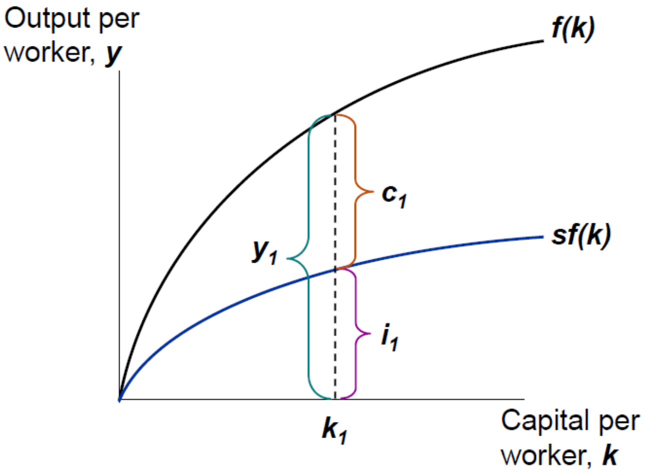


Figure: Output, consumption and investment.

What is the link between capital and investment?

3.1. The Solow model - Output and Capital

What is the link between capital and investment?

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ is the rate at which capital depreciates.

Next period capital is equal to the proportion of current capital that will be usable next period plus the current investment in capital.

3.1. The Solow model - Output and Capital

We can rewrite this law of motion of capital $K_{t+1} = (1-\delta)K_t + I_t$ as

$$K_{t+1} - K_t = I_t - \delta K_t$$

or

$$K_{t+1} - K_t = sY_t - \delta K_t$$

or

$$k_{t+1} = (1-\delta)k_t + i_t$$

or

$$k_{t+1} = (1-\delta)k_t + sy_t$$

or

$$\Delta k_{t+1} = k_{t+1} - k_t = sf(k_t) - \delta k_t$$

An increase in output leads to an increase in capital next period.

3.1. The Solow model - Output and Capital

$$\Delta k_{t+1} = k_{t+1} - k_t = sf(k_t) - \delta k_t$$

- When $sf(k_t) > \delta k_t$, investment exceeds the depreciation of capital, the change in capital per worker is positive, output per worker increases
- When $sf(k_t) < \delta k_t$, the depreciation of capital exceeds investment, the change in capital per worker is negative, output per worker decreases
- When $sf(k_t) = \delta k_t$, the depreciation of capital equals investment:
    - capital per worker does not change
    - from the production function, output per worker does not change
    - **steady state**

3.1. The Solow model - Output and Capital

Therefore, the steady state is such that:

$$sf(k_t) = \delta k_t$$

We will denote the steady state by a star:

$$sf(k^*) = \delta k^*$$

This equation gives us the steady state value of capital per worker.

Using the steady state value of capital per worker, we get the steady state value of output per worker:

$$y^* = f(k^*)$$

3.1. The Solow model - Output and Capital

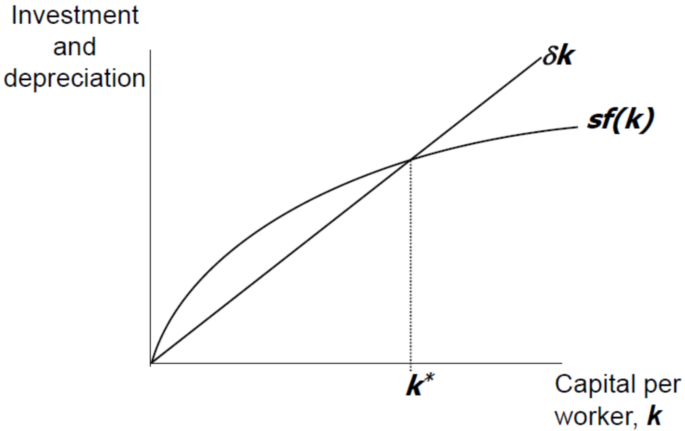
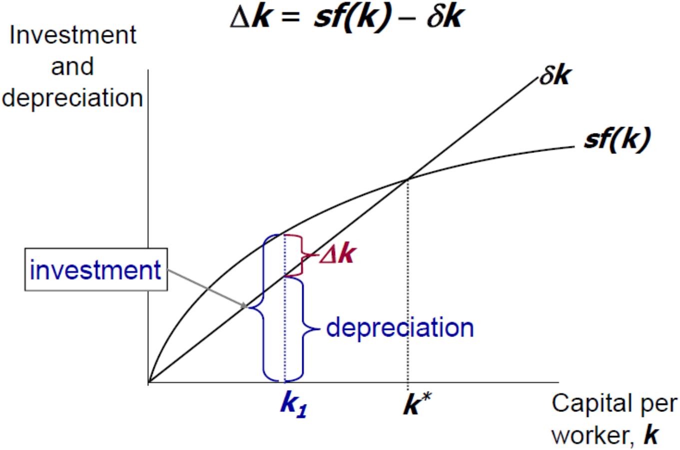


Figure: Output, consumption and investment.

3.1. The Solow model - Output and Capital



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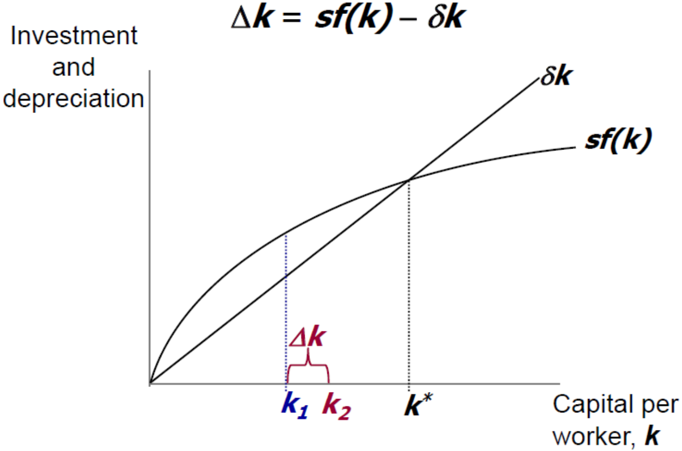

