Multiple Comparisons

STAT 401 - Statistical Methods for Research Workers

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SAS code and output for one-way ANOVA

```
DATA mice;
  INFILE 'U:\401A\Sleuth Datasets\CSV\case0501.csv' DSD FIRSTOBS=2;
  INPUT lifetime diet $;

PROC GLM DATA=mice;
  CLASS diet;
  MODEL lifetime = diet;
  LSMEANS diet / ADJUST=T;
  RUN;
```

```
                                        The GLM Procedure

Dependent Variable: lifetime

                                                Sum of
       Source                      DF          Squares     Mean Square    F Value    Pr > F
       Model                        5      12733.94181      2546.78836      57.10    <.0001
       Error                      343      15297.41532        44.59888
       Corrected Total            348      28031.35713
```

What have we learned?

SAS code and output for one-way ANOVA

```
                          The GLM Procedure
                         Least Squares Means

                           lifetime      LSMEAN
                   diet      LSMEAN      Number

                   N/N85  32.6912281          1
                   N/R40  45.1166667          2
                   N/R50  42.2971831          3
                   NP     27.4020408          4
                   R/R50  42.8857143          5
                   lopro  39.6857143          6


                 Least Squares Means for effect diet
                 Pr > |t| for H0: LSMean(i)=LSMean(j)

                     Dependent Variable: lifetime

  i/j           1          2          3          4          5          6

     1                <.0001     <.0001     <.0001     <.0001     <.0001
     2     <.0001                0.0166     <.0001     0.0731     <.0001
     3     <.0001     0.0166                <.0001     0.6223     0.0293
     4     <.0001     <.0001     <.0001                <.0001     <.0001
     5     <.0001     0.0731     0.6223     <.0001                0.0117
     6     <.0001     <.0001     0.0293     <.0001     0.0117

NOTE: To ensure overall protection level, only probabilities associated with pre-planned
      comparisons should be used.
```

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$$1-(1-0.05)^{15}$$

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But we wanted the probability of making a mistake to be 0.05.

Multiple comparison adjustments

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Scheffé	$\sqrt{(\mathrm{I}-1)F_{(\mathrm{I}-1,n-\mathrm{I})}(1-\alpha)}$	All contrasts
Dunnett		Compare all groups to control
LSD	$t_{n-\mathrm{I}}(1-\alpha)$	After significant F-test (no adjustment)
Bonferroni	$t_{n-\mathrm{I}}(1-\alpha/2k)$ $k = \mathrm{I}(\mathrm{I}-1)/2$	k tests (most generic)

SAS code and output for one-way ANOVA

```
                The GLM Procedure
               Least Squares Means
  Adjustment for Multiple Comparisons: Bonferroni

                        lifetime      LSMEAN
           diet           LSMEAN      Number

           N/N85      32.6912281           1
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          Least Squares Means for effect diet
          Pr > |t| for H0: LSMean(i)=LSMean(j)

             Dependent Variable: lifetime

i/j              1             2             3             4             5             6

   1                      <.0001        <.0001        0.0009        <.0001        <.0001
   2        <.0001                      0.2488        <.0001        1.0000        0.0002
   3        <.0001        0.2488                      <.0001        1.0000        0.4402
   4        0.0009        <.0001        <.0001                      <.0001        <.0001
   5        <.0001        1.0000        1.0000        <.0001                      0.1751
   6        <.0001        0.0002        0.4402        <.0001        0.1751
```

SAS code and output for one-way ANOVA

```
                    The GLM Procedure
                   Least Squares Means
      Adjustment for Multiple Comparisons: Tukey-Kramer

                          lifetime      LSMEAN
              diet          LSMEAN      Number

              N/N85     32.6912281           1
              N/R40     45.1166667           2
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   2       <.0001                      0.1565        <.0001        0.4684        0.0002
   3       <.0001        0.1565                      <.0001        0.9964        0.2460
   4       0.0008        <.0001        <.0001                      <.0001        <.0001
   5       <.0001        0.4684        0.9964        <.0001                      0.1168
   6       <.0001        0.0002        0.2460        <.0001        0.1168
```

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If you wanted the longest lifetime, which diet should you prefer?

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Does it really make a difference?