45. The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying *joint probability table* gives the proportions of individuals in the various ethnic group-blood group combinations.

Blood Group

		O	A	B	AB
	1	.082	.106	.008	.004
Ethnic Group	**2**	.135	.141	.018	.006
	3	.215	.200	.065	.020

Suppose that an individual is randomly selected from the population, and define events by A = {type A selected}, B = {type B selected}, and C = {ethnic group 3 selected}.

a. Calculate $P(A)$, $P(C)$, and $P(A \cap C)$.

b. Calculate both $P(A|C)$ and $P(C|A)$, and explain in context what each of these probabilities represents.

c. If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

a) $P(A) = .106 + .141 + .200 = .447$

$P(C) = .215 + .200 + .065 + .020 = .50$

$P(A \cap C) = .2$

b) $P(A|C) = \frac{.2}{.5} = .4$

$P(C|A) = \frac{.2}{.447} = .447$

c) $P(B') = 1 - P(B) = .909$ $\quad \frac{.106 + .082 + .004}{.909} = .2112$

47. Return to the credit card scenario of Exercise 12 (Section 2.2), where A = {Visa}, B = {MasterCard}, $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$. Calculate and interpret each of the following probabilities (a Venn diagram might help).

a. $P(B|A)$ **b.** $P(B'|A)$

c. $P(A|B)$ **d.** $P(A'|B)$

e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?

a) $\frac{.25}{.5} = .5$ b) $(1 - .5) = .5$

c) $\frac{.25}{.4} = .625$ d) $= (1 - .625) = .375$

e $\frac{.5}{.625} = .769$

49. The accompanying table gives information on the type of coffee selected by someone purchasing a single cup at a particular airport kiosk.

	Small	Medium	Large
Regular	14%	20%	26%
Decaf	20%	10%	10%

a) SC = .34
Decaf = .40

Consider randomly selecting such a coffee purchase.

a. What is the probability that the individual purchased a small cup? A cup of decaf coffee?

b. If we learn that the selected individual purchased a small cup, what now is the probability that he/she chose decaf coffee, and how would you interpret this probability?

b) $\frac{.2}{.34} = .588$

c. If we learn that the selected individual purchased decaf, what now is the probability that a small size was selected, and how does this compare to the corresponding unconditional probability of (a)?

c) $\frac{.2}{.4} = .5$

51. One box contains six red balls and four green balls, and a second box contains seven red balls and three green balls. A ball is randomly chosen from the first box and placed in the second box. Then a ball is randomly selected from the second box and placed in the first box.

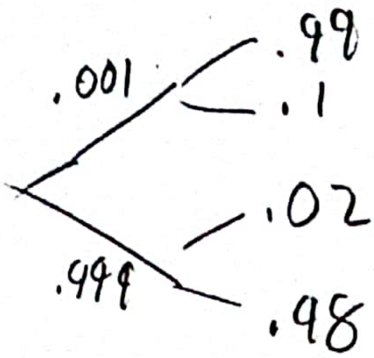
a. What is the probability that a red ball is selected from the first box and a red ball is selected from the second box?

$\frac{6}{10} \cdot \frac{8}{11} =$

$\frac{24}{55}$

~~**b.** At the conclusion of the selection process, what is the probability that the numbers of red and green balls in the first box are identical to the numbers at the beginning?~~

Incidence of a rare disease. Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur 99% of the time, whereas an individual without the disease will show a positive test result only 2% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?



$$.001 \cdot .99 = .00099$$

$$.999 \cdot .02 = .01998$$

$$+ \overline{\quad .02097 \quad}$$

$$\frac{.00099}{.02097} = .047$$