QMDA Homework 5

Generalized least squares

Consider a problem of the form $\mathbf{Gm} = \mathbf{d}$ and of size $N = M = 101$. The model parameters are a discretized function $m(x)$. The data kernel $\mathbf{G}$ computes data $d(x)$ that are a moving average of $m(x)$. However, this moving average differs from those presented in the lectures by having a "blur" that increases with x. Thus, $d(x{=}5)$ is a blurred version of $m(x)$ over the range $x{=}5\pm2$, whereas $d(x{=}90)$ is a blurred version of $m(x)$ for the range of x=90±10.

A. Load into Matlab 'QMDA_HW_05.mat' (in "Files/Homework" on CourseWorks), which defines variables 'N', 'M', 'x', 'G', and 'dobs'. Plot the data kernel $\mathbf{G}$ as a 2D image, describe how it looks and why it has the averaging properties described above. Plot the observed data $\mathbf{d}_{\text{obs}}$ as a function of x and describe them.

B. Estimate the standard deviation of the data errors σ_{e} by examining $\mathbf{d}_{\text{obs}}$ and assuming that the small-scale fluctuations of the data are mainly due to noise (these small-scale, random fluctuations are more prominent for $x > 50$). Assume also that all the data have the same error, so that the covariance of the data errors is $\mathbf{\Sigma}_{\text{e}} = \sigma_{\text{e}}^2\, \mathbf{I}$.

C. Apply generalized least squares to compute the posterior mean vector $\boldsymbol{\mu}_{\text{post}}$ and the covariance matrix $\mathbf{\Sigma}_{\text{post}}$. Use a prior mean of zero and a prior covariance matrix that is diagonal as in $\mathbf{\Sigma}_{\text{prior}} = \sigma_{\text{m}}^2\, \mathbf{I}$. The prior standard deviation σ_{m} quantifies how large are the expected fluctuations of the solution. Choose an initial value of σ_{m} based on the observed data $\mathbf{d}_{\text{obs}}$, which are just a smoothed version of $\mathbf{m}$. Compute the generalized least squares solution and plot two figures with

1. The posterior mean $\boldsymbol{\mu}_{\text{post}}$ as a function of x and two lines that encompass the 95% credible interval around the posterior mean (based on the posterior variances in the diagonal of $\mathbf{\Sigma}_{\text{post}}$);
2. The observed data $\mathbf{d}_{\text{obs}}$ and the data predicted by the posterior mean $\mathbf{d}_{\text{pred}} = \mathbf{G}\boldsymbol{\mu}_{\text{post}}$ as a function of x.

D. Experiment by changing the assumed standard deviation of the data errors σ_{e} and the prior standard deviation σ_{m}. Comment on what happens if you make σ_{e} very large or very small or if you make σ_{m} very large or very small.

E. Conclude by choosing final values of σ_{e} and σ_{m} that give a reasonably smooth posterior mean $\boldsymbol{\mu}_{\text{post}}$ and predicted data $\mathbf{d}_{\text{pred}}$ that are a good fit to $\mathbf{d}_{\text{obs}}$. Plot again the two figures described in step C for your final values of σ_{e} and σ_{m}.