Complexity

Please complete this problem set by December 9, 2021 at 11:59PM. I would recommend reading Chapter 8 of the Algorithms textbook as a reference: https://people.eecs.berkeley.edu/ vazirani/algorithms/chap8.pdf.

Problem 0 – Complexity hierarchy (25%)

Consider the decision version of the job scheduling problem from the exam. We have a number of jobs to be run that all require exclusive use of a shared resource. The i^{th} job has a deadline d_i, a profit p, and takes 1 unit of time for which the job requires exclusive access of the shared resource. Only one job can use the resource at a time and you can schedule any jobs at any time before their deadline; assume that time starts at 0. Is there a scheduling of jobs with profit at least P?

1. Is it true that the decision version of Job Scheduling $\geq_p$ Node Cover? Either say it is unequivocally true (and why), it unequivocally is not true (and why), or that you cannot be 100% sure (and why).
2. Is it true that the decision version of Job Scheduling $\leq_p$ Node Cover? Either say it is unequivocally true (and why), it unequivocally is not true (and why), or that you cannot be 100% sure (and why).

Problem 1 – Hamiltonian cycles and paths (25%)

Given a graph $G(V, E)$, a Hamiltonian path is a path in G that passes through every vertex exactly once. Given a graph $G(V, E)$, a Hamiltonian cycle is a path in G that starts and ends at the same vertex and visits every vertex exactly once (besides the start/end vertex which it visits twice). Show that the problem of finding a Hamiltonian path $\geq_p$ the problem of finding a Hamiltonian cycle. Show that the problem of finding a Hamiltonian cycle $\geq_p$ the problem of finding a Hamiltonian path.

Problem 2 – Math Camp (25%)

You are put in charge of recruiting counselors for UConn's annual math camp. There are n areas of mathematics for which you need a counselor who is knowledgeable to help instruct

the participants. You receive applications from m counselors; each counselor is qualified to assist with a subset of the n mathematics areas. Put in another way, each of the n areas of mathematics have a subset of counselors qualified to assist students.

Show that the following problem is NP-complete. For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified for each of the n areas of mathematics?

Problem 3 – Kernels of hardness (25%)

There are n processes on a cluster that has m distinct resources (cpu1, cpu2, I/O, disk1, disk2, and so on). A process may request multiple resources, but a resource can only be allocated to a single process at a time. If a job is granted access to all of the resources it requests, it may run; otherwise, it is stuck in a pending state.

You are charged with writing an algorithm that allocates resources to processes. **Problem**: Given a set of n processes, m resources, the set of requested resources for each process, and an integer k, is it possible to allocate the resources to processes so that at least k processes are active?

For the following list of problems, give a polynomial-time algorithm or prove that the problem is NP-Complete.

1. The general problem as defined above. If you need a hint, click here.
2. The special case of the problem where $k = 2$.
3. The special case where there are two types of resources: CPU and memory access. Each process requires 0 or 1 of each resource type.
4. The special case where each resource is requested by 0, 1, or 2 processes.