Teaching *p*-values

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ASA Statement on *p*-values: *p*-values can indicate how incompatible the data are with a specified statistical model.

Outline

- Bayesians vs Frequentists - not the point of today's talk
- ASA Statement on *p*-values
- STAT 226 hypothesis testing recipe
- Hypothesis testing false dichotomy
- Interpreting a *p*-value through Bayes rule

Bayesianism and Frequentism

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https://stats.stackexchange.com/questions/230097/think-like-a-bayesian-check-like-a-frequentist-what-does-that-mean:

> *Think like a Bayesian, check like a frequentist.*

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2. p-values do **NOT** measure the probability that the studied hypothesis is true, or the probability that the data were produced by random chance alone.
3. Scientific conclusions and business or policy decisions should **NOT** be based only on whether a p-value passes a specific threshold.
4. Proper inference requires full reporting and transparency.
5. A p-value, or statistical significance, does **NOT** measure the size of an effect or the importance of a result.
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Bold-face and capitalization have been added for emphasis; the original article bold-faced these sentences.

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     Conclude that the data most likely came from a population that has a mean different than μ_0.
   - If p-value is not small enough, **fail to reject null hypothesis**.

     The data lack sufficient evidence to conclude that the population mean is different than μ_0.

False dichotomy

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The false dichotomy is that the only two possibilities are

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- mean is μ_0.

We need to evaluate these assumptions before we conclude $\mu \neq m_0$ for the population of interest.

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```
shiny::runGitHub('jarad/pvalue')
```

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Summary

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This slides are available

- `https://github.com/jarad/pvalue2019`
- `http://www.jarad.me/research/presentations.html`