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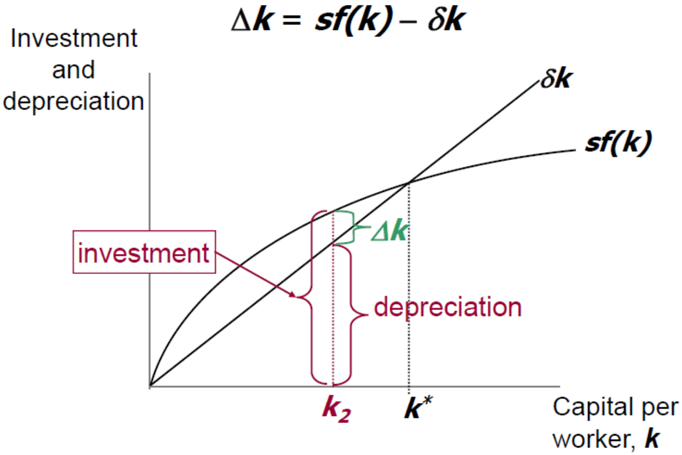

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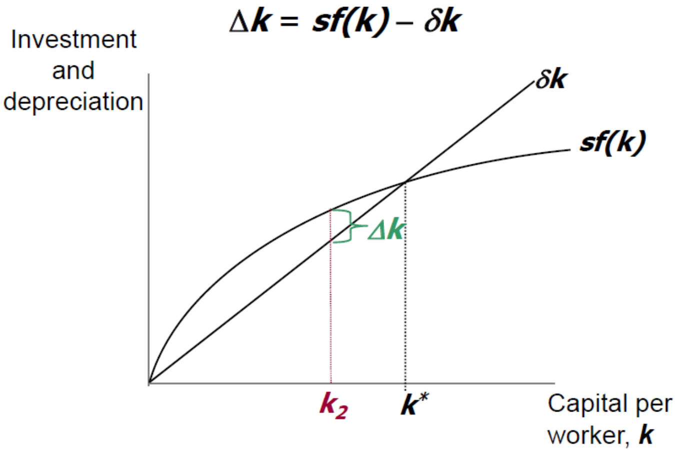

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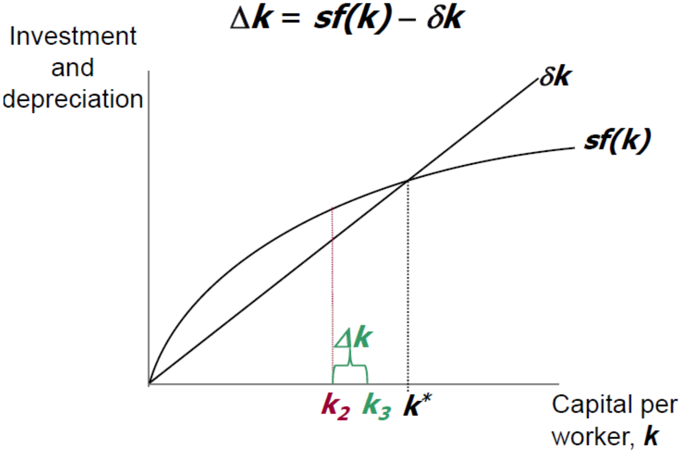

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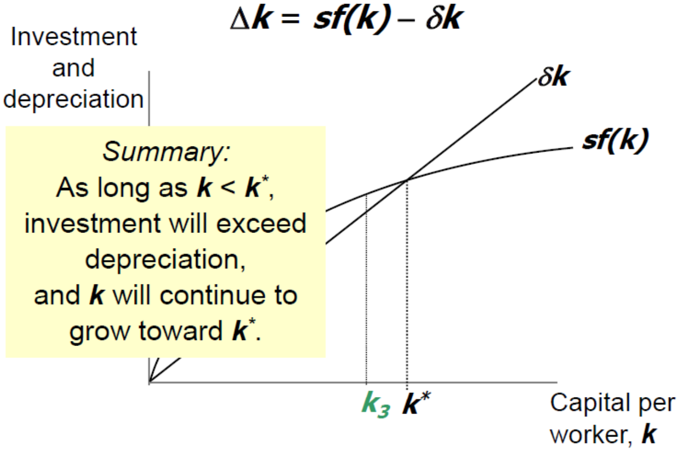


Figure: Output, consumption and investment.

To sum up:

When investment per worker exceeds depreciation per worker, capital per worker increases and so does output per worker. We move rightward along the production curve.

As capital increases, depreciation increases faster than investment (the depreciation curve is steeper than the investment curve).

Depreciation catches up with investment and we reach the steady state.

Inversely:

When depreciation per worker exceeds investment per worker, capital per worker decreases and so does output per worker. We move leftward along the production curve.

As capital decreases, depreciation decreases faster than investment (the depreciation curve is steeper than the investment curve).

Investment catches up with depreciation and we reach the steady state.

3.1. The Solow model - Output and Capital

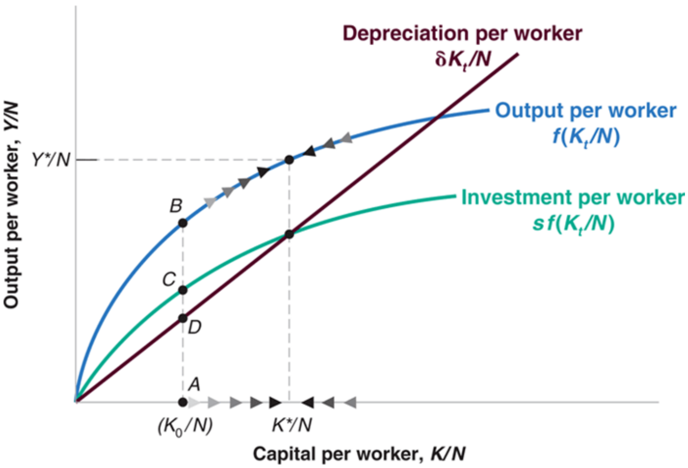


Figure: Output, consumption and investment.

