Agda lecture 4 Q&A (Solved)

HoTTEST Summer School 2022

The HoTTEST TAs

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Q1 why is the sphere $\mathbb{S}^2$?

The circle is $\mathbb{S}^1$, a one-dimensional sphere. Topologically, $\mathbb{S}^n$ is the set of points at unit distaance from the origin in $\mathbb{R}^{n+1}$.

This HIT description of $\mathbb{S}^1$ does not seem to say anything about a set of points at a given distance? Is it equivalent somehow?

The HIT is a different representation that does not use point-set topology. But the HIT really is the circle in the sense that it has the right properties (we'll get there in due time).

The "set of points" description is analytic, while the HIT description is synthetic. Think of it as the difference between doing geometry in vector spaces and using Euclid's axioms

After 2000 years of thinking we found out what is the "homotopical quintessence of $\mathbb{S}^1$" and now we can tell agda what $\mathbb{S}^1$ is.

Q2 Does higher inductive types need identity types? I don't understand from an abstract point of view why equality should be so distinguished from all other 'lower' inductive types

Yes, because a higher inductive type introduces *path* constructors, so the target types of those constructors will be identity types.

$=$ is just a type inductively defined by $X \to X \to$ Type. What if I defined "my-relation" as $X \to Y \to Z \to$ Type with some constructor, and then tried to define a type using this trinary relation, would it be a HIT?

I'd say we're treating equality specially since it is, after all, one of the defining elements of homotopy type theory.

It would certainly be some sort of HIT in the sense that it extends regular inductive types.

Q3 why is necessary this —rewriting?

The rewriting flag makes the equality you use it on behave like a definitional equality

It will become necessary later to make our lives easier. I'm pretty sure Dan will explain it at that time, but if not, then we can ask for a clarification at that point. (Looking ahead, it will give us a definitional computation rule for S1.)

Q4 How do we know our postulates are consistent?

We don't.

Q5 could we eliminate the use of postulate?

If we use cubical Agda we can :)

Why are we not using cubical agda (yet)?

Because it's useful to (try to) understand things without having to introduce or relying on cubical machinery first.

Q6 Is loop distinct from refl? How does agda know this?

At this point: maybe. See also Q16.

Q7 What is the difference between groupoids and infinity-groupoids?

Groupoids have structure up to "dimension one" meaning roughly that there is a set of morphisms and that operations on morphisms are strict, whereas in higher groupoids there can be interesting ways to show that two morphisms are equal

So in a 2-groupoid, the type of equalities between two parallel arrows is itself a 1-groupoid

Is there a simple example of a 2-groupoid?

i don't know so here's an nlab link https://ncatlab.org/nlab/show/2-groupoid#examples

Q8 Do the square brackets do anything or are they just for emphasis?

They're just for emphasis

(they make the type of the elements on each side of $\equiv$ explicit)

(This comes from a new syntax declaration in new-prelude; it wouldn't have worked in the previous lectures.)

Makes sense, it's just like writing a subscript on the equality when doing a pencil and paper proof

Q10 Might have missed this but how do you read the dot out loud? i.e. $p \bullet q$ is "p — q"

"dot", "before", "concatenated with", "composed with", or "p then q"

Q11 That looks tedious. In Lean I'd tag those lemmas as '@[simp]' and then use 'by simp'. Is there something similar in Agda?

Agda does not use tactics much, but the IAS Agda HoTT library has an advanced version of equational reasoning for path algebra

I wrote my own extended vesion of this: https://github.com/FrozenWinters/pathalg/blob/main/path-alg-tests.agda

Q15 Would you want to add a rewrite rule for S1-rec-loop for symmetry? Or is rewriting only relevant for S1-rec-base?

It just doesn't turn out to be that useful to have the rewrite rule for S1-rec-loop. (Dan briefly mentioned this earlier, I believe.)

Q16 Is the Q&A comment (i.e. (Q) is loop distinct from refl? (A) Maybe) at the bottom of the file wrong now?

Once we add univalence, you can prove that loop is different from refl.

Recursor on it's own is not sufficient?

Without univalence, you can prove that if loop = refl, then every type must be a set. And this contradicts univalence.

Q17 are we not passing loop and loop for w and e because otherwise we would do two loop walks, where we just want to do one?

Yes

Q21 Are HITs validated by all models of MLTT?

Definitely not! Not by any set models!

No, not by all, but they are by some.