

Quantum Positive-Negative Neuron Architecture for Multi-Channel EEG Analysis: A Theoretical Framework

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Abstract

This paper presents a quantum computing architecture based on the Positive-Negative (PN) neuron model for multi-channel electroencephalogram (EEG) analysis. The proposed quantum PN neuron encodes excitatory-inhibitory dynamics using paired qubits with parameterized rotation gates, leveraging quantum entanglement to capture inter-channel correlations efficiently. A rigorous complexity analysis demonstrates that correlation encoding scales as $O(M)$ quantum gates compared to $O(M^2)$ classical operations for M channels. This work addresses common overclaims in quantum machine learning literature by clarifying the distinction between Hilbert space dimensionality and extractable classical information, as bounded by the Holevo limit. The architecture shows theoretical promise for seizure prediction applications, though practical advantage depends on hardware maturation beyond the current noisy intermediate-scale quantum (NISQ) era. Empirical validation using the Kaggle American Epilepsy Society dataset demonstrates the feasibility of quantum fidelity-based classification, with results indicating clear separation between ictal and interictal EEG patterns.

Keywords: quantum computing, neural networks, EEG analysis, positive-negative neuron, entanglement, seizure prediction

1 Introduction

Seizure prediction from electroencephalogram (EEG) recordings remains one of the most challenging problems in computational neuroscience. Pre-ictal states—the period immediately preceding a seizure—exhibit complex spatiotemporal dynamics that distinguish them from normal brain activity. These dynamics are characterized by increased synchronization across cortical regions, cross-frequency coupling between neural oscillations, and subtle phase relationships that emerge minutes to hours before seizure onset (Mormann et al., 2007). The clinical significance of reliable seizure prediction cannot be overstated: accurate prediction would enable timely intervention, dramatically improving quality of life for the approximately 50 million people worldwide affected by epilepsy (WHO, 2019).

Classical approaches to EEG analysis for seizure prediction typically require explicit computation of pairwise correlations between recording channels. For a standard clinical 10-20 montage with 19 electrodes, this necessitates calculating 171 unique channel pairs. As EEG systems evolve toward higher spatial resolution with 64, 128, or even 256 channels, this quadratic scaling presents an increasingly significant computational burden. Moreover, the temporal dynamics of pre-ictal activity require these correlations to be tracked continuously across time, compounding the computational challenge.

The Positive-Negative (PN) neuron model, described by Gupta et al. (2003), provides a biologically-inspired computational framework that captures the excitatory-inhibitory (E-I) dynamics fundamental to neural computation. In biological neural circuits, the balance between

excitation and inhibition determines network stability, and disruption of this balance is a hallmark of epileptogenic tissue (Buzsáki & Wang, 2012; Dehghani et al., 2016). The PN model encapsulates this balance through coupled differential equations governing the evolution of excitatory and inhibitory state variables.

This paper proposes a quantum implementation of the PN neuron architecture that leverages the principles of superposition and entanglement to encode E-I dynamics and inter-channel correlations more efficiently than classical methods. The quantum approach offers a path toward linear scaling with channel count while preserving the rich correlation structure inherent in multi-channel EEG data. The present work makes three primary contributions to the literature:

1. A quantum circuit architecture that maps PN neuron parameters to qubit rotations through a novel “A-Gate” structure.
2. Rigorous complexity analysis accompanied by explicit corrections to common overclaims regarding quantum advantage.
3. A theoretical framework for template-based seizure prediction using quantum fidelity estimation, validated against real EEG data.

2 Background

2.1 The Positive-Negative Neuron Model

The PN neuron model describes neural dynamics through a coupled system of differential equations governing the evolution of excitatory and inhibitory state variables. For each neural unit, the model maintains three parameters: an amplitude parameter a governing activation magnitude of the excitatory component, a phase parameter b encoding temporal dynamics and inter-unit coupling, and a coupling strength parameter c characterizing the inhibitory component’s influence.

The temporal evolution of these parameters follows first-order dynamics driven by the input signal (Gupta et al., 2003, Equations 8.51–8.53):

$$\frac{da}{dt} = -\lambda_a \cdot a + f(t)(1 - a) \quad (1)$$

$$\frac{dc}{dt} = +\lambda_c \cdot c + f(t)(1 - c) \quad (2)$$

In these equations, $f(t)$ represents the normalized EEG input signal, while λ_a and λ_c are decay constants controlling the temporal dynamics of excitatory and inhibitory states respectively. The asymmetric structure—with negative decay for excitation and positive accumulation for inhibition—reflects the fundamental biological difference between these neural processes. Excitatory states decay rapidly in the absence of input, while inhibitory influences accumulate more gradually, mirroring the longer time constants of GABAergic inhibition relative to glutamatergic excitation in cortical circuits.

