CSE2500 - Discrete Mathematics	March 10, 2021
Problem Set 4	Due Date: March 19, 11:59 pm, EST

For full credit, please adhere to the following:

- Unsupported answers receive no credit.

All of the following questions are in the textbook for the course, *Discrete Mathematics with Applications (Fifth Edition)*, by Susanna Epp.

1. (10 points) Exercise Set 4.3, Question 10
   Assume that m and n are both integers and that $n \neq 0$. Explain why $\frac{(5m+12n)}{(4n)}$ must be a rational number.

2. (10 points) Exercise Set 4.3, Question 28
   Suppose a, b, c, and d are integers and $a \neq c$.Suppose also that x is a real number that satisfies the equation
   $$\frac{ax+b}{cx+d} = 1$$
   Must x be rational? If so, express x as a ratio of two integers.

3. (12 points) Exercise Set 4.3, Question 11 & 30

   (a) (4 points) Exercise Set 4.3, Question 11
   Prove that the negative of any rational number is rational.

   (b) (8 points) Exercise Set 4.3, Question 30
   Use the statement you proved in part (a) to prove that if one solution for a quadratic equation of the form $x^2 + bx + c = 0$ is rational (where b and c are rational), then the other solution is also rational. (Use the fact that if the solutions of the equation are r and s, then $x^2 + bx + c = (x - r)(x - s)$).

4. (10 points) Exercise Set 4.4, Question 5
   Is $6m(2m + 10)$ divisible by 4? Give reasoning for your answer. Assume m is an integer.

5. (12 points) Exercise Set 4.4, Question 29
   Determine whether the following statement is true or false. If true, prove the statement directly from definitions. If false, give a counterexample.

   For all integers a and b, if $a|b$ then $a^2|b^2$.

6. (12 points) Exercise Set 4.4, Question 45
   Prove that if n is any nonnegative integer whose decimal representation ends in 5, then $5|n$.

7. (10 points) Exercise Set 4.5, Question 18(a)
   Prove that the product of any two consecutive integers is even.

8. (12 points) Exercise Set 4.5, Question 35
   Prove the following statement:
   The fourth power of any integer has the form $8m$ or $8m+1$ for some integer m.

9. (12 points) Exercise Set 4.5, Question 50
   Prove that if m, d, and k are integers and $d > 0$, then $(m + dk) \bmod d = m \bmod d$.