HoTTEST Summer School

Agda Lecture 4-6:

Higher inductive types

4) Examples, non-dependent elim

5) Dependent elim, proving some type equivalences

6) $\pi_1(S^1) \simeq \mathbb{Z}$

Types are ∞-groupoids:

points

paths

paths-between-paths

....

Inductive types have point constructors

true false

$\mathbb{B}$

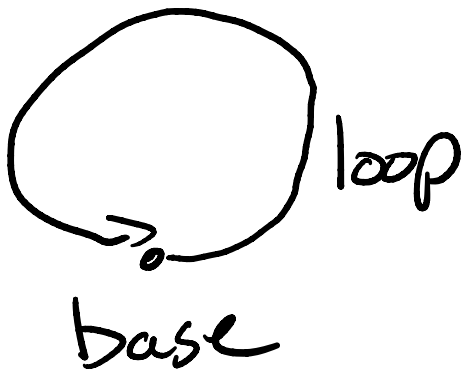
0 1 2 3 4 5 ...

$\mathbb{N}$

Higher inductive types can have point + path + path-between-path... constructors

[Bauer, Lumsdaine Shulman, Warren 2011]

Circle S^1

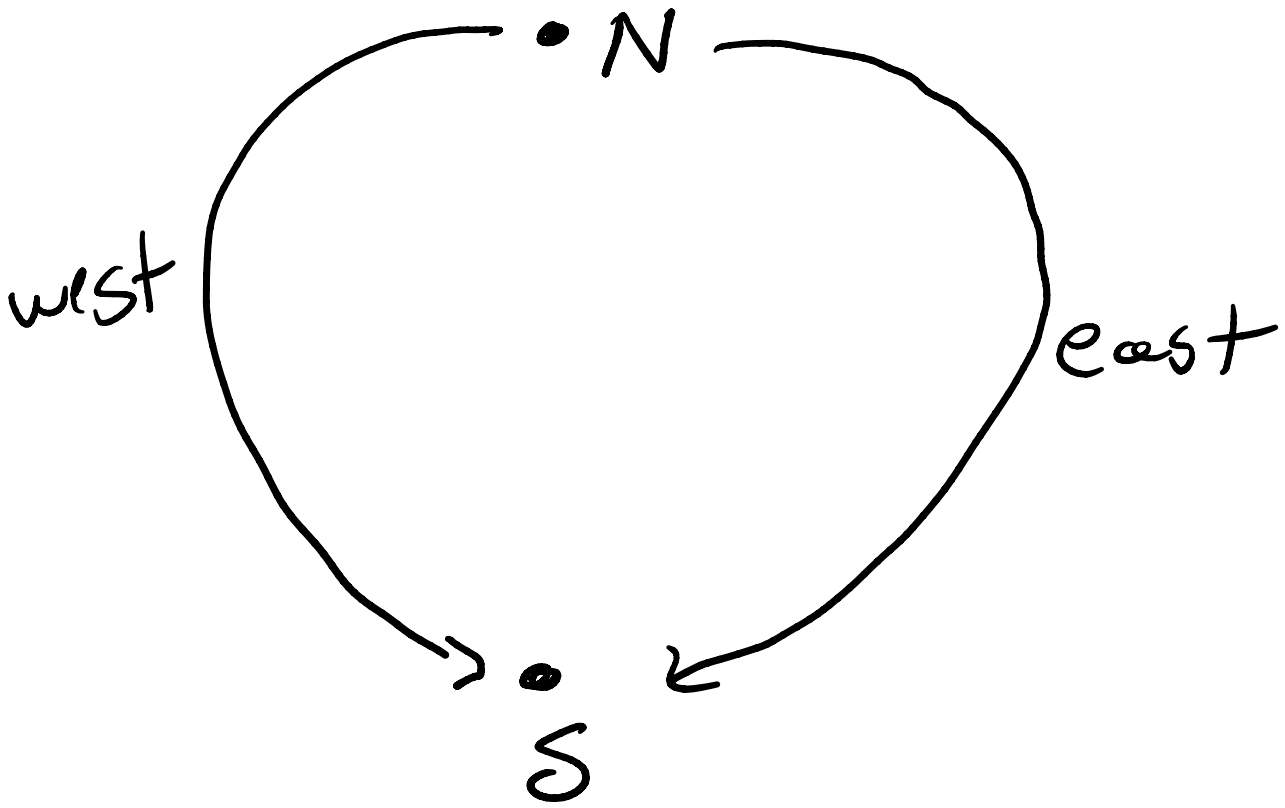


$$\begin{cases} \text{base} : S^1 \\ \text{loop} : \text{base} \equiv_{S^1} \text{base} \end{cases}$$

