GPU Teaching Kit

Accelerated Computing

Module 10.1 – 10.4 Parallel Computation Patterns (scan)

Prefix Sum, A Work-Inefficient scan kernel, A Work-Efficient Parallel Scan Kernel, & More on Parallel Scan

Part 1: Prefix Sum

- To master parallel scan (prefix sum) algorithms
  - Frequently used for parallel work assignment and resource allocation
  - A key primitive in many parallel algorithms to convert serial computation into parallel computation
  - A foundational parallel computation pattern
  - Work efficiency in parallel code/algorithms

- Reading –Mark Harris, Parallel Prefix Sum with CUDA
  - https://developer.nvidia.com/gpugems/GPUGems3/gpugems3_ch39.html

Inclusive Scan (Prefix-Sum) Definition

Definition: *The* scan *operation takes a binary associative operator* $\oplus$ (pronounced as circle plus), *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then scan operation on the array would return

[3 1 7 0 4 1 6 3],
[3 4 11 11 15 16 22 25].

An Inclusive Scan Application Example

- Assume that we have a 100-inch sandwich to feed 10 people
- We know how much each person wants in inches
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut the sandwich quickly?
- How much will be left?

- Method 1: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.

- Method 2: calculate prefix sum:
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences from sequential:

```
out[0]=f(0);
for(j=1;j<n;j++)
  out[j] = out[j-1] + f(j);
```

  - Into parallel:

```
forall(j) { temp[j] = f(j) };
 scan(out, temp);
```

- Useful for many parallel algorithms:
  - Radix sort
  - Quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms, ….

A Work Efficient C Implementation

```
void scanAdd(int *x, int y*, int n) {
  y[0] = x[0];
  for (int i = 1; i < n; i++) y[i] = y [i-1] + x[i];
}
```

Computationally efficient:

N additions needed for N elements - O(N)!

Only slightly more expensive than sequential reduction.

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$$y_0 = x_0$$

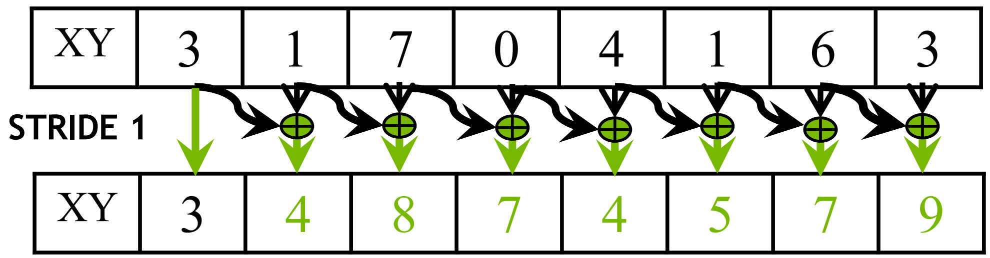
$$y_1 = x_0 + x_1$$

$$y_2 = x_0 + x_1 + x_2$$

"Parallel programming is easy as long as you do not care about performance."

A Better Parallel Scan Algorithm

1. Read input from device global memory to shared memory
2. Iterate log(n) times; stride from 1 to n-1: double stride each iteration



ITERATION = 1

STRIDE = 1

- Active threads *stride* to n-1 (n-stride threads)
- Thread *j* adds elements *j* and *j-stride* from shared memory and writes result into element j in shared memory
- Requires barrier synchronization, once before read and once before write

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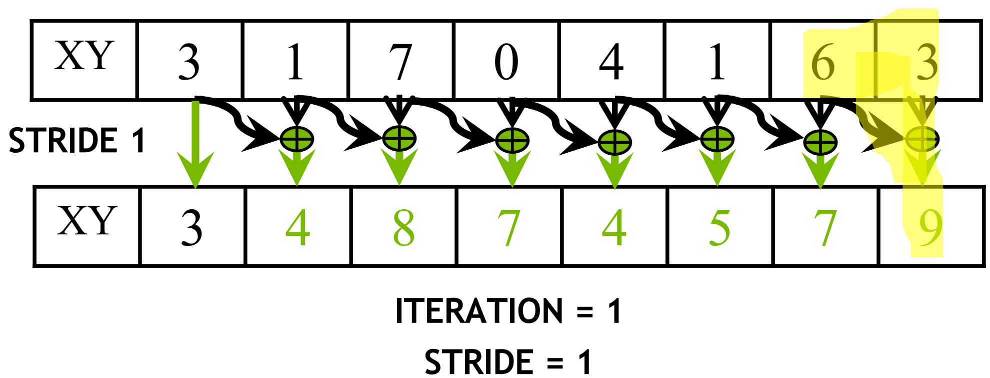

