1. a) $\forall x \in \mathbb{R}$, if $2 < x < 5$, then $x^3 < 125$

i. converse: $\forall x \in \mathbb{R}$, if $x^3 < 125$, then $2 < x < 5$

ii. inverse: $\forall x \in \mathbb{R}$, if $2 \geq x \geq 5$, then $x^3 \geq 125$

iii. contropositive: $\forall x \in \mathbb{R}$, if $x^3 \geq 125$, then $2 \geq x \geq 5$

iiii. negation: $\exists x \in \mathbb{R}$, if $2 < x < 5$ and $x^3 < 125$

1.b)

i. if the product of two real numbers, a and b is non-zero, the a and b are both non-zero.

At least one of the real numbers, a or b, equals 0

$\therefore$ a ~~and~~ or b are not both non-zero, therefore their product is not non-zero Universal Modus Tollens

ii

$\therefore$ either both a and b are even

or

both a and b are odd by Universal Modus Ponens

2a)i) Negation:

$\exists x \in \mathbb{Z}$ $m, n \mid 2m+n$ is odd and m and n are Both not ~~both~~ odd

ii) Disprove by counter example:

$m = 8$ $n = 7$ $2m + n = 2(8) + 7 = 23$

but m, n are not odd

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2 bi) n, if n^2 is even, n is even

Contrapositive Let $n = 2k$
Substitute $(2k)^2$
$\frac{4k^2}{2}$
Of form $2(k) = 2(2k^2) \rightarrow$ always even

Contrapositive:
if n is odd, then n^2 is odd
$(2k+1)^2 = 4k^2+4k+1$
$= 2(2k^2+2k)+1$
or
T $2(r)+1$ where $r = 2k^2+2k$
since we proved the contrapositive, True

2bii) A number is odd if $2k+1$ for some $k \in \mathbb{Z}$
~~Assume $2k+1$~~

~~$(2k+1)^2 = 4k^2+4k+1 = 4(k^2+k)+1$~~ ~~odd~~
~~Therefore, we've reached a contradiction, no even numbers for odd [illegible]~~

Let n^2 be even and n be odd.
$n = 2k+1$ - odd
So $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$ odd
we've reached a contradiction

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3 a) i) when $n = 1$:

$2n^2 + 2n = 2(1)^2 + 2(1) = 4$ T holds for $n=1$

ii) if it holds for $n = k$, $(4(1+2+3+\dots+k) = 2k^2+2k$

$4+8+12+\dots+4k+4(k+1) = 2k^2+2k+4(k+1)$

$= 2k^2+2k+4k+4$

$= 2k^2+4k+2+2k+4$

$= 2(k+1)^2+2(k+1) = 2k^2+2k$

Therefore it holds for $n=k+1$

QED

strong induction

3b) i) $P(0)$ holds since $a_0 = 2$

$5\cdot3^0 - 3\cdot2^0 = 2$ T

ii) $P(1)$ holds since $a_1 = 9$

$5\cdot3^1 - 3\cdot2^1 = 15 - 6 = 9$ T

iii) Assume that $p(i)$ is true for all i within $0 < i < n$, that is, $a_i = 5\cdot3^i - 3\cdot2^i$ for all $0 < i < n$ where $n > 1$ show that $p(n)$ is true $a_n = 5\cdot3^n - 3\cdot2^n$

$a_n = 5a_{n-1} - 6a_{n-2}$: by def of sequence

$= 5(5\cdot3^{n-1} - 3\cdot2^{n-1}) - 6(5\cdot3^{n-2} - 3\cdot2^{n-2})$

$= 3+2(5\cdot3^{n-1} - 3\cdot2^{n-1}) - 3\cdot2(5\cdot3^{n-2} - 3\cdot2^{n-2})$

$= 3\cdot5\cdot3^{n-1} - 3\cdot3\cdot2^{n-1} + 2\cdot5\cdot3^{n-1} - 3\cdot2\cdot2^{n-1} - 3\cdot2\cdot5\cdot3^{n-2} + 3\cdot3\cdot2\cdot2^{n-2}$

$= 3\cdot5\cdot3^{n-1} - 3\cdot3\cdot2^{n-1} + 2\cdot5\cdot3^{n-1} - 3\cdot2\cdot2^{n-1} - \frac{3\cdot2\cdot5\cdot3\cdot3^{n-2}}{3} + \frac{3\cdot2\cdot3\cdot2^{n-2}}{2}$

$= 3\cdot5\cdot3^{n-1} - 3\cdot3\cdot\cancel{2^{n-1}} + 2\cdot5\cdot\cancel{3^{n-1}} - 2\cdot3\cdot2^{n-1} - 2\cdot5\cdot\cancel{3^{n-1}} + 3\cdot3\cdot\cancel{2^{n-1}}$

$= \frac{3\cdot5\cdot3^n}{3} - \frac{2\cdot3\cdot2^n}{2}$

$a_n = 5\cdot3^n - 3\cdot2^n$ Thus, $P(n)$ is shown true for all integers $n \geq 0$ by strong induction

4 a) i) NP → BC : 3   BD → SL : 2
BC → BD : 2

$3 \times 2 \times 2 = 12$

ii) NP → BC : 3
BC → SL : 4

$4 \cdot 3 = 12$

4b) i) $2 \times 2 \times 2 \times 2$ or $2^4 = 16$

ii) P(2 Heads) $\frac{6}{16}$ or $\frac{3}{8} = .375$ or 37.5%

P(exactly 1 Head) = $\frac{4}{16} = \frac{1}{4} = .25 = 25\%$

4 c) n objects can be arranged as $(n-1)!$

$(5-1)! = (4)! = 24$

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4d) $P(n+1,2) = \frac{(n+1)!}{(n+1-2)!} = \frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)(n-1)!}{(n-1)!}$

$= (n+1)n = \underline{n^2+n}$

$P(n,2) = \frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1)$

$= n^2 - n$

$P(n+1,2) - P(n,2) = (n^2+n) - (n^2-n)$

$= n^2 + n - n^2 + n$

$= 2n$

$= 2P(n,1)$