

Main Heading

Review Material

TRANSPORTATION MODELING

(pp. 730–731)

The transportation models described in this module prove useful when considering alternative facility locations *within the framework of an existing distribution system*. The choice of a new location depends on which will yield the minimum cost for the entire system.

■ **Transportation modeling**—An iterative procedure for solving problems that involves minimizing the cost of shipping products from a series of sources to a series of destinations.

Origin points (or *sources*) can be factories, warehouses, car rental agencies, or any other points from which goods are shipped.

Destinations are any points that receive goods.

To use the transportation model, we need to know the following:

1. The origin points and the capacity or supply per period at each.
2. The destination points and the demand per period at each.
3. The cost of shipping one unit from each origin to each destination.

The transportation model is a type of linear programming model.

A *transportation matrix* summarizes all relevant data and keeps track of algorithm computations. Shipping costs from each origin to each destination are contained in the appropriate cross-referenced box.

FROM \ TO	DESTINATION 1	DESTINATION 2	DESTINATION 3	CAPACITY
Source A				
Source B				
Source C				
Demand				

Concept Questions:
1.1–1.4

DEVELOPING AN INITIAL SOLUTION

(pp. 732–734)

Two methods for establishing an initial feasible solution to the problem are the northwest-corner rule and the intuitive lowest-cost method.

■ **Northwest-corner rule**—A procedure in the transportation model where one starts at the upper-left-hand cell of a table (the northwest corner) and systematically allocates units to shipping routes.

The northwest-corner rule requires that we:

1. Exhaust the supply (origin capacity) of each row before moving down to the next row.
2. Exhaust the demand requirements of each column before moving to the next column to the right.
3. Check to ensure that all supplies and demands are met.

The northwest-corner rule is easy to use and generates a feasible solution, but it totally ignores costs and therefore should be considered only as a starting position.

■ **Intuitive method**—A cost-based approach to finding an initial solution to a transportation problem.

The intuitive method uses the following steps:

1. Identify the cell with the lowest cost. Break any ties for the lowest cost arbitrarily.
2. Allocate as many units as possible to that cell, without exceeding the supply or demand. Then cross out that row or column (or both) that is exhausted by this assignment.
3. Find the cell with the lowest cost from the remaining (not crossed out) cells.
4. Repeat Steps 2 and 3 until all units have been allocated.

Concept Questions:
2.1–2.4

Problems: C.1–C.3, C.15

THE STEPPING-STONE METHOD

(pp. 734–737)

■ **Stepping-stone method**—An iterative technique for moving from an initial feasible solution to an optimal solution in the transportation method.

The stepping-stone method is used to evaluate the cost-effectiveness of shipping goods via transportation routes not currently in the solution. When applying it, we test each unused cell, or square, in the transportation table by asking: What would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route? We conduct the test as follows:

1. Select any unused square to evaluate.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used (only horizontal and vertical moves are permissible). You may, however, step over either an empty or an occupied square.

Concept Questions:
3.1–3.4

Problems: C.4–C.13

Virtual Office Hours
for Solved Problem: C.1

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	<p>3. Beginning with a plus (+) sign at the unused square, place alternative minus signs and plus signs on each corner square of the closed path just traced.</p> <p>4. Calculate an improvement index by first adding the unit-cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign.</p> <p>5. Repeat Steps 1 through 4 until you have calculated an improvement index for all unused squares. If all indices computed are <i>greater than or equal to zero</i>, you have reached an optimal solution. If not, the current solution can be improved further to decrease total shipping costs.</p> <p><i>Each negative index represents the amount by which total transportation costs could be decreased if one unit was shipped by the source–destination combination. The next step, then, is to choose that route (unused square) with the largest negative improvement index. We can then ship the maximum allowable number of units on that route and reduce the total cost accordingly. That maximum quantity is found by referring to the closed path of plus signs and minus signs drawn for the route and then selecting the smallest number found in the squares containing minus signs. To obtain a new solution, we add this number to all squares on the closed path with plus signs and subtract it from all squares on the path to which we have assigned minus signs. From this new solution, a new test of unused squares needs to be conducted to see if the new solution is optimal or whether we can make further improvements.</i></p>
SPECIAL ISSUES IN MODELING (pp. 737–738)	<p><i>Dummy sources</i>—Artificial shipping source points created when total demand is greater than total supply to effect a supply equal to the excess of demand over supply.</p> <p><i>Dummy destinations</i>—Artificial destination points created when the total supply is greater than the total demand; they serve to equalize the total demand and supply.</p> <p>Because units from dummy sources or to dummy destinations will not in fact be shipped, we assign cost coefficients of zero to each square on the dummy location. If you are solving a transportation problem by hand, be careful to decide first whether a dummy source (row) or a dummy destination (column) is needed.</p> <p>When applying the stepping-stone method, <i>the number of occupied squares in any solution (initial or later) must be equal to the number of rows in the table plus the number of columns minus 1</i>. Solutions that do not satisfy this rule are called <i>degenerate</i>.</p> <p>■ Degeneracy—An occurrence in transportation models in which too few squares or shipping routes are being used, so that tracing a closed path for each unused square becomes impossible.</p> <p>To handle degenerate problems, we must artificially create an occupied cell: That is, we place a zero (representing a fake shipment) into one of the unused squares and <i>then treat that square as if it were occupied</i>. Remember that the chosen square must be in such a position as to allow all stepping-stone paths to be closed.</p>

Concept Questions:
4.1–4.4

Problems: C.14–C.18

Virtual Office Hours for
Solved Problem: C.2

Self Test

■ **Before taking the self-test**, refer to the learning objectives listed at the beginning of the module and the key terms listed at the end of the module.

LO C.1 With the transportation technique, the initial solution can be generated in any fashion one chooses. The only restriction(s) is that:

- the solution be optimal.
- one use the northwest-corner method.
- the edge constraints for supply and demand be satisfied.
- the solution not be degenerate.
- all of the above.

LO C.2 The purpose of the stepping-stone method is to:

- develop the initial solution to a transportation problem.
- identify the relevant costs in a transportation problem.
- determine whether a given solution is feasible.
- assist one in moving from an initial feasible solution to the optimal solution.
- overcome the problem of degeneracy.

LO C.3 The purpose of a *dummy source* or a *dummy destination* in a transportation problem is to:

- provide a means of representing a dummy problem.
- obtain a balance between total supply and total demand.
- prevent the solution from becoming degenerate.
- make certain that the total cost does not exceed some specified figure.
- change a problem from maximization to minimization.

LO C.4 If a solution to a transportation problem is degenerate, then:

- it will be impossible to evaluate all empty cells without removing the degeneracy.
- a dummy row or column must be added.
- there will be more than one optimal solution.
- the problem has no feasible solution.
- increase the cost of each cell by 1.

Answers: LO C.1. c; LO C.2. d; LO C.3. b; LO C.4. a.