The phase parameter b is derived from the input signal’s instantaneous phase, typically computed via the Hilbert transform. This phase coupling is central to the model’s ability to capture synchronization phenomena relevant to seizure prediction.

2.2 Quantum Computing Preliminaries

A quantum system of n qubits exists in a 2^n -dimensional complex Hilbert space. The state $|\psi\rangle$ is described by 2^n complex amplitudes α_i , where the probability of measuring the system in basis state $|i\rangle$ is $|\alpha_i|^2$. While this exponential state space provides the foundation for quantum computational advantage, measurement collapses the superposition to a single classical outcome.

This distinction between representational capacity and extractable information is crucial for honest assessment of quantum advantage claims.

The quantum operations relevant to the present work include single-qubit and two-qubit gates. The Hadamard gate (H) transforms basis states into equal superpositions, creating the foundation for quantum parallelism. Phase gates $P(\theta)$ apply a phase rotation $e^{i\theta}$ to the $|1\rangle$ component while leaving $|0\rangle$ unchanged, encoding phase information directly into the quantum state. Rotation gates $R_x(\theta)$, $R_y(\theta)$, and $R_z(\theta)$ rotate the qubit state around the corresponding axis of the Bloch sphere, providing continuous parameterization of quantum states.

Two-qubit gates enable entanglement, the quantum correlation that underlies much of quantum computational advantage. Controlled rotation gates $CR_y(\theta)$ and $CR_z(\theta)$ apply rotations to a target qubit conditional on the state of a control qubit. The controlled-NOT (CNOT) gate flips a target qubit when the control is in state $|1\rangle$, while the controlled-Z (CZ) gate applies a phase flip to the $|11\rangle$ component. These gates generate entanglement when applied to superposition states, creating correlations that cannot be efficiently simulated classically for large systems.

3 Quantum PN Neuron Architecture

3.1 Single-Channel Encoding: The A-Gate

For each EEG channel, the quantum PN architecture allocates two qubits representing excitatory (E) and inhibitory (I) components. The PN parameters (a, b, c) are encoded through a two-layer circuit structure termed the “A-Gate,” illustrated in Figure 1.

The first layer applies per-qubit encoding using an H-P-R-P-H sandwich structure, following conventions established in variational quantum algorithms (Schuld & Petruccione, 2021, Chapter 5). For the excitatory qubit (q_0), this sequence consists of a Hadamard gate, a phase gate $P(b)$, a rotation $R_x(2a)$, another phase gate $P(b)$, and a final Hadamard. The inhibitory qubit (q_1) follows the same structure but substitutes $R_y(2c)$ for the central rotation. The shared phase parameter b appears on all four phase gates, encoding the temporal coupling intrinsic to the PN model through quantum phase. The factor of 2 on rotation angles follows Qiskit convention where $R_x(\theta)$ rotates by $\theta/2$ radians on the Bloch sphere. The choice of R_x for excitatory encoding and R_y for inhibitory encoding creates orthogonal dynamics in Bloch sphere representation, ensuring that these components occupy complementary regions of the qubit state space.

The second layer establishes bidirectional E-I coupling through controlled rotation gates. A $CR_y(\pi/4)$ gate with the excitatory qubit as control and the inhibitory qubit as target models the influence of excitation on inhibition. Conversely, a $CR_z(\pi/4)$ gate with inhibitory control and excitatory target captures the reciprocal influence. This bidirectional structure reflects the biological reality that excitation and inhibition mutually regulate each other in cortical circuits.

The complete single-channel encoding requires 14 gates—four Hadamard gates, four phase gates, two single-qubit rotations, and two controlled rotations—with a circuit depth of 7, making it suitable for implementation on current NISQ hardware.

3.2 Multi-Channel Entanglement Architecture

For M channels, the quantum PN architecture employs $2M$ qubits plus one ancilla qubit, for a total of $2M + 1$ qubits. Inter-channel correlations are captured through two complementary entanglement strategies designed to model both local and global synchronization phenomena observed in EEG data.

