Contents

Part 1

Prerequisites

1 Vectors

In this chapter we'll give an introduction to vectors, and some results in geometry that you may find useful. Vectors are not introduced rigorously here, but we do use some facts from linear algebra.

1.1 Co-ordinate Free Vectors

We first describe vectors as being co-ordinate free. And really, even though we will introduce co-ordinates later, you should still think of vectors as just existing out somewhere, with or without co-ordinates.

You can think of a vector as a *directed line segment*. Vectors are usually denoted using boldface, $\mathbf{A}$.

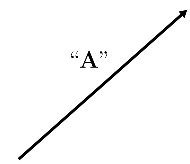


Figure 1.1: A directed line segment, the vector $\mathbf{A}$.

Because they're directed line segments, only their length matters and we're not concerned with the location of their end points. This means that a vector can be freely translated, without altering it.

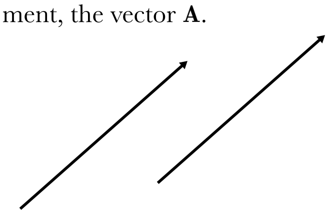


Figure 1.2: Two identical vectors.

The magnitude of the vector is its length, which we call it's *norm*, and write as $\|\mathbf{A}\|$ or simply A. You may also see it being written as $|\mathbf{A}|$.

A unit vector, $\hat{\boldsymbol{\gamma}}$ ("gamma hat") is a vector whose magnitude is unity. We use the unit vector to often denote the direction of a vector by multiplying the unit vector with its magnitude. For instance, the unit vector that point is the direction of $\mathbf{A}$, $\hat{\mathbf{A}}$ can be calculated as,

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{A}.$$

Further, vectors have certain operations defined on them. The base operations are *Scalar Multiplication* and *Vector Addition*.

Scalar Multiplication

Scalar Multiplication refers to multiplying a vector by a *scalar*. A scalar for our purposes refers to a real number. Consider a vector $\mathbf{v}$. Multiplying it by a scalar, $\alpha \in \mathbb{R}$ produces a vector $\alpha\mathbf{v}$ parallel to the original vector and changes the magnitude of the vector so that $\|\alpha\mathbf{v}\|$ is $|\alpha|$ times greater than $\|\mathbf{v}\|$, thus $\|\alpha\mathbf{v}\| = |\alpha|\|\mathbf{v}\|$. $|\alpha|$ can be smaller or greater than 1 which accordingly increases/decreases the lenght of $\mathbf{v}$.

If $\alpha > 0$, the vector produced is in the same direction as the original vector. If $\alpha < 0$, the direction of the vector is reversed.

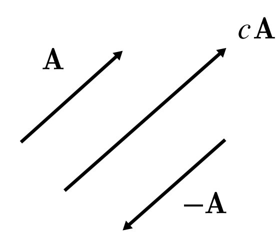


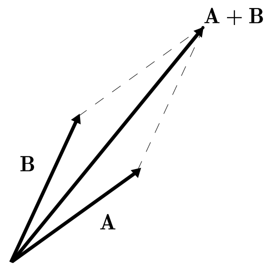
Figure 1.3: Scalar multiplication of a vector $\mathbf{A}$ by $c > 1$ and -1.

Vector Addition

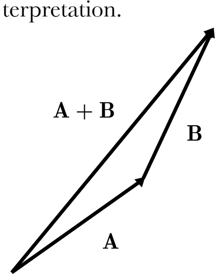
Adding two vectors, $\mathbf{A}$ and $\mathbf{B}$ produces another vector $\mathbf{A}+\mathbf{B}$. Geometrically, vector addition is done by placing the tail of $\mathbf{B}$ on the head of $\mathbf{A}$, and then the vector joining the tail of $\mathbf{A}$ and head of $\mathbf{B}$ is the vector $\mathbf{A}+\mathbf{B}$ as in fig. 1.3a.

It can also be interpreted as the diagonal of the parallelogram made by placing the tails of the two vectors together, and producing two sides parallel to them as show in fig. 1.3b.

Subtraction of vectors, $\mathbf{A}-\mathbf{B}$ is equivalent to multiplying $\mathbf{B}$ by -1 and then adding it with $\mathbf{A}$.



(a) Adding vectors, parallelogram interpretation.



(b) Adding vectors, triangle interpretation.

Figure 1.4: Addition of two vectors, $\mathbf{A}$ and $\mathbf{B}$ produces another vector, $\mathbf{A}+\mathbf{B}$.

1.2 Introducing Co-ordinates

There are an infinite number of vectors in 3d space, of course, but that makes it harder to deal with them. Since vectors are just lengths with some direction, could we perhaps assign them co-ordinates?

Consider a plane, P, and two non-parallel vectors in it, $\mathbf{v}$, $\mathbf{w}$. Suppose we wanted

Example 1.1

For example $\mathbf{e}_x = (1,0)$ and $\mathbf{e}_y = (0,1)$ form a basis of $\mathbb{R}^2$.