

For full credit, please adhere to the following:

- Unsupported answers receive no credit.
- All answers can be typed or handwritten, and should be readable.
- Submit the assignment in one file (.pdf, .doc, etc.) via HuskyCT.

All of the following questions are in the textbook for the course, *Discrete Mathematics with Applications (5th Edition)*, by Susanna Epp.

I encourage you to learn how to typeset documents with LATEX. You can download Texmaker at

. It is available for most platforms. Another

option is to use online service . There is a wealth of information online on how to format documents with LATEX and you can always post a question on Moodle's Technical Forum. Leslie Lamport has authored a book that provides a nice introduction to the basic features of LATEX which you can read about here. Most scholarly articles in Mathematics and Computer Science, and even many books, are typeset with this tool.

1. (10 points) Exercise Set 1.1, Question 4. Fill in the blanks using a variable or variables to rewrite the given statement.

Given any real number, there is a real number that is greater.

- a. Given any real number  $r$ , there is  $\exists$   $s$  such that  $s$  is  $> r$ .  $\exists (r) \in \mathbb{R}$
- b. For any  $\mathbb{R}$ ,  $\exists \in$  such that  $s > r$

2. (10 points) Exercise Set 1.1, Question 11. Fill in the blanks to rewrite the given statement.

Every positive number has a positive square root.

- a. All positive numbers  $\mathbb{R}$ .
- b. For every positive number  $e$ , there is  $e^2$  for  $e$ .
- c. For every positive number  $e$ , there is a positive number  $r$  such that  $r \in e \cdot e$   $\forall e \in \mathbb{R}$

3. (10 points) Exercise Set 1.2, Question 4.

- a. Is  $2 \in \{2\}$ ? Yes, 2 is contained in the set  $\{2\}$
- b. How many elements are in the set  $\{2, 2, 2, 2\}$ ? 4 elements
- c. How many elements are in the set  $\{0, \{0\}\}$ ? 2 elements

- d. Is  $\{0\} \in \{\{0\}, \{1\}\}$  Yes, the set  $\{\{0\}, \{1\}\}$  contains element  $\{0\}$   
 e. Is  $0 \in \{\{0\}, \{1\}\}$  no,  $\{0\} \neq 0$

4. (10 points) Exercise Set 1.2, Question 7.

Use the set-roster notation to indicate the elements in each of the following sets.

- a.  $S = \{n \in \mathbb{Z} \mid n = (-1)^k, \text{ for some integer } k\}$   $\{1, -1\} = S$   
 b.  $T = \{m \in \mathbb{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$   $\{0, 2\} = T$   
 c.  $U = \{r \in \mathbb{Z} \mid 2 \leq r \leq -2\}$   $U = \{-2, -1, 0, 1, 2\}$   
 d.  $V = \{s \in \mathbb{Z} \mid s > 2 \text{ or } s < 3\}$   $V = \mathbb{Z}$   
 e.  $W = \{t \in \mathbb{Z} \mid 1 < t < -3\}$   $\{-4, -3, -2, -1, 0, 1, 2, \dots\} = W$   
 f.  $X = \{u \in \mathbb{Z} \mid u \leq 4 \text{ or } u \geq 1\}$   $\{4, 5, 6, \dots\}$

5. (10 points) Exercise Set 1.2, Question 8.

Let  $A = \{c, d, f, g\}$ ,  $B = \{f, j\}$ ,  $C = \{d, g\}$ . Answer each of the following questions. Give reasons for your answers.

- a. Is  $B \subseteq A$ ? no, not all elements of  $B \in A$   
 b. Is  $C \subseteq A$ ? yes, all elements of  $C \in A$   
 c. Is  $C \subseteq C$ ? yes, all elements of  
 d. Is  $C$  a proper subset of  $A$ ? yes,  $A$  contains other elements  $| C \neq A$

6. (10 points) Exercise Set 1.2, Question 12.

Let  $S = \{2, 4, 6\}$  and  $T = \{1, 3, 5\}$ . Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set.

- a.  $S \times T = \{(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)\}$  9  
 b.  $T \times S = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$  9  
 c.  $S \times S = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$  9  
 d.  $T \times T = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$  9

7. (10 points) Exercise Set 1.2, Question 14.

Let  $R = \{a\}$ ,  $S = \{x, y\}$ , and  $T = \{p, q, r\}$ . Find each of the following sets.

- a.  $R \times (S \times T) = \{(a, (x, p)), (a, (x, q)), (a, (x, r)), (a, (y, p)), (a, (y, q)), (a, (y, r))\}$   
 b.  $(R \times S) \times T = \{((a, x), p), ((a, x), q), ((a, x), r), ((a, y), p), ((a, y), q), ((a, y), r)\}$   
 c.  $R \times S \times T$

8. (10 points) Exercise Set 1.3, Question 4

Let  $G = \{-2, 0, 2\}$  and  $H = \{4, 6, 8\}$  and define a relation  $V$  from  $G$  to  $H$  as follows:

For all  $(x, y) \in G \times H$ ,  $(x, y) \in V$  means that  $\frac{x-y}{4}$  is an integer.

$$\{(-2, 4), (-2, 6), (-2, 8), (0, 4), (0, 6), (0, 8), (2, 4), (2, 6), (2, 8)\}$$



- yes      no      no      no
- a. Is  $2V6$ ? Is  $(-2)V(-6)$ ? Is  $(0,6) \in V$ ? Is  $(2,4) \in V$ ?  
 b. Write  $V$  as a set of ordered pairs.  
 c. Write the domain and co-domain of  $V$ .  
 d. Draw an arrow diagram for  $V$ .



9. (10 points) Exercise Set 1.3, Question 8.

Let  $A = \{2, 4\}$  and  $B = \{1, 3, 5\}$  and define relations  $U$ ,  $V$ , and  $W$  from  $A$  to  $B$  as follows:

For every  $(x, y) \in A \times B$ :

- $(x, y) \in U$  means that  $y - x > 2$   $(2, 5)$
- $(x, y) \in V$  means that  $y - 1 = \frac{x}{2}$
- $W = \{(2, 5), (4, 1), (2, 3)\}$

- a. Draw arrow diagrams for  $U$ ,  $V$ , and  $W$ .  
 b. Indicate whether any of the relations  $U$ ,  $V$ , and  $W$  are functions.

10. (10 points) Exercise Set 1.3, Question 20.

Define functions  $H$  and  $K$  from  $\mathbf{R}$  to  $\mathbf{R}$  by the following formulas: For every  $x \in \mathbf{R}$ ,  $H(x) = (x - 2)^2$  and  $K(x) = (x - 1)(x - 3) + 1$ . Does  $H = K$ ? Explain.

$$H = (x - 2)(x - 2) \quad K = (x - 1)(x - 3) + 1$$

$$x^2 - 4x + 4 = x^2 - 4x + 4$$