Generates paths like

$$\text{loop} \cdot \text{loop}$$
$$\text{loop}^{-1}$$
$$\text{loop} \cdot \text{loop}^{-1}$$

2 point circle



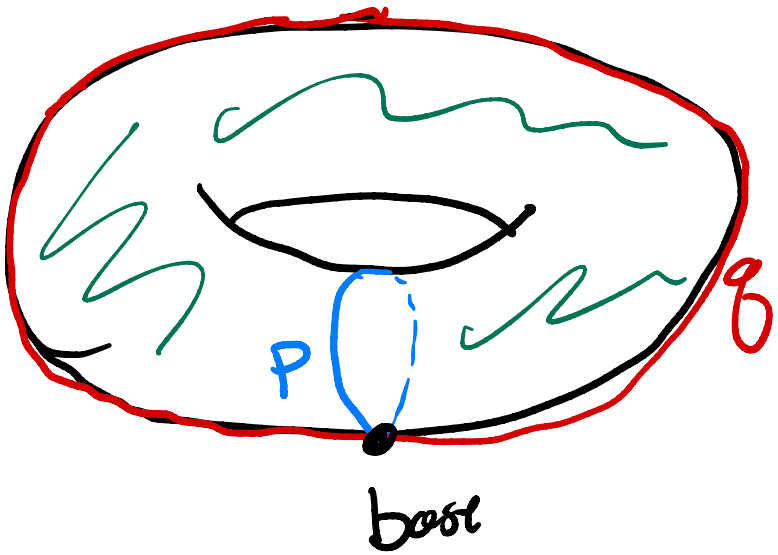
$N : \text{Circle}_2$

$S : \text{Circle}_2$

east: $N \equiv S$

west: $N \equiv S$

Torus



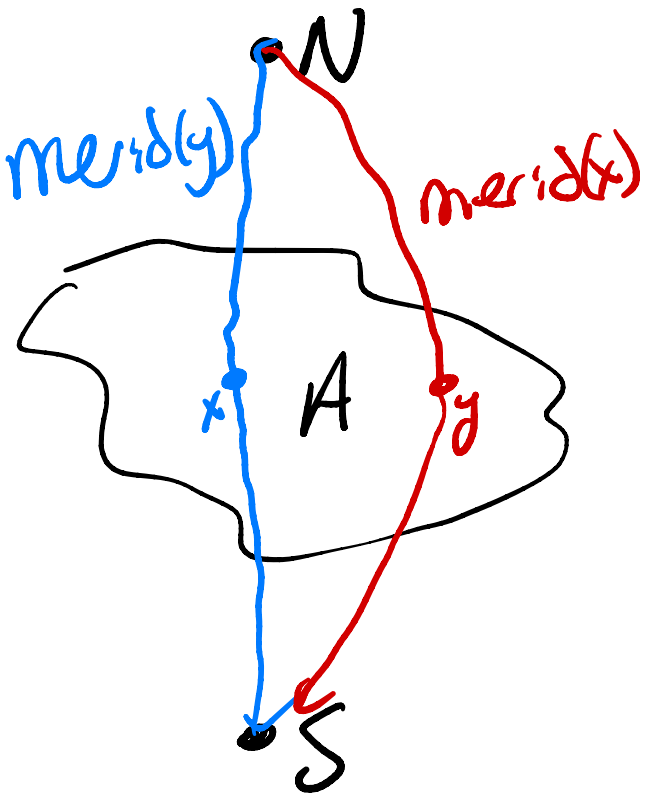
$$baseT : Torus$$

$$p : base \equiv base$$

$$q : base \equiv base$$

$$S : p \cdot q \equiv q \cdot p$$

Suspension

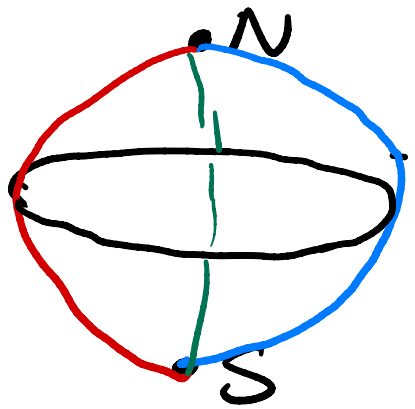


$N : \mathrm{Susp}\, A$

$S : \mathrm{Susp}\, A$

$\mathrm{merid} : (x : A) \to N \equiv S$

E.g. $\mathbb{S}^2 = \mathrm{Susp}\, \mathbb{S}^1$



Pushout

$$\mathrm{inl} : A \to A +_C B$$

$$\mathrm{inr} : A \to A +_C B$$

$$\mathrm{glue} : (x : A) \to \mathrm{inl}(fx) \equiv \mathrm{inr}(gx)$$