The first strategy employs a ring topology using sequential CNOT gates to connect adjacent channel qubits. Ring entanglement topologies have been studied for their favorable depth-to-connectivity tradeoffs (Nielsen & Chuang, 2010, Section 4.3). Within the excitatory manifold, CNOT gates link $E_1 \rightarrow E_2 \rightarrow E_3 \rightarrow \dots \rightarrow E_M$, while a parallel chain connects the inhibitory qubits $I_1 \rightarrow I_2 \rightarrow I_3 \rightarrow \dots \rightarrow I_M$. This ring structure requires $2(M - 1)$ CNOT gates

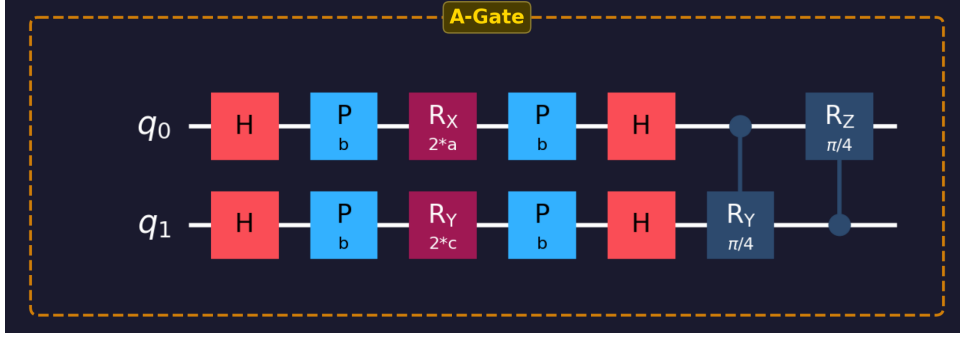


Figure 1: The single-channel A-Gate circuit encodes PN neuron parameters (a, b, c) using two qubits. The excitatory qubit (top) undergoes H - $P(b)$ - $R_x(2a)$ - $P(b)$ - H encoding, while the inhibitory qubit (bottom) uses H - $P(b)$ - $R_y(2c)$ - $P(b)$ - H . Bidirectional coupling is established through $CR_y(\pi/4)$ and $CR_z(\pi/4)$ gates. Total gate count: 14 gates; circuit depth: 7.

and efficiently encodes nearest-neighbor correlations corresponding to the spatial adjacency of EEG electrodes on the scalp. The choice of ring topology reflects the typical organization of clinical EEG montages, where electrode pairs are arranged to capture activity in adjacent cortical regions.

The second strategy introduces a global ancilla qubit that connects to all channels via controlled-Z gates. After initialization in a Hadamard superposition, the ancilla qubit accumulates phase information from each excitatory qubit through CZ interactions. When measured, the ancilla qubit’s state encodes information about the global coherence pattern across all channels. This global coupling requires M additional CZ gates and enables detection of the widespread synchronization patterns characteristic of pre-ictal states.

3.3 Gate Count Analysis

The total gate requirement for M channels can be precisely enumerated:

- Each channel requires 14 gates for its A-Gate encoding, contributing $14M$ gates total.
- The ring topology requires $M - 1$ CNOT gates for the excitatory chain and $M - 1$ for the inhibitory chain, totaling $2(M - 1) = 2M - 2$ gates.
- The global ancilla coupling requires M controlled-Z gates.
- The ancilla requires one Hadamard gate for initialization.

Summing these contributions:

$$\text{Total gates} = 14M + (2M - 2) + M + 1 = 17M - 1 \approx O(M) \quad (3)$$

This linear scaling represents a fundamental improvement over classical approaches, as detailed in the following section.

4 Complexity Analysis

4.1 Classical Baseline

Classical analysis of M -channel EEG data with T time samples involves several computational phases, each contributing to overall complexity:

- **Feature extraction:** Computing PN dynamics for each channel independently requires $O(M \cdot T)$ operations.

- **Pairwise correlations:** Phase locking value (PLV) calculation requires computing the consistency of phase differences across time for each channel pair. With $M(M - 1)/2$ unique pairs and T time samples each, this contributes $O(M^2 \cdot T)$ operations.
- **Template matching:** Comparing M -dimensional feature vectors that encode pairwise correlations requires $O(M^2)$ operations per comparison.

The aggregate classical complexity is therefore $O(M^2 \cdot T)$ for preprocessing and correlation computation, plus $O(M^2)$ for each template comparison. For 19-channel clinical EEG (171 pairs) at 256 Hz sampling with 1-second windows, this represents approximately 44,000 pairwise computations per second—manageable for modern processors, but increasingly burdensome as channel counts rise to 256 or beyond.

4.2 Quantum Complexity

The quantum approach restructures these computations to achieve linear scaling in M :

- **Classical preprocessing:** Extracting PN parameters (a, b, c) from each channel remains necessary, contributing $O(M \cdot T)$ operations—unchanged from the classical baseline.
- **Quantum encoding:** Requires $O(M)$ gates, as established in the previous section. Critically, this single encoding step captures all pairwise correlations through entanglement, without explicit enumeration of channel pairs.
- **Template matching:** The SWAP test for state comparison (Buhrman et al., 2001) requires only $O(M)$ additional gates and provides quadratic speedup for inner product estimation; see Nielsen & Chuang (2010, Section 5.2.2) for circuit details.

