1. a) $\begin{matrix} 1 & 2 \\ 1 & 1 \end{matrix} \qquad \begin{matrix} 1 & 1 \\ 0 & 3 \end{matrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & \frac{2}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

so then $\underset{C \leftarrow B}{P} = \begin{bmatrix} \frac{2}{3} & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

b) $P \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

2, a) $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 3 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \to$ RREF $\begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

b) 1 free variable, so the solution set is 1 dimensional aka a line

$$\begin{cases} x_3 = \text{free} \\ x_1 = -x_3 \\ x_2 = +2x_3 \end{cases}$$

$$\begin{matrix} -x_3 \\ 2x_3 \\ x_3 \end{matrix} \quad \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

3. a) $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -3 & -1 \\ 2 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1. $R_2 = R_2 + R_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & 8 & 3 \end{bmatrix}$
2. $R_3 = R_3 - 2R_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$
3. $R_1 = R_1 + 2R_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$
4. $R_3 = R_3 + 4R2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5. $R_2 = -1 \cdot R_2$

$R_1 = R_1 - R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

or $A^{-1} = \begin{bmatrix} 1 & -2 & -1 \\ -1 & -1 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

b)

$\begin{bmatrix} 1 & -2 & -1 & 1 \\ -1 & -1 & 0 & 1 \\ 2 & 4 & 1 & 1 \end{bmatrix} \rightarrow$

$\begin{bmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & 13 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 13 \end{bmatrix}$

$\begin{bmatrix} 4 \\ -5 \\ 13 \end{bmatrix} = \vec{X}$

4) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

- orthogonal? yes!

$$\frac{1}{\sqrt{5}^2}(2\ 0\ 1) = \left(\frac{2}{5}\ 0\ \frac{1}{5}\right)$$

$$+$$

$$\left(\frac{4}{21}\ -\frac{1}{21}\ \frac{2}{21}\right)$$

$$\begin{bmatrix} \frac{62}{105} \\ \frac{-5}{105} \\ \frac{31}{105} \end{bmatrix}$$

b) $= 72.5^{\theta}$

5)

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$$(a-\lambda)(d-\lambda) - \cancel{b}\cdot 0 \quad 0$$

$ad - a\lambda - d\lambda + \lambda^2$ or roots are

eigenvalues → $(\lambda - a), (\lambda - d)$

Eigenspace 1 = $A - (\lambda - a)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Eigenspace 2 = $A - (\lambda - d)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. a) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\lambda = 2, -1, 1$

$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

b) $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

7) a) $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix}$

$d = \sqrt{(4-1)^2 + (-4-1)^2 + (0-2)^2}$

$\sqrt{38} \approx 6.164$

b) $\left\langle \begin{matrix} 1 \cdot 4 \\ + \; 1 \cdot -4 \\ 2 \cdot 0 \end{matrix} \right\rangle = 0$

c) $\sqrt{6} = |u|$

$4\sqrt{2} = |v|$

d) $\cos(\theta) = \frac{0}{\sim}$

$\arccos(0) = \frac{\pi}{2}$

or

90°

8) $\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \rightarrow \begin{matrix} 0 \\ 2c_1 + 2c_2 \\ 3c_1 - c_2 \end{matrix}$ ☆

$$T_{rel} = \left\{ \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

4) a) $\begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

$$U_1 = V_1 = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$$

$$\frac{U \cdot V}{U \cdot U} U$$

$$U_2 = V_2 - \frac{U_1 \cdot V_2}{U_1 \cdot U_1} U_1 = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$U_3 = V_3 - \frac{U_1 V_3}{U_1 \cdot U_1} U_1 - \frac{U_2 \cdot V3}{U_2 \cdot U_2} U_2 = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

5) $\frac{U_1}{\sqrt{U_1 \cdot U_1}} = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$ $\frac{U_2}{\sqrt{U_2 \cdot U_2}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{U3}{\sqrt{U_3 \cdot U_3}} = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

10. a) an eigenvector is the vector by which a transformation can occur which will either linearly stretch the vector, or flip it, but not change it. Eigenvectors are like scalars. An Eigenvalue is the factor by which a vector is stretched by an eigenvector. If an eigenvector stretches a vector to twice its size, the eigenvalue will be 2.

b. i) = 5A

ii) -12A

iii) 14A

11. False

12 True

13 False

14 True

15 True

16 False

17 True

18 False

19 True

20 False

21 True,

22 If $b \cdot c > ad$

23 If they (i)rent 0, 0 is counterexample

24 True

25 False, subspace is set which is also a vector space

Bonus: Good kidd Madd City