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BA 64018: Quantitative Management Modeling
Assignment: Module 2 – The LP Module

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

    a. Clearly define the decision variables
        i. x = the # of Collegiate backpacks to produce per week.
        ii. y = the # of Mini backpacks to produce per week.
    b. What is the objective function?
        i. Our objective is to maximize the profit. The profit for each Collegiate backpack is $32, & for each Mini backpack it is $24. Meaning, the objective maximize function is:
            1. $Z = 32x + 24y$
    c. What are the constraints?
        i. *Material Constraint:* The nylon total square footage used should not go past 5000 square feet each week.
            1. $3x + 2y \leq 5000$
        ii. *Sales Forecast Constraints:* The # of Collegiate backpakcs made should not go past 1000, & the # of Mini backpacks made should not go past 1200 per week.
            1. $x \leq 1000$
            2. $y \leq 1200$
        iii. *Labor Constraint:* Total labor hours worked should not go past the hours of availability.
            1. $45x + 40y \leq 35 \times 40 \times 60$ (converting hours to minutes)
        iv. Non-negativity Constraint:
            1. $x \geq 0$
            2. $y \geq 0$
    d. Write down the full mathematical formulation for this LP problem.

    Maximize:

    $Z = 32x + 24y$

Subject to:

Material Constraint:

$3x + 2y \leq 5000$

Sales Forecast Constraints:

$x \leq 1000$

$y \leq 1200$

Labor Constraint:

$45x + 40y \leq 35 \times 40 \times 60$

Non-negativity Constraints:

$x \geq 0$

$y \geq 0$

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

   a. Define the decision variables.
      i. x_1 = the # of large units made by Plant 1
      ii. x_2 = the # of medium units made by Plant 1
      iii. x_3 = the # of small units made by Plant 1
      iv. y_1 = the # of large units made by Plant 2
      v. y_2 = the # of medium units made by Plant 2

vi. y_3 = the # of small units made by Plant 2
vii. z_1 = the # of large units made by Plant 3
viii. z_2 = the # of medium units made by Plant 3
ix. z_3 = the # of small units made by Plant 3

b. Formulate a linear programming model for this problem.

Maximize:

$Z = 420(x_1 + y_1 + z_1) + 360(x_2 + y_2 + z_2) + 300(x_3 + y_3 + z_3)$

Subject to:

Production Capacity Constraints:

$x_1 + x_2 + x_3 \leq 750$

$y_1 + y_2 + y_3 \leq 900$

$z_1 + z_2 + z_3 \leq 450$

Storage Space Constraints:

$20x_1 + 15x_2 + 12x_3 \leq 13000$

$20y_1 + 15y_2 + 12y_3 \leq 12000$

$20z_1 + 15z_2 + 12z_3 \leq 5000$

Forecast Constraints:

$x_1 + y_1 + z_1 \leq 900$

$x_2 + y_2 + z_2 \leq 1200$

$x_3 + y_3 + z_3 \leq 750$

Non-negativity Constraints:

$x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \geq 0$