

Lack-of-fit F-test

STAT 401A - Statistical Methods for Research Workers

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October 6, 2013

Testing Composite hypotheses

Comparing two models

- H_0 : (reduced)
- H_1 : (full)

Do the following

1. Calculate extra sum of squares.
2. Calculate extra degrees of freedom
3. Calculate

$$\text{F-statistic} = \frac{\text{Extra sum of squares} / \text{Extra degrees of freedom}}{\hat{\sigma}_{full}^2}$$

4. Compare this to an F-distribution with

- numerator degrees of freedom = extra degrees of freedom
- denominator degrees of freedom = degrees of freedom in estimating $\hat{\sigma}_{full}^2$

Simple Linear Regression

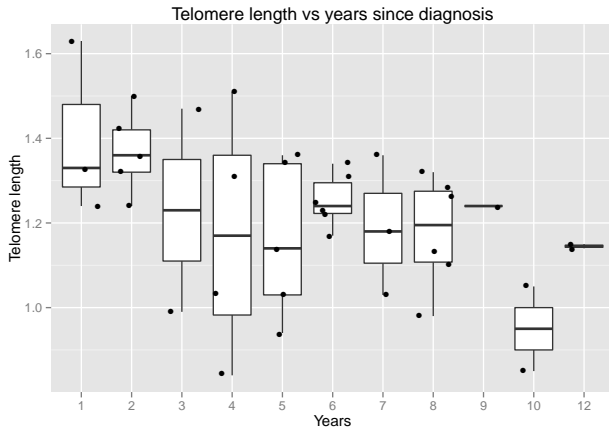
Two models:

ANOVA: $Y_{ij} \stackrel{\text{ind}}{\sim} N(\mu_i, \sigma^2)$ (full)

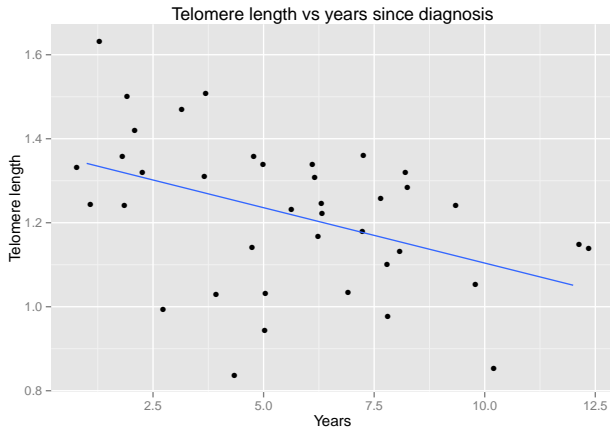
Regression: $Y_{ij} \stackrel{\text{ind}}{\sim} N(\beta_0 + \beta_1 X_i, \sigma^2)$ (reduced)

- Regression model is reduced:
 - ANOVA has I parameters for the mean
 - Regression has 2 parameters for the mean
 - $\mu_i = \beta_0 + \beta_1 X_i$
- Small pvalues indicate a lack-of-fit, i.e. the reduced model is not adequate.
- Lack-of-fit F-test requires multiple observations at a few X_i values.

Telomere length



Telomere length



SAS code

```
DATA t;
  INFILE 'telomeres.csv' DSD FIRSTOBS=2;
  INPUT years length;

PROC REG DATA=t;
  MODEL length = years / CLB LACKFIT;
  RUN;
```

The REG Procedure
 Model: MODEL1
 Dependent Variable: length

Number of Observations Read	39
Number of Observations Used	39

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.22777	0.22777	8.42	0.0062
Error	37	1.00033	0.02704		
Lack of Fit	9	0.18223	0.02025	0.69	0.7093
Pure Error	28	0.81810	0.02922		
Corrected Total	38	1.22810			

Indicates no evidence for a lack of fit, i.e. regression seems adequate.

Summary

- Lack-of-fit F-test tests the assumption of linearity
- Needs multiple observations at various explanatory variable values
- Small pvalue indicates a lack-of-fit, i.e. means are not linear
 - Transform response, e.g. log
 - Transform explanatory variable
 - Add other explanatory variables