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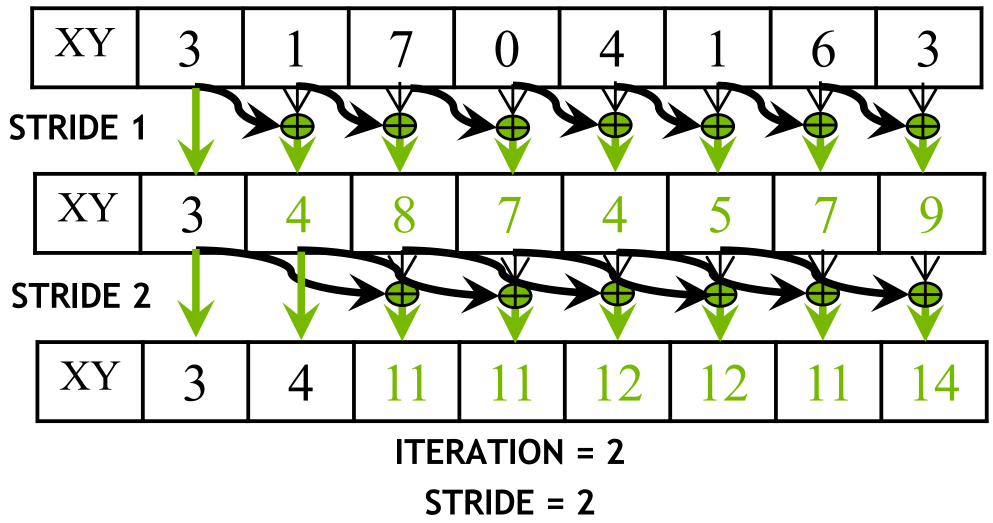

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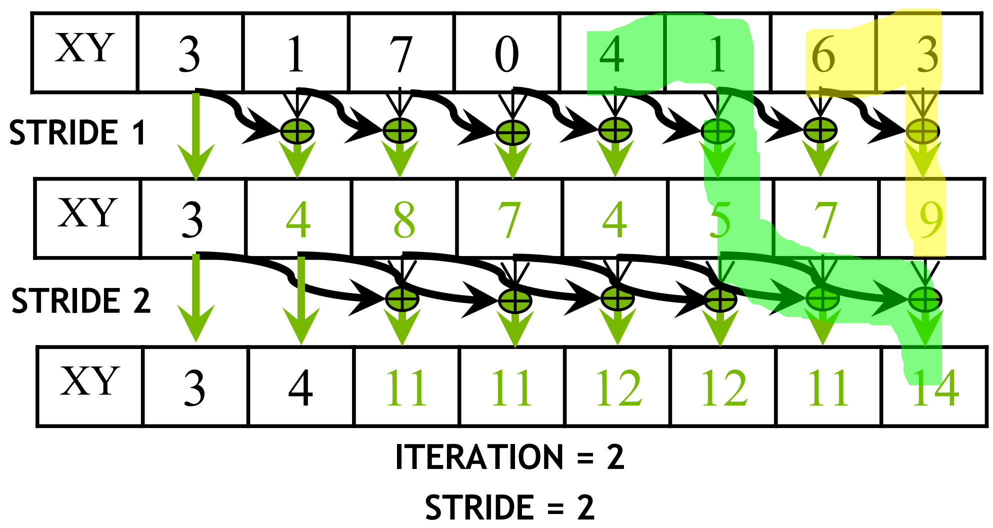
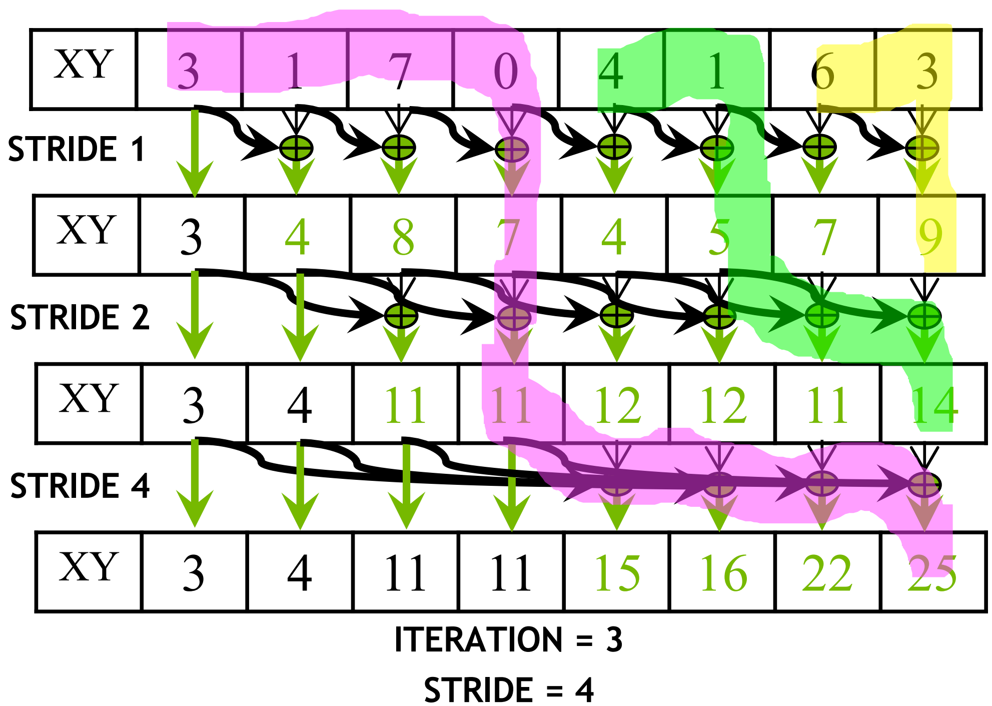


