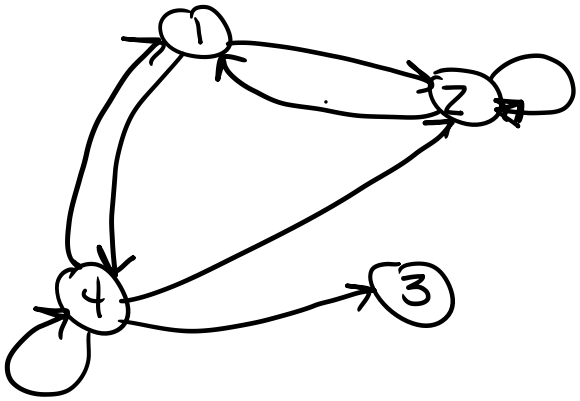
SECTION 9 ASSIGNMENT (108 POINTS) – BOOLEAN FUNCTIONS

To receive credit, you must either show your work on the worksheet or explain how you got the answer.

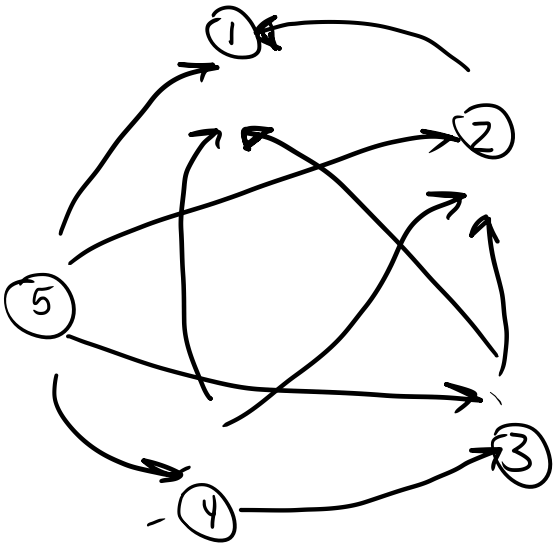
1. (12 points) Draw the arrow diagram and the matrix representation for each relation.

   a. (6 pts) The domain for relation R is {1, 2, 3, 4}
   R = { (1, 2), (1, 4), (2, 2), (2, 1), (4, 1), (4, 2), (4, 3), (4, 4)}



	1	2	3	4
1	0	1	0	1
2	1	1	0	0
3	0	0	0	0
4	1	1	1	1

   b. (6 pts) The domain of relation P is {1, 2, 3, 4, 5}
   For x, y in the domain, xPy if $\left\lceil \frac{x}{y} \right\rceil > 1$



	1	2	3	4	5
1	0	0	0	0	0
2	1	0	0	0	0
3	1	1	0	0	0
4	1	1	1	0	0
5	1	1	1	1	0

2. (18 points) For each relation, indicate whether the relation is:

- reflexive, anti-reflexive, neither,
- symmetric, anti-symmetric, neither, both
- transitive or not transitive

a. (6 pts)

	1	2	3	4
1	0	1	1	1
2	1	1	0	0
3	0	0	1	0
4	1	0	1	1

Neither reflexive nor anti-reflexive

Neither symmetric nor anti-symmetric

Not transitive

b. (6 pts)

	1	2	3	4
1	1	1	0	1
2	1	1	0	0
3	0	0	1	1
4	1	0	1	1

reflexive
symmetric
not transitive

c. (6 pts)

	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

reflexive
symmetric and anti-symmetric
transitive

3. (6 points) Determine whether the relation R on the set A is an equivalence relation
   A = {a, b, c}, R = { (a, a), (a, c), (b, b), (c, a), (c, c)}

   equivalence

4. (10 points) Figure 1 shows a directed graph (digraph).

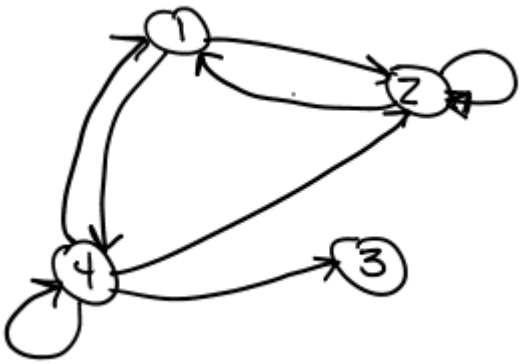


Figure 1

   a. (2 pts) What is the in-degree of vertex 4?
      2

   b. (2 pts) What is the out-degree of vertex 4?
      4

   c. (2 pts) What is the head of edge (4, 3)?
      Vertex 3

   d. (2 pts) What is the tail of edge (4, 2)?
      Vertex 4

   e. (2 pts) List all self-loops in the graph, if any:
      (2, 2), (4, 4)

zyBook defines a path as a succession of edges and vertices with no repeating edge or vertex. (Section 9.3). The textbook indicates the vertices <and the edges for that matter> don't have to be distinct. (Chapter 4.3).