The steady state is an absorbing state. Once the economy reaches its steady state level, it remains at this steady state level.

- *Why?* Because with a level of capital k^*, the economy produces y^* and invests sy^* which is equal to δk^*: the economy invests just enough to cover depreciation, therefore capital does not increase.

To sum up:

- When $k < k^*$, investment exceeds the depreciation of capital, capital per worker and output per worker increase, at a decreasing rate, until they reach their steady state levels
  - The growth rate is positive and decreases towards zero as the economy reaches its steady state.
- When $k > k^*$, the depreciation of capital exceeds investment, capital per worker and output per worker decrease, at a decreasing rate, until they reach their steady state levels
  - The growth rate is negative and decreases (in absolute value) towards zero as the economy reaches its steady state.
- The more distant the economy is from its steady state, the higher the growth rate (in absolute value).
- In the long run, the economy is at its steady state and the growth rate is equal to zero.

The production function is assumed to be equal to: $Y = K^{\frac{1}{2}} N^{\frac{1}{2}}$.

a. Does this function exhibits decreasing, increasing or constant returns to scale?

Suppose that $s = 0.3$ and $\delta = 0.1$.

b. What are the steady state values of k, y and c?

The current capital per worker is equal to 4.

c. What will be the level of capital per worker in $t + 1$?
d. Will capital increase or decrease over time? Will output increase or decrease over time?

3. The Solow model

- 3.1. Output and capital
- **3.2. Savings**
- 3.3. Population growth
- 3.4. Technological progress

3.2. The Solow model - Savings

How does the saving rate affect the growth rate of the economy as well as its steady state?

3.2. The Solow model - Savings

How does the saving rate affect the **growth rate** *of the economy?*

Remember:

- Investment per worker: $sf(k)$
- Depreciation per worker: δk

An increase in the savings rate leads to an increase in investment per worker.

An increase in the savings rate does not affect depreciation per worker.

$\Rightarrow$ The immediate effect of an increase in the savings rate is a **jump in the growth rate** of capital per worker, and therefore of output per worker.

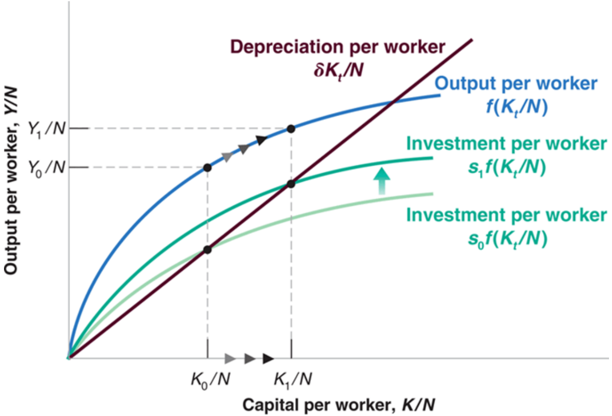
3.2. The Solow model - Savings

If the economy is at its steady state, an increase in the savings rate leads to a temporary positive growth rate.

If the economy is below its steady state, an increase in the savings rate leads to a temporary increase in the growth rate.

If the economy is above its steady state, an increase in the savings rate leads to a temporary decrease in the growth rate, in absolute value.

3.2. The Solow model - Savings



Increase in the savings rate, starting from a steady state level.

3.2. The Solow model - Savings

How does the saving rate affect the **steady state** *of an economy?*

3.2. The Solow model - Savings

Remember:

At the steady state, capital per worker (and therefore output per worker) is constant.

$$k_{t+1} - k_t = sf(k_t) - \delta k_t = 0$$

$$sf(k^*) = \delta k^*$$

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}$$

If $f(k) = k^{1/2}$, then:

$$k^{*1/2} = \frac{s}{\delta}$$

$$k^* = (\frac{s}{\delta})^2$$

k^* is a function of s. The higher s, the higher the steady state level of capital per worker.

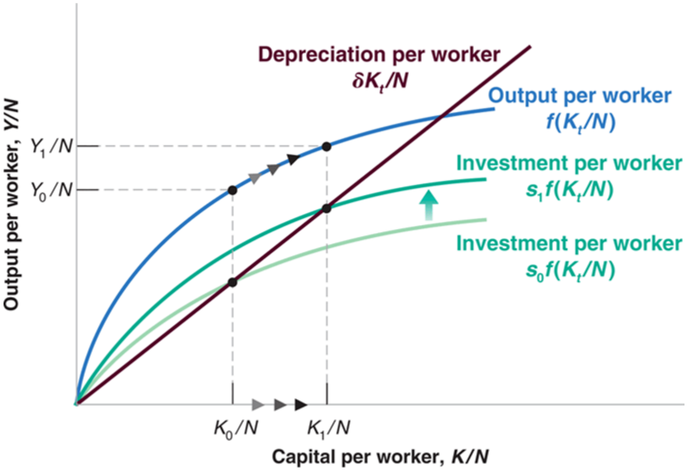
(The result can be generalized to all types of production functions.)

3.2. The Solow model - Savings

After an increase in s, the economy grows towards a higher steady state value.

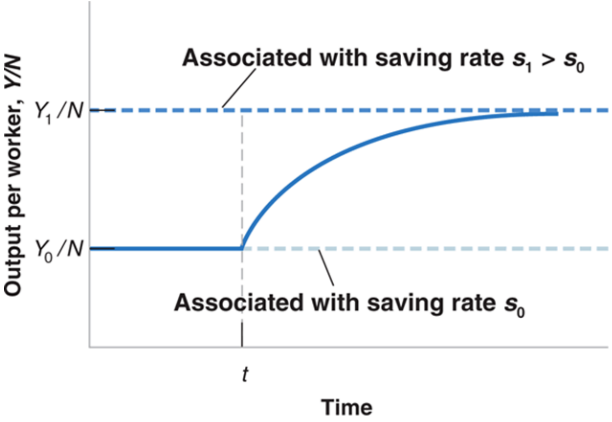
$\Rightarrow$ s affects the level of the steady state: A higher savings rate leads to a higher level of capital per worker, and therefore to a higher level of output per worker, in the long run.