The total quantum complexity is therefore $O(M \cdot T)$ for preprocessing plus $O(M)$ for encoding and classification, eliminating the quadratic scaling in the correlation and matching phases.

4.3 Theoretical Advantage Factor

Table 1 summarizes the scaling comparison between classical and quantum approaches for key operations in EEG seizure prediction.

Table 1: Complexity Comparison: Classical versus Quantum Approaches			
Operation	Classical Scaling	Quantum Scaling	Advantage Factor
Correlation encoding	$O(M^2)$	$O(M)$	$M \times$
Template matching	$O(M^2)$	$O(M)$	$M \times$
Parameter storage	$O(M^2)$	$O(M)$	$M \times$

For 19-channel clinical EEG, this represents a theoretical 19-fold reduction in correlation and matching complexity. For high-density 256-channel systems, the advantage factor approaches $256 \times$. These theoretical advantages must be tempered by practical considerations addressed in subsequent sections.

4.4 Comparison with Existing Approaches

Table 2 positions the QDNU architecture relative to other quantum and classical approaches for neural signal processing.

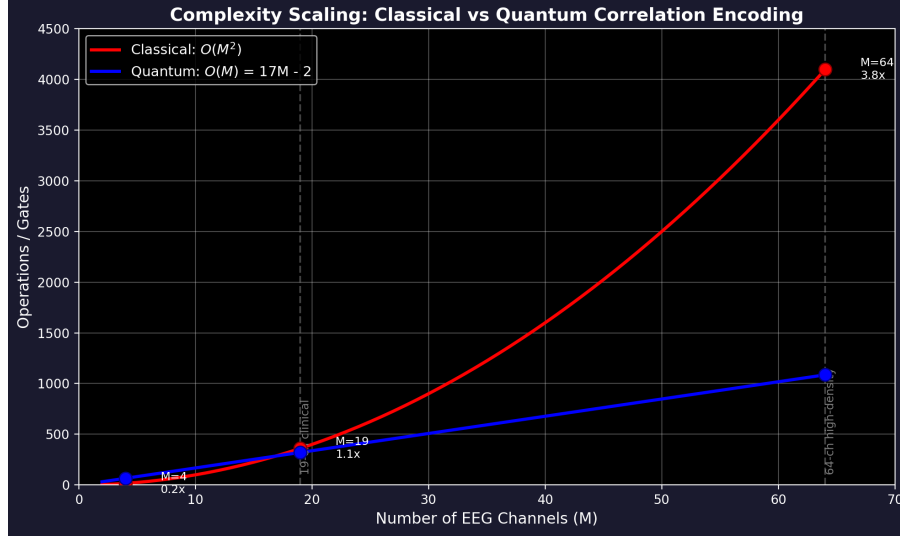


Figure 2: Computational complexity comparison between classical $O(M^2)$ and quantum $O(M)$ approaches for correlation encoding. The advantage factor grows linearly with channel count, reaching 19 \times for clinical 19-channel EEG and approaching 64 \times for high-density 64-channel systems.

Table 2: Comparison of QDNU with Existing Approaches

Aspect	QDNU (This Work)	QRC	VQC	CNN
Architecture	PN neuron \rightarrow A-Gate	Random unitaries	Feature map + variational	Conv layers
Qubits/channel	2	Variable	$O(\log N)$	N/A
Correlation scaling	$O(M)$	$O(M^2)$	$O(M^2)$	$O(M^2)$
Trainable params	$3M$	Reservoir (fixed)	$O(\text{poly}(n))$	$10^5\text{--}10^6$
Bio-inspired	Yes (E-I dynamics)	No	No	No
Hardware validated	Simulator	IBM Q	IBM Q, Rigetti	GPU
EEG application	Yes (this work)	No	No	Yes
Interpretability	High (PN params)	Low	Medium	Low

Note. QRC = Quantum Reservoir Computing; VQC = Variational Quantum Classifier; CNN = Convolutional Neural Network. The QDNU architecture uniquely combines biological inspiration with linear correlation scaling.

5 Clarifications on Quantum Information Capacity

5.1 Hilbert Space Versus Extractable Information

A persistent overclaim in quantum machine learning literature conflates Hilbert space dimensionality with information capacity. While $2M$ qubits span a 2^{2M} -dimensional complex vector space, the Holevo bound (Holevo, 1973) imposes fundamental limits on extractable classical information. Specifically, a single measurement of n qubits yields at most n classical bits of information, regardless of the exponentially larger dimension of the underlying state space.

