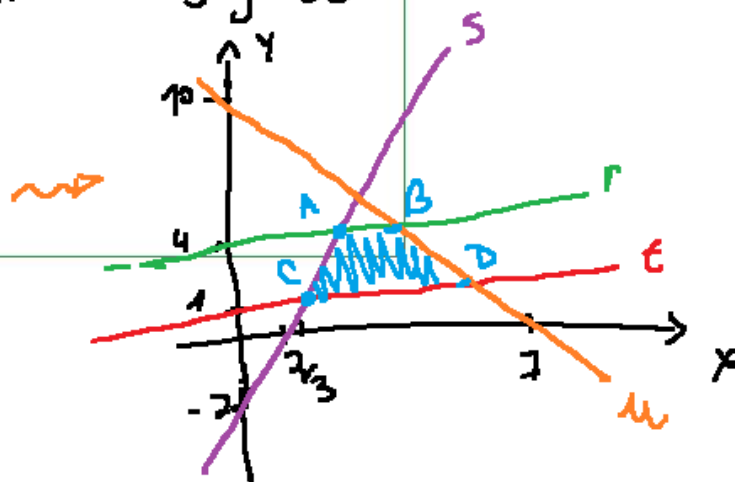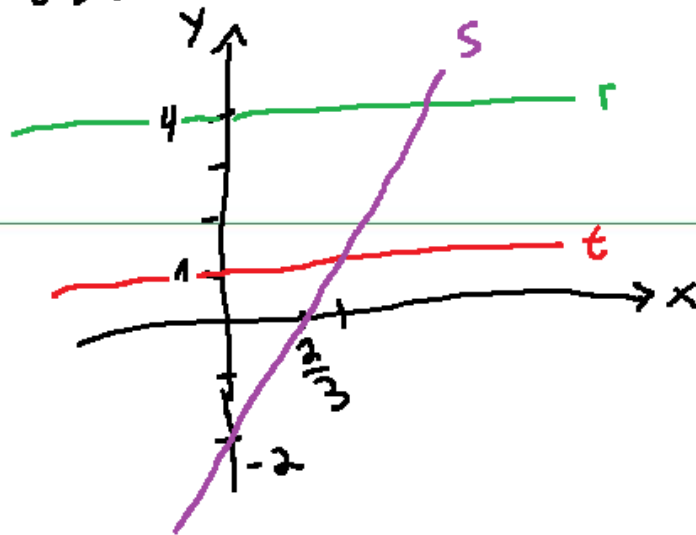(UPE - SSA) Qual é a medida da área do quadrilátero limitado pelas retas (r) $y = 4$; (s) $3x - y - 2 = 0$; (t) $y = 1$ e (u) $3x + 2y - 20 = 0$?

$x = 0 \Rightarrow y = -2$

$y = 0 \Rightarrow x = \frac{2}{3}$

- a. 11
- b. $10,5$
- c. $7,5$
- d. 12
- e. $9,0$

↳ $x = 0 \Rightarrow y = 10$

↳ $y = 0 \Rightarrow x = \frac{20}{3}$

Vamos esboçar a figura:

Podemos encontrar a área do quadrilátero ao dividirmos ele nos triângulos $\triangle ABC$ e $\triangle BDC$; calcularmos suas áreas e somarmos. Para isso, vamos encontrar A, B, C e D.

