

There are 7 problems, each carries 5 points. Total 35.

1)

13. A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
$p(x)$.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events.

- {at most three lines are in use}
- {fewer than three lines are in use}
- {at least three lines are in use}
- {between two and five lines, inclusive, are in use}

2)

24. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X = the number of months between successive payments. The cdf of X is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \leq x < 3 \\ .40 & 3 \leq x < 4 \\ .45 & 4 \leq x < 6 \\ .60 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

- What is the pmf of X ?
- Using just the cdf, compute $P(3 \leq X \leq 6)$ and $P(4 \leq X)$.

3)

29. The pmf of the amount of memory X (GB) in a purchased flash drive was given in Example 3.13 as

x	1	2	4	8	16
$p(x)$.05	.10	.35	.40	.10

Compute the following:

- $E(X)$
- $V(X)$ directly from the definition
- The standard deviation of X
- $V(X)$ using the shortcut formula

4)

30. An individual who has automobile insurance from a certain company is randomly selected. Let Y be the number of moving violations for which the individual was cited during the last 3 years. The pmf of Y is

y	0	1	2	3
$p(y)$.60	.25	.10	.05

- Compute $E(Y)$.
- Suppose an individual with Y violations incurs a surcharge of $\$100Y^2$. Calculate the expected amount of the surcharge.

5)

32. An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let X = the amount of storage space purchased by the next customer to buy a freezer. Suppose that X has pmf

x	13.5	15.9	19.1
$p(x)$.2	.5	.3

- a. Compute $E(X)$, $E(X^2)$, and $V(X)$.
- b. If the price of a freezer having capacity X cubic feet is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?
- c. What is the variance of the price $25X - 8.5$ paid by the next customer?
- d. Suppose that although the rated capacity of a freezer is X , the actual capacity is $h(X) = X - .01X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

6)

33. Let X be a Bernoulli rv with pmf as in Example 3.18.
 - a. Compute $E(X^2)$.
 - b. Show that $V(X) = p(1 - p)$.

7)

37. The n candidates for a job have been ranked $1, 2, 3, \dots, n$. Let X = the rank of a randomly selected candidate, so that X has pmf

$$p(x) = \begin{cases} 1/n & x = 1, 2, 3, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

(this is called the *discrete uniform distribution*). Compute $E(X)$ and $V(X)$ using the shortcut formula. [Hint: The sum of the first n positive integers is $n(n + 1)/2$, whereas the sum of their squares is $n(n + 1)(2n + 1)/6$.]