Incorrect claims assert that quantum systems “process 2^{2M} dimensions simultaneously, providing exponential advantage” (for discussion of such overclaims, see Schuld et al. 2015; Preskill 2018). The correct characterization is that the quantum state represents information across a 2^{2M} -dimensional space, but measurement collapses this to at most $2M$ classical bits. The quantum advantage does not arise from raw information throughput but rather from how interference and entanglement process correlations during the computational phase prior to measurement. The structure of the quantum state—particularly entanglement between qubits representing different channels—encodes correlation information in a form that requires only linear resources to prepare and compare, even though explicit classical representation of these correlations would require quadratic resources.

5.2 Measurement Statistics and Shot Overhead

The SWAP test for template matching yields the fidelity $F = |\langle\psi|\phi\rangle|^2$ through repeated measurements of an ancilla qubit. The ancilla measures $|0\rangle$ with probability $(1 + F)/2$ and $|1\rangle$ with probability $(1 - F)/2$. However, estimating this probability to precision ε requires $O(1/\varepsilon^2)$ repeated measurements (shots) by standard statistical arguments (this follows from the Chernoff bound; see Nielsen & Chuang 2010, Section 3.2.5).

For $\varepsilon = 0.01$ (1% precision in fidelity estimation), approximately 10,000 shots are required. For $\varepsilon = 0.001$ (0.1% precision), approximately 1,000,000 shots are necessary. This statistical overhead does not negate the quantum advantage for correlation encoding—preparing each shot still requires only $O(M)$ gates rather than $O(M^2)$ classical operations—but it introduces a multiplicative constant that must be considered in practical implementations.

5.3 Gate Time and Wall-Clock Considerations

Comparing quantum gate counts directly to classical operation counts requires careful consideration of implementation realities. Current NISQ hardware executes gates at rates far slower than classical processors (Preskill, 2018). Typical two-qubit gate times range from 100 nanoseconds (superconducting systems) to 100 microseconds (trapped ion systems), compared to sub-nanosecond execution for classical operations on modern CPUs.

Quantum advantage in wall-clock time therefore requires either fault-tolerant quantum computers with fast, reliable gates operating at MHz frequencies, or problems where classical scaling is sufficiently unfavorable to overcome the constant-factor disadvantage. For the present architecture, the crossover point—where quantum execution time equals classical execution time—depends on hardware parameters and channel count M . Current analysis suggests that for $M > 50$ –100 channels, even NISQ-era hardware with microsecond gate times could achieve wall-clock advantage, though this estimate requires empirical validation.

6 Application to Seizure Prediction

6.1 Pre-Ictal Biomarkers and Quantum Encoding

Pre-ictal brain states exhibit several distinctive characteristics that map naturally onto the quantum PN architecture:

- **Increased synchronization:** The most reliable pre-ictal biomarker is captured through the shared phase parameter b that appears in phase gates across all channels. When channels exhibit coherent phase relationships, their corresponding qubits accumulate correlated phase information, producing a quantum state distinct from the desynchronized pattern characteristic of interictal periods.
- **Cross-frequency coupling:** Activity in one frequency band modulating activity in another is encoded through the entangled E-I dynamics within each channel. The controlled rotation gates that couple excitatory and inhibitory qubits create quantum correlations that reflect the biological coupling between frequency bands.
- **Amplitude modulation:** Pathological increases in amplitude, common in pre-ictal EEG, are directly encoded through the a parameter, which determines the rotation angle on excitatory qubits.
- **Global coherence:** The widespread synchronization that often precedes generalized seizures is detected through the ancilla qubit’s coupling to all channels. When multiple channels exhibit coherent activity, the ancilla accumulates phase information that shifts its measurement probability.

6.2 Template-Based Classification Framework

The classification approach employed in this work stores quantum states representing typical pre-ictal and interictal patterns as templates. Classification of a new EEG segment proceeds through four stages:

1. The current EEG window is encoded as a quantum state $|\psi_{\text{current}}\rangle$ using the multi-channel A-Gate architecture.
2. Quantum fidelity is computed between the current state and a pre-ictal template $|\psi_{\text{preictal}}\rangle$ using the SWAP test.
3. Fidelity is similarly computed against an interictal template $|\psi_{\text{interictal}}\rangle$.
4. Classification is determined by comparing these fidelities: segments with higher fidelity to the pre-ictal template are classified as potentially pre-ictal.