But s does not affect the long run growth rate: in the long run, the growth rate is equal to zero, for any savings rate.



Increase in the savings rate, starting from a steady state level.

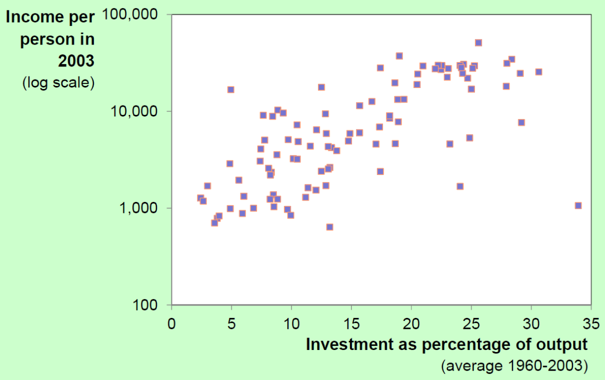
3.2. The Solow model - Savings



Effect of an increase in the savings rate on output per worker, starting from a steady state level.

The Solow model therefore predicts that countries that have higher savings and investment rates will have higher levels of capital and output per worker in the long run.

3.2. The Solow model - Savings



3.2. The Solow model - Savings

To sum up:

- Different levels of savings rates lead to different steady state levels of capital per worker.
- Different steady state levels of capital per worker lead to different steady state levels of output per worker.
- Different steady state levels of output per worker lead to different steady state levels of consumption per worker.

To sum up:

- Different levels of savings rates lead to different steady state levels of capital per worker.
- Different steady state levels of capital per worker lead to different steady state levels of output per worker.
- Different steady state levels of output per worker lead to different steady state levels of consumption per worker.

Is there a level of savings rate that maximizes the steady state level of consumption per worker?

3.2. The Solow model - Savings

Consumption: $c = (1 - s)y$

- if s is high:
  - the share of output which is consumed $(1 - s)$ is low: it pushes consumption down
  - investment is high, capital is high, output is high: it pushes consumption up
- if s is low:
  - the share of output which is consumed $(1 - s)$ is high: it pushes consumption up
  - investment is low, capital is low, output is low: it pushes consumption down

3.2. The Solow model - Savings

In order to understand which savings rate maximizes the steady state level of consumption per worker, let's rewrite the equilibrium condition on the goods market:

$$y = c + i$$

$$c = y - i$$

$$c^* = y^* - i^*$$

$$c^* = f(k^*) - sf(k^*)$$

$$c^* = f(k^*) - \delta k^*$$

Consumption is therefore what is left of steady state output after paying for steady state depreciation.

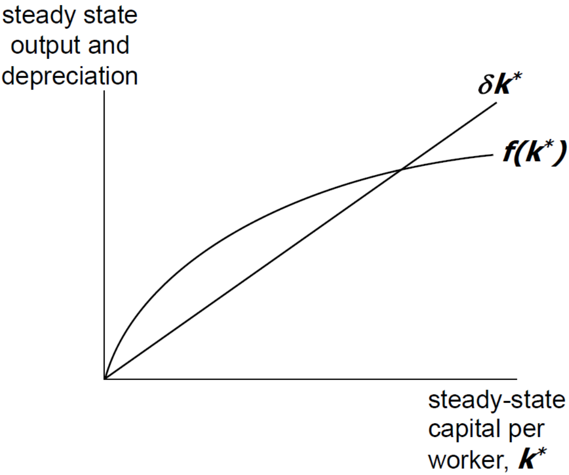
3.2. The Solow model - Savings

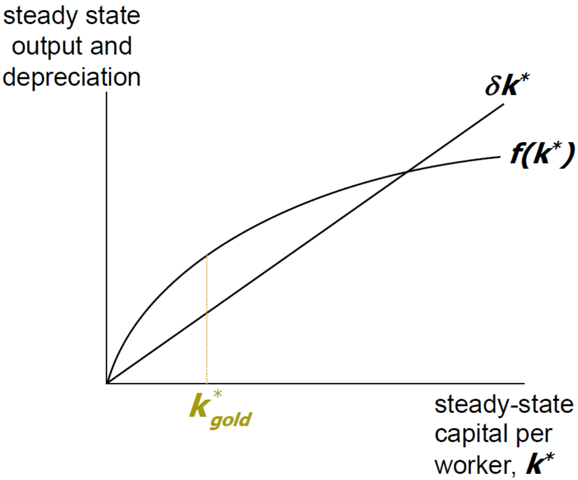
$$c^* = f(k^*) - \delta k^*$$

And we know that k^* is a function of s.
(Example, when $f(k) = k^{1/2}$, $k^* = (\frac{s}{\delta})^2$)

Therefore, consumption is maximized when the savings rate is such that the difference between steady state output and steady state depreciation is the largest.
At this point, the savings rate and steady state capital are equal to their golden rule levels.

3.2. The Solow model - Savings

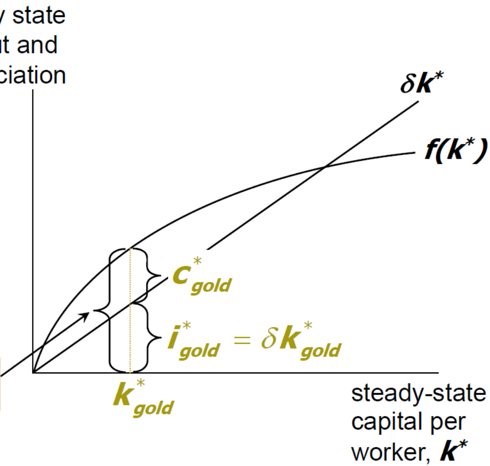



steady state output and depreciation
δk^*
$f(k^*)$
k^*_{gold}
steady-state capital per worker, k^*

3.2. The Solow model - Savings

Then, graph $f(k^*)$ and δk^*, look for the point where the gap between them is biggest.

$$y^*_{gold} = f(k^*_{gold})$$



When $s = s_{golden}$, $k^* = k^*_{golden}$ and consumption is maximized.