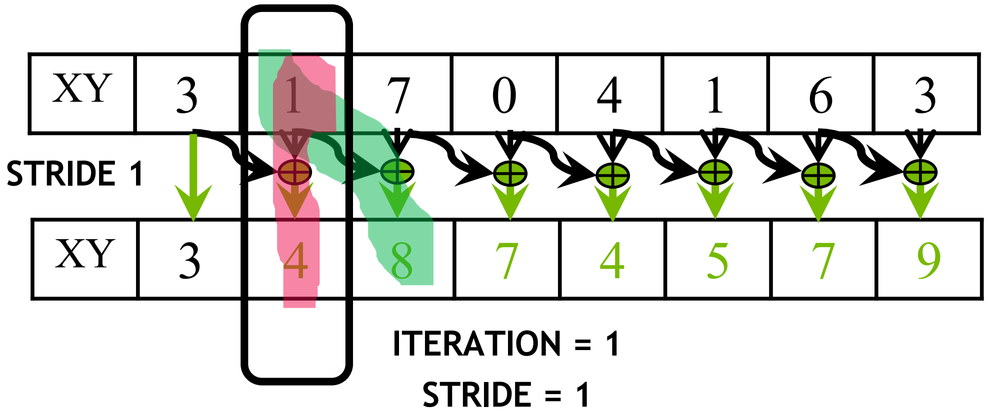
A Better Parallel Scan Algorithm

1. Read input from device to shared memory
2. Iterate log(n) times; stride from 1 to n-1: double stride each iteration
3. Write output from shared memory to device memory



Handling Dependencies

- During every iteration, each thread can overwrite the input of another thread
  - Barrier synchronization to ensure all inputs have been properly generated
  - All threads secure input operand that can be overwritten by another thread
  - Barrier synchronization is required to ensure that all threads have secured their inputs
  - All threads perform addition and write output



A Work-Inefficient Scan Kernel

```
__global__ void work_inefficient_scan_kernel(float *X, float *Y, int InputSize) {
    __shared__ float XY[SECTION_SIZE];
    int i = blockIdx.x * blockDim.x + threadIdx.x;

    if (i < InputSize)  XY[threadIdx.x] = X[i]; else XY[threadIdx.x] 0.0f;

    for (unsigned int stride = 1; stride < blockDim.x; stride *= 2) {
      __syncthreads();
      float in1;
      if (threadIdx.x >= stride)  in1 = XY[threadIdx.x - stride];
      __syncthreads();
      if (threadIdx.x >= stride) XY[threadIdx.x] += in1;
    }

    __ syncthreads();
   If (i < InputSize)  Y[threadIdx.x] = in1;
}
```

SECTION_SIZE == blockDim.x
1 thread per element of the data worked on: (n-1) threads active in the first iteration of the for loop

Work Efficiency Considerations

- This Scan executes log(n) parallel iterations
  - The iterations do (n-1), (n-2), (n-4),..(n- n/2) adds each
  - Total adds: n * log(n) - (n-1) → O(n*log(n)) work