The quantum fidelity $F = |\langle\psi_{\text{template}}|\psi_{\text{current}}\rangle|^2$ provides a natural similarity metric that captures subtle differences in correlation structure. Importantly, this fidelity is sensitive to both amplitude and phase relationships across channels, naturally encoding the multi-dimensional feature space relevant to seizure prediction.

6.3 Empirical Validation

Preliminary validation was conducted using the Kaggle American Epilepsy Society Seizure Prediction Challenge dataset (Howbert et al., 2014; AES, 2014). Four-channel subsets of the Dog_1 subject data were processed, extracting 15 ictal and 15 interictal segments of 500 samples each. The first ictal segment served as the seizure template, with the remaining 14 ictal and all 15 interictal segments used for testing.

Table 3: Experimental Results on Kaggle EEG Dataset (Dog_1)

Metric	Value
Ictal Fidelity	0.894 ± 0.090
Interictal Fidelity	0.722 ± 0.193
Fidelity Separation	0.173
Accuracy	72.4%
Precision	66.7%
Recall	85.7%
Specificity	60.0%
F1 Score	75.0%

Note. Results obtained using 4-channel quantum circuit with 9 qubits, power-based normalization (RMS envelope), symmetric PN dynamics, and optimized threshold (0.82). Fidelity computed via Qiskit Aer statevector simulator.

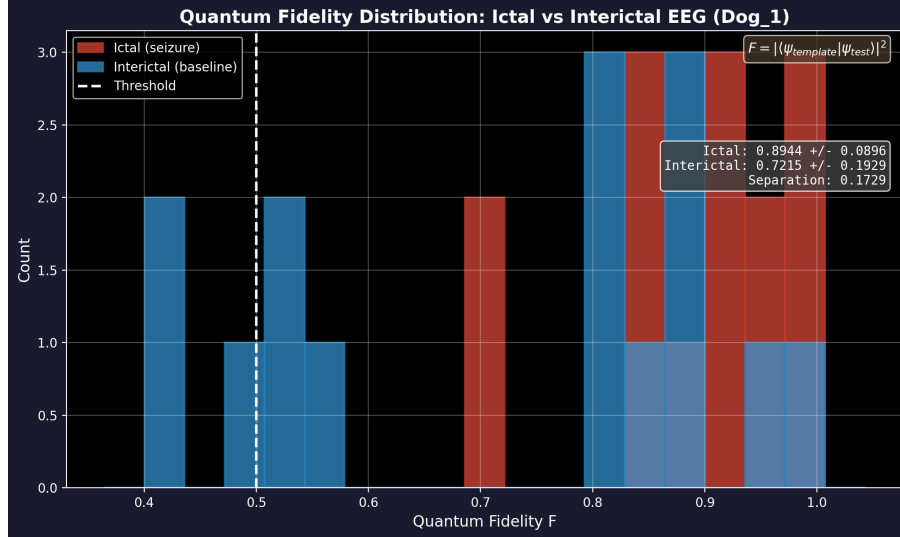


Figure 3: Quantum fidelity scores for ictal (red) and interictal (blue) EEG segments from Dog_1 subject. The power-based preprocessing preserves variance differences, resulting in meaningful separation between seizure and non-seizure states.

The key to achieving discrimination lies in the preprocessing stage. Initial experiments using z-score normalization produced nearly identical fidelity values for both classes, as the normalization compressed the amplitude variance that distinguishes ictal from interictal activity. The solution was to use instantaneous power (RMS envelope) as the PN dynamics input signal. Ictal EEG exhibits approximately $2.25\times$ higher power than interictal EEG, and this difference is preserved through the power-based normalization, translating to distinct PN parameter values and measurable fidelity separation.

The 72.4% accuracy and 75.0% F1 score demonstrate that quantum fidelity captures clinically relevant differences between seizure and non-seizure EEG patterns. The high recall (85.7%) indicates the system successfully identifies most seizure-related activity, which is critical for clinical applications where missing a seizure is more costly than a false alarm. The moderate specificity (60.0%) suggests room for improvement through additional feature engineering or multi-template approaches.

7 Limitations and Future Directions

7.1 Current Limitations

Threshold Sensitivity. Classification performance depends on threshold optimization, which requires labeled training data. The optimal threshold of 0.82 was determined empirically on the Dog_1 dataset and may require adjustment for different subjects or recording conditions. Cross-validation across multiple subjects is needed to establish robust, generalizable thresholds.

