Rank-Sum Test

STAT 401 - Statistical Methods for Research Workers

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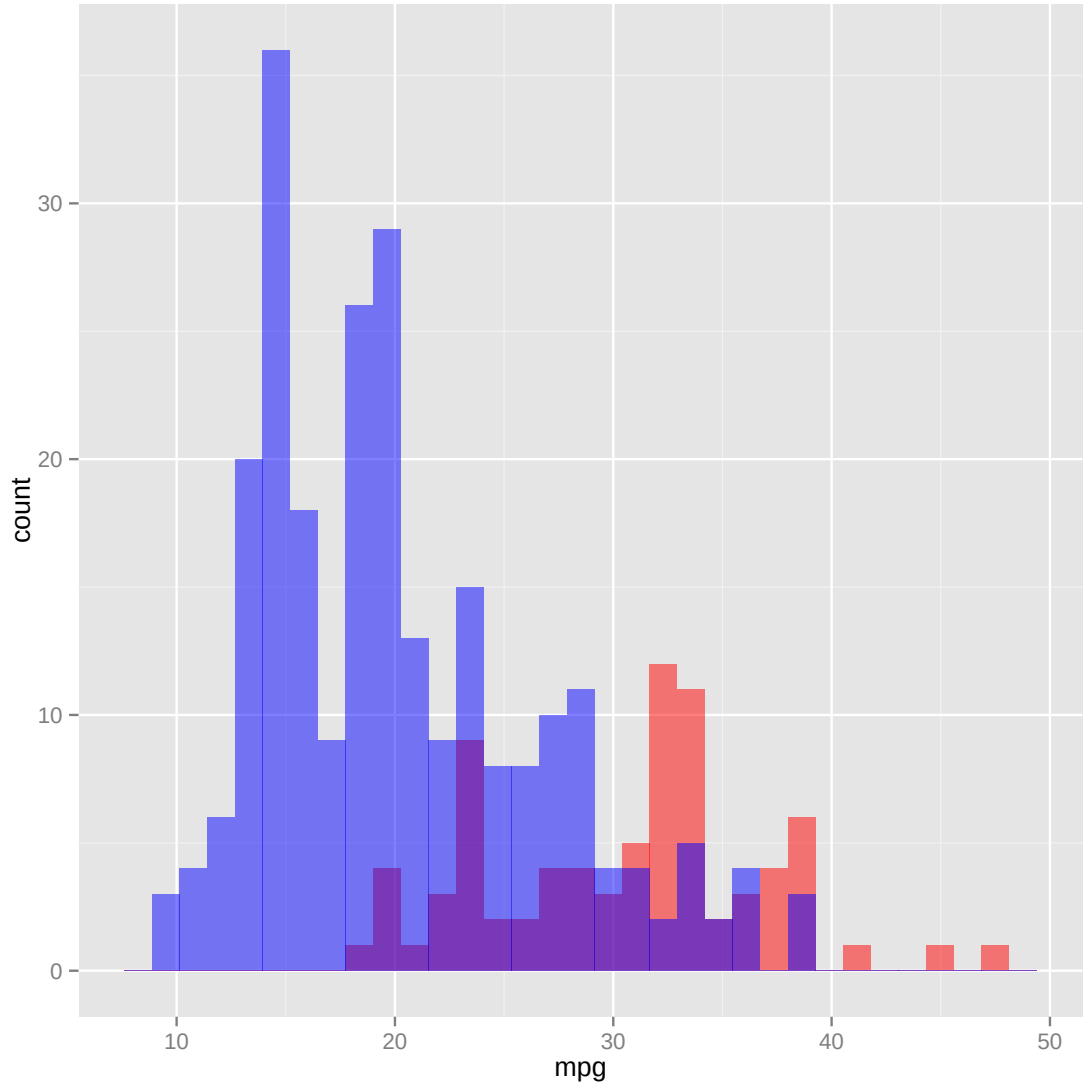
Do these data look normal?

Raw histogram of mpg for US and Japanese cars.



