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Midterm Exam Math 2210Q, Section 12

Instructions: You have 70 minutes to take this exam and 5 minutes to upload a clear and legible copy of your answers to HuskyCT. On each question show all your work and explain your reasoning. You may refer to your textbook during the exam, but no other outside assistance is permitted.

(1) Solve the following system of equations: x + y + z = 9, 2x + 3y = 8, x - y - z = -7.



(2) Find all solutions to the matrix equation $\begin{bmatrix} 2 & 8 & 0 \\ 0 & 0 & c \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0}.$

(3) For which value(s) of c does the above equation have a unique solution?

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$$0.-1.|\frac{3-3}{3-6}|+0.-1.|\frac{1-2}{0-3}|=$$

$$2|\frac{1-2}{3-6}| ad-bc = -6--6=0$$

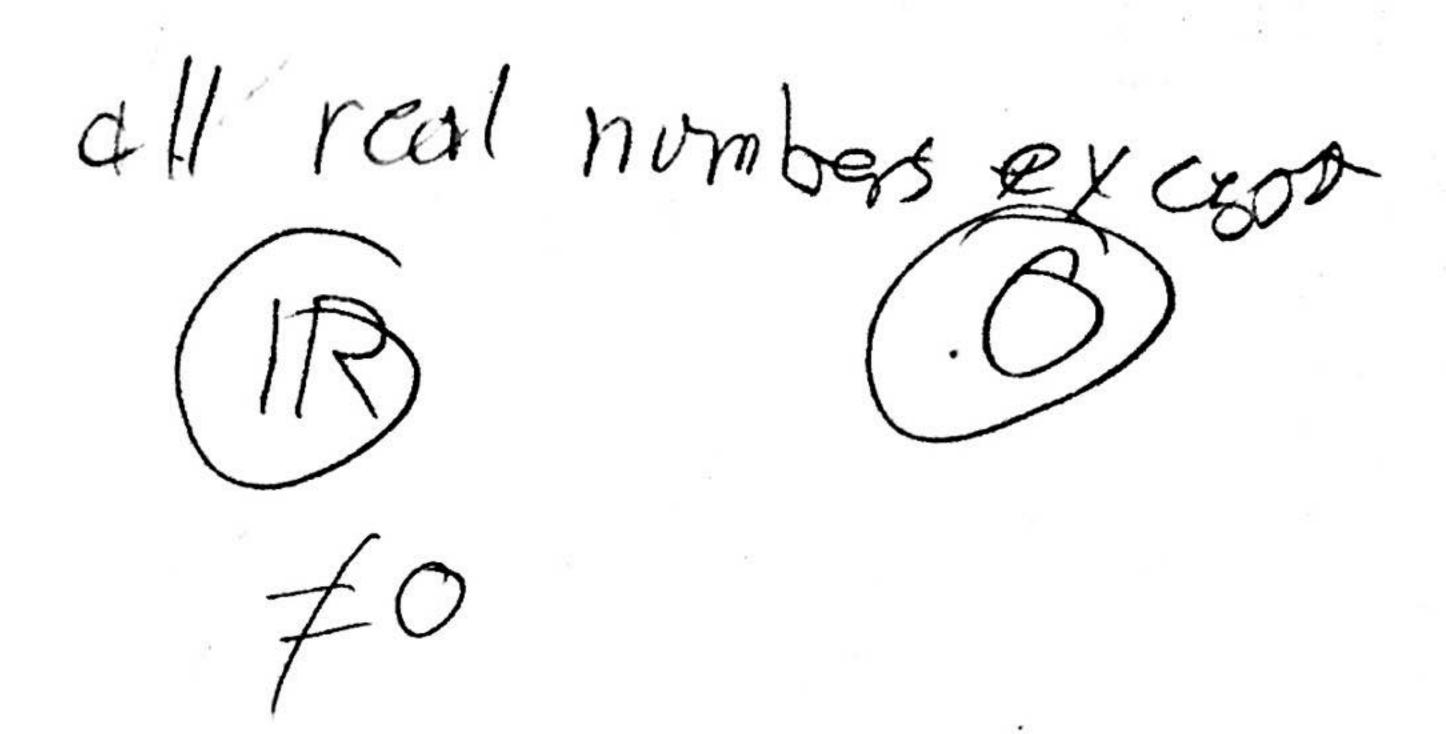
$$\det = 0$$

Vectors de dejondro Strice determinate 15 0

(5) Is
$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$
 in Span $\left(\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \\ 12 \end{bmatrix} \right)$?

1 is not in span

(6) Let A be the matrix representing the linear transformation from \mathbb{R}^2 to \mathbb{R}^2 given by $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 7 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ c \end{bmatrix}$. For what value(s) of c does A have an inverse?



(7) Supposing c is such that A is invertible, what is the matrix A^{-1} ?

For the following statements let A be an $n \times n$ matrix. Determine whether each statement is True or False. Only answer True if the statement is always true.

(8) A is invertible if and only if the diagonal is non-zero.

True

Irve

(9) If the determinant is A is non-zero, then the linear transformation represented by A is one-to-one.

(10) The image of A forms a subset of \mathbb{R}^n , but not always a subspace.

False

(11) The kernel of A forms a subspace of the codomain.

Folse

(12) A maps all non-zero vectors to a non-zero vectors if and only if A is invertible.

Truc

(13) The columns of A are linearly dependent if and only if the determinant is non-zero.

Table

(14) A is row equivalent to the identity matrix if and only if A is invertible.

Tru

(15) If A is invertible then, $\det(A^{-1}) = -\det(A)$.

Fulse

(16) A may be onto, but not one-to-one.

Truc

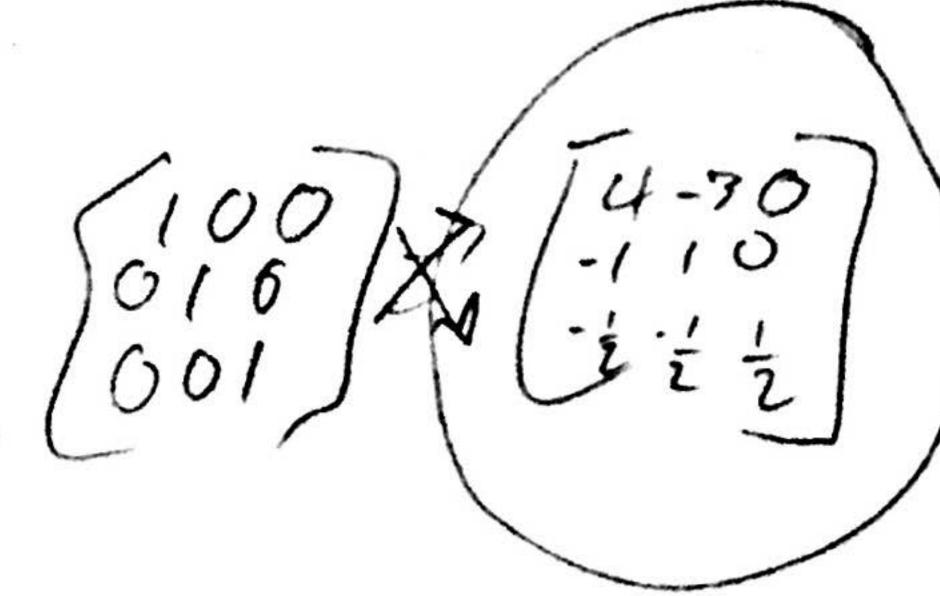
(17) If n = 2 and det(A) = 3, then A sends $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} \sqrt{3} \\ 0 \end{bmatrix}$

(°°) 3

Use cofactor expansion along the 2^{nd} row of A to compute det(A). Check your work with the cofactor expansion along the 3^{rd} column of A.

(19) A sphere in \mathbb{R}^3 has volume $\frac{4}{3}\pi r^3$. Compute the volume of the image of the sphere under the linear transformation corresponding to the matrix A from (18).

(20) Let
$$A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 4 & 0 \\ 2 & 7 & 2 \end{bmatrix}$$
. Compute A^{-1} .



(21) Suppose I tell you that an $n \times n$ matrix B is invertible. Write down at least three non-trivial things that you now know about B and the linear transformation corresponding to B.

Let $A = \begin{bmatrix} -3 & 3 \\ 1 & 2 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 3 & 1 \end{bmatrix}$. For each of the following questions compute the given quantity or explain why it is not possible to compute.

compute the given quantity or explain why it is not possible to compute.

(22)
$$BA$$

ROWS OF B X COIS OF A

(23) AB ROWS OF A X COIS OF A

(24) A^{TB}

(25) $(BA)^{-1}$

(26) $B^{T}A^{T}$

ROW B T. COI AT

(26) $B^{T}A^{T}$

ROW B T. COI AT

(27) Construct a
$$3\times3$$
 matrix that has image (column space) equal to Span $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{bmatrix} 7 \\ -1 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$.

(28) Construct a
$$3\times3$$
 matrix that has kernel (null space) equal to Span $\left(\begin{bmatrix}1\\-1\\0\end{bmatrix},\begin{bmatrix}7\\-1\\6\end{bmatrix},\begin{bmatrix}2\\0\\2\end{bmatrix}\right)$.

(29) Consider the set $H := \left\{ \begin{bmatrix} 0 \\ b \\ c \end{bmatrix} : b, c \in \mathbb{R} \right\}$. Prove H is a subspace under the vector space operations inherited from \mathbb{R}^3 .

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(30) Define new vector space operations so that H is still a vector space, but no longer a subspace of R³.

Time

¹The symbol ':=' is read 'defined to be equal to.' It is apparently borrowed from computer science.

Bonus: Given an invertible linear transformation $A: \mathbb{R}^2 \to \mathbb{R}^2$, describe the possible images of a parabola under A.

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