Finding pvalues (and critical values)

STAT 401 - Statistical Methods for Research Workers

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Hypotheses

Recall

Definition

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The difference between one-sided and two-sided hypotheses is that they affect the region.

Z-statistics

Let's assume, we have

- calculated a test statistic z that
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$F(c) = P(Z < c)$ is the cumulative distribution function for the standard normal.

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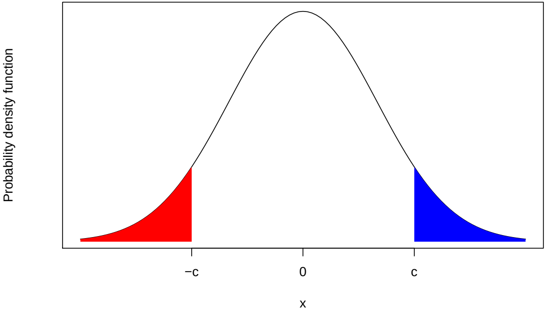

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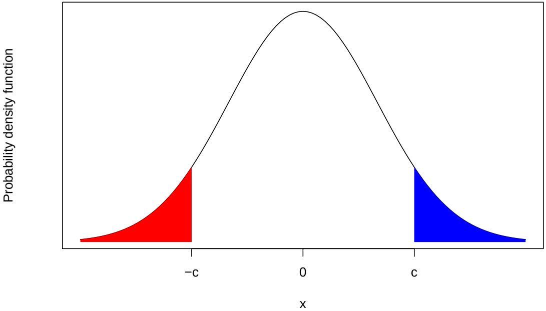

$$P(Z > c) = P(Z < -c) \quad \text{blue area is equal to red area}$$

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- which has a t distribution with 7 degrees of freedom

So we need to calculate

$$P(t_7 > 2.43) = 1 - P(t_7 < 2.43)$$

where t_7 represents a t-distribution with 7 degrees of freedom.

Using SAS or R

In SAS,

```
PROC IML;
  p = 1-CDF('T', 2.43, 7);
  PRINT p;
  QUIT;
```

In R,

```
p = 1-pt(2.43,7)
```

Both obtain p=0.0227.

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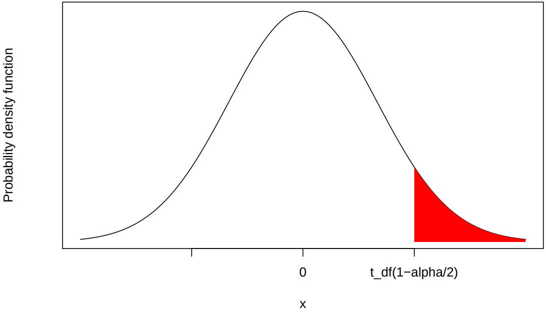

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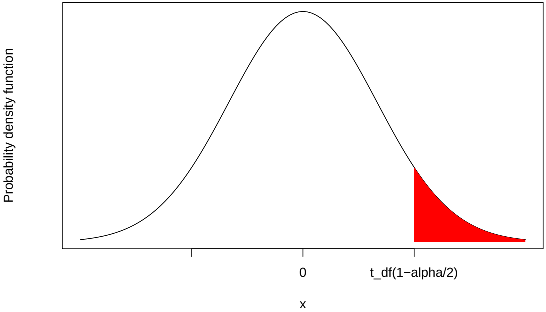

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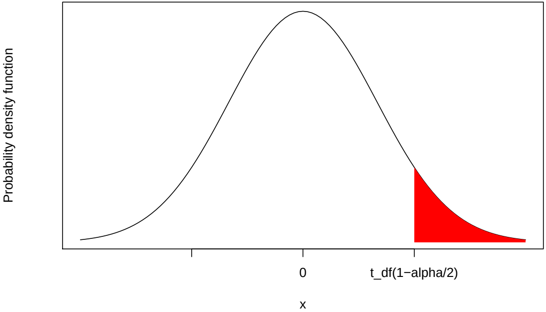

The red area is $\alpha/2$. Let $c = t_{df}(1-\alpha/2)$, then we need $P(t_{df} < c) = 1-\alpha/2$.

Using SAS or R

If $\alpha = 0.05$, then $1 - \alpha/2 = 0.975$.

In SAS,

```
PROC IML;
  q = QUANTILE('T', 0.975, 7);
  PRINT q;
  QUIT;
```

In R,

```
q = qt(0.975,7)
```

Both obtain q=2.364.