Finding pvalues (and critical values)

STAT 401 - Statistical Methods for Research Workers

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Hypotheses

Recall

Definition

A pvalue is the probability of observing a test statistic as or more extreme than that observed, if the null hypothesis is true.

Three key features:

- a test statistic calculated from data
- a sampling distribution for the test statistic under the null hypothesis
- a region that is as or more extreme

The difference between one-sided and two-sided hypotheses is that they affect the region.

Z-statistics

Let's assume, we have

- calculated a test statistic z that
- has a $Z \sim N(0,1)$ sampling distribution if the null hypothesis is true.

We could easily replace z with t and have a sampling distribution that is t_{df} for some df.

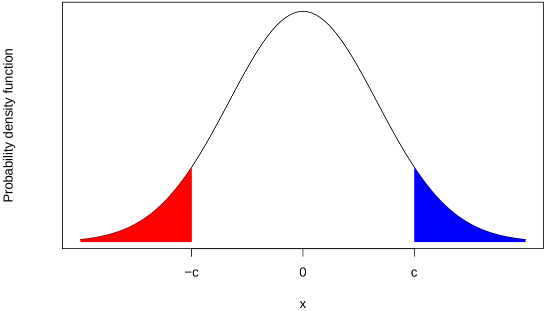
Now, we can have one of three types of hypotheses:

- Two-sided: $P(|Z| > |z|) = P(Z > |z|) + P(Z < -|z|) = 2P(Z < -|z|)$
- One-sided:
  - $P(Z > z) = P(Z < -z)$
  - $P(Z < z)$

$F(c) = P(Z < c)$ is the cumulative distribution function for the standard normal.

Symmetric distributions

The standard normal and t distribution are both symmetric around zero.



$$P(Z > c) = P(Z < -c)$$ blue area is equal to red area

Paired t-test example

In the minilecture on the paired t-test, we had

- a one-sided hypothesis, namely the difference is greater than zero
- a test statistic $t = 2.43$
- which has a t distribution with 7 degrees of freedom

So we need to calculate

$$P(t_7 > 2.43) = 1 - P(t_7 < 2.43)$$

where t_7 represents a t-distribution with 7 degrees of freedom.

Using SAS or R

In SAS,

```
PROC IML;
  p = 1-CDF('T', 2.43, 7);
  PRINT p;
  QUIT;
```

In R,

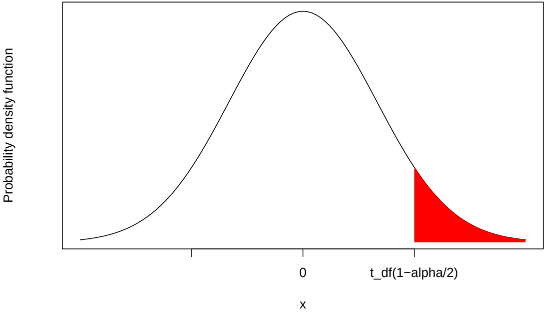
```
p = 1-pt(2.43,7)
```

Both obtain p=0.0227.

Critical values

A related quantity are critical values for confidence interval construction, e.g.

$$(\overline{Y}_2 - \overline{Y}_1) \pm t_{df}(1-\alpha/2) SE(\overline{Y}_2 - \overline{Y}_1).$$



The red area is $\alpha/2$. Let $c = t_{df}(1-\alpha/2)$, then we need $P(t_{df} < c) = 1 - \alpha/2$.

Using SAS or R

If $\alpha = 0.05$, then $1 - \alpha/2 = 0.975$.

In SAS,

```
PROC IML;
  q = QUANTILE('T', 0.975, 7);
  PRINT q;
  QUIT;
```

In R,

```
q = qt(0.975,7)
```

Both obtain q=2.364.