- $s = 0$: $i^* = 0 \rightarrow k^* = 0 \rightarrow y^* = 0 \rightarrow c^* = 0$
- $s = 1$: $1 - s = 0 \rightarrow c^* = (1 - s)y^* = 0$

3.2. The Solow model - Savings

When $s = s_{golden}$, $k^* = k^*_{golden}$ and consumption is maximized.

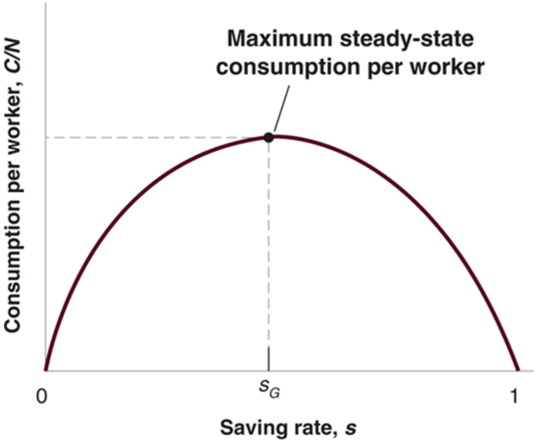
- $s = 0$: $i^* = 0 \rightarrow k^* = 0 \rightarrow y^* = 0 \rightarrow c^* = 0$
- $s = 1$: $1 - s = 0 \rightarrow c^* = (1 - s)y^* = 0$
- $0 < s_{golden} < 1$

3.2. The Solow model - Savings

When $s = s_{golden}$, $k^* = k^*_{golden}$ and consumption is maximized.

- $s < s_{golden}$: $k^* < k^*_{golden}$ and c^* is below its maximum: an increase in s.s. capital leads to an increase in s.s. output that exceeds the related increase in s.s. depreciation. S.s. consumption increases.
- $s > s_{golden}$: $k^* > k^*_{golden}$ and c^* is below its maximum: an increase in s.s. capital leads to an increase in s.s. output that is lower than the related increase in s.s. depreciation. S.s. consumption decreases.

3.2. The Solow model - Savings



NB1:

Even developed countries with high capital levels are usually below the golden rule.

NB2: dynamic perspective

- The economy does not have a tendency to move toward the golden rule steady state.
- Achieving the golden rule requires policymakers to adjust s.
- An increase in s will
  - decrease $1 - s$, the share of income that is consumed, immediately
  - increase output and consumption with a delay

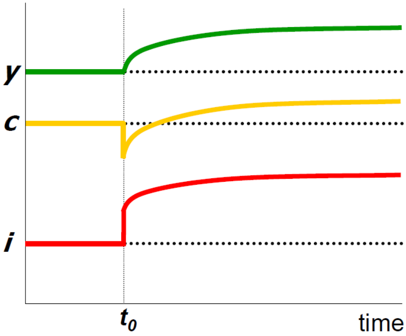
It might be that the positive effect of an increase in s comes too late (for the next generation) and that policy makers and voters will only look at the immediate negative effect. Inter-generational issues $\Rightarrow$ difficult to increase s and get closer to the golden rule.

This is an extension that we won't take into account in this class. We will always assume that, starting from a point below the golden rule level, an increase in s leads to an immediate increase in c^*.

3.2. The Solow model - Savings

If $\boldsymbol{k}^* < \boldsymbol{k}^*_{gold}$
then increasing $\boldsymbol{c}^*$ requires an increase in $\boldsymbol{s}$.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Some multiple choice questions before to go further.
Go on `www.responseware.eu` and enter the session ID.

3.2. Exercise II

3.2. Exercise II

The production function is $Y = K^{1/3}N^{2/3}$.

a. Does the production function exhibit decreasing, constant or increasing returns to scale?

b. Write the production function in per worker terms.
[Remember: $Z^x/Z = Z^{x-1}$]

c. Write down the steady state condition.

Let suppose that $s = 0.1$ and $\delta = 0.2$.

d. Find the steady state values of k, y and c.

e. Find the golden rule value of the savings rate

f. Find the golden rule values of k, y and c

3.2. Exercise II

g. What is the effect you expect an increase in s will have on steady state consumption?

h. Check that an increase in s from 0.1 to 0.2 will lead to an increase in s.s consumption.

i. If s would be equal to 0.5, what do you expect regarding the s.s value of consumption?

j. Check your prediction.

k. What is the effect you expect an increase in s from 0.5 to 0.6 will have on steady state consumption?

l. Check that an increase in s from 0.5 to 0.6 will lead to an decrease in s.s consumption.

3. The Solow model
   - 3.1. Output and capital
   - 3.2. Savings
   - **3.3. Population growth**
   - 3.4. Technological progress

What happen to the steady state when we allow for the population to grow?

Let's denote by g_N the growth rate of the population:

$$g_N = \frac{\Delta Population}{Population}$$

If the number of workers is a fixed proportion of the population, the number of workers grow at rate g_N too:

$$g_N = \frac{\Delta N}{N}$$

3.3. The Solow model - Population growth

What is the level of investment that is needed to maintain the level of capital per worker?

To keep K/N constant, investment I must include:

- δK: to replace the capital that wears out (depreciation).
- $g_N K$: to equip the new workers with capital

If investment would only cover depreciation, the ratio of capital per worker would decrease due to the increase in the number of workers.

