Name [illegible]

Quiz 2

Math 2210Q, Section 12

Consider the matrix equation

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 3 & 3 & 4 & 0 \\ 7 & 7 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 9 \\ 25 \\ 55 \\ 63 \end{bmatrix}. \quad \text{One solution is} \quad \begin{bmatrix} 9 \\ 0 \\ 7 \\ 8 \end{bmatrix}.$$

(1) Write the full solution set as a linear combination of vectors, using free variables as parameters.

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$x_1 + x_2 = 0$

$x_3 = 0$

$x_4 = \text{Free}$

Wrong

$$\begin{matrix} 1 & 1 & 0 & 0 & 9 \\ 2 & 2 & 1 & 0 & 25 \\ 3 & 3 & 4 & 0 & 55 \\ 7 & 7 & 0 & 0 & 63 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$x_1 \cancel{+x_2} = 9 - x_2$ $-x_2$

$x_2 = \text{Free}$

$x_3 = 7$

$x_4 = \text{Free}$

Real, no contradiction

$x_1 = 9 - x_2$

$x_2 = \text{Free}$

$x_3 = 7$

$x_4 = \text{free}$

(2) Show the following vectors are linearly independent.

$$\begin{bmatrix} 8 \\ 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}$$

Independ

$$\begin{bmatrix} 8 & 6 & 7 & 0 \\ 5 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

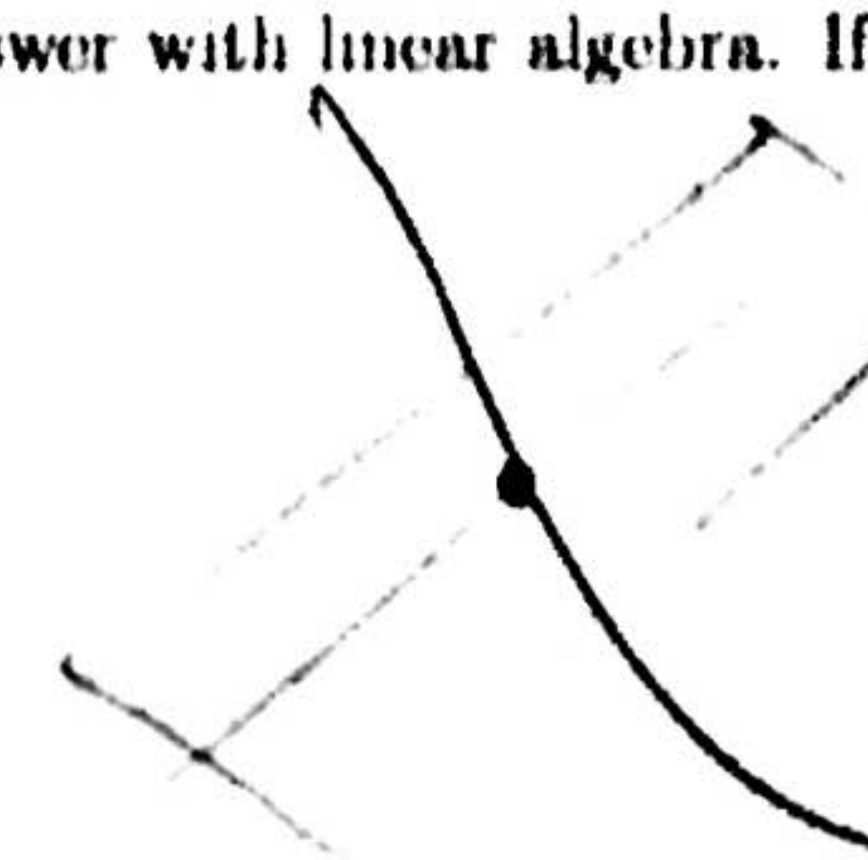
So yes

$\alpha_1 = 0$
$\alpha_2 = 0$
$\alpha_3 = 0$

Only the trivial solution

Independ

Bonus: A *hyperplane* in $\mathbb{R}^n$ is an $n-1$ dimensional linear subspace cut out by one non-trivial linear equation. A *line* is a one dimensional linear subspace cut out by $n-1$ appropriately chosen linear equations. If the line doesn't lie on the hyperplane, what do you expect the dimension of the intersection of the line and the plane to be? Justify your answer with linear algebra. If you don't have time to answer here, answer on Piazza.



$V = \text{Span}(v)$

Contains V but no $w \notin V$ for $n-1$ subspace

$\text{Sub}_1 \cap \text{Sub}_2$