**Momento**

Like it was thoroughly substantiated in the last assignment, there are two types of options, Call options which allow the owner to buy the underlying asset at strike price stated in the contract within a specific timeframe. On the other hand, Put options allow the owner to sell the underlying asset at the strike price stated in the contract within a specific timeframe.

A plethora of method exist to price options, for instance, as previously studied in the previous lectures and assignment, the Binomial tree calculates the value of an asset over a series of time steps. In this stepwise process, the asset price can go up or down based on the up and down probability. The value of the option is sequentially computed at each point in the tree, that is from the final to its initial point.

In this vein, the objective of this assignment is based on the applications to the pricing of an American or Exotic option, using the Monte Carlo Simulations by the means of Excel and VBA. For our case, our focal point will be laid on American options.

***Foundations of the option pricing model***

According to McDonald, R.L (2013), the Brownian stochastic process is a random process that is a function of time. This stochastic process, that is a building block for derivative pricing models evolve continuously in time, that is, with continuous movements. Moreover, depending on how quick stock prices will change over time frames, it can be either Arithmetic Brownian Motion, or Geometric Brownian Motion.

With the principles of the Brownian motion taken into account, the kind of movement that the stock price follows is evaluated by the model. It takes the daily prices depending on the time frame for instance 30 days and generates a linear and exponential regression, following which depending on the , will depending the dynamics of the prices. For example, the dynamics are considered geometric dynamics when the linear regression has a higher than the arithmetic one. The same applies the other way round.

If the price dynamic is determined, the model (for both the Arithmetic and Brownian motion) estimates the stock prices for the required number of days using the formula below:

Where:

: expected change in the daily returns of the stock

: expected return

: standard deviation of the stock

: period of time change

: random component using a normal distribution;

Furthermore, the law of large numbers state that the mean of a large number of trails should be approximately equal to the expected value. As long as the number of trails will increase, the expected value will be closer to the real one. Thus, the model will generate with a simulation of the stock prices for a period of 30 days. Using the Monte Carlo simulations, it will simulate 2000 random realisations of the stock price for a period of 30 days.

**Call option:**

Where:

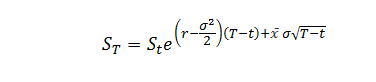
* : premium of the call option
* : expected spot price of the spot
* : strike price
* : exponential constant
* : continuously compounded risk-free rate
* : time length

**Put option:**

Where:

* : premium of the put option
* : expected spot price of the spot
* : strike price
* : exponential constant
* : continuously compounded risk-free rate
* : time length

In addition, the Monte Carlo method, used for pricing path-dependent options can be used to generate future stock prices. The equation below shows how a stock price fluctuates over time considering the Weiner process.



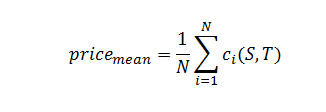
Where:

* : stock price at expiry date
* : stock price at current time
* : time to expiry
* : current time
* : continuously compounded risk-free rate
* : volatility
* : number sampled from a normal distribution

Many values of are generated across time steps, giving rise to many future price paths, and calculated for each. The payoff is then computed once the is generated. The expected present value of the call and put options, where K is the strike price are computed as follows:

The exponential term from the two equations discounts the price from time T to expiry date T. For the case of an Asian option, is replaced with an average price over the whole path.

Thus, considering a whole set of and , the mean option price is being calculated. For instance, for a call option, the mean price is shown below:



***Model of the Geometric Brownian Motion in the Black-Scholes model***

Geometric Brownian Motion is used to model the stock prices in the Black-Scholes model. It is also considered as the most widely used model of stock price behaviour. For a more detailed explanation on the Black-Scholes, feel free to visit the Github page of our previous assignment. (<https://github.com/jardieljr/derivatives/tree/main/assignment_1>)

Advantages of using the GBM to model stock prices:

* The expected returns of GBM are independent of the value of the process (stock price)
* A GBM process only assumes positive values, just like real stock prices
* A GBM process shows the same kind of 'roughness' in its paths as we see in real stock prices
* Calculations with GBM processes are relatively easy

Constraints of using the GBM to model stock prices:

* In real stock prices, volatility changes over time (stochastically), but in GBM, volatility is assumed constant
* In real life, stock prices often show jumps caused by unpredictable events or news, but in GBM, the path is continuous (no discontinuity)

*References:*

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