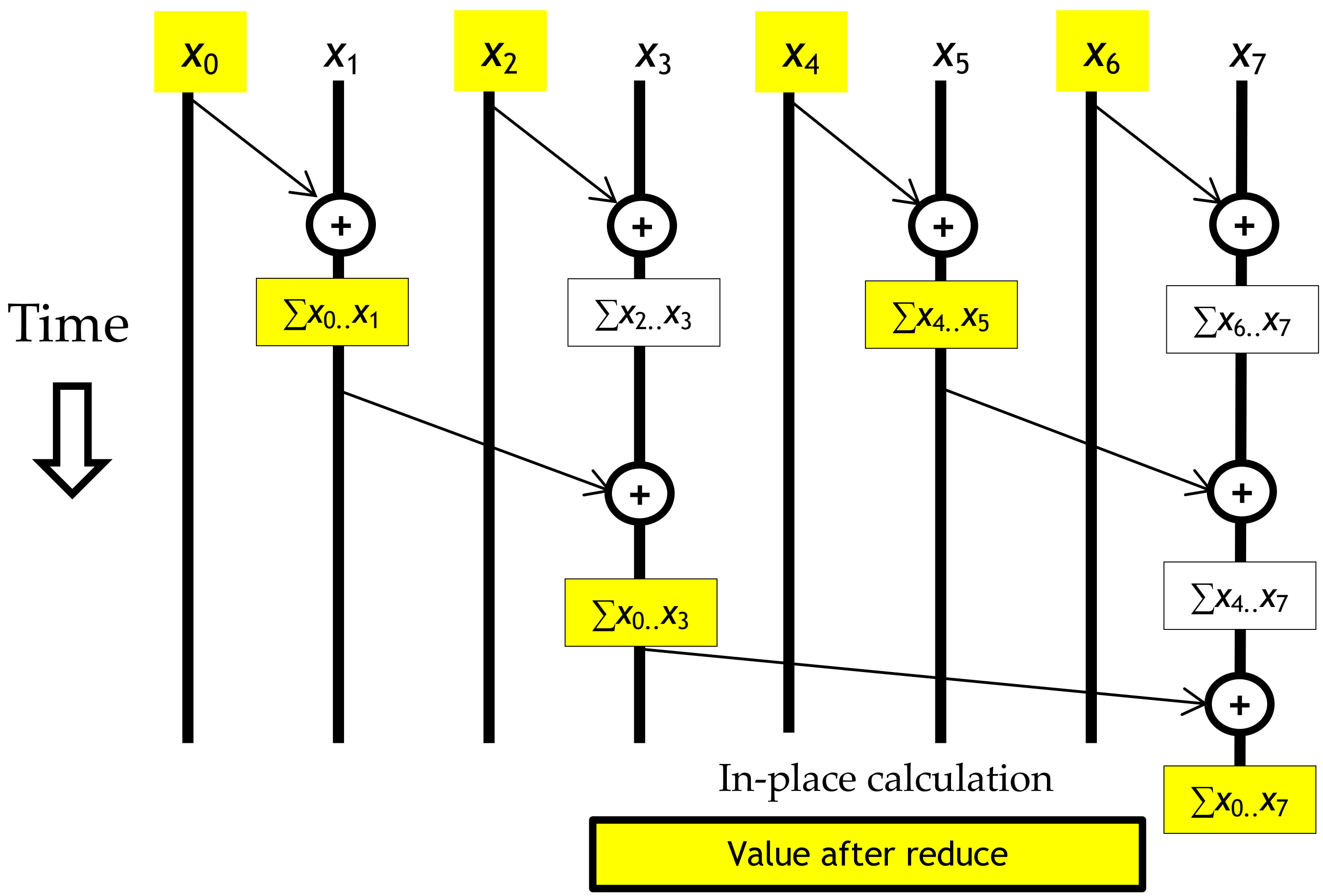
- This scan algorithm is not work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) can hurt: 10x for 1024 elements!

- A parallel algorithm can be slower than a sequential one when execution resources are saturated from low work efficiency

Improving Efficiency

- *Balanced Trees*
  - Form a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step
- For scan:
  - Traverse down from leaves to the root building partial sums at internal nodes in the tree
    - The root holds the sum of all leaves
  - Traverse back up the tree building the output from the partial sums