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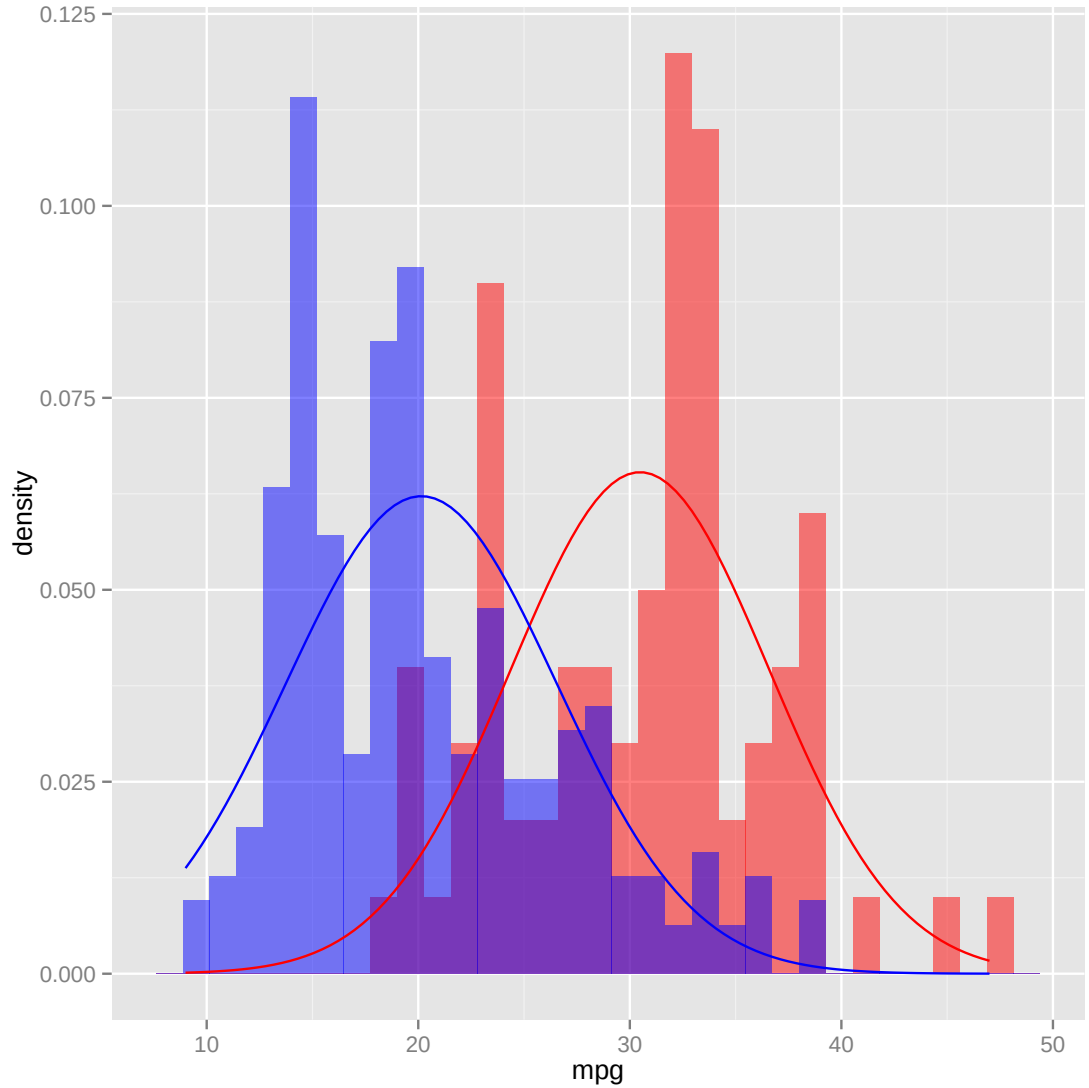
Compared to best fitting normal curves.



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  - s_R: standard deviation of the ranks
- Calculate $Z = (U + c - E[U])/SD(U)$ where c, the continuity correction, is either 0.5 or -0.5.
- Determine the pvalue using a standard normal distribution.

Example on a small dataset

	mpg	country	rank
1	13	US	1
2	15	US	2
3	17	US	3
4	22	US	4
5	26	Japan	5.5
6	26	US	5.5
7	28	US	7
8	32	Japan	8
9	33	Japan	9

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- $U = 22.5$

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- $E[U] = 15$

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- $SD[U] = 3.86$

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- $E[U] = 15$
- $SD[U] = 3.86$
- $z = 1.81$ (appropriate continuity correction is -0.5)

