

# I06 - $p$ -values

STAT 5870 (Engineering)  
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# $p$ -value

A  $p$ -value is the probability of observing a statistic as or more extreme than observed if the model is true.

A  $p$ -value is the probability of observing a statistic as or more extreme than *the one you* observed if the model is true *when the data are considered random*.

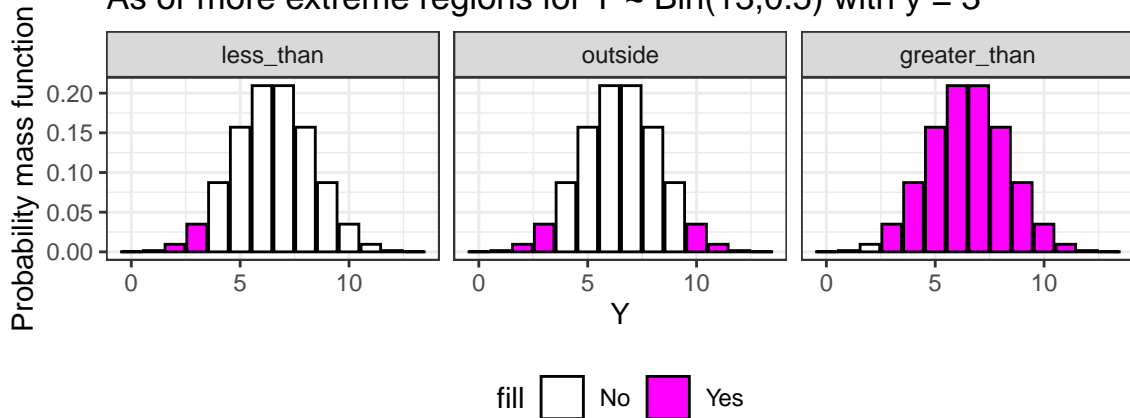
# Binomial model

Let  $H_0 : Y \sim \text{Bin}(13, 0.5)$  and observe  $y = 3$ .

Choose

- statistic is 3,
- its sampling distribution *when the model is true* is  $Y \sim \text{Bin}(13, 0.5)$ , and
- there are three *as or more extreme regions*:
  - $Y \leq 3$
  - $Y \geq 3$
  - $|Y - 13 \cdot 0.5| \geq |3 - 13 \cdot 0.5|$

## as or more extreme regions

As or more extreme regions for  $Y \sim \text{Bin}(13, 0.5)$  with  $y = 3$ 

# R Calculation

One-sided  $p$ -values:

- $P(Y \leq y)$ :

```
pbinom(y, size = n, prob = p)
[1] 0.04614258
```

- $P(Y \geq y) = 1 - P(Y < y) = 1 - P(Y \leq y - 1)$ :

```
1-pbinom(y-1, size = n, prob = p)
[1] 0.9887695
```

Two-sided  $p$ -value:

$$P(|Y - n\theta| \leq |y - n\theta|) = 2P(Y \leq y)$$

```
2*pbinom(y, size = n, prob = p)
[1] 0.09228516
```

## Normal model

Let  $H_0 : Y_i \sim N(3, 4^2)$  for  $i = 1, \dots, 6$  and you observe  $\bar{y} = 6.3$ ,  $s = 4.1$ , and

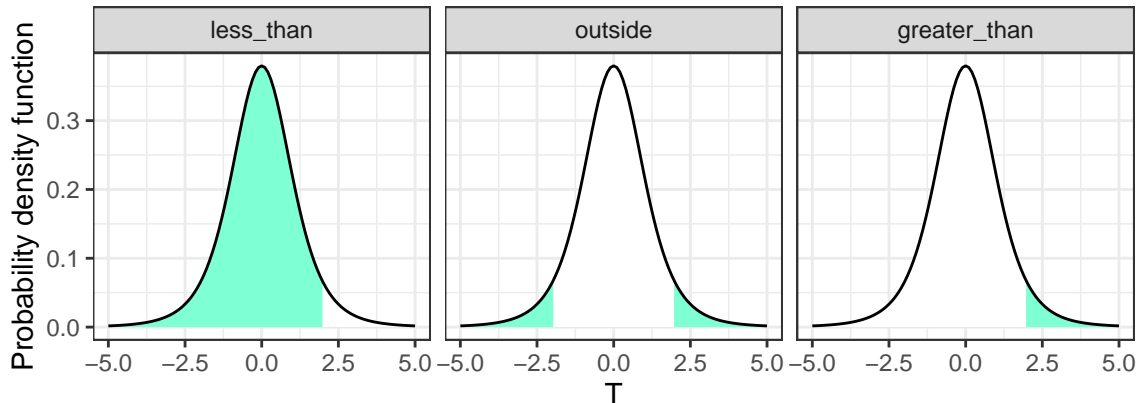
$$t = \frac{\bar{y} - 3}{s/\sqrt{n}} = \frac{6.3 - 3}{4.1/\sqrt{6}} = 1.97.$$

Choose

- $t$ -statistic  $t = 1.97$ ,
- its sampling distribution *when the model is true* is  $T_5 \sim t_5$ , and
- there are three *as or more extreme regions*:
  - $T_5 \leq 1.97$
  - $T_5 \geq 1.97$
  - $|T_5| \geq |1.97|$

## as or more extreme regions

As or more extreme regions for  $t = 1.97$  with 5 degrees of freedom



# R Calculation

- One-sided  $p$ -values:

- $P(T_5 \leq t)$ :

```
pt(t, df = n-1)
[1] 0.9471422
```

- $P(T_5 \geq t) = 1 - P(T_5 < t) = 1 - P(T_5 \leq t)$ :

```
1-pt(t, df = n-1)
[1] 0.05285775
```

- Two-sided  $p$ -value:

$$P(|T_5| \geq |t|) = 2P(T_5 \geq t)$$

```
2*(1-pt(t, df = n-1))
[1] 0.1057155
```



# Interpretation

Small  $p$ -values provide evidence that the data are incompatible with the model.

Recall

$$Y_i \stackrel{\text{ind}}{\sim} N(\mu, \sigma^2)$$

indicates the data

- are independent,
- are normally distributed,
- have a common mean, and
- have a common variance.

# Summary

- $p$ -value: the probability of observing a statistic as or more extreme than observed if the model is true
- small  $p$ -values provide evidence that the data are incompatible with the model