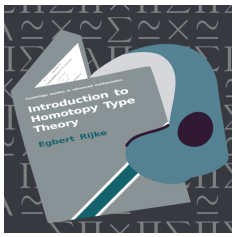

Worksheet 1

HoTTEST Summer School 2022

The HoTTEST TAs , and

4th July 2022

1 (⋆)

State the introduction and elimination rules for

1. $\times$-types
2. $\to$-types
3. $\prod$-types

2 (⋆)

Fill in this proof tree:

$$\dfrac{\dfrac{\dfrac{}{a : A, b : B \vdash a : A} \qquad \dfrac{}{}}{ \vdash : A \times B}}{\dfrac{a : A \vdash \lambda(b : B).(a, b) : }{\cdot \vdash \lambda(a : A).\lambda(b : B).(a, b) : A \to B \to A \times B}}$$

3 $(\star\star)$

Write a proof tree ending with a term of type $A \times B \to B \times A$ in the empty context.

4 $(\star\star)$

For problems 2 and 3, what is the *logical* content of the proof tree? That is, under the "types are theorem" interpretation of Curry-Howard, what theorems have we proven?

Next, what is the *computational* content of the proof tree? That is, under the "programs are proofs" interpretation of Curry-Howard, what programs have we written?

5 $(\star\star\star)$

Define the **swap** function $\sigma_{A,B}$ of type

$$\sigma_{A,B} : \left(\prod_{x:A}\prod_{y:B} C(x,y)\right) \to \left(\prod_{y:B}\prod_{x:A} C(x,y)\right)$$

and show that $\sigma_{B,A} \circ \sigma_{A,B}$ is (definitionally) equal to the identity.