The level of investment that is needed to maintain the level of capital per worker is therefore:

$$I^r = \delta K + g_N K$$

$$I^r = (\delta + g_N)K$$

or in per worker terms:

$$i^r = (\delta + g_N)k$$

I^r is called the break-even investment or required investment.

3.3. The Solow model - Population growth

And the law of motion of capital is:

$$\Delta K = I - I^r$$

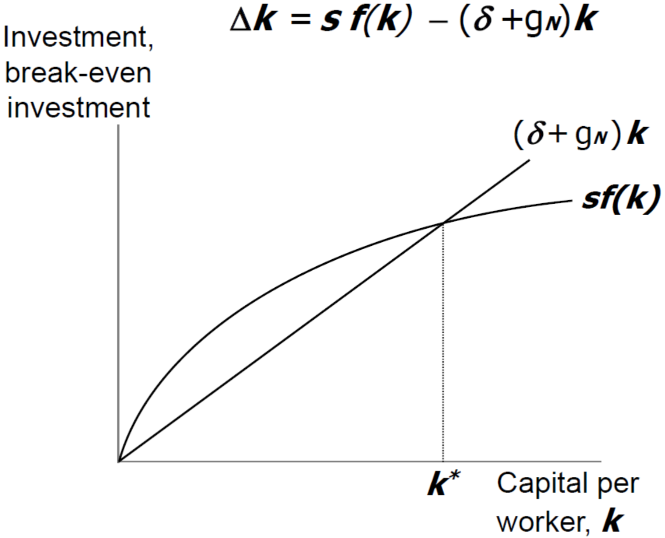
$$\Delta K = I - (\delta + g_N)K$$

$$\Delta K = sF(K, N) - (\delta + g_N)K$$

or in per worker terms:

$$\Delta k = sf(k) - (\delta + g_N)k$$

3.3. The Solow model - Population growth



At the steady state, K/N is constant, which means that Y/N is constant.

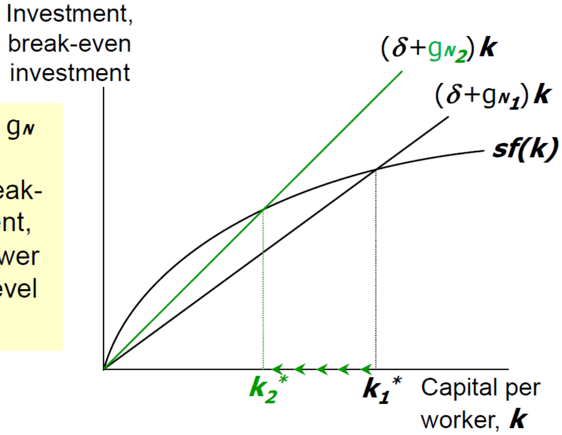
$\Rightarrow$ because N grows at the rate g_N, Y also grows at the rate g_N.

Said differently, the steady state is characterized by the growth rate of output being equal to the growth rate of population.

What is the effect of an increase in the population growth rate on the steady state?

3.3. The Solow model - Population growth

An increase in g_N causes an increase in break-even investment, leading to a lower steady-state level of ***k***.

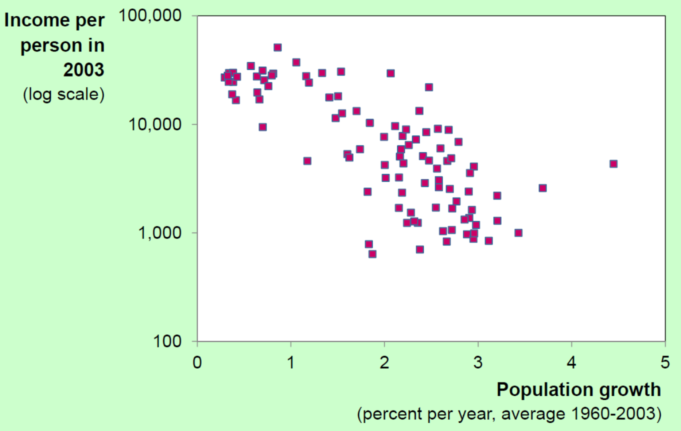


3.3. The Solow model - Population growth

An increase in the population growth rate leads to a lower steady state level of capital per worker and therefore of output per worker.

The Solow model predicts that countries with higher population growth rates will have lower levels of capital and output per worker in the long run.

3.3. The Solow model - Population growth



3. The Solow model

- 3.1. Output and capital
- 3.2. Savings
- 3.3. Population growth
- **3.4. Technological progress**

3.4. The Solow model - Technological progress

Until now, we did not look at the state of technology and technological progress. We were assuming that the state of technology was constant.

Let's relax this assumption.

3.4. The Solow model - Technological progress

Remember: the determinants of output are:

- the **factors of production**
  - labor
  - capital
- the **state of technology** A

Therefore, we can write the production function as:

$$Y = F\underbrace{(A, K, N)}_{(+,+,+)}$$

A captures everything that affects output without affecting the stock of capital and the number of workers.
An increase in A leads to an increase in output, keeping K and N constant.

A might capture total productivity, or the productivity of capital, or the productivity of labor.

We will **assume** that A only captures the productivity of labor:

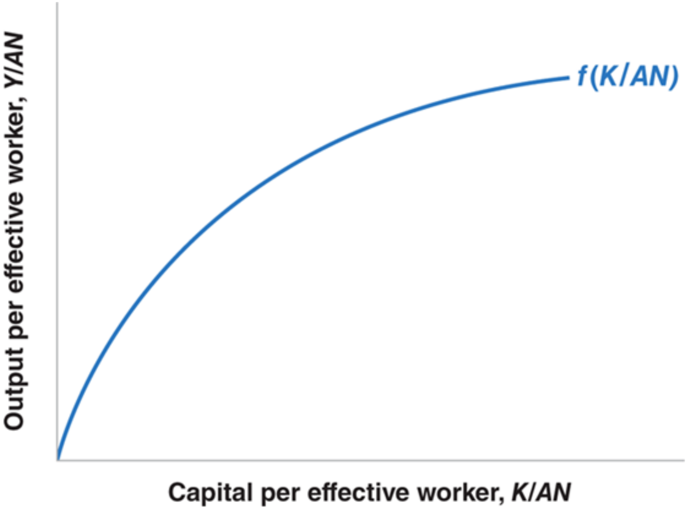
$$Y = F(K, AN)$$

- AN is the amount of **effective workers**
- if A doubles, it is as if the number of workers would double
- if A doubles, output increases for a given number of workers
- if A doubles, firms can produce the same level of output with half of the number of workers

Now, instead of expressing capital, investment, output and consumption in per worker terms, lets express these variables in per effective worker terms.

