Module D *Rapid* Review

Main Heading	Review Material	MyOMLab
QUEUING THEORY (pp. 748–749)	■ **Queuing theory**—A body of knowledge about waiting lines. ■ **Waiting line (queue)**—Items or people in a line awaiting service.	Concept Questions: 1.1–1.3
CHARACTERISTICS OF A WAITING-LINE SYSTEM (pp. 749–752)	The three parts of a waiting-line, or queuing, system are: *Arrivals or inputs to the system; queue discipline, or the waiting line itself;* and *the service facility.* ■ **Unlimited,** or **infinite, population**—A queue in which a virtually unlimited number of people or items could request the services, or in which the number of customers or arrivals on hand at any given moment is a very small portion of potential arrivals. ■ **Limited, or finite, population**—A queue in which there are only a limited number of potential users of the service. ■ **Poisson distribution**—A discrete probability distribution that often describes the arrival rate in queuing theory: $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ *for* $x = 0, 1, 2, 3, 4, \ldots$ (D-1) A queue is *limited* when it cannot, either by law or because of physical restrictions, increase to an infinite length. A queue is *unlimited* when its size is unrestricted. *Queue discipline* refers to the rule by which customers in the line are to receive service: ■ **First-in, first-out (FIFO) rule**—A queue discipline in which the first customers in line receive the first service. ■ **Single-server (single-channel) queuing system**—A service system with one line and one server. ■ **Multiple-server (multiple-channel) queuing system**—A service system with one waiting line but with more than one server (channel). ■ **Single-phase system**—A system in which the customer receives service from only one station and then exits the system. ■ **Multiphase system**—A system in which the customer receives services from several stations before exiting the system. ■ **Negative exponential probability distribution**—A continuous probability distribution often used to describe the service time in a queuing system.	Concept Questions: 2.1–2.4
QUEUING COSTS (pp. 753–754)	Operations managers must recognize the trade-off that takes place between two costs: the cost of providing good service and the cost of customer or machine waiting time.	Concept Questions: 3.1–3.4
THE VARIETY OF QUEUING MODELS (pp. 754–765)	***Model A:*** *Single-Server System (M/M/1):* *Queuing Formulas:* λ = mean number of arrivals per time period μ = mean number of people or items served per time period L_s = average number of units in the system = $\lambda/(\mu - \lambda)$ W_s = average time a unit spends in the system = $1/(\mu - \lambda)$ L_q = average number of units waiting in the queue = $\lambda^2/[\mu(\mu - \lambda)]$ W_q = average time a unit spends waiting in the queue = $\lambda/[\mu(\mu - \lambda)] = L_q/\lambda$ ρ = utilization factor for the system = λ/μ P_0 = probability of 0 units in the system (i.e., the service unit is idle) = $1 - (\lambda/\mu)$ $P_{n>k}$ = probability of > k units in the system = $(\lambda/\mu)^{k+1}$ ***Model B:*** *Multiple-Server System (M/M/S):* $P_0 = \frac{1}{\left[\sum_{n=0}^{M-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{M!}\left(\frac{\lambda}{\mu}\right)^M \frac{M\mu}{M\mu - \lambda}}$ *for* $M\mu > \lambda$ $L_s = \frac{\lambda\mu(\lambda/\mu)^M}{(M-1)!(M\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$ $W_s = L_s/\lambda \quad L_q = L_s - (\lambda/\mu) \quad W_q = L_q/\lambda$ ***Model C:*** *Constant Service (M/D/1):* $L_q = \lambda^2/[2\mu(\mu - \lambda)] \qquad W_q = \lambda/[2\mu(\mu - \lambda)]$ $L_s = L_q + (\lambda/\mu) \qquad W_s = W_q + (1/\mu)$	Concept Questions: 4.1–4.4 Problems: D.1–D.14, D.16–D.21, D.24–D.39 Virtual Office Hours for Solved Problems: D.1–D.4 **ACTIVE MODELS D.1, D.2, D.3**

MyOMLab

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	Little's Law A useful relationship in queuing for any system in a steady state is called Little's Law: $L_s = \lambda W_s$ (which is the same as $W_s = L_s/\lambda$) (D-2) $L_q = \lambda W_q$ (which is the same as $W_q = L_q/\lambda$) (D-3) ***Model D:** Finite Population (M/M/1 with finite source)* With a limited, or finite, population, there is a *dependent* relationship between the length of the queue and the arrival rate. As the *waiting* line becomes longer, the *arrival rate* drops. N = size of the popuation $P_0 = \dfrac{1}{\sum_{n=0}^{N} \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n}$ $L_q = N - \left(\dfrac{\lambda+\mu}{\lambda}\right)(1-P_0)$ $L_s = L_q + (1 - P_0)$ $W_q = \dfrac{L_q}{(N-L_s)\lambda}$ $W_s = W_q + \dfrac{1}{\mu}$ $P_n = \dfrac{N!}{(N-n)!}\left(\dfrac{\lambda}{\mu}\right)^n P_0$ for $n = 0, 1, \ldots, N$	
OTHER QUEUING APPROACHES (p. 765)	Often, *variations* of the four basic queuing models are present in an analysis. Many models, some very complex, have been developed to deal with such variations.	Concept Question: 5.1

Self Test

■ **Before taking the self-test,** refer to the learning objectives listed at the beginning of the module and the key terms listed at the end of the module.

LO D.1 Which of the following is *not* a key operating characteristic for a queuing system?
- **a)** Utilization rate
- **b)** Percent idle time
- **c)** Average time spent waiting in the system and in the queue
- **d)** Average number of customers in the system and in the queue
- **e)** Average number of customers who renege

LO D.2 Customers enter the waiting line at a cafeteria's only cash register on a first-come, first-served basis. The arrival rate follows a Poisson distribution, while service times follow an exponential distribution. If the average number of arrivals is 6 per minute and the average service rate of a single server is 10 per minute, what is the average number of customers in the system?
- **a)** 0.6
- **b)** 0.9
- **c)** 1.5
- **d)** 0.25
- **e)** 1.0

LO D.3 In performing a cost analysis of a queuing system, the waiting time cost is sometimes based on the time in the queue and sometimes based on the time in the system. The waiting cost should be based on time in the system for which of the following situations?
- **a)** Waiting in line to ride an amusement park ride
- **b)** Waiting to discuss a medical problem with a doctor
- **c)** Waiting for a picture and an autograph from a rock star
- **d)** Waiting for a computer to be fixed so it can be placed back in service

LO D.4 Which of the following is *not* an assumption in a multiple-server queuing model?
- **a)** Arrivals come from an infinite, or very large, population.
- **b)** Arrivals are Poisson distributed.
- **c)** Arrivals are treated on a first-in, first-out basis and do not balk or renege.
- **d)** Service times follow the exponential distribution.
- **e)** Servers each perform at their own individual speeds.

LO D.5 If everything else remains the same, including the mean arrival rate and service rate, except that the service time becomes constant instead of exponential:
- **a)** the average queue length will be halved.
- **b)** the average waiting time will be doubled.
- **c)** the average queue length will increase.
- **d)** we cannot tell from the information provided.

LO D.6 A company has one computer technician who is responsible for repairs on the company's 20 computers. As a computer breaks, the technician is called to make the repair. If the repairperson is busy, the machine must wait to be repaired. This is an example of:
- **a)** a multiple-server system.
- **b)** a finite population system.
- **c)** a constant service rate system.
- **d)** a multiphase system.
- **e)** all of the above.

Answers: LO D.1. e; LO D.2. c; LO D.3. d; LO D.4. e; LO D.5. a; LO D.6. b.