1) m and n are both integers with $n \neq 0$ $\frac{5m+12n}{4n}$

- 5 is integer and m is integer, therefore there product will be an integer
- Since 12 is integer and n is integer, there product will be an integer
- Since 5m is an integer and 12n is an integer, their product will be an integer
- Since 4 is an integer and n is a integer, 4n is an inter
- $\frac{5m+12n}{4n} = \frac{int}{int}$ = ratio of ints
- $\frac{5m+12n}{4n}$
- n does not equal 0, means the denom cannot equal 0
- $\frac{5m+12n}{4n}$ is a rational number, the num and denom are both non-0 .. integers

2) a,b,c,d int, $a \neq c$ x is real num

$\frac{ax+b}{cx+d} = 1$

$ax+b = cx+d$
$-cx \quad -cx$
$ax-cx+b = d-b$
$-b$

$ax-cx = d-b$

$x(a-c) = d-b$

$x = \frac{d-b}{a-c}$ ← int ← non 0, int

x is rational, repeat

$\frac{d-b}{a-c}$

3) a) Prove
$\forall$ real numbers, ~~m, if m is an integer, then m is a rational number.~~

- ~~m is int~~
- $m = \frac{m}{1}$ ← nonzero ints

$\therefore$ $m = \frac{m}{1}$ is a rational number,

Every integer is a rational number

3 b) • $x^2+bx+c=(x-r)(x-s)$ Prove if soln. is rational then form is rational

- $x^2+bx+c = x^2-rx-sx+rs$
- $x^2+bx+c = x^2+x(-r-s)+rs$

coefficients must be equal

- $b=(-r-s)$ $+s$ $+s$
- $b+s=-r-b$ $-b$
- $s=-r-b$ $-r$ is rational, b is rational
- the difference of two rational numbers is rational

$\therefore$ if one solution for the quadratic equation "$x^2+bx+c=0$" is rational the other solution is also rational.

4) Prove • $6m(2m+10)$ is devisible by 4

- $6m(2(m+5))$
- $12m(m+5)$
- $4 \cdot 3 \cdot m(m+5)$ $\therefore$ $\frac{n}{4} = 3m(m+5)$ is devisible

5) Prove: $\forall$ int a and b, if $a|b$ then $a^2|b^2$

- assume a and b are ints and $a|b$
- x such that $b = ax$
- $b^2 = (ax)^2$
- $b^2 = a^2x^2$
- since the square of an int is an int, x^2 is int

$\therefore$ There exists an int k, ($k = x^2$), such that $b^2 = a^2k$ and b^2 is then divisible by a^2, noted as $a^2 | b^2$

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6) Prove: n is non neg int whose decimal ends in 5 then $5|n$

- $n = d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \ldots d_2 \cdot 10^2 + d_1 \cdot 10 + d_0$
- $d_0 = 5$
- n = any non neg integer ending in 5
- $10^k = (2 \cdot 5)^k$
- $(2 \cdot 5)^k = 2^k \cdot 5^k$
- $5(d^k \cdot (2^k \cdot 5^{k-1}) + d^k{-1}(2^{k-1} \cdot 5^{k-2}) + \ldots + d_2 \cdot (2^2 \cdot 5) + d_1 \cdot 2 + 1)$

$\therefore$ $\frac{5(\text{"} \quad \text{"})}{5}$ since we can factor out a 5, we have $n = 5(\text{int})$

$5|n$

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7) def of even = $n=2k$

- $n(n+1) = 2k(2k+1$
- $= 2k \cdot 2k + 2k$
- $= 4k^2+2k$
- $= 2(2k^2+k$

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8) - $n = 2k+1$ if integer is odd

- $n^4 = (2k+1)^4$
- $n^4 = (2k+1)^2(2k+1)^2$
- $n^4 = (4k^2+4k+1)(4k^2+4k+1)$
- $n^4 = 16k^4+32k^3+24k^2+8k+1$ ← $\overset{m}{\text{is an integer}}$

- $8(2k^4+4k^3+3k^2+k)+1$
- Therefore $8m+1$ if m is integer

if even

- $n=2k$
- $n^4=(2k)^4$
- $n^4=2^4k^4$
- $n^4=16k^4$
- $n^4=8(2k^4)$
- $n^4=8m$

$m=2k^4$

$\boxed{n \text{ is of form } 8m \text{ or } 8m+1 \text{ of any case}}$

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4) To prove: $\forall$ int m, d, k, if $d > 0$, then $(m+dk) \bmod d = m \bmod d$

- m, d, k int such that $d > 0$
- $r = m \bmod d$
- $m = dq + r$
- $m + dk = (dq + r) + dk$
- $= dq + r + dk$
- $= d(q+k) + r$
- $q+k$ is an integer since q and k are integers
- there exists $m = q+k$ | $(m+dk) = dm + r$ by the quotient remainder theorem
- $(m+dk) \bmod d = m = q+k$
- $(m+dk)$ div $d = r$
- Since, $r = m \bmod d$, $(m+dk)$ div $d = r = m \bmod d$

For