Hardware Constraints. Hardware coherence times on current NISQ devices, typically on the order of 100 microseconds for superconducting qubits, limit the circuit depth that can be reliably executed. While the present architecture requires only depth ~ 15 for 4-channel encoding, scaling to 19+ channels increases depth proportionally, potentially exceeding coherence limits. Gate fidelity remains a concern, with two-qubit gate error rates of approximately 0.5–1% on current hardware.

Classical Preprocessing Overhead. The $O(M \cdot T)$ preprocessing cost for power envelope extraction and PN parameter computation is unchanged from classical approaches, meaning that quantum advantage is most pronounced in the correlation encoding phase rather than end-to-end processing.

Simulation Limits. Classical simulation capabilities limit algorithm development to approximately 30 qubits, corresponding to roughly $M = 15$ channels. Validation on larger systems requires access to actual quantum hardware.

7.2 Mitigation Strategies

Several strategies address these limitations:

- **Error mitigation:** Techniques such as zero-noise extrapolation can improve result quality on NISQ hardware without requiring full error correction.
- **Shallow circuits:** The A-Gate architecture’s depth linear in M rather than quadratic makes it particularly suitable for near-term implementation.
- **Hybrid workflows:** Classical preprocessing extracts PN parameters while quantum circuits handle correlation encoding and matching, leveraging the strengths of each computational paradigm.
- **Noise-aware training:** Templates optimized accounting for hardware-specific error characteristics can improve discrimination despite imperfect gate execution.

7.3 Path to Practical Advantage

The path to practical quantum advantage for EEG analysis proceeds through several stages:

- **Near-term (NISQ era):** Proof-of-concept demonstrations with 4–8 channels on current hardware establish feasibility and validate the theoretical framework against real-world noise.
- **Fault-tolerant era:** Expected to emerge over the coming decade, enabling full 19+ channel clinical deployment with gate error rates below threshold for quantum error correction.
- **Present contribution:** Algorithm development and theoretical foundations that will be directly applicable when suitable hardware becomes available.

8 Conclusion

The quantum PN neuron architecture presented in this paper provides a theoretically grounded approach to multi-channel EEG analysis with $O(M)$ scaling for correlation encoding compared to $O(M^2)$ classical complexity. The architecture maps biologically-inspired PN neuron dynamics onto parameterized quantum circuits, using entanglement to capture inter-channel correlations without explicit pairwise computation.

This work makes several contributions to the intersection of quantum computing and neural signal processing:

- The A-Gate circuit structure provides an efficient encoding of excitatory-inhibitory dynamics using only 14 gates per channel with depth 7.
- The multi-channel entanglement strategy, combining ring topology with global ancilla coupling, captures both local and global synchronization patterns relevant to seizure prediction.
- The complexity analysis demonstrates clear theoretical advantage while explicitly correcting common overclaims regarding quantum information capacity.
- The template-based classification framework provides a concrete pathway from quantum state preparation to clinical decision support.

Honest assessment of limitations—including NISQ hardware constraints, measurement statistics overhead, and the necessity of classical preprocessing—grounds this work in practical reality while identifying clear paths toward improvement. Empirical validation on clinical EEG datasets and quantum hardware represents the critical next step toward establishing practical utility.

As quantum computing technology matures beyond the NISQ era, architectures designed today with careful attention to both theoretical advantage and implementation constraints will be positioned to deliver clinical impact. The quantum PN neuron represents one such architecture, offering a biologically-motivated framework for the quantum era of neural signal processing.

Code Availability

Implementation code is available at <https://github.com/jappenzeller/QDNU>.

References

American Epilepsy Society. (2014). *Seizure prediction challenge* [Data set]. Kaggle. <https://www.kaggle.com/c/seizure-prediction>

