Inverse gamma distribution

STAT 587 (Engineering)
Iowa State University

March 30, 2021

Inverse gamma distribution

The random variable X has an inverse gamma distribution with

- shape parameter $\alpha > 0$ and
- scale parameter $\beta > 0$

if its probability density function is

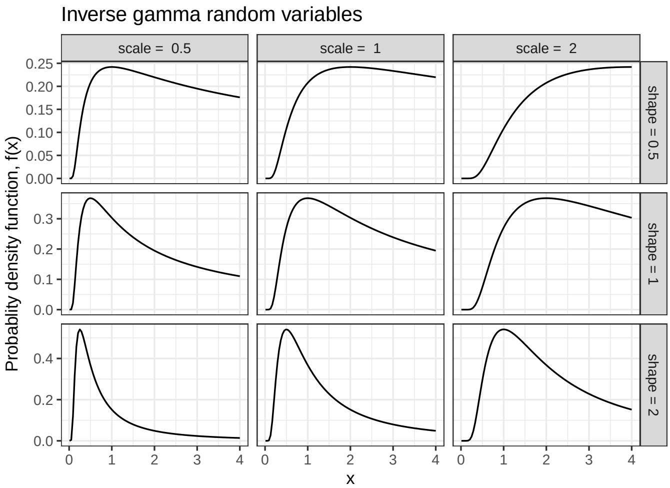
$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x} \, \mathrm{I}(x > 0).$$

where $\Gamma(\alpha)$ is the gamma function,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$

We write $X \sim IG(\alpha, \beta)$.

Inverse gamma probability density function



Inverse gamma mean and variance

If $X \sim IG(\alpha, \beta)$, then

$$E[X] = \int_0^\infty x \, \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x} dx = \cdots = \frac{\beta}{\alpha-1}, \quad \alpha > 1$$

and

$$\begin{aligned} Var[X] &= \int_0^\infty \left(x - \frac{\beta}{\alpha-1}\right)^2 \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x} dx \\ &= \cdots = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}, \quad \alpha > 2. \end{aligned}$$

Relationship to gamma distribution

If $X \sim Ga(\alpha, \lambda)$ where λ is the rate parameter, then

$$Y = \frac{1}{X} \sim IG(\alpha, \lambda).$$

Summary

Inverse gamma random variable

- $X \sim IG(\alpha, \beta)$, $\alpha, \beta > 0$
- $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x}, x > 0$
- $E[X] = \frac{\beta}{\alpha-1}$, $\alpha > 1$
- $Var[X] = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$