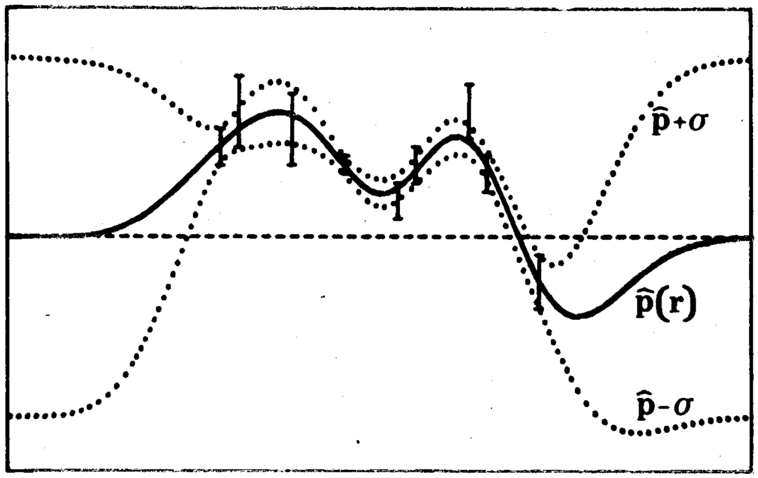
QMDA Homework 10

Interpolation by generalized least squares

The Courseworks file 'Files/Homework/QMDA_HW_10.mat' contains vectors 'd' (data values), 'xd' (x-coordinates of data points), and 'sdeve' (standard deviation of the measurement errors) that correspond to the $N = 9$ data points in Fig. 1 of 'Files/Lecture Notes/interpolation.pdf', reproduced below:



In this assignment, you will compute the generalized least squares interpolation of these data with different assumptions.

A) Write a program that calculates the generalized least squares interpolation. The interpolated values should be computed for $M = 500$ points with x evenly spaced between 0 and 1. Assume a prior mean for the interpolated values equal to zero, a prior variance equal to 1, a correlation length equal to 0.1, and a Gaussian autocovariance function. You should fill the matrices **A** and **B** described in 'interpolation.pdf' using the Matlab function 'acvf.m' to compute the autocovariance at a given lag. For example,

```
B(i,j) = acvf(xd(i)-xd(j), 1, 0.1, 'G');
```

Plot the interpolated posterior mean, its value ± one posterior standard deviation, the data points and their error bars as a function of x. Your plot should match the figure above.

B) Decrease the correlation length to 0.05 and repeat the calculation. How does the interpolation change compared to your result in A)? Explain considering the change in the autocovariance function.

C) Keeping the correlation length to 0.05, repeat the calculation with an exponential autocovariance (call `acvf()` with `'E'` as the last argument). How does the interpolation change compared to your result in B)? Explain considering the change in the autocovariance function.

D) For a Gaussian autocovariance and a correlation length of 0.05, repeat the calculation assuming that the data have zero measurement errors. Show that the interpolated posterior mean goes through each data point while the posterior standard deviation goes to zero at the data points.