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Quiz 1

Math 2210Q, Section 12

Consider

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 3 \\ 5 & 20 & 15 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}.$$

(1) Write above equation as a vector equation.

$Ax = b$ $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 3 \\ 5 & 20 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 2 \\ 2 \\ 10 \end{pmatrix}$

(2) Write it as a a system of linear equations.

$1x_1 + 1x_2 = 2$

$x_1 + 4x_2 + 3x_3 = 3$

$5x_1 + 20x_2 + 15x_3 = 10$

(3) Solve it any way you like.

$R_2 - R_1 \rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 3 & 6 \\ 5 & 20 & 15 & 10 \end{bmatrix}$

$R_3 - 5 \cdot R_1$

$\begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 3 & 6 \\ 0 & 15 & 15 & 0 \end{pmatrix}$

$R_3 - 5R_2 =$

$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Free

(4) Write the solution parametrically.

$X = \begin{pmatrix} 2 + x_3 \\ -x_3 \\ x_3 \end{pmatrix}$

(5) What is the dimension of the solution set, i.e., how many free variables are there?

x_3 Free

1 Free variable

(6) Is the following matrix in reduced echelon form?

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

no

(7) If the matrix above is the augmented matrix for a linear system, what are the solutions to the system?

$R2 - R1$
$$\begin{pmatrix} 1 & 0 & 5 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x + 5x_3 = 0$ $\quad x_1 = 0$

$-5x_3 = 0 \rightarrow x_3 = 0 \quad x_2$ free

$x \begin{Bmatrix} 0 \\ x_2 \\ 0 \end{Bmatrix}$

Bonus: The *transpose* of a matrix A, denoted A^T, is obtained by "flipping the entries across the diagonal," e.g., the transpose of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$. If $\begin{bmatrix} e \\ f \end{bmatrix} \neq \vec{0}$ is a solution to $A\vec{x} = \vec{0}$, what is a non-trivial solution to $\begin{bmatrix} x_1 & x_2 \end{bmatrix} A^T = \vec{0}$.

Need more time