Multivariate Temporal Modeling of Crime with Dynamic Linear Models

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https://forensicstats.org/

Outline

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- Chicago Crime Data
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  - Posterior (partial) correlations
- Discussion

Background

https://www.chicagomag.com/Chicago-Magazine/May-2014/Chicago-crime-rates/:

- Anecdotal evidence of misreporting
- Pressure to down-grade crimes
- Selective reporting

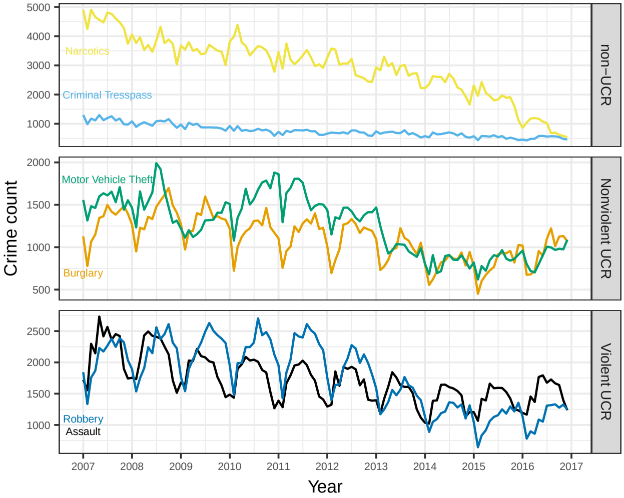
https://www.economist.com/democracy-in-america/2014/05/22/deceptive-numbers: specific hypothesis:

> *burglaries (tracked in UCR) are being misclassified as criminal trespasses (non-UCR)*

FBI's Uniform Crime Reporting (UCR) program:

> *[crime] data received from more than 18,000 city, university and college, county, state, tribal, and federal law enforcement agencies voluntarily participating in the program.*

Chicago crime data



Multivariate dynamic linear model

Let $Y_t = (Y_{1,t}, \ldots, Y_{C,t})$ where $Y_{c,t}$ is the log count for crime type c in month t.

Multivariate dynamic linear model:

$$\begin{array}{lrll} \text{observation:} & Y_t & = F_t\theta_t + \epsilon_t, & \epsilon_t \stackrel{ind}{\sim} N_6\left(0, \Sigma_\epsilon\right) \\ \text{evolution:} & \theta_t & = G_t\theta_{t-1} + \delta_t, & \delta_t \stackrel{ind}{\sim} N_*\left(0, \Sigma_\delta\right). \end{array}$$

where

$$\begin{array}{llll} F_t & = F & = & I_{6\times 6} \otimes (1,0,1,0,...,1,0)_{(2+2q)\times 1} \\ G_t & = G & = & I_{6\times 6} \otimes \text{blockdiag}(G_0, G_1, ..., G_q)_{(2+2q)\times(2+2q)}. \end{array}$$

The evolution is decomposed into a

$$\text{Linear trend: } G_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

and

$$\text{Seasonality: } G_j = \begin{bmatrix} \cos(\omega_j) & \sin(\omega_j) \\ -\sin(\omega_j) & \cos(\omega_j) \end{bmatrix}, \qquad \omega_j = 2\pi j/s$$

where $s = 12$ (due to 12 months) and $q = 4$ is the number of harmonics.

Bayesian Estimation in Stan

Decompose covariance matrices: $\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_6)\Omega\text{diag}(\sigma_1, \ldots, \sigma_6)$.

Priors (mutually independent):

- $\sigma_j \sim Ca^+(0, 1)$
- $p(\Omega) \propto 1$ implies slight peak at 0 correlation
- $\theta_0 \sim N_{6(2+2q)}(0, 10^7\text{I})$

Full posterior:

$$p(\theta, \Sigma_\epsilon, \Sigma_\delta | Y) \propto \left[\prod_{t=1}^{T} N(Y_t; F\theta_t, \Sigma_\epsilon) N(\theta_t; G\theta_{t-1}, \Sigma_\delta)\right] \left[\prod_j Ca^+(\sigma_j; 0, 1)\right] N(\theta_0; 0, 10^7 I)$$

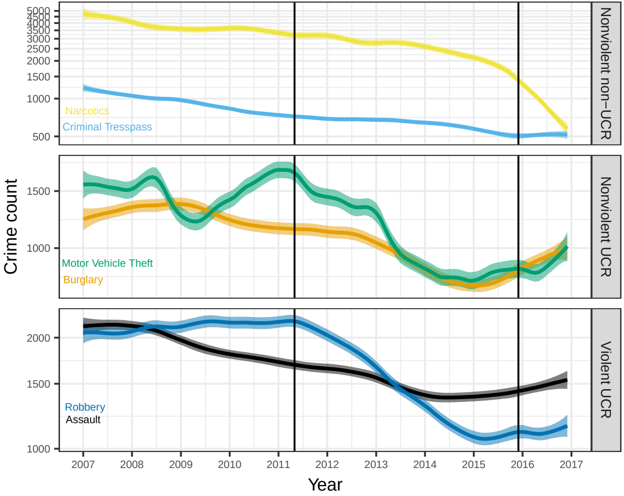
Stan (`rstan`): No-U-turn HMC sampler targets:

$$p(\Sigma_\epsilon, \Sigma_\delta | Y) = \int p(\theta, \Sigma_\epsilon, \Sigma_\delta | Y) d\theta$$

