7a) $\hat{v} = \bar{x} = 120.7$

7c) $X \sim \text{Binomial}(10, \theta)$ $\quad \theta = \frac{8}{10} = .8$

8 a) $\frac{80-12}{80} = .85$

9) a) $E\left(\frac{1}{150}\sum_{i=1}^{150} x_i\right)$ $\quad \frac{1}{150}\left[37 + 2\times42 + 3\times30 + 4\times13 + 5\times7 + 6\cdot2 + 7\right]$

$\frac{1}{150} E(x_i)$

$\frac{317}{150} = 2.1133$

$\frac{150 \cdot \mu}{150} = \mu$ so

9 b) $Var(\bar{x}) = Var\left(\frac{1}{150}\sum_{i=1}^{150} x_i\right) = \left(\frac{1}{150}\right)^2 Var\left(\sum_{i=1}^{150} x_i\right)$

$\sqrt{\frac{\mu}{150}} = \sqrt{\frac{2.1133}{150}} = 0.1187$

Scanned with CamScanner

10. a) ① $E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \frac{1}{n} = \mu$

② $var(\bar{x}) = \frac{1}{n}\sigma^2$

③ $E(\bar{x^2}) = \frac{1}{n}\sigma^2 + \mu^2 \neq \mu^2$

Because ① ② $\neq$ ③ $\bar{x}^2$ is not an unbiased estimator for μ

10b) $E(\bar{x}^2 - ks^2) = E(\bar{x}^2) - kE(s^2)$

$\frac{1}{n}\sigma^2 + \mu^2 - kE(s^2)$

Therefore $= \frac{1}{n}\sigma^2 + \mu - k\sigma^2$

noting $E(\bar{x} - ks^2) = \mu^2$ if $k = \frac{1}{n}$

11a) $E(x_1) = n_1 p_1$ $E(x_1/n_1) = E(x_2/p_2) =$

$E(x_2) = n_2 p_2$

$p_1 - p_2$

$\frac{1}{n_1}E(x_1) - \frac{1}{n_2}E(x_2) = p_1 - p_2$

12a) $S_p^2 \quad \dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$

$\downarrow$

$E(S_p^2)=\sigma^2$

$E(S_p^2)=E\,\dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$

$E(S_p^2)=\dfrac{(n_1-1)E(S_1^2)+(n_2-1)E(S_2^2)}{n_1+n_2-2}$

$E(S_p^2)=\dfrac{(n_1-1)+(n_2-1)}{n_1+n_2-2}\,\sigma^2=\sigma^2$

---

13 $\quad E(x)=(.5)+(x/\theta)$

$E(\bar{x})=\frac{1}{n}(.5\,1+\cancel{\theta}x) \quad -1\leq x\leq 1$

where $-1\leq\theta\leq 1$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_{-1}^{1} x \cdot .5 \cdot (1 + \theta \cdot x)\, dx = \frac{1}{2}\left(\frac{x^2}{2} + \frac{\theta \cdot x^3}{3}\right)\Big|_{-1}^{1}$$

$$E(X) = \frac{1}{2}\left(\left[\frac{1^2}{2} + \frac{\theta \cdot 1^3}{3}\right] - \left[\frac{-1^2}{2} + \frac{\theta \cdot (-1)^3}{3}\right]\right) = \frac{\theta}{3}$$

$$E(\hat{\theta}) = E(3\bar{X}) = 3\,E(X) = 3 \cdot \frac{\theta}{3} = \theta$$

θ is unbiased estimate for $E(X)$