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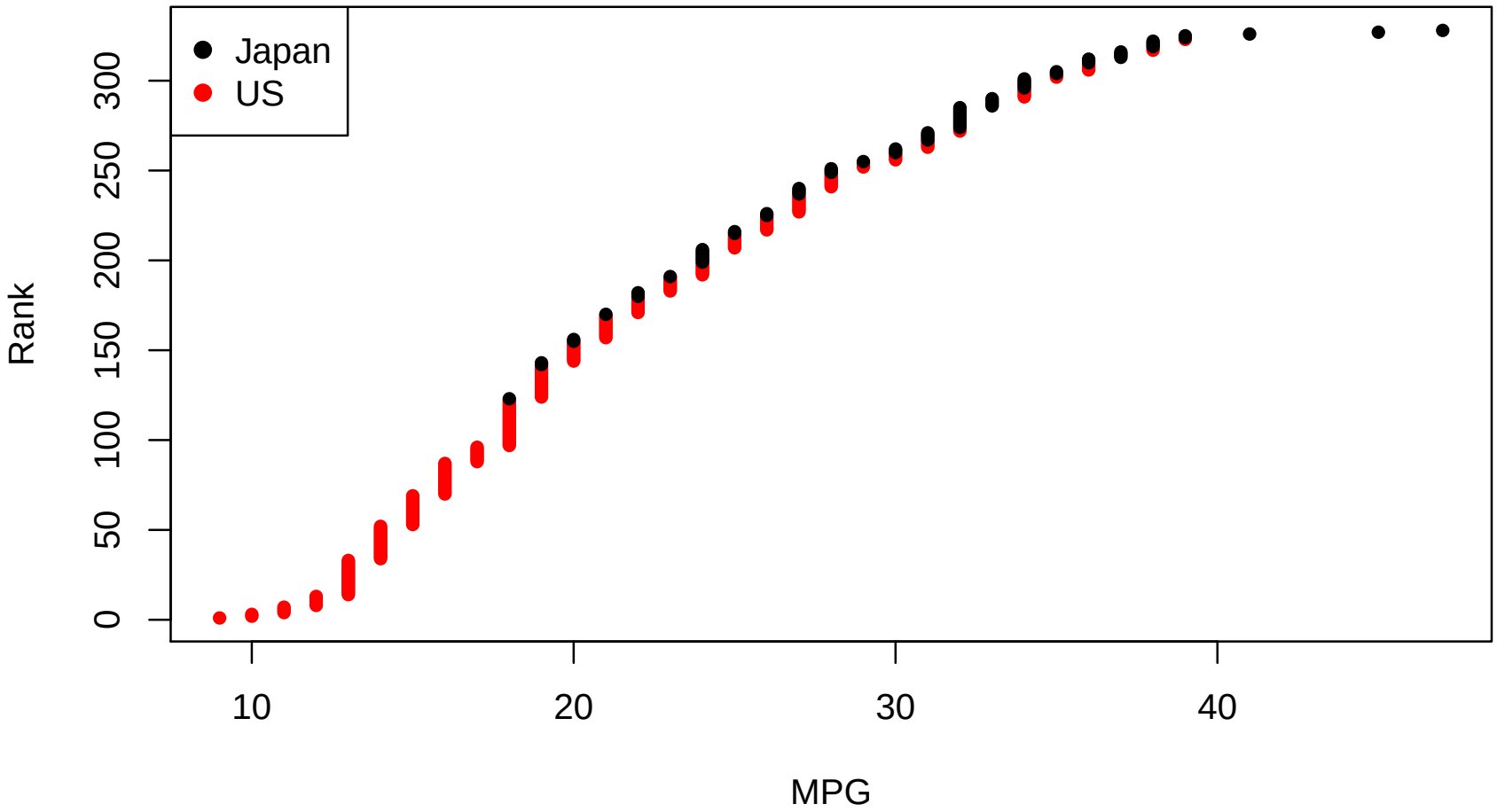
- $U = 22.5$
- $E[U] = 15$
- $SD[U] = 3.86$
- $z = 1.81$ (appropriate continuity correction is -0.5)
- $p = 0.07$

Transform data to ranks



SAS code for Wilcoxon rank sum test

```
DATA mpg;
    INFILE 'mpg.csv' DELIMITER=',' FIRSTOBS=2;
    INPUT mpg country $;

PROC NPAR1WAY DATA=mpg WILCOXON;
    CLASS country;
    VAR mpg;
    RUN;
```

The NPAR1WAY Procedure

Wilcoxon Scores (Rank Sums) for Variable mpg
Classified by Variable country

country	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
US	249	33646.50	40960.50	733.579091	135.126506
Japan	79	20309.50	12995.50	733.579091	257.082278

Average scores were used for ties.

Wilcoxon Two-Sample Test

Statistic	20309.5000
Normal Approximation	
Z	9.9696
One-Sided Pr > Z	<.0001
Two-Sided Pr > |Z|	<.0001
t Approximation	
One-Sided Pr > Z	<.0001
Two-Sided Pr > |Z|	<.0001

Z includes a continuity correction of 0.5.

Kruskal-Wallis Test

Chi-Square	99.4068
DF	1
Pr > Chi-Square	<.0001

Conclusion

Average miles per gallon of Japanese cars are significantly different than average miles per gallon of American cars (Wilcoxon rank sum test, $p < 0.0001$).