- $\tilde{k} = \frac{K}{AN}$
- $\tilde{i} = \frac{I}{AN}$
- $\tilde{y} = \frac{Y}{AN} = f(\tilde{k})$
- $\tilde{c} = \frac{C}{AN} = (1 - s)\tilde{y}$

3.4. The Solow model - Technological progress



The production function in per effective worker terms.

3.4. The Solow model - Technological progress

Because there is decreasing marginal returns to capital, the curve representing the relation between output per effective worker and capital per effective worker is concave (increasing curve with decreasing slope).

3.4. The Solow model - Technological progress

Let's denote by g_A the rate of technological progress.

$$g_A = \frac{\Delta A}{A}$$

What is the level of investment that is needed to maintain the level of capital per effective worker?
To keep $K/(AN)$ constant, investment I must include:

- δK: to replace the capital that wears out (depreciation).
- $g_N K$: to equip the new workers with capital
- $g_A K$: to equip with capital the new effective workers created by technological progress

If investment would only cover depreciation, the ratio of capital per effective worker would decrease due to the increase in the number of effective workers (effective workers grow at the rate $g_N + g_A$).

The level of investment that is needed to maintain the level of capital per worker is therefore:

$$I^r = \delta K + g_N K + g_A K$$

$$I^r = (\delta + g_N + g_A) K$$

or in per effective worker terms:

$$\tilde{i}^r = (\delta + g_N + g_A)\tilde{k}$$

Remember, I^r is called the break-even investment or required investment.

And the law of motion of capital is:

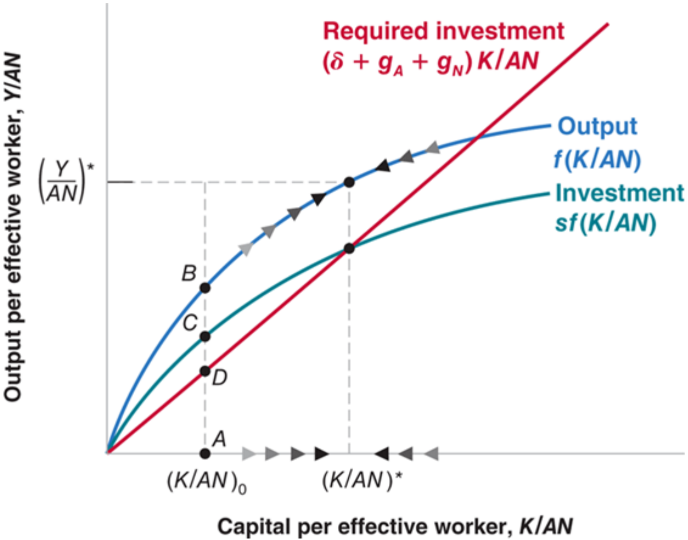
$$\Delta K = I - I^r$$

$$\Delta K = I - (\delta + g_N + g_A)K$$

$$\Delta K = sF(K, N) - (\delta + g_N + g_A)K$$

or in per effective worker terms:

$$\Delta k = sf(\tilde{k}) - (\delta + g_N + g_A)\tilde{k}$$


Required investment
$(\delta + g_A + g_N)\,K/AN$
Output
$f(K/AN)$
Investment
$sf(K/AN)$
Output per effective worker, Y/AN
$\left(\frac{Y}{AN}\right)^*$
B
C
D
A
$(K/AN)_0$
$(K/AN)^*$
Capital per effective worker, K/AN

3.4. The Solow model - Technological progress

At the steady state, $K/(AN)$ is constant, which means that $Y/(AN)$ is constant.

- $\Rightarrow$ because AN grows at the rate $g_N + g_A$, K also grows at the rate $g_N + g_A$.
- $\Rightarrow$ because AN grows at the rate $g_N + g_A$, Y also grows at the rate $g_N + g_A$.

Said differently, the steady state is characterized by the growth rate of output being equal to the growth rate of capital, itself being equal to the sum of the growth rate of population and the rate of technological progress.

The steady state exhibits **balanced growth**: capital, output and effective labor grow at the same rate $g_N + g_A$.

Also, capital per worker and output per worker grow at the same rate g_A.

Steady state = long run = balanced growth path.

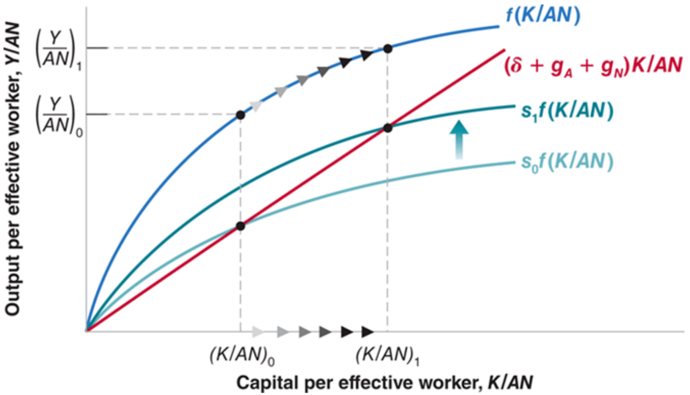
		Growth Rate:
1	Capital per effective worker	0
2	Output per effective worker	0
3	Capital per worker	g_A
4	Output per worker	g_A
5	Labor	g_N
6	Capital	$g_A + g_N$
7	Output	$g_A + g_N$