Parallel Scan - Reduction Phase



Reduction Phase Kernel Code

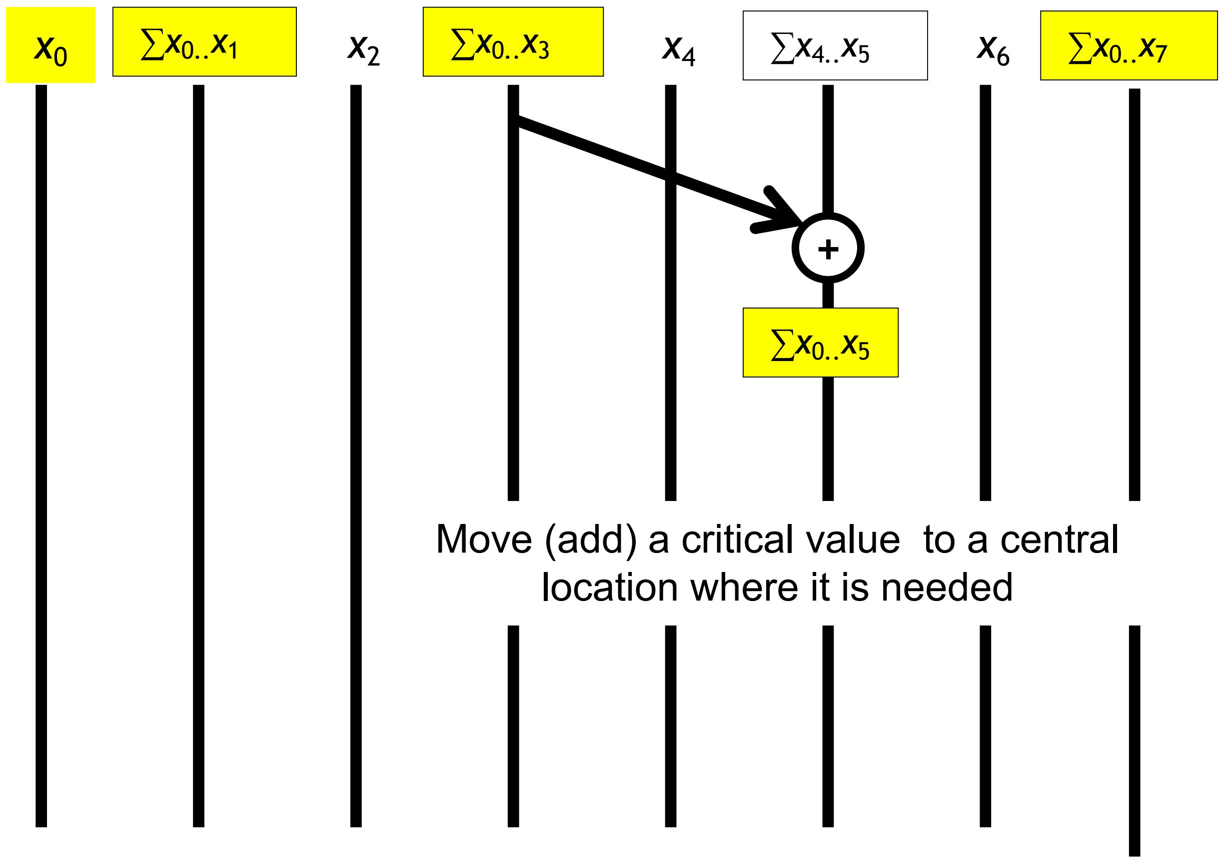
```
// XY[*BLOCKSIZE] is in shared memory SECTION_SIZE = 2*BLOCKSIZE

for (unsigned int stride = 1;stride <= BLOCK_SIZE; stride *= 2) {
    int index = (threadIdx.x+1)*stride*2 - 1;
    if(index < 2*BLOCK_SIZE)
        XY[index] += XY[index-stride];
    __syncthreads();
}
```

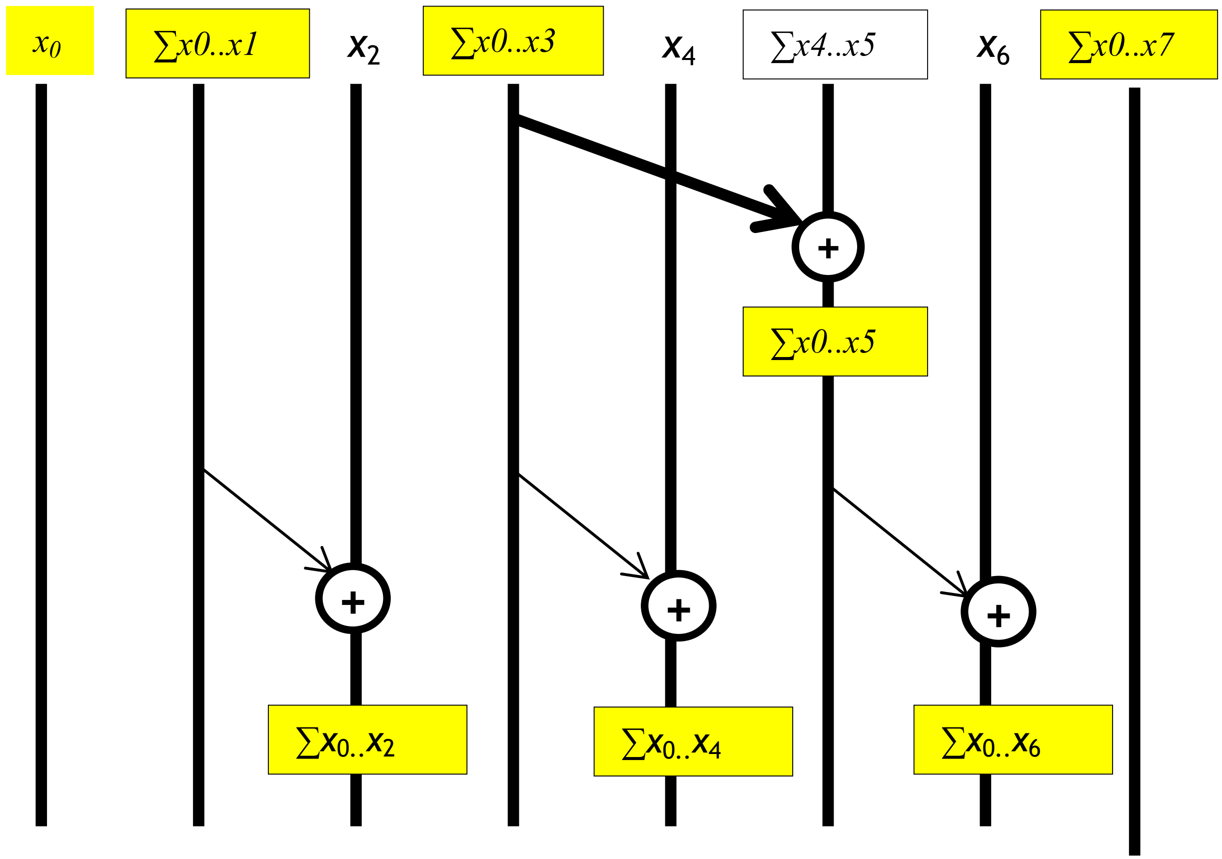
SECTION_SIZE == 2*blockDim.x
1 thread per two elements of the data worked on:
threads active in the first iteration of the for loop

