I06b - Correspondence between p-values and confidence intervals

STAT 5870 (Engineering)
Iowa State University

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p-values and confidence intervals

From the ASA statement on p-values:

> *a p-value is the probability under a specified statistical model that a statistical summary of the data would be equal to or more extreme than its observed value.*

A $100(1-a)\%$ confidence interval contains the true value of the parameter in $100(1-a)\%$ of the intervals constructed using the procedure.

Both are based on the sampling distribution.

Let $H_0 : \theta = \theta_0$,

- if p-value $< a$, then $100(1-a)\%$ CI will not contain θ_0 but
- if p-value $> a$, then $100(1-a)\%$ CI will contain θ_0.

Normal model

Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$ with $H_0 : \mu = \mu_0 = 1.5$.

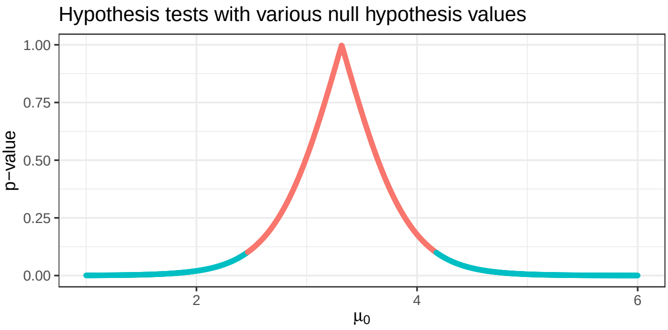
```
y = rnorm(10, mean = 3, sd = 1.5)
a = 0.05
t = t.test(y, mu = mu0, conf.level = 1-a)
t$p.value

[1] 0.003684087

round(as.numeric(t$conf.int),2)

[1] 2.26 4.37
```

```
a = 0.001
t = t.test(y, mu = mu0, conf.level = 1-a)
t$p.value

[1] 0.003684087

round(as.numeric(t$conf.int),2)

[1] 1.08 5.55
```

Explanation

Values for μ_0 that fail to reject H_0 at significance level a are precisely the $100(1-a)\%$ confidence interval.

```
a = 0.1
ci = t.test(y, conf.level = 1-a)$conf.int; round(as.numeric(ci),2)

[1] 2.46 4.17
```



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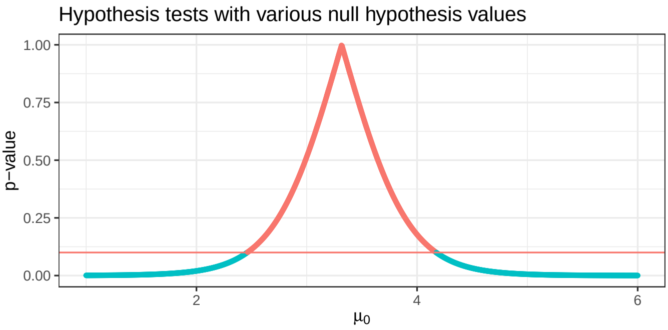

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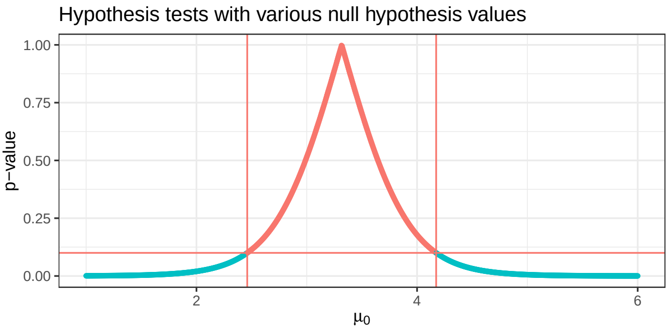

Importance

The population mean was significantly different than 1.5 (p = 0.004).

A 90% confidence interval for the population mean was (2.46, 4.17).

From the second statement, you know

- the p-value is less than 0.1 for any value outside the interval,
- a range of reasonable values for the population mean is given by the interval, and
- a measure of uncertainty given by the interval width and confidence level.