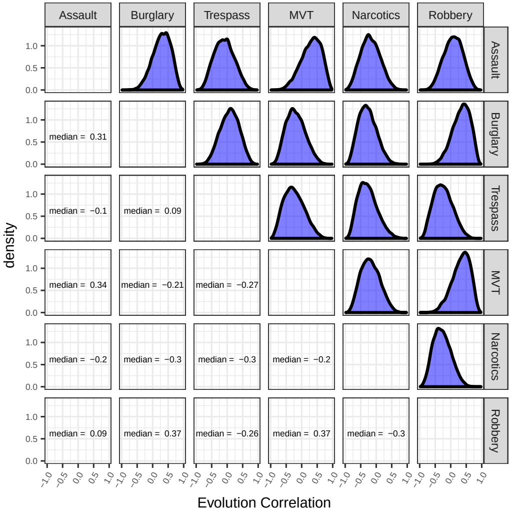
Use forward-filtering, backward-sampling (FFBS) algorithm (`dlm`) to sample

$$p(\theta | \Sigma_\epsilon, \Sigma_\delta, Y).$$

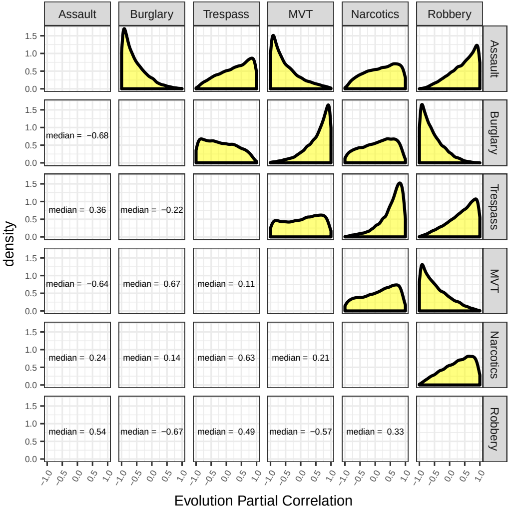
Standard crime trend analysis



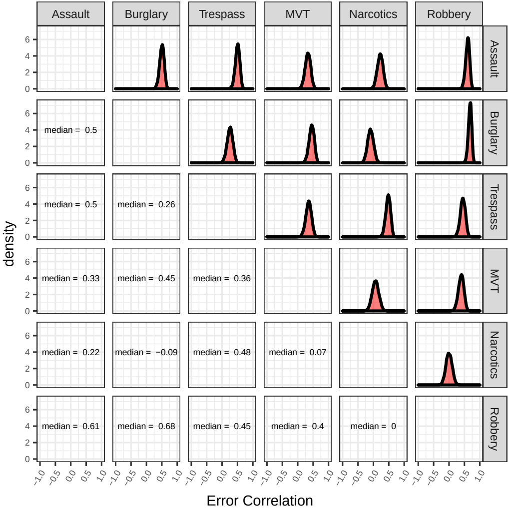
Evolution correlation



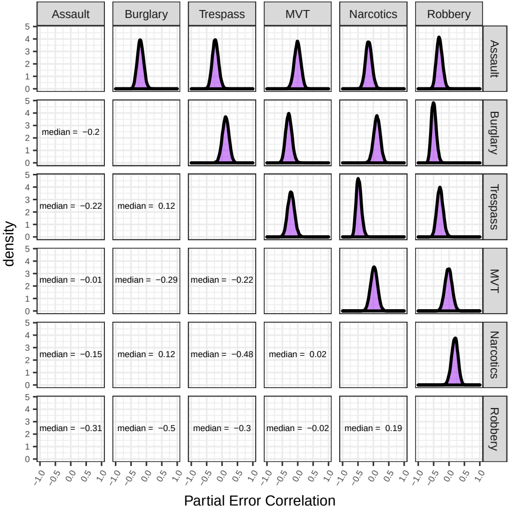
Evolution partial correlation



Error correlation



Error partial correlation



Summary

Built a multivariate DLM to model Chicago crime data to investigate a hypothesis that burglaries were being mis-classified as criminal trespasses, but did not find much evidence for this hypothesis.

These slides are available at

- https://github.com/jarad/JSM2020
- http://www.jarad.me/research/presentations.html
- https://github.com/330nategarton13/CrimeDLM.RPackage

Thank you!

Other links:

- https://www.youtube.com/jaradniemi
- https://twitter.com/jaradniemi