NB:

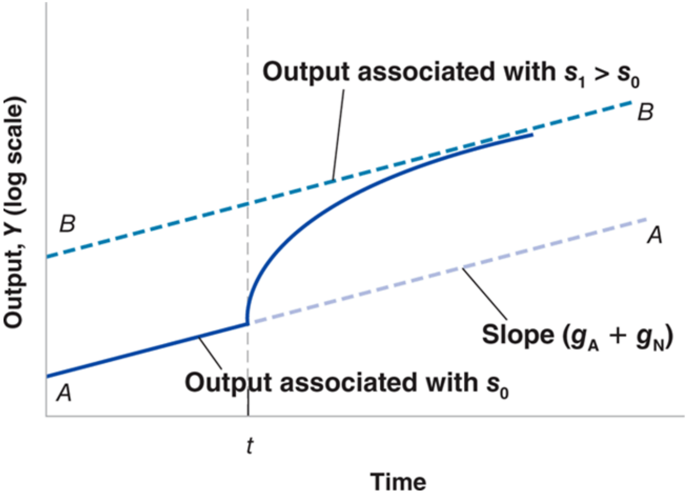
In steady state, both capital and output grow at the rate $g_N + g_A$.

$\Rightarrow$ Also in the extended model with technological progress, the savings rate does not affect the growth rate in the long run. But the savings rate affects the long run level of output per effective worker.

3.4. The Solow model - Technological progress



3.4. The Solow model - Technological progress



The Solow model states that, in the long run:

- capital, output and effective labor grow at the rate $g_N + g_A$.
- capital per worker and output per worker grow at the rate g_A.

$\Rightarrow$ Importance of policies focusing on R&D.

3.4. The Solow model - Technological progress

Are rich countries at their steady state?

Are rich countries at their steady state?

$\rightarrow$ Let's check if output per worker grows at the rate of technological progress.

3.4. The Solow model - Technological progress

	Rate of Growth of Output per Worker (%) 1985–2009	Rate of Technological Progress (%) 1985–2009
France	1.9	1.6
Japan	1.8	2.1
United Kingdom	2.1	1.6
United States	1.9	1.3
Average	1.9	1.7

Source: Calculations from the OECD Productivity Statistics

These 4 countries seem to be at their long run levels (balanced growth path).

Is China at its steady state level?

Is China at its steady state level?

$\rightarrow$ Let's check if output per worker grows at the rate of technological progress.

3.4. The Solow model - Technological progress

Period	Rate of Growth of Output (%)	Rate of Growth of Output per Worker (%)	Rate of Technological Progress (%)
1978–1995	10.2	8.6	7.8
1995–2007	9.9	9.4	6.0

Source: Barry Bosworth and Susan M. Collins, "Accounting for Growth: Comparing China and India," *Journal of Economic Perspectives*, 2008 22(No. 1): p. 49.

3.4. The Solow model - Technological progress

China seems not to be *anymore* on a balanced growth path.

The Solow model also states that, before an economy reaches its steady state:

- capital, output and effective labor grow at a rate that is higher than $g_N + g_A$. This rate is higher the further the economy is from its steady state. This rate also depends on the savings rate (an increase in the savings rate leads to a temporary increase in the growth rate).
- capital per worker and output per worker grow at a rate that is higher than g_A. This rate is higher the further the economy is from its steady state. This rate also depends on the savings rate (an increase in the savings rate leads to a temporary increase in the growth rate).

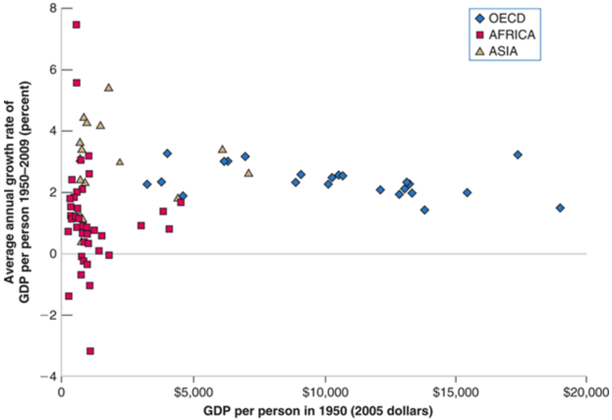
The Solow model therefore predicts that poor countries, that are far from their steady states, should grow faster than rich countries, that are at/close to their steady state levels: **convergence process**.

3.4. The Solow model - Technological progress

The Solow model therefore predicts that poor countries, that are far from their steady states, should grow faster than rich countries, that are at/close to their steady state levels: **convergence process**.

Is it the case?

3.4. The Solow model - Technological progress



Source: See Table 10-1.

Growth rate of GDP per person since 1950 versus GDP per person in 1950.
Did poor countries grow faster? No.

Why don't we observe convergence?

3.4. The Solow model - Technological progress

Why don't we observe convergence?

Because countries have different savings rates, rates of technological progress, and population growth.

Imagine a poor country with a low savings rate. We know that the s.s. level of capital and output is higher the higher s. In this country, the steady state will therefore be low.

Even if the country is poor, with a low level of capital, it might be that the economy is not far from its steady state (because the steady state itself is low):
$\Rightarrow$ because the economy is not far from its steady state, the growth rate is low.

Even a poor country can have a low growth rate.

3.4. The Solow model - Technological progress

Imagine a rich country with a high rate of technological progress. This country will have a high growth rate even if it is at its steady state level.

A rich country might have a growth rate higher than a poor country if the difference in the rates of technological progress is sizable.

There is much more to learn on this topic of long-run growth (human capital, endogenous growth...) but no time for this.

If you are interested in the topic, you should have a look at Chapter 13 that we won't cover in this course (in the next class, we will solve the mock exam).

End of the long run.

More multiple choice questions.
Go on `www.responseware.eu` and enter the session ID.