S: 3x−y−2=0

r: y=4

t: y=1
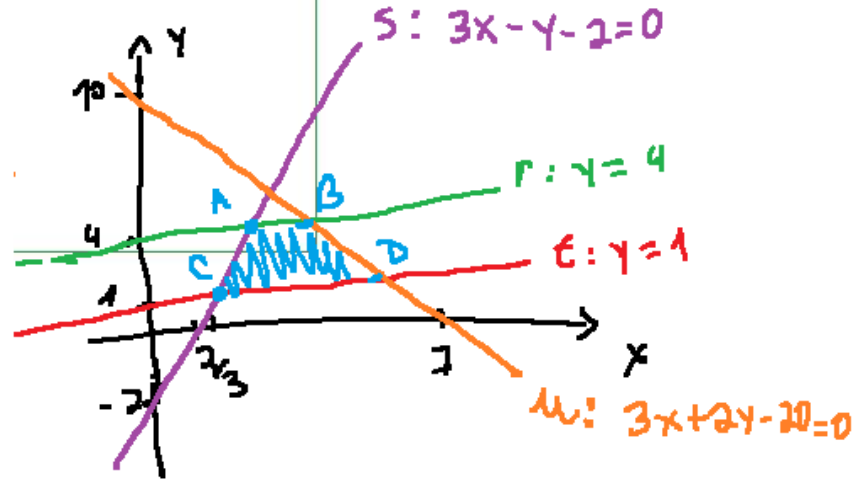
u: 3x+2y−20=0

Por conta de nossa construção,

$$A = s \cap r, \quad B = u \cap r, \quad C = s \cap t, \quad D = u \cap t$$

$$\begin{cases} 3x-y-2=0 \\ y=4 \end{cases}$$

$$\rightsquigarrow 3x-4-2=0$$
$$3x=6$$
$$\therefore x=2$$
$$A(2,4)$$

$$\begin{cases} 3x+2y-20=0 \\ y=4 \end{cases}$$

$$\rightsquigarrow 3x+2\cdot4-20=0$$
$$3x-12=0$$
$$3x=12$$
$$\therefore x=4$$
$$B(4,4)$$

$$\begin{cases} 3x-y-2=0 \\ y=1 \end{cases}$$

$$\rightsquigarrow 3x-1-2=0$$
$$3x=3$$
$$\therefore x=1$$
$$C(1,1)$$

$$\begin{cases} 3x+2y-20=0 \\ y=1 \end{cases}$$

$$\rightsquigarrow 3x+2\cdot1-20=0$$
$$3x-18=0$$
$$3x=18$$
$$\therefore x=6$$
$$D(6,1)$$

s: 3x − y − 2 = 0

r: y = 4

t: y = 1

u: 3x + 2y − 20 = 0

Por conta de nossa construção,

$$A = s \cap r \quad, \quad B = u \cap r \quad, \quad C = s \cap t \quad, \quad D = u \cap t$$
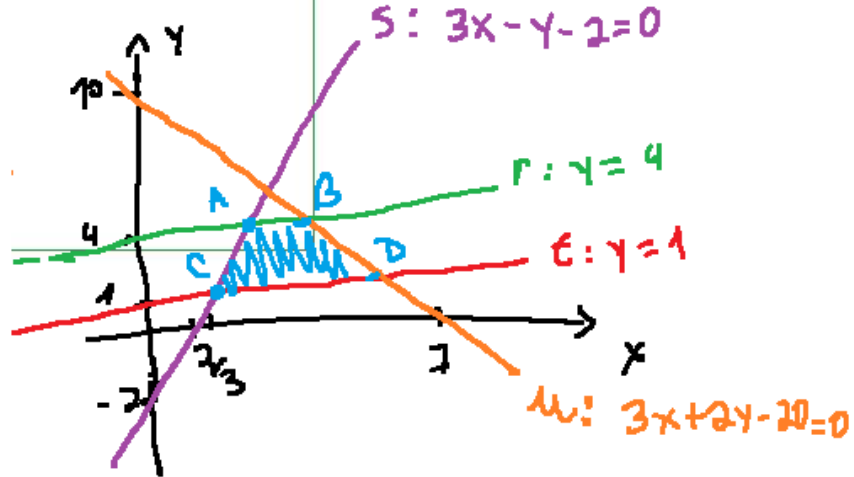
$$A(2, 4) \qquad B(4, 4) \qquad C(1, 1) \qquad D(6, 1)$$

$$\Delta_1 = \begin{vmatrix} 2 & 4 & 1 \\ 4 & 4 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 8 + 4 + 4 - 4 - 2 - 16$$
$$= -6$$

$$\Delta_2 = \begin{vmatrix} 4 & 4 & 1 \\ 1 & 1 & 1 \\ 6 & 1 & 1 \end{vmatrix} = 4 + 24 + 1 - 6 - 4 - 4$$
$$= 15$$

$$A_{\triangle ABC} = \frac{1}{2}|\Delta_1| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$A_{\triangle BCD} = \frac{1}{2}|\Delta_2| = \frac{1}{2}|15| = \frac{15}{2} = 7,5$$

$$A = A_{\triangle ABC} + A_{\triangle BCD} = 3 + 7,5 = 10,5$$