Section 5- Propositions (120 points)

To receive credit, you must show your work on the worksheet.

1. (21 points) Determine if the following statements are true or false *given p: true, q: true, r: false, s: false*

    a. (3 pts) $p \oplus q \wedge \neg p$

    b. (4 pts) $(p \wedge \neg p) \wedge (q \vee \neg r)$

    c. (4 pts) $((p \vee q) \wedge \neg s)$

    d. (5 pts) $((r \vee s) \rightarrow \neg q) \rightarrow \neg p$

    e. (5 pts) $(((r \rightarrow q) \oplus p) \vee \neg s)$

2. (30 points) Write the truth table for the following expressions:

    a. (5 pts) $\neg(p \vee q)$

b. (10 pts) $(p \wedge \neg p) \wedge (q \vee \neg r)$

c. (15 pts) $(((r \rightarrow q) \oplus p) \vee \neg s)$

3. (8 points) State the **converse** of the following implications

   a. (4 pts) If it snows this weekend, then I will go skiing.

   b. (4 pts) The river will freeze over if Texas has a heat wave.

4. (8 points) State the **contrapositive** of the following implications

   a. (4 pts) If the DJ has a deep voice then there is another song to play

   b. (4 pts) Hockey is a great sport if frogs have fleas.

5. (6 points) If $p \rightarrow q$ is false, can you determine the truth value of the following? Explain your answer.
    a. $(\neg p) \vee (p \leftrightarrow q)$

6. (6 points) If $p \rightarrow q$ is true, can you determine the truth value of the following? Explain your answer.
    a. $(\neg p \rightarrow q) \wedge \neg p$

7. (30 points) Use a truth table to demonstrate that the following is a tautology, a contradiction (absurdity), or neither.
    a. (10 pts) $(\neg p \wedge (p \vee q) \rightarrow p)$

    b. (10 pts) $(q \wedge r) \wedge (\neg(p \vee q))$

c. (10 pts) $((p \rightarrow (q \wedge r)) \leftrightarrow ((p \wedge q) \rightarrow p)$

8. (15 points) Use the laws of logic to show whether the following is equivalent:

a. $p \leftrightarrow (p \wedge r) \equiv \neg p \vee r$