

Exponential distribution

STAT 5870 (Engineering)
Iowa State University

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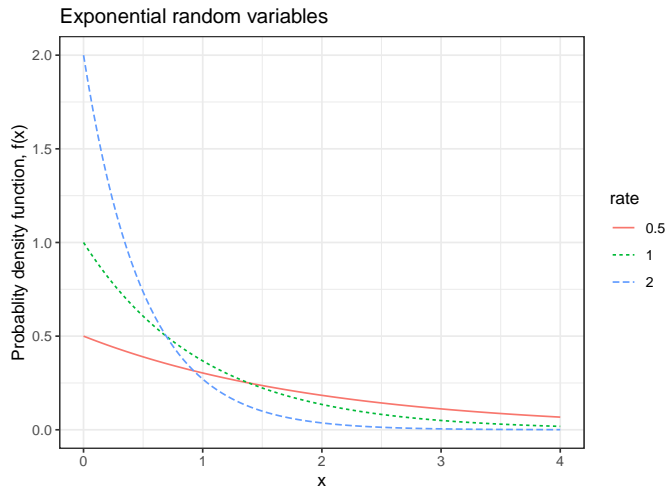
Exponential distribution

The random variable X has an **exponential distribution** with **rate parameter** $\lambda > 0$ if its probability density function is

$$p(x|\lambda) = \lambda e^{-\lambda x} \mathbf{I}(x > 0).$$

We write $X \sim \text{Exp}(\lambda)$.

Exponential probability density function



Exponential mean and variance

If $X \sim \text{Exp}(\lambda)$, then

$$E[X] = \int_0^{\infty} x \lambda e^{-\lambda x} dx = \dots = \frac{1}{\lambda}$$

and

$$\text{Var}[X] = \int_0^{\infty} \left(x - \frac{1}{\lambda}\right)^2 \lambda e^{-\lambda x} dx = \dots = \frac{1}{\lambda^2}.$$

Exponential cumulative distribution function

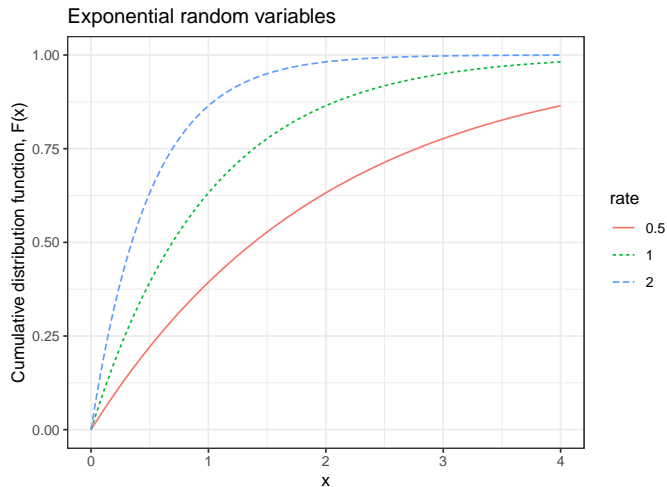
If $X \sim \text{Exp}(\lambda)$, then its cumulative distribution function is

$$F(x) = \int_0^x \lambda e^{-\lambda t} dt = \dots = 1 - e^{-\lambda x}.$$

The inverse cumulative distribution function is

$$F^{-1}(p) = \frac{-\log(1-p)}{\lambda}.$$

Exponential cumulative distribution function - graphically



Memoryless property

Let $X \sim \text{Exp}(\lambda)$, then

$$P(X > x + c | X > c) = P(X > x).$$

Parameterization by the scale

A common alternative parameterization of the exponential distribution uses the **scale** $\beta = \frac{1}{\lambda}$. In this parameterization, we have

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \mathbf{I}(x > 0)$$

and

$$E[X] = \beta \quad \text{and} \quad \text{Var}[X] = \beta^2.$$

Summary

Exponential random variable

- $X \sim \text{Exp}(\lambda), \lambda > 0$
- $f(x) = \lambda e^{-\lambda x}, x > 0$
- $F(x) = 1 - e^{-\lambda x}$
- $F^{-1}(p) = \frac{-\log(1-p)}{\lambda}$
- $E[X] = \frac{1}{\lambda}$
- $\text{Var}[X] = \frac{1}{\lambda^2}$