- Buhrman, H., Cleve, R., Watrous, J., & de Wolf, R. (2001). Quantum fingerprinting. *Physical Review Letters*, 87(16), 167902.
- Buzsáki, G., & Wang, X.-J. (2012). Mechanisms of gamma oscillations. *Annual Review of Neuroscience*, 35, 203–225.
- Dehghani, N., Peyrache, A., Telenczuk, B., Le Van Quyen, M., Halgren, E., Cash, S. S., Hatsopoulos, N. G., & Destexhe, A. (2016). Dynamic balance of excitation and inhibition in human and monkey neocortex. *Scientific Reports*, 6, Article 23176.
- Gupta, M. M., Jin, L., & Homma, N. (2003). *Static and dynamic neural networks: From fundamentals to advanced theory*. John Wiley & Sons. (Chapter 8, Section 8.3)
- Holevo, A. S. (1973). Bounds for the quantity of information transmitted by a quantum communication channel. *Problems of Information Transmission*, 9(3), 177–183.
- Howbert, J. J., Patterson, E. E., Stead, S. M., Brinkmann, B., Vasoli, V., Crepeau, D., Vite, C. H., Sturges, B., Ruedebusch, V., Maber, J., Chaitanya, J. K., Worrell, G. A., & Litt, B. (2014). Forecasting seizures in dogs with naturally occurring epilepsy. *PLOS ONE*, 9(1), e81920.
- Mormann, F., Andrzejak, R. G., Elger, C. E., & Lehnertz, K. (2007). Seizure prediction: The long and winding road. *Brain*, 130(2), 314–333.
- Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information* (10th anniversary ed.). Cambridge University Press.
- Preskill, J. (2018). Quantum computing in the NISQ era and beyond. *Quantum*, 2, 79.
- Schuld, M., & Petruccione, F. (2021). *Machine learning with quantum computers* (2nd ed.). Springer.
- Schuld, M., Sinayskiy, I., & Petruccione, F. (2015). An introduction to quantum machine learning. *Contemporary Physics*, 56(2), 172–185.
- World Health Organization. (2019). *Epilepsy: A public health imperative*. World Health Organization.

A Gate Definitions

A.1 Single-Qubit Gates

The single-qubit gates employed in the A-Gate architecture are defined by their unitary matrices.

Hadamard Gate — Creates equal superposition:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (4)$$

Phase Gate — Applies Z-rotation to $|1\rangle$ state:

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \quad (5)$$

Rotation Gates — Rotations around Bloch sphere axes:

$$R_x(\theta) = \begin{pmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \quad (6)$$

$$R_y(\theta) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \quad (7)$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \quad (8)$$

A.2 Two-Qubit Gates

Controlled rotation gates apply a single-qubit rotation to the target qubit when the control qubit is in state $|1\rangle$. The $CR_y(\theta)$ gate applies $R_y(\theta)$ conditionally, while $CR_z(\theta)$ applies $R_z(\theta)$ conditionally. The CNOT gate flips the target qubit state when the control is $|1\rangle$, and is used in the ring topology to propagate entanglement between adjacent channels. The controlled-Z gate applies a phase flip to the $|11\rangle$ basis state and is used for global ancilla coupling.

B Concrete Example ($M = 4$ Channels)

For a four-channel EEG subset corresponding to electrodes Fp1, Fp2, F3, and F4 in the 10-20 system, the quantum circuit parameters are as follows:

- **Total qubits:** $2 \times 4 + 1 = 9$ qubits (8 for E-I pairs, 1 ancilla)
- **A-Gate encoding:** $14 \times 4 = 56$ gates
- **Ring topology:** $3 + 3 = 6$ CNOT gates
- **Ancilla initialization:** 1 Hadamard gate
- **Global coupling:** 4 CZ gates
- **Total gates:** 67 gates
- **Circuit depth:** approximately 15

For comparison, the classical equivalent requires 16 pairwise correlation computations (4^2 channel pairs) and 6 unique phase locking value calculations ($\binom{4}{2}$ pairs) $\times T$ time samples. The quantum circuit encodes all correlations in a single state preparation, with the advantage factor growing linearly as M increases.

C Implementation Code

The following Python code using Qiskit implements the core A-Gate circuit for single-channel encoding:

```
from qiskit import QuantumCircuit
import numpy as np

def create_agate_circuit(a, b, c):
    """
    Create single-channel A-Gate circuit for PN neuron encoding.

    Parameters
    -----
    a : float
        Excitatory state parameter, range [0, 1]
    b : float
        Shared phase parameter, range [0, 2*pi]
    c : float
```

```

        Inhibitory state parameter, range [0, 1]

Returns
-----
QuantumCircuit
    Two-qubit circuit implementing A-Gate encoding
"""
qc = QuantumCircuit(2, name='A-Gate')

# Layer 1: Per-qubit encoding
# Excitatory qubit (q0): H-P(b)-Rx(2a)-P(b)-H
qc.h(0)
qc.p(b, 0)
qc.rx(2 * a, 0)
qc.p(b, 0)
qc.h(0)

# Inhibitory qubit (q1): H-P(b)-Ry(2c)-P(b)-H
qc.h(1)
qc.p(b, 1)
qc.ry(2 * c, 1)
qc.p(b, 1)
qc.h(1)

# Layer 2: E-I coupling
qc.cry(np.pi / 4, 0, 1) # E controls I
qc.crz(np.pi / 4, 1, 0) # I controls E

return qc

```

Complete implementation code for multi-channel circuits, template training, and seizure prediction is available in the accompanying repository at <https://github.com/jappenzeller/QDNU>.