Mathematical Model

Our goal is to minimize the number of classes that each student will not be able to attend.

$$z := \min_{\psi,\omega,\sigma} \left\{ \sum_{(i,j)\in\mathbb{T}} \sigma_{ij} \right\}$$

Subject to the following constraints:

In summary, the equations below express the constraints of the problem, such as the non-overlapping of teachers in classrooms with identical schedules or not exceeding certain upper limits such as the maximum number of sessions, maximum number of students per session, and maximum number of classes that teachers can teach.

$$\Phi_{\min} \leq \sum_{j\in C_i^T} \sum_{\xi\in\Xi_j} \sum_{k\in P} \psi_{ij\xi k} \leq \Phi_{\max} \qquad \forall(i, C_i^T) \in T \tag{1}$$

$$\sum_{j\in C_i^T} \sum_{\xi\in\Xi_j} \psi_{ij\xi k} \leq 1 \qquad \forall((i, C_i^T), k) \in T \times P \tag{2}$$

$$\sum_{i\in\mathbb{V}_j} \psi_{ij\xi k} \leq 1 \qquad \forall(j, \xi, k) \in \mathbb{Y} \tag{3}$$

$$\sum_{i\in\mathbb{V}_j} \sum_{k\in P} \psi_{ij\xi k} \leq 1 \qquad \forall(j, \xi) \in \mathbb{D} \tag{4}$$

$$\sum_{\xi\in\Xi_j} \sum_{k\in P} \omega_{ij\xi k} + \sigma_{ij} = 1 \qquad \forall(i, j) \in \mathbb{K} \tag{5}$$

$$\sum_{j\in C_i^S} \sum_{\xi\in\Xi_j} \omega_{ij\xi k} = 1 \qquad \forall((i, C_i^S), k) \in S \times P \tag{6}$$

$$\sum_{i\in\mathbb{W}_j} \omega_{ij\xi k} \leq L_{\max} \sum_{i\in\mathbb{V}_j} \psi_{ij\xi k} \qquad \forall(j, \xi, k) \in \mathbb{Y} \tag{7}$$

Where:

$$\psi\_, \omega\_, \sigma\_ \in \{0, 1\} \tag{8}$$

$$\mathbb{V}_j = \{i : C_i^T \cap \{j\} \neq \emptyset\} \tag{9}$$

$$\mathbb{Y} = \bigcup_{i\in C} (\{i\} \times \Xi_i \times P) \tag{10}$$

$$\mathbb{D} = \bigcup_{i\in C} (\{i\} \times \Xi_i) \tag{11}$$

$$\mathbb{K} = \bigcup_{(i,C_i^S)\in S} (\{i\} \times C_i^S) \tag{12}$$

$$\mathbb{W}_j = \{i : C_i^S \cap \{j\} \neq \emptyset\} \tag{13}$$