

1. The current in a certain circuit as measured by an ammeter is a continuous random variable  $X$  with the following density function:

$$f(x) = \begin{cases} .075x + .2 & 3 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- Graph the pdf and verify that the total area under the density curve is indeed 1.
  - Calculate  $P(X \leq 4)$ . How does this probability compare to  $P(X < 4)$ ?
  - Calculate  $P(3.5 \leq X \leq 4.5)$  and also  $P(4.5 < X)$ .
2. Suppose the reaction temperature  $X$  (in  $^{\circ}\text{C}$ ) in a certain chemical process has a uniform distribution with  $A = -5$  and  $B = 5$ .
- Compute  $P(X < 0)$ .
  - Compute  $P(-2.5 < X < 2.5)$ .
  - Compute  $P(-2 \leq X \leq 3)$ .
  - For  $k$  satisfying  $-5 < k < k + 4 < 5$ , compute  $P(k < X < k + 4)$ .
3. The error involved in making a certain measurement is a continuous rv  $X$  with pdf

$$f(x) = \begin{cases} .09375(4 - x^2) & -2 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Sketch the graph of  $f(x)$ .
- Compute  $P(X > 0)$ .
- Compute  $P(-1 < X < 1)$ .
- Compute  $P(X < -.5 \text{ or } X > .5)$ .

4. Let  $X$  denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. The article "Blade Fatigue Life Assessment with Application to VAWTS" (*J. of Solar Energy Engr.*, 1982: 107–111) proposes the Rayleigh distribution, with pdf

$$f(x; \theta) = \begin{cases} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

as a model for the  $X$  distribution.

- Verify that  $f(x; \theta)$  is a legitimate pdf.
  - Suppose  $\theta = 100$  (a value suggested by a graph in the article). What is the probability that  $X$  is at most 200? Less than 200? At least 200?
  - What is the probability that  $X$  is between 100 and 200 (again assuming  $\theta = 100$ )?
  - Give an expression for  $P(X \leq x)$ .
5. A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let  $X$  = the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of  $X$  is

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- Find the value of  $k$  and draw the corresponding density curve. [*Hint:* Total area under the graph of  $f(x)$  is 1.]
- What is the probability that the lecture ends within 1 min of the end of the hour?