threadIdx.x+1 = 1, 2, 3, 4….
stride = 1,
index = 1, 3, 5, 7, …

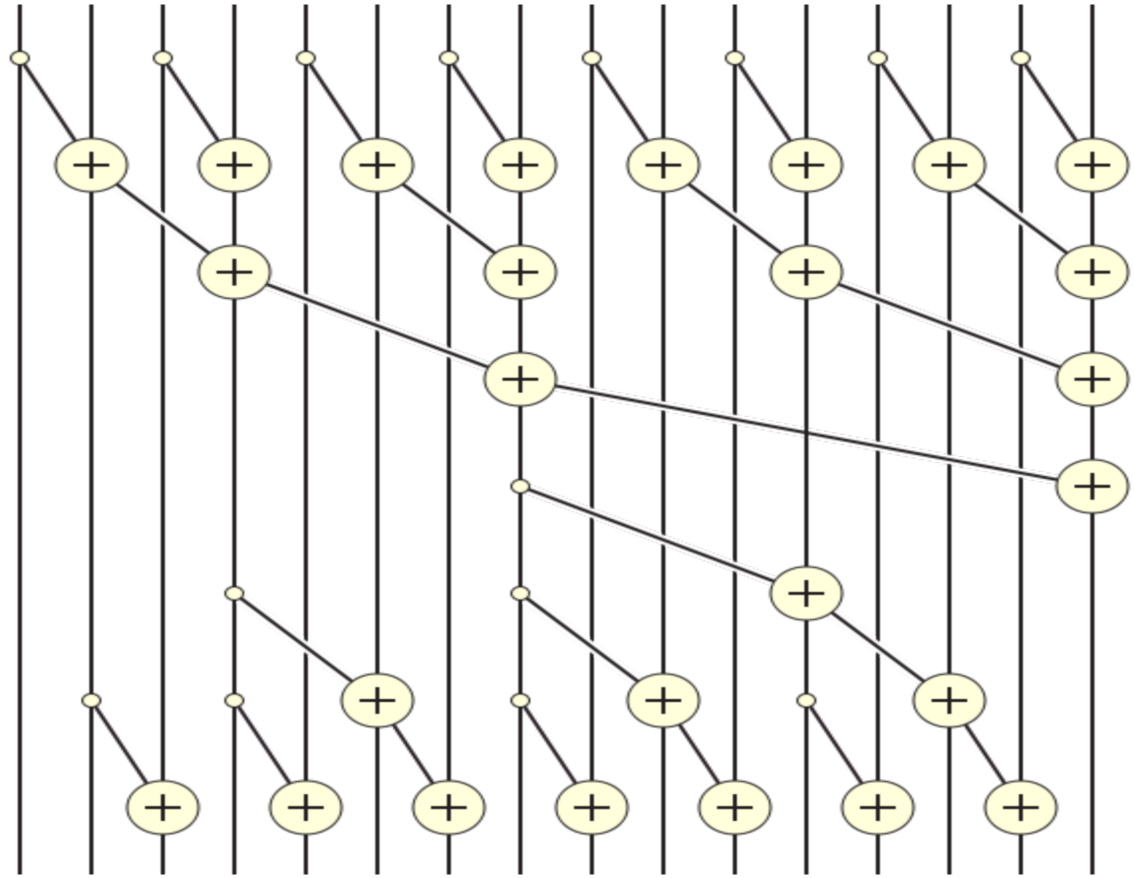
Parallel Scan - Post Reduction Reverse Phase



