

Section 8 Assignment (72 points) – Advanced Counting

To receive credit, you must either show your work on the worksheet or explain how you got the answer.

1. (6 points) If 27 people are assembled in a room, show that at least two of them must have last names that start with the same letter.

$$= \left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

there must be at least 2 last names with the same letter.

2. (6 points) 14 people volunteer for a four-person committee. Every possible committee of four that can be formed from these 14 names are written on a slip of paper, one slip for each possible committee, and the slips are put in 100 hats. Show that at least one hat contains 11 or more slips.

$$= \left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

there must be at least 11 slips in one hat

3. (6 points) Describe the sample space: Three numbers are selected at random from the numbers 2, 4, 6, 8. The numbers are chosen at random and repeating elements are allowed.

$$S = \{222, 224, 226, 228, 246, 248, 268$$

$$444, 442, 446, 448, 468$$

$$666, 662, 664, 668$$

$$888, 882, 884, 886\}$$

4. (6 points) Describe the sample space: A box contains 14 items, five of which are defective. An item is chosen at random and not replaced. This is continued until all five defective items have been selected. The total number of items selected is recorded.

$$S = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$$

5. (18 points) A card is chosen from a standard deck of 52 cards. Given the following events:

E_1 : the card drawn is a number card (i.e. card with a number on it)

E_2 : the card drawn is a spade

E_3 : the card drawn has an odd number on it

E_4 : the card drawn is a black card

Compute each of the following

a) (5 pts) $p(E_4)$

$$= \frac{26}{52} \text{ or } \frac{1}{2}$$

b) (6 pts) $p(E_2 \cap E_3)$

$$= \frac{5}{52}$$

c) (7 pts) $p(\overline{E_3} \cup E_2)$

$$= \frac{36}{52}$$

6. (24 points) When a certain defective die is tossed, the numbers from 1-6 will be on the top face with the following probabilities:

$$p_1 = \frac{5}{20}, \quad p_2 = \frac{5}{20}, \quad p_3 = \frac{2}{20}, \quad p_4 = \frac{1}{20}, \quad p_5 = \frac{3}{20}, \quad p_6 = \frac{4}{20},$$

Find the probability that

a) (6 pts) An even number is on top

$$10/20 = 1/2$$

b) (6 pts) A non-prime number is on top

$$5/20 = 1/4$$

c) (6 pts) A number less than 4 is on top

$$12/20 = 3/5$$

d) (6 pts) A number greater than 2 is on top.

$$10/20 = 1/2$$

7. (6 points) A fair coin is tossed seven times. What is the probability of obtaining three heads and four tails?

$$= \frac{35}{128}$$