| CSE 2500 - Introduction to Discrete Systems | Spring 2021 |
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| Assignment 2 | Due date: Feb 17th, 11:59pm, EST |

For full credit, please adhere to the following:

- Unsupported answers receive no credit.
- All answers can be typed or handwritten, and should be readable.
- Submit the assignment in one file (.pdf, .doc, etc.) via HuskyCT.

All of the following questions are in the textbook for the course, *Discrete Mathematics with Applications (5th Edition)*, by Susanna Epp.

1. Exercise Set 2.1

   (a) (5 points) Question 28
   Use De Morgan's laws to write negations for the statement "The train is late or my watch is fast." $\neg(TL \vee WF) \rightarrow (\neg TL) \wedge (\neg WF)$

   $A = 0 > x$

   (b) (5 points) Question 37
   Assume $x$ is a particular real number and use De Morgan's laws to write the negation of $0 > x \geq -7$. $\neg(A \wedge B) \rightarrow \neg A \vee \neg B$

   $B = x \geq -7$

2. (10 points) Exercise Set 2.1, Question 46(c)
   In Example 2.1.4, the symbol $\oplus$ was introduced to denote exclusive or, so $p \oplus q \equiv (p \vee q) \wedge \sim (p \wedge q)$. Hence the truth table for exclusive or is as follows:

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

| p | q | r | $p \oplus q$ | $q \oplus r$ | $(p \oplus q) \oplus R$ | $p \oplus (q \oplus r)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T |
| T | T | F | F | T | F | F |
| T | F | T | T | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | T | F | F | F |
| F | T | F | T | T | T | T |
| F | F | T | F | T | T | T |
| F | F | F | F | F | F | F |

   same

   Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? Justify your answer.

3. (10 points) Exercise Set 2.1, Question 52
   Use Theorem 2.1.1 to verify the logical equivalence:

$$\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$$

   Supply a reason for each step.

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3. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$ Distribution

$(\sim p \wedge \sim(\sim q)) \vee (\sim p \wedge \sim q)$ Double Negation

$(\sim p \wedge q) \vee (\sim p \wedge \sim q)$ Distributive

$\sim p \wedge (q \vee \sim q)$ Negation Law

$\sim p \wedge T$ Identity Law

$\sim p$

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| Name | Equivalence |
|---|---|
| Identity Laws | $p \vee c \equiv p$ <br> $p \wedge t \equiv p$ |
| Domination Laws | $p \vee t \equiv t$ <br> $p \wedge c \equiv c$ |
| Idempotent Laws | $p \vee p \equiv p$ <br> $p \wedge p \equiv p$ |
| Double Negation Law | $\sim(\sim p) \equiv p$ |
| Universal Bound Laws | $p \vee t \equiv t$ <br> $p \wedge c \equiv c$ |
| Negations of **t** and **c** | $\sim t \equiv c$ <br> $\sim c \equiv t$ |
| Commutative Laws | $p \vee q \equiv q \vee p$ <br> $p \wedge q \equiv q \wedge p$ |
| Associative Laws | $p \vee (q \vee r) \equiv (p \vee q) \vee r$ <br> $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ |
| Distributive Laws | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ <br> $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| Absorption Law | $p \vee (p \wedge q) \equiv p$ <br> $p \wedge (p \vee q) \equiv p$ |
| Negation Law | $p \vee \sim p \equiv t$ <br> $p \wedge \sim p \equiv c$ |

4. (10 points) Exercise Set 2.2, Question 21 if p then q ≠ T
Suppose that $p$ and $q$ are statements so that $p \to q$ is false. Find the truth values of each of the following:
T F

(a) $\sim p \to q$ True

(b) $p \vee q$ True

(c) $q \to p$ True

5. (10 points) Exercise Set 2.2, Question 27
Use truth tables to establish the truth of the statement "The converse and inverse of a conditional statement are logically equivalent to each other." Back

6. (10 points) Exercise Set 2.2, Question 43
Use the contrapositive to rewrite the statements in if-then form in two ways. Doing homework regularly is a necessary condition for Jim to pass the course.

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7. (10 points) Exercise Set 2.2, Question 50
Use the logical equivalences $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\leftrightarrow$, and (b) use the

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5.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $q \to p$ | $\sim p \to \sim q$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | F | T | T | T |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

6. If Jim does his homework regularly he may pass the course.

If Jim does not do his homework regularly, he will not pass the course.

7. a) $\equiv (p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ $\quad p \to q \equiv \sim p \vee q$

$\equiv (p \to (\sim q \vee r)) \leftrightarrow$ ''

$\equiv (\sim p \vee (\sim q \vee r)) \leftrightarrow$ '' $\quad$ ''

$\equiv (\sim p \vee (\sim q \vee r)) \leftrightarrow (\sim (p \wedge q) \vee r)$ $\quad$ ''

$\equiv [\sim(\sim p \vee (\sim q \vee r)) \vee (\sim(p \vee q) \vee r)] \wedge [\sim(\sim(p \wedge q) \vee r) \vee (\sim p \vee (\sim q \vee r)]$ $\quad p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

b) $\equiv [\sim(\sim p \vee (\sim q \vee r)) \vee (\sim(p \vee q) \vee r)] \wedge [\sim(\sim(p \wedge q) \vee r) \vee (\sim p \vee (\sim q \vee r))]$

$\equiv [\sim(\sim p \vee \sim(\sim(\sim q) \wedge \sim r)) \vee \sim(\sim(\sim(p \wedge q)) \wedge \sim r)] \wedge [\sim(\sim(\sim(\sim(p \wedge q)) \wedge \sim r) \vee$ $(\sim p \vee \sim(\sim(\sim q) \wedge \sim r)$

$\equiv [\sim(\sim p \vee \sim(q \vee \sim r)) \vee \sim((p \wedge q) \wedge \sim r)] \wedge [((p \wedge q) \wedge \sim r) \vee (\sim p \vee \sim(q \wedge \sim r))]$

$\equiv [(p \wedge (q \wedge \sim r)) \wedge \sim(\sim((p \wedge q) \wedge \sim r))] \wedge [\sim(\sim((p \wedge q) \wedge \sim r)) \wedge \sim(\sim(p \wedge (q \wedge \sim r)$

$\equiv [\sim(\sim(p \wedge (q \wedge \sim r)) \wedge ((p \wedge q) \wedge \sim r)] \wedge [\sim(\sim((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)$

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logical equivalence $p \vee q \equiv \sim (\sim p \wedge \sim q)$ to rewrite each statement form using only $\wedge$ and $\sim$.

$$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$$

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8. (10 points) Exercise Set 2.3, Question 16
Use truth tables to show that the argument form below is valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid.

$p \wedge q$
$\therefore\ p$

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9. (10 points) Exercise Set 2.3, Question 28
Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.

$P \to q \not\equiv q \to P$

Neither of these two numbers is divisible by 6.

Inverse Error

$\therefore$ The product of these two numbers is not divisible by 6.

10. (10 points) Exercise Set 2.3, Question 40
Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

L a. Socko: Lefty killed Sharky.
L b. Fats: Muscles didn't kill Sharky.
L c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
T d. Muscles: Lefty didn't kill Sharky.

Who did kill Sharky?

Muscles killed sharky



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8.

| $p$ | $q$ | $p \wedge q$ | $p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | F |
| F | F | F | F |

True premesis leads to true conclusion

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