

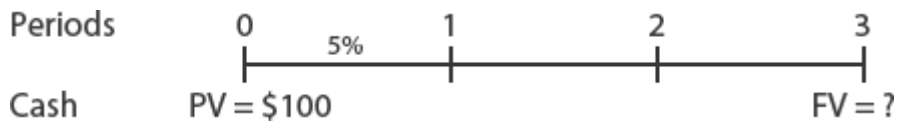
# Chapter 5 Notes

---

## Time Value of Money

---

### Time Lines



- Ticks represent units of time
- Number underneath the line represent cashflows at a given period
- ?'s are used to determine the points at which we need to solve for

### Future Values

The process of going from a **present value (PV)** to a **Future Value (FV)** is called **compounding**.

#### Compound Interest Formula

$$FV_N = PV(1 + I)^N$$

Where **I** = interest and **N** = Number of Periods

**Simple Interest** on the other hand, does not earn interest on the interest.

- Usually used only in legal proceedings because the law moves slowly

$$FV = PV + PV(I)(N)$$

### Working with a Financial Calculator

Five basic keys on a financial calculator for calculating time value of money:

- N (Number of Periods)
- I/YR (interest rate per period)
- PV (Present Value)
- PMT (payment, used when there are equal or constant payments)
- FV (Future Value)

### Working With Spreadsheets

- use the FV function
- FV(rate, nper(N), pmt, pv, type)
- Use the PV function
- PV(rate, nper, pmt, fv, type)

- type is referring to when payments come out (type=0, end of year payments; type=1, beginning of year payments)

## Discounting

Finding **Present Values** is called **Discounting**. The formula is a reorganization of the Future Value Formula

$$PV = \frac{FV_N}{(1+I)^N}$$

- Spreadsheets use PV(rate,nper,pmt,fv,type)

## Finding the Number of Years

- Spreadsheets use NPER(rate,pmt,pv,[fv],[type])

## Annuities

---

- **Ordinary Annuities** are payments that occur annually at the *end* of each year (type=0)
- **Annuity Due** are payments that occur at the *beginning* of each year

Formula

## Finding Future Value

$$FVA_N = PMT(I + 1)^{N-1} + PMT(I + 1)^{N-2} + \dots = PMT \left[ \frac{(1 + I)^N - 1}{I} \right]$$

- FV(rate,nper,pmt,pv,type)
- $FVA_{due} = FVA_{ordinary}(1 + I)$

## Finding Present Value

- $PVA_N = PMT / (I + 1)^1 + PMT / (I + 1)^2 = PMT \left[ 1 - \frac{1}{(1+I)^N} \right] / I$
- PV(rate,nper,pmt,[fv],[type])

## Finding Annuity Payments

-PMT(rate,nper,pv,[fv],[type])

- type 0 for end of year
- type 1 for beginning of year

## Finding the number of Periods

- NPER(rate,pmt,pv,[fv],[type])

## Finding the Interest Rate

- RATE(rate,pmt,pv,[fv],[type],[guess])

## Perpetuities

---

A Perpetuity is an annuity with no end date.

$$PV of Perpetuity = \frac{PMT}{I}$$

## Uneven Cashflows

---

In two separate categories:

1. a stream of annuity payments with a final lump sum
2. All other uneven streams

Examples:

- Common stock (dividends increase over time)
- Capital Equipment investments

$CF_t$  is the term for Cashflow

Formula:

Find PV of each, then Sum

$$PV = \frac{CF_1}{(1+I)^1} + CF_2(1+I)^2 + \dots \text{or } \sum_{t=1} \frac{CF_t}{(1+I)^t}$$

Find FV of each, then sum

Find Interest Rate

IRR

## Semi-Annual Compounding Periods

---

- Interest compounded once a year is **Annual Compounding**
- Interest compounded twice a year or more is **Semi-Annual** compounding interest

Step by step process:

1. Convert Interest rate to "Periodic Rate"
2. Convert number of years to number of periods

Formula's:

$$\text{Periodic Rate } (I_{PER}) = \frac{I}{\text{num.Payments Per Year}}$$

$$\text{Number of Periods} = (\text{Number of years})(\text{Periods per Year}) = NM$$

## Comparing Interest Rates

---

- **Nominal Interest Rate** ( $I_{NOM}$ ) is also called **Annual Percentage Rate** and is the quoted rate of lenders, but not the true rate (a monthly APR equivalent to a Quarterly APR costs more)
- **Effective Annual Rate** or  $EFF$  is the rate that would produce the same future value under annual compounding

- If a loan is annual compounding, the EFI and APR are the same
- if a loan compounds more than once a year, the EFF% is higher than the APR

Formula's:

$$\text{Effective annual Rate (EFF\%)} = \left[1 + \frac{I_{NOM}}{M}\right]^M - 1.0$$

- where M = periods per year  
EFFECT(nomina\_rate,npery)

## Amortized Loans

---

- A loan that must be repaid on a monthly, quarterly, or annual basis is called an **amortized loan**