

There are 7 problems, each carries 5 points. Total 35.

1)

13. A mail order computer business has six telephone lines. Let  $X$  denote the number of lines in use at a specified time. Suppose the pmf of  $X$  is as given in the accompanying table.

$x$	0	1	2	3	4	5	6
$p(x)$	.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events.

- (at most three lines are in use)  $\bullet 7$
- (fewer than three lines are in use)  $\bullet 45$
- (at least three lines are in use)  $\bullet 55$
- (between two and five lines, inclusive, are in use)  $\bullet 71$

2)

24. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let  $X$  = the number of months between successive payments. The cdf of  $X$  is as follows.

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \leq x < 3 \\ .40 & 3 \leq x < 4 \\ .45 & 4 \leq x < 6 \\ .60 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

- What is the pmf of  $X$ ?
- Using just the cdf, compute  $P(3 \leq X \leq 6)$  and  $P(4 \leq X)$ .

3) pmf =

a)

1	3	4	6	12
.30	.10	.15	.15	.40

b) .4

29. The pmf of the amount of memory  $X$  (GB) in a purchased flash drive was given in Example 3.13 as

$x$	1	2	4	8	16
$p(x)$	.05	.10	.35	.40	.10

Compute the following:

- $E(X)$  6.45
- $V(X)$  directly from the definition 15.6475
- The standard deviation of  $X$  3.95557
- $V(X)$  using the shortcut formula 15.6475

4)

30. An individual who has automobile insurance from a certain company is randomly selected. Let  $Y$  be the number of moving violations for which the individual was cited during the last 3 years. The pmf of  $Y$  is

$y$	0	1	2	3
$p(y)$	.60	.25	.10	.05

- Compute  $E(Y)$ . 0.6
- Suppose an individual with  $Y$  violations incurs a surcharge of \$100<sup>Y</sup>. Calculate the expected amount of the surcharge.

\$121

$$110 \{ (1^2 \cdot .25) + (4 \cdot .1) + (9 \cdot .05) \}$$

5)

32. An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let  $X$  = the amount of storage space purchased by the next customer to buy a freezer. Suppose that  $X$  has pmf

$x$	13.5	15.9	19.1
$p(x)$	.2	.5	.3

compute  $E(X)$ ,  $E(X^2)$ , and  $V(X)$ .

The price of a freezer having capacity  $X$  cubic feet is  $25X - 8.5$ . What is the expected price paid by the next customer to buy a freezer.

- c. What is the variance of the price  $25X - 8.5$  paid by the next customer?
- d. Suppose that although the rated capacity of a freezer is  $X$ , the actual capacity is  $h(X) = X - .01X^2$ . What is the expected actual capacity of the freezer purchased by the next customer?

6)

35. Let  $X$  be a Bernoulli rv with pmf as in Example 3.18.

- a. Compute  $E(X^2) = p$
- b. Show that  $V(X) = p(1-p)$ .

7)

37. The  $n$  candidates for a job have been ranked  $1, 2, 3, \dots, n$ . Let  $X$  = the rank of a randomly selected candidate, so that  $X$  has pmf

$$p(x) = \begin{cases} 1/n & x = 1, 2, 3, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

(this is called the *discrete uniform distribution*). Compute  $E(X)$  and  $V(X)$  using the shortcut formula. [Hint: The sum of the first  $n$  positive integers is  $n(n+1)/2$ , whereas the sum of the squares of the first  $n$  positive integers is  $n(n+1)(2n+1)/6$ .]

$$\begin{aligned} c) \quad E(X) &= 16.38 \\ E(X^2) &= 272.3 \\ V(X) &= 3.944 \end{aligned}$$

$$b) = 401$$

$$c) = 2495.624$$

$$d) 16.38 - .01 \cdot 272.3 = 13.66$$

$$p(1-p) (p - p^2) \text{ which is } V(X) = (p - p^2)$$

$$E(X) = (n+1)/2$$

$$E(X^2) = (n+1)(n+2)/6$$