Question 5 and 6 uses the zyBook definition:

5. (6 points) WALK/TRAIL/PATH in Figure 1
   a. (3 pts)
      Is ⟨1, 2, 4, 1⟩ a walk in the graph? **No**
      Is it a trail? **No**
      Is it a path? **No**

   b. (3 pts)
      Is ⟨4, 2, 2, 1⟩ a walk in the graph? **Yes**
      Is it a trail? **Yes**
      Is it a path? **No**

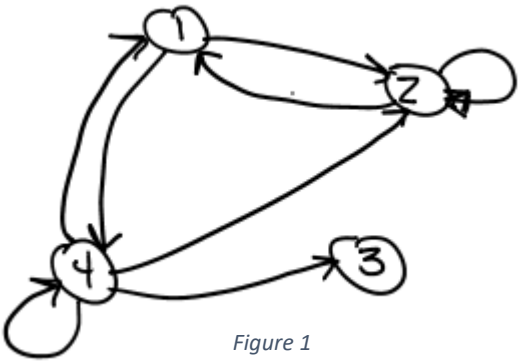


Figure 1

6. (6 points) CIRCUIT/CYCLE in Figure 1
   a. (2 pts)
      Is ⟨2, 1, 4, 2⟩ a circuit in the graph? **Yes**
      Is it a cycle? **Yes**

   b. (2 pts)
      Is ⟨2, 1, 2, 1, 2, 1⟩ a circuit in the graph? **No**
      Is it a cycle? **No**

   c. (2 pts)
      Is ⟨4, 4, 2, 2, 1, 4⟩ a circuit in the graph? **Yes**
      Is it a cycle? **No**

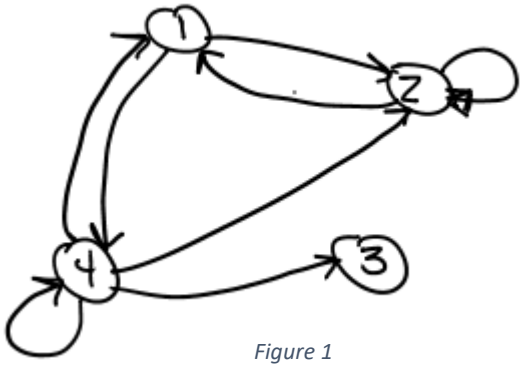


Figure 1

7. (20 points) Figure 2 shows a graph G. Draw G^2, G^3, and G^4. Then take the union of all of the graphs (including G) to get G^+

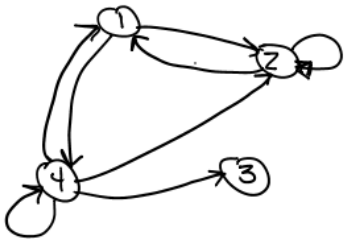
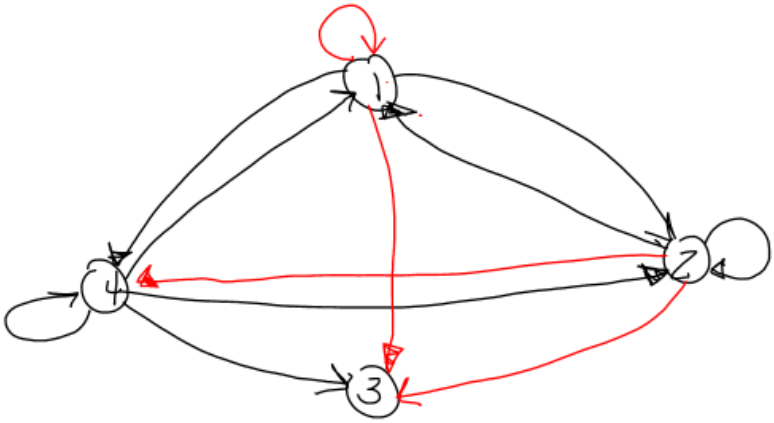


Figure 2

$G^+ = G + G^2 + G^3 + G^4$



zyBook defines a path as a succession of edges and vertices with no repeating edge or vertex. (Section 9.3). The textbook indicates the vertices <and the edges for that matter> don't have to be distinct. (Chapter 4.3).

8. (10 points) For the digraph in Figure 5,

