Q Don't we run into impredicativity issues by taking power sets like that?

Great question! If we're sticking with dProp, then we never will as we can always stay in $\mathcal{U}_0$, because dProp w.r.t. any universe is equivalent to 2. In general, it depends what you do. Just because your powersets live in universe up, doesn't necessarily get you into trouble.