1) Property $\frac{x^{k}-1}{x-1} = x^{k-1}+x^{k-2}+\dots+x^2+x+1$

Therefore:

$$ar^m+ar^{m+1}+\dots+ar^{m+n} = ar^m+ar^m\cdot r+ar^m\cdot r^2+\dots+ar^m\cdot r^n$$

$$= ar^m(1+r+r^2+\dots+r^n)$$

$k-1=n \mid n=0$ and k is natural number

$1+r+r^2+\dots+r^n = \frac{r^{n+1}-1}{r-1}$ therefore

$$ar^m+ar^{m+1}+\dots+ar^{m+n} = ar^m\left(\frac{r^{n+1}-1}{r-1}\right)$$

---

2) $P(n) = e_n = 5\cdot 3^n + 7\cdot 2^n$ $\quad e_k = 5e_{k-1} - 6e_{k-2} \quad k \geq 2$

$e_0 = 12$
$e_1 = 29$

i. $e_0 = 5\cdot 3^0 + 7\cdot 2^0 = 5+7 = 12$ T

ii. $e_1 = 5\cdot 3^1 + 7\cdot 2^1 = 15+14 = 29$ T

iii. $P(0)$ and $P(1)$ are true

IV. $e_i = 5\cdot 3^i + 7\cdot 2^i \rightarrow e_{k+1} = 5\cdot 3^{k+1} + 7\cdot 2^{k+1} = P(k+1)$

V. $e_{k+1} = 5e_k - 6e_{k-1}$ by def of e_0, e_1, e_2

$e_{k+1} = 5(5\cdot 3^k + 7\cdot 2^k) - 6(5\cdot 3^{k-1} + 7\cdot 2^{k-1})$

" " $= 25\cdot 3^k + 35\cdot 2^k - 30\cdot 3^{k-1} - 42\cdot 2^{k-1}$

" " $= 25\cdot 3^k + 35\cdot 2^k - 10\cdot 3\cdot 3^{k-1} - 21\cdot 2\cdot 2^{k-1}$

" " $= 25\cdot 3^k + 35\cdot 2^k - 10\cdot 3^k - 21\cdot 2^k$

" " $= 15\cdot 3^k + 14\cdot 2^k$

" " $= 5\cdot 3\cdot 3^k + 2\cdot 7\cdot 2^k$

" " $= \boxed{5\cdot 3^{k+1} + 7\cdot 2^{k+1}}$ There fore property is true by strong mathematical induction

3) $g_k = \frac{1}{1+\frac{2}{g_{k-1}}}$ or $\frac{1}{g^k} = 1 + \frac{2}{g_{k-1}}$

$q_k = \frac{1}{g_k} = 1 + 2q_{k-1}$

$q = 1$

$q_2 = 1+2$

$q_3 = 2(2+1)+1 = 2^2+2+1$

$q_4 = 2(2^2+2+1)+1 = 2^3+2^2+2+1$

$q_k = 2^{k-1}+2^{k-2}+\ldots+2+1$

$q_k = \frac{2^{k}-1}{2-1} = 2^{k}-1$ for all $k \geq 1$

$g_k = \frac{1}{2^k-1}$ for all $k \geq 1$

4) $P_1 = 2$

$P_2 = P_1 + 2\cdot 3^2$

$= 2 + 2\cdot 3^2$

$= 2(1+3^2)$

$P_3 = P_2 + 2\cdot 3^3$

$= 2(1+3^2)+2\cdot 3^3$

$= 2(1+3^2+3^3)$

$P_4 = P_3 + 2\cdot 3^4$

$2(1+3^2+3^3)+2\cdot 3^4$

$2(1+3^2+3^3+3^4)$

$P_n = 3^{n+1}-7$

Guess: $P_n = 2(1+3^2+3^3+\ldots+3^n)$

$= 2(1+3^2+3^3+\ldots+3^n) - 6$

$= 2\left[\frac{3^{n+1}-1}{3-1}\right] - 6$

$= \cancel{2}\left[\frac{3^{n+1}-1}{\cancel{2}}\right] - 6$

$= 3^{n+1}-1-6$

$= \boxed{3^{n+1}-7}$

5)
- a_k = No, Coefficients are not Constant
- b_k = Yes, $A=-1$, $B=7$
- c_k = No, Not homogeneous because there is a second term
- d_k = no, not linear because $(d_{k-1})^2$
- r_k = No, not second order because $k-3$ not $k-2$
- s_k = Yes, $A=1$, $B=10$

b_k and s_k are second order homogeneous reccurence relations with constant coefficients

6) $A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$
$B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$
$C = \{x \in \mathbb{R} / 3 \leq x < 9$

$A \cup B = \{x \in \mathbb{R} / 0 < x < 4\}$

$A \cap B = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$

$A^c = \{x \in \mathbb{R} \mid \neg(0 < x \leq 2)\} = \{x \in \mathbb{R} / x \leq 0 \text{ or } x > 2\}$

$A \cup C = \{x \in \mathbb{R} \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

$A \cap C = \emptyset$ disjoint sets

$B^c = \{x \in \mathbb{R} \mid x < 1 \text{ or } 4 \leq x\}$

$A^c \cap B^c = \{x \in \mathbb{R} \mid x \leq 0 \text{ or } x \geq 4\}$

$A^c \cup B^c = \{x \in \mathbb{R} \mid x < 1 \text{ or } x > 2\}$

$(A \cap B)^c = \{x \in \mathbb{R} / x < 1 \text{ or } x > 2\}$

7) Let $x \in (A-B) \cup (C-B) = x \in (A-B)$ or $x \in (C-B)$

Case 1: $x \in (A-B)$
by definition of the difference $x \in A$, $x \notin B$
by def of union $x \in C$, $\therefore x \in A \cup C$
by def of difference $x \in A \cup C$ and $x \notin B$,
$x \in (A \cup C) - B$ T

case 2: $x \in C-B$
by def od difference $x \in C$, $x \notin B$
by def of union : $x \in C$: $x \in A \cup C$
by def of difference $x \in A \cup C$ and $x \notin B$
$x \in A \cup C - B$ T

Let $x \in (A \cup C) - B$
by def of difference $(x \in A$ or $x \in C)$
$x \notin B$

Case 1: $x \in A$ and $x \notin B$
by def of difference : $x \in A - B$
by def of union $x \in A - B$
$x \in (A-B) \cup (C-B)$ T

Case 2: $x \in C$ and $x \notin B$
by def of difference: $x \in C - B$
by def of union, $x \in C - B$
$x \in (A-B) \cup (C-B)$ T

Since they are subsets of each other, they must be equal

8) $A = (A - B) \cup (A \cap B)$

$= (A \cap B^c) \cup (A \cap B)$ - set difference law

$= A \cap (B^c \cup B)$ - Distributive Law

$= A \cap (B \cup B^c)$ - Commutative Law for $\cup$

$= A \cap U$ - Complement law for $\cup$

$= A$ - Identity law for $\cap$