Parallel Scan - Post Reduction Reverse Phase



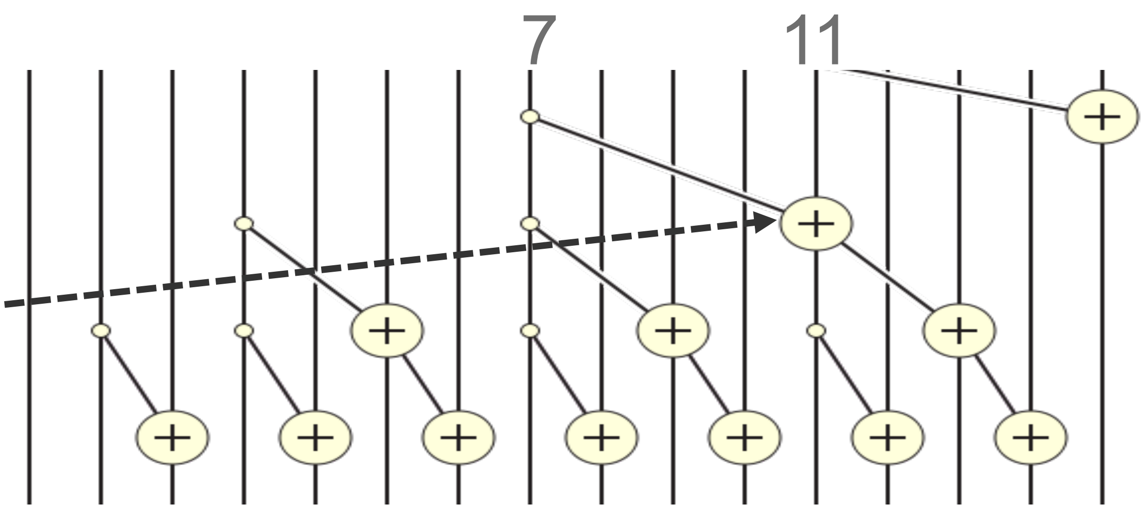
Putting it Together



Post Reduction Reverse Phase Kernel Code

```
for (unsigned int stride = BLOCK_SIZE/2; stride > 0; stride /= 2) {
       __syncthreads();
       int index = (threadIdx.x+1)*stride*2 - 1;
       if(index+stride < 2*BLOCK_SIZE) {
         XY[index + stride] += XY[index];
       }
}
  __syncthreads();
if (i < InputSize) Y[i] = XY[threadIdx.x];
```

First iteration for 16-element section
BLOCKSIZE=8
stride = BLOCK_SIZE/2 = 8/2 = 4
For threadIdx.x = 0
  index = (0+1) * (2*4) - 1 = 8-1 = 7
  If (7+4< 2*8) { XY[11] += XY[7] }
For threadIdx.x = 1
  index = (1+1) * (2*4) - 1 = 15
  (15+4 < 2 *8) NOPE
So only thread 0 gets through on first iteration and updates XY[11] += XY[7]



Work Analysis of the Work Efficient Kernel

- The work efficient kernel executes log(n) parallel iterations in the reduction step
  - The iterations do n/2, n/4,..1 adds
  - Total adds: (n-1) → O(n) work

- It executes log(n)-1 parallel iterations in the post-reduction reverse step
  - The iterations do 2-1, 4-1, …. n/2-1 adds
  - Total adds: (n-2) – (log(n)-1) → O(n) work

