Section 3 – Sequences (65 points)

Show your work, if possible, on the worksheet. If the answer is wrong and you've shown your work, you can receive partial credit. But if the answer is wrong and you haven't shown your work, there will be no credit for that question.

1. (15 points) Give the first FIVE terms for the following explicit sequences:

   a. (5 pts) $A = (n+2)^2 \text{ MOD } 3n$ for $n >= 1$

   $\boxed{0, 4, 7, 0, 4}$

   b. (5 pts) $A = \lfloor -(\frac{1}{3})\, n \rfloor + \lceil (n)/3 \rceil$ for $n >= 1$

   $\boxed{0, 0, 0, 0, 0}$

   c. (5 pts) $A = \sqrt{(2n^2 - n)}$ for $n >= 1$. The answers can be left in radical form

   $\boxed{A = 1, \sqrt{6}, \sqrt{15}, \sqrt{28}, \sqrt{45}}$

2. (18 points) Give the first SIX terms for the following geometric and arithmetic sequences:

   a. (4 pts) A geometric sequence S_n with a first value of -4 and a common ratio of 1/3.

   $\boxed{S = -4, -\frac{4}{3}, -\frac{4}{9}, -\frac{4}{27}, -\frac{4}{81}, -\frac{4}{243}}$

   You can also represent the fractions in decimal form

   $\boxed{S = -4, -1.\overline{333}, -0.\overline{444}, -0.1\overline{48}\overline{148}, -0.049, -0.016}$

   b. (5 pts) A geometric sequence defined by the function
   $A(n) = 3(A_{n-1}) * \lceil (A_{n-2})/5 \rceil$ for $n >= 1$ where the first value $A_1 = 2$, $A_2 = 3$

   $\boxed{A = 2, 3, 9, 27, 162, 2916}$

c. (4 pts) An arithmetic sequence S_n in which the first value is 13 and the common difference is 3.

$\boxed{S = 13, 16, 19, 22, 25, 28}$

d. (5 pts) An arithmetic sequence defined by the function $A(n) = (A_{n-2}) + n!$ for $n >= 1$ where the first value $A_1 = 1$ and $A_2 = 3$

$\boxed{A = 1, 3, 7, 27, 127, 747}$

3. (20 points) Give the ***next*** five terms <after those given> for the following recursive sequences:

a. (5 pts) $A = 2(A_{n-1}) - 4(A_{n-2}) - 3$ $n >= 1$ where $A_1 = -1$, $A_2 = 0$

$\boxed{(1, -1, -9, -17, -1)}$

b. (5 pts) $A = \lceil n/3 * (A_{n-2}) \rceil$ for $n >= 1$ where $A_1 = 1$, $A_2 = 2$,

6

$\boxed{(\underbrace{1, 2}_{\text{first } 2}, 1, 3, 2, 5)}$

Change index:

c. (5 pts) $A = n(A_{n-2})^n$ MOD 5 for $n >= 0$ where $A_0 = -2$, $A_1 = 1$

$\boxed{(\underbrace{-2, 1}_{\text{first } 2}, 3, 3, 4, 0, 1)}$

d. (5 pts) $A = 2(A_{n-1}) + 3(A_{n-2}) - 4(A_{n-3})$ for $n >= 0$ where $A_0 = 2$, $A_1 = 3$, $A_2 = 4$

$\boxed{(\underbrace{2, 3, 4}_{\text{first } 3}, 9, 18, 47, 112, 293)}$

4. (12 points) Find a formula for the following sequences and tell if it's explicit (E) or recursive (R). Use A for the sequence and use n for the element in the sequence.

For instance

- *for the infinite sequence S = 3, 6, 9, 12, 15, 18…*
  - *The formula is **A(n)** = 3n, for n >=1 and it's explicit (E)*
  - *Can also be written as* $\boldsymbol{A_n}$ *= 3n, for n >=1 and it's explicit (E)*

- *for the infinite sequence* S_1 *= 100, 96, 87, 71, 46…*
  - *The formula is* $A(n) = A_{n-1} - n^2$ *for n >= 1 where* $A_1 = 100$ *and it's recursive (R)*

There can be more than one function that generates a sequence. There can even be separate explicit and recursive functions that generate the same sequence.

If your answer is not the same as mine, show me how you generate the 5 elements in the sequence using YOUR function. Make sure the writing is legible.

a. (6 pts) S_1 = 2, 9, 28, 65, 126, 217…
   $S_n = n^3 + 1$, for n>=1, explicit

b. (6 pts) S_2 = 0, 1/2, 1, 3/2, 2, 5/2, 3
   $S_n = (1/2)n$ for n>=0 and n<=6, explicit