1. $n=2k+1$

$$\frac{n^2}{4} = \frac{(2k+1)^2}{4} = \frac{2k^2+2(2k)+1^2}{4} = \frac{4k^2+4k+1}{4} = k^2+k+\frac{1}{4}$$

$$k^2+k < k^2+k+\frac{1}{4} < k^2+k+1$$

$$\frac{n^2}{4} = k^2+k = 4(k+1) =$$

Proof

$$\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+\frac{2}{2}\right) = \left(\frac{n-1+2}{2}\right)\left(\frac{n-1}{2}\right) = \left(\frac{n+1}{2}\right)\left(\frac{n-1}{2}\right)$$

2. froom proof 1.

$$k^2+k < k^2+k+\frac{1}{4} < k^2+k+1$$

$$k^2+k < \frac{n^2}{4} < k^2+k+1$$

$$\frac{n^2}{4} = k^2+k+1$$

$$\left(\frac{n-1}{2}\right)^2 + \frac{n-1}{2} + 1$$

$$\frac{n^2-2n+1}{4} + \frac{2n-2}{4} + \frac{4}{4}$$

$$\frac{n^2-2n+1+2n-2+4}{4} = \frac{n^2+3}{4}$$

$\therefore$

3. $a^2 = (2k+1)^2$

$4k^2 + 1 + 4k$

$2(2k^2 + 2k) + 1$

$2(\text{int}) + 1$ $a^2 + b^2 + c^2$ must be odd

$a^2 + b^2 = \text{odd} + \text{odd}$ odd + odd = even

Proof if a and b are odd, $c \neq$ odd

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4. + 5.? $m = 2k+1$

$n = 2x+1$ • $(2k+1) \cdot (2x+1) =$

• $4kl + 2k + 2x + 1$

• $2(\underset{\text{int}}{2kx + k + x}) + 1$

Proof: • $(2z+1) = \text{odd}$

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6. Prove
• $a^2 - 3 = 4k$
• $a^2 = 9k + 3$
• $a^2 = 3(3k+1) \Rightarrow a^2$ is divisible by 3
• $a^2 = (3b)^2$

• $a^2 = 9b^2$
• $= 3(3b^2)$

The assumption is false, since $a^2 = 3(3b)$ and $a^2 = 3(3k+1)$
for all integers $n \neq \frac{1}{2}$

6. cont. since $d^2 = 3(3b^2)$ and $d^2 = 3(3k+1)$

$n = 3q_1 + r_1$ and $n = 3q_2 + r_2$ $r_1 \neq r_2$,

The assumption d is an integer such that $9 \mid (d^2-3)$ is false, implying for all integers d, $9 \nmid (d^2-3)$ ∴

7. 1. $\boxed{n = 12 = 4 \cdot 3 + 0 \cdot 7}$

2. for $n \geq 12$, $n = 3a + 7b$ $a, b \geq 0$,

i. if $a \geq 2$, then $3(a-2) + 7(b+1) = n+1$

ii. if $a \leq 1$, $3a + 7b = n \geq 12$

$7b \geq 9$ so $3(a+5) + 7(b-2) = n+1$

$b \geq 2$

There fore, $n+1$ cents can be made with 3 or 7 cent stamps.

8. • $1 \cdot 2^1 = 2$ and $0 \cdot 2^2 + 2 = 2$

• Let $k \geq 0$ and suppose $\sum_{i=1}^{k+1} i \cdot 2^i = k \cdot 2^{k+2} + 2$

$\quad '' \quad = (k+1) \cdot 2^{k+3} + 2$

8. cont. $= \left(\sum_{i=1}^{k+1} i \cdot 2^i\right) + \left((k+2) \cdot 2^{k+2}\right)$

$'' = \left(k \cdot 2^{k+2} + 2\right) + \left((k+2) \cdot 2^{(k+2)}\right)$

$'' \; '' = 2^{k+2} \cdot (2k+2) + 2$

$= 2^{k+2} \cdot 2(k+1) + 2$

$= (k+1) \cdot 2^{k+3} + 2$ $\therefore$ proof

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9. $n^3 - n / 6$ for $n \geq 0$

$n=2$ $\quad 2^3 - 2 = 6$

- for all $k \geq 2$, propex is true for $n = k$,
  " " $n = k+1$
- $(k+1)^3 - (k+1) / 6$
- $k^3 + 3k^2 + 3k + 1 - k - 1$
- $(k^3 - k) + 3(k^2 + k)$
- $(k^3 - k) + 3(k(k+1))$
- $k^3 - k = 6r$
- $k(k+1) = 2s$
- $6r + 3 \cdot 2s = 6(r+s)$ divisible by 6