- Together phases perform up to no more than 2x(n-1) adds

- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

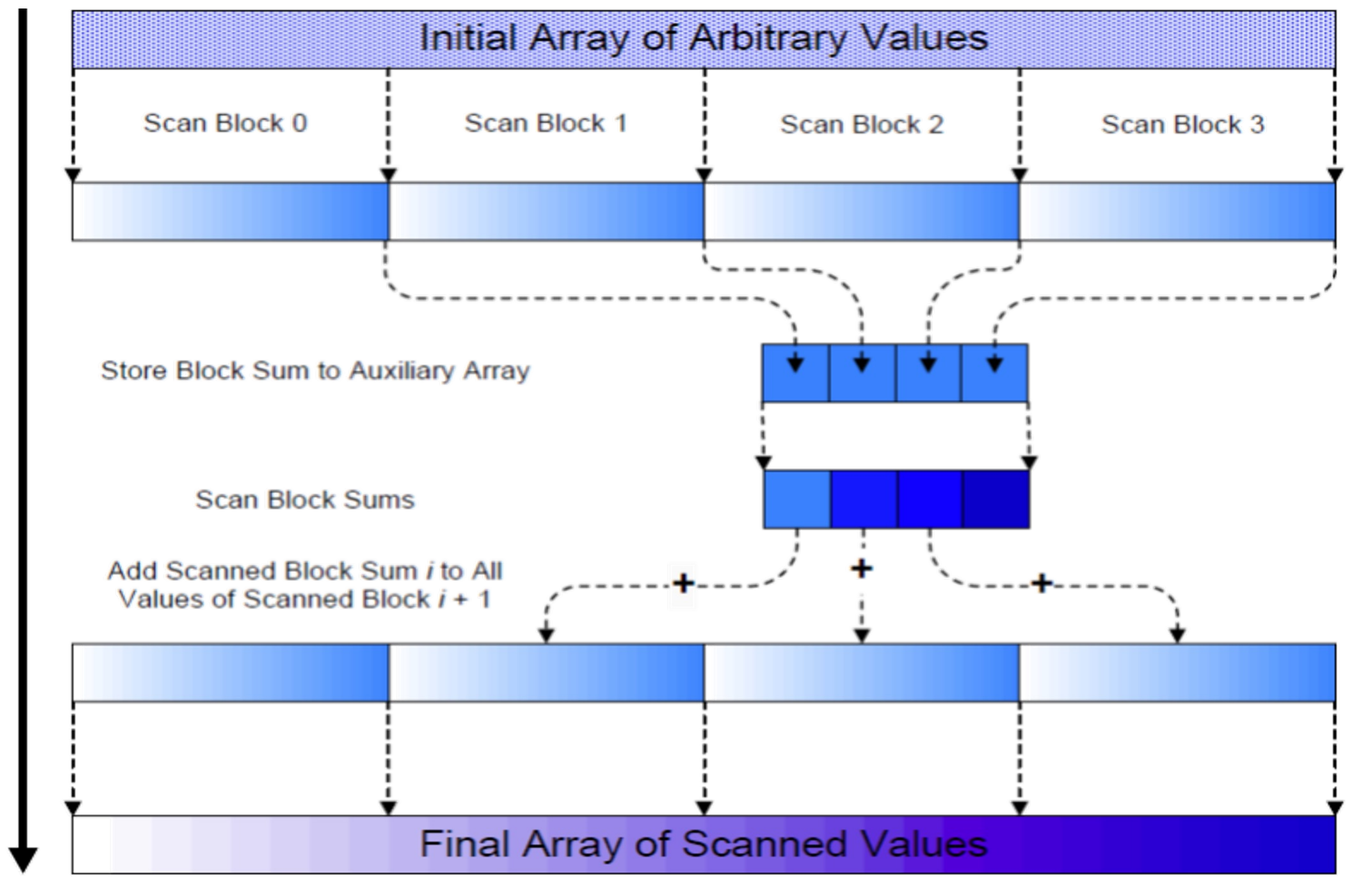
Some Tradeoffs

- The work efficient scan kernel is normally more desirable
  - Better Energy efficiency
  - Less execution resource requirement
- However, the work inefficient kernel could be better for absolute performance due to its single-phase nature (forward phase only)
  - There is sufficient execution resource

Handling Large Input Vectors

- Have each section of 2*blockDim.x elements assigned to a block
  - Perform parallel scan on each section
- Have each block write the sum of its section into a Sum[] array indexed by blockIdx.x
- Run the scan kernel on the Sum[] array
- Add the scanned Sum[] array values to all the elements of corresponding sections

Overall Flow of Complete Scan



Exclusive Scan Definition

Definition: *The* exclusive scan *operation takes a binary associative operator* $\oplus$, *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation

on the array [3 1 7 0 4 1 6 3],

would return [0 3 4 11 11 15 16 22].

Why Use Exclusive Scan?

– To find the beginning address of allocated buffers

– Inclusive and exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

A Simple Exclusive Scan Kernel

- Adapt an inclusive, work inefficient scan kernel
- Block 0:
  - Thread 0 loads 0 into XY[0]
  - Other threads load X[threadIdx.x-1] into XY[threadIdx.x]
- All other blocks:
  - All thread load X[blockIdx.x*blockDim.x+threadIdx.x-1] into XY[threadIdex.x]
- Similar adaption for work efficient scan kernel but ensure that each thread loads two elements
  - Only one zero should be loaded
  - All elements should be shifted to the right by only one position

Read the Harris article (Parallel Prefix Sum with CUDA) for a more intellectually interesting approach to exclusive scan kernel implementation.

GPU Teaching Kit

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