CSE 2500 - Introduction to Discrete Systems	Spring 2021
Assignment 3	Due date: February 26, 11:59pm, EST

For full credit, please adhere to the following:

- Note that for Assignment 3, **solutions will be posted on March 1.**
- Unsupported answers receive no credit.
- All answers can be typed or handwritten, and should be readable.
- Submit the assignment in one file (.pdf, .doc, etc.) via HuskyCT.

All of the following questions are in the textbook for the course, *Discrete Mathematics with Applications (5th Edition)*, by Susanna Epp.

1. Exercise Set 3.1
   (a) (5 points) Question 32 part (d)
   Let $\mathbf{R}$ be the domain of the predicate variable x. Which of the following are true and which are false? Give counter examples for the statements that are false
   (d) $x^2 > 4 \Leftrightarrow |x| > 2$ True, $x^2>4 \equiv |x|>2$
   (b) (5 points) Question 33 part (d)
   Let $\mathbf{R}$ be the domain of the predicate variables a, b, c, and d. Which of the following are true and which are false? Give counterexamples for the statements that are false.
   (d) $a < b$ and $c < d \Rightarrow ac < bd$ False, $a=-1$ $b=2$ $c=-6$ $d=1$ $\not\Rightarrow$ $6<2$

2. (10 points) Exercise Set 3.2, Question 19
   Write a negation for the following statement.

   $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$. $\exists n \in \mathbf{Z}$ such that n is prime, n is even, and $n \neq 2$.

3. (10 points) Exercise Set 3.2, Question 29
   Write the converse, inverse, and contrapositive. Indicate as best as you can which among the statement, its converse, its inverse, and its contrapositive are true and which are false. Give a counterexample for each that is false. C: if $d=3$ then, $6/d$ is an integer, T

   $\forall$ integer d, if $6/d$ is an integer then $d = 3$. C: If $6/d$ is not an integer, then $d \neq 3$, T
   if $d \neq 3$, then $6/d$ is not an integer, F
   2

4. (10 points) Exercise Set 3.2, Question 47
   The computer scientists Richard Conway and David Gries once wrote:

   The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.

A program is not reasonably correct if it has no error messages during translation.

Rewrite this statement without using the words necessary or sufficient.

5. (10 points) Exercise Set 3.3, Question 12
Let $D = E = \{-2, -1, 0, 1, 2\}$. Write negations for each of the following statements and determine which is true, the given statement or its negation.

$\exists x \in D$ such that $\forall y \in E$, $xy < y$, False

$\forall x$ in D, $\exists y$ in E such that $xy \geqslant y$

if $y = -2$, $x = 2$, $-4 \geq -2$

6. (10 points) Exercise Set 3.3, Question 43
The following is the definition for $\lim_{x \to a} f(x) = L$:
For all real numbers $\epsilon > 0$, there exists a real number $\sigma > 0$ such that for all real numbers x, if $a - \sigma < x < a + \sigma$ and $x \neq a$ then $L - \epsilon < f(x) < L + \epsilon$. Write what it means for $\lim_{x \to a} f(x) \neq L$. In other words, write the negation of the definition.

$\exists \epsilon > 0$ such that $\forall \delta > 0$, $\exists x$ such that $a - \delta < x < a + \delta$ and $x \neq a$ and either $f(x) \leq L - \epsilon$ or $f(x) \geq L + \epsilon$

7. (10 points) Exercise Set 3.3, Question 50
Let $P(x)$ and $Q(x)$ be predicates and suppose D is the domain of x. For the statement forms in each pair, determine whether (a) they have the same truth value for every choice of $P(x)$, $Q(x)$, and D, or (b) there is a choice of $P(x)$, $Q(x)$, and D for which they have opposite truth values. a) not always same truth value

b) There is such a choice

$\exists x \in D, (P(x) \wedge Q(x))$, and ~~$(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$~~

8. (10 points) Exercise Set 3.4, Question 18
Some of the arguments in 7-18 are valid by universal modus ponens or universal modus tollens; others are invalid and exhibit the converse or the inverse error. State which are valid and which are invalid. Justify your answers.

- If an infinite series converges, then its terms go to 0.
- The terms of the infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ do not go to 0.
- $\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ does not converge.

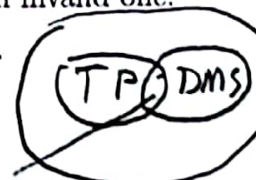
Universal modus Tollens, Valid

$\forall x$, if $P(x)$ then $Q(x)$
$\sim Q(x)$
$\sim P(x)$

$P \to Q$
not Q
therefore not P

9. (10 points) Exercise Set 3.4, Question 22
Indicate whether the arguments in 21-27 are valid or invalid. Support your answers by drawing diagrams.

- $\overbrace{\text{All discrete mathematics students}}^{\text{DMS}}$ $\overbrace{\text{can tell a valid argument from an invalid one.}}^{\text{X}}$
- TP All thoughtful people can tell a valid argument from an invalid one.
- $\therefore$ All discrete mathematics students are thoughtful



invalid, not all students must be thoughtful

Both

10. (10 points) Exercise Set 3.4, Question 32
In exercises 28-32, reorder the premises in each of the arguments to show that the

conclusion follows as a valid consequence from the premises. It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives. Exercises 31 and 32 are adapted from Symbolic Logic by Lewis Carroll.

1. When I work a logic example without grumbling, you may be sure it is one I understand.
2. The arguments in these examples are not arranged in regular order like the ones I am used to.
3. No easy examples make my head ache.
4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.
5. I never grumble at an example unless it gives me a headache.

∴ These examples are not easy.

2. These examples are not arranged in regular order.
4. for all x, if x is not arranged in order, then I don't get x.
1. If I don't get x, I work on x with grumbling
5. for all x, if I grumble on x, it gives me a headache
3. for all x, if x makes my head ache, x is not easy
∴ These examples are not easy