1. a) i. $6m+8n=2(3m+4n)$ $\quad 2k=$ even by def of even

a) ii. $10mn+7=2(5mn+3)+1 = 2k+1$ odd by def of odd

a) iii. $m>n>0$, m^2-n^2 composite?

$m^2-n^2=(m-n)(m+n)$ if $m=5$ and $n=4$

divisors

can be 1 and m^2-n^2

$25-16=9$, 9 not prime

It can be prime but is possible to be not composite

b) yes, it is the ratio of $60583/100$

$a=\frac{b}{c}$

c) • $b=an$ • $c=am$ • $5b+3c=(5an+3am)$

def of 1 $\quad$ def of 1

• $a(5b+3c)=a(5n+3m)$

QED $\therefore$ • $a \mid 5b+3c=a(\text{int})$

2. d) i $\forall n,\ n^2-n+3$ is odd

Case I. n=Even $n=2k$

- $n^2-n+3=(2k)^2-2k+3=4k^2-2k+3$
- $(4k^2-2k+2)+1$
- $2(2k^2-k+1)+1$ since $2k^2-k+1$ is closed by addition, mutiplication, and subtraction, n^2-n+3 is odd.

d) ii. Case 2: n = odd $n=2k+1$

- $(2k+1)^2-(2k+1)+3$
- $(2k)^2+2(2k)+1^2-2k-1+3$
- $4k^2+2k+3$
- $2(2k^2+k)+3$ since $2k+k$ is closed under sum and product, $2k+k$ is in $\mathbb{Z}$

n^2-n+3 is odd

2b.

Floor: $n \leq x < n+1$

~~Suppose x is int~~: $\lfloor x - 2 \rfloor =$

~~$n \leq x-2 < n+1$~~

$n = \lfloor x \rfloor$

$\lfloor x \rfloor - 2 = n \leq x-2 < n+1$

by: def of floor

- $n \leq x < n+1$ to add all elements
- $n-2 \leq x-2 \leq n-1$ since n is integer and $(x-2)$ is int closed under subtraction
- $\lfloor x+m \rfloor = n+m$

QED $\therefore$ $\boxed{\lfloor x+2 \rfloor = \lfloor x \rfloor + 2}$

2c) $n^3 - n = \underline{n(n-1)(n+1)}$ by def of even, closed by even

$n^3 = 2k$ ~~$P \to Q$ $\neg Q \to \neg P$~~

$2(4k^3) = \text{even}$ ~~$2(n^2 - 2n + n)$~~

$\boxed{n^3 \text{ is even if } n \text{ is even}}$

2d.) contrapositive Assume n = odd

$P \to Q$

$\neg Q \to \neg P$

$\underline{n = 2k+1}$

$(2k+1)^3 = 8k^3 + 12k^2 + 6k + 1$

$2(4k^3 + 6k^2 + 3k) + 1$

Therefore, if n^3 is even, n is even

3. a) P(3) $n = 3$ $(n-2)(n+3)/2 = \frac{(3-2)(3+3)}{2} = 3$

LHS = RHS P(3) is true

3 b) $n = k+1$

Supp. That $\frac{((k+1)+2)((k+1)+3)}{2}$

Let k be any integer that is greater than or equal to 3 and suppose that K is true. we must show that K+1 is true

3c) $\dfrac{((k+1)-2)((k+1)+3)}{2}$

Time