STATISTICAL METHODS (STAT 3025)

Final Exam

Total points 100

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Instructions

No point will be given for without showing the necessary steps

Each Q carrying 10 *points.Mixing up questions is not allowed.*

Q1: For any two events A and B with $P(A) > 0$ and $P(B) > 0$. Answer the following questions.

a) Show that $P(A)+P(B)-1 \leq P(A\cup B\ ) \leq P(A)+P(B)$. (Hint: A and B are NOT mutually exclusive) (6 points)

b) Let's say that A and B are independent but NOT mutually exclusive. It is given that $P(A) = 0.3, P(B/A) = 0.4$ Find $P(A \cup B\ )$.(4 points)

Q2: For any two events A and B with $P(A) > 0$ and $P(B) > 0$. Answer the following questions.

a) Show that the events A' and B' are independent **if it is given that A and B are independent**. (5 points)

b) If it is given that the events A and B are independent, $P(A) = 0.8$ and $P(B) = 0.5$. Find $P(A^{'} \cup B^{'})$ and $P(A^{'}/B^{'})$ (5 points)

Q3: In data scince R and SAS are the two softwares which are most frequently used. In a recent study in a particular public university suggests that 45% of the UG students know R. It has been found that of the students who know R, 70% know SAS and of the students who don't know R, 20% know SAS. Based on this study answer the following questions.

a) Find the probability that any randomly selected student does not know R (3 points)

b) Find the probability that any randomly selected student knows SAS. (3 points)

c) If you know that any student knows SAS find the probability that the student knows R. (4 points)

Q4: Consider two boxes B1 and B2. B1 has 10 red and 3 green balls where B2 has 6 red and 4 green balls.

You are selecting one ball at random from B1 (Without replacement) and adding that to B2.Finally selecting one ball from B2. Find the following probabilities

a) What is the probability of selecting a red ball from B1? (3 points)

b) What is the probability of selecting a red ball from B2? (7 points)

Q5:A recent study in the city Willimantic (CT) indicates that 50% of the residents staying in Willimantic go to "Stop and Shop" for groecry shopping. A newly appointed analyst wants to use this report for her study.She chooses 25 residents at random. Let X be the random variable that denotes how many of them go to "Stop and Shop" for groecry shopping out of 25 selected residents.

Based on the information answer the following questions.

a) What do you think about the distribution of X? (1 point)

b) What is the probability that the number of "Stop and Shop" shoppers is at least 8? (3 points)

c) What is the probability that the number "Stop and Shop" shoppers is less than 5? (3 points)

d) What is the probability that the number of "Stop and Shop" shoppers is between 2 and 9 both inclusive? (3 points)

Q6: Let $X_1, X_2.....X_n$ are independent random sample from $U(0, \theta)$.

a) Show that both $2X_1$ and $2\overline{X}$ both are Unbiased estimators (UE) of θ. (5 points)

b) Compute the MSE of both the estimators and tell me which one is better? (5 points)

(Hint: What is the MSE of any UE? Recall the expectation and variance formula of uniform distribution)

Q7: Let $X_1, X_2.....X_n$ are independent random sample from Poisson distribution with parameter λ $(P(\lambda))$

a) Find the MOM estimate of λ.(4 points)

b) Write down the likelihood function of λ explicitly. (2 points)

c) Find the MLE of λ and λ^2. (4 points)

(Show each step)

Q8: Let $X_1, X_2.....X_n$ are independent random sample from $U(0, \theta)$.

a) Find the MOM estimate of θ.(3 points)

b) Let $T = Max(X_1, X_2.....X_n)$.The PDF of T is given as below.

$$\begin{aligned} f_T(t) &= \frac{nt^{n-1}}{\theta^n} \text{ if } 0 \leq t \leq \theta \\ &= 0 \ o.w \end{aligned}$$

Here T is the random variable and t is any particular value taken by T
Show that T is not UE of θ (Show that $E(T) = \frac{n}{n+1}\theta$) (5 points)

c) Make an UE of θ based on T. (2 points)

Q9: Let $X_1, X_2.....X_n$ are independent random sample from $N(\mu, \sigma^2)$.

a) What is the distribution of the sample mean $\overline{X}$? (1 point)

b) Show that X_1, $\frac{X_1+2X_2}{3}$ and $\overline{X}$ all are UE of μ.(4 points)

c) Compare the MSE of all the 3 estimators and choose the best.(5 points) (Consider $n \geq 5$)

(Show every step)

Q10: Let $X_1, X_2.....X_n$ are independent random sample from $N(\mu, \sigma^2)$. Recall the MLE of μ and σ^2.
a) Find the MLE of $\mu + \sigma$ and $\frac{\mu}{\sigma}$ (5 points)

b) Find the MLE of $P(\overline{X} \geq 5)$. (5 points)

(Show every step)