Agda lecture 6 Q&A (Solved)

HoTTEST Summer School 2022

The HoTTEST TAs

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Q1 Why is it `reflbase` = `reflbase` instead of e.g. `loop` = `loop`?

The loopspace operation is defined on spaces with a basepoint and produces a based space. The basepoint of $\Omega(X, a)$ is $\mathtt{refl}_\mathtt{a}$

Q2 Do we always do '! stuff' to get the inverse of something?

That's Dan's notation for the "inverse-of-path" operator. You may also see it called "sym" in other places. In the Cubical Library, we use $(-)^{-1}$.

Q3 Does the need to write out the computation rules explicitly come from having these as module parameters? Is there a way to avoid having to do that?

Yes it does. Having it as a module parameter just says: 'I have something that I call Z-rec of this type', but you have to specify how it behaves. You *could* write your proof against a concrete implementation — then it would compute as a normal Agda program (i.e.: in a wonky way). But here we're abstracting out the integers, and, as a trade-off, we lose the nice, automatic computation

Q4 I don't understand how you somehow combine the successor and predecessor, could someone explain it again please?

Sure! The idea is that an equivalence, phrased as a coherently-invertible map, comes with a forwards and backwards map, as well as homotopies and a coherence. In particular, the inverse that you can get is pred. The idea is that the integers don't *just* have a x+1 operation; they have a "slide to the right" operation (sending e.g. 0 to 1), which has an inverse, "slide to the left", sending 1 to 0 (and 0 to -1, etc)

Q5 Dan said the induction principle implies the uniqueness we just defined—is the converse true as well?

Yes. This was shown in work by Sojakova, Awodey and Gambino: "Homotopy-initial algebras in type theory"

Q6 If I remember my algebraic topology correctly, the universal cover of the circle is the reals—in Agda, can we work with this Cover we defined as if it were the reals? It seems like a kind of synthetic construction of the reals, which acts like it topologically but where you can't really get at anything apart from the integers; are there other definitions of the reals in the setting of HoTT/Agda, and how does this relate?

The universal cover that we constructed is commonly known as the "homotopy reals" in type theory. The total space of this fibration is contractible, so there's not much information there. To "construct the reals" we would want a non-contractible type that is capable of encoding each distinct definable real.
The definition we're using is actually equivalent to a classical definition that you can find in e.g. Hatcher. For nice-enough spaces, there is a Galois correspondence between covering spaces of X (in the point-set definition), presheaves from the fundamental groupoid of X to Sets, and $\pi_1(X)$-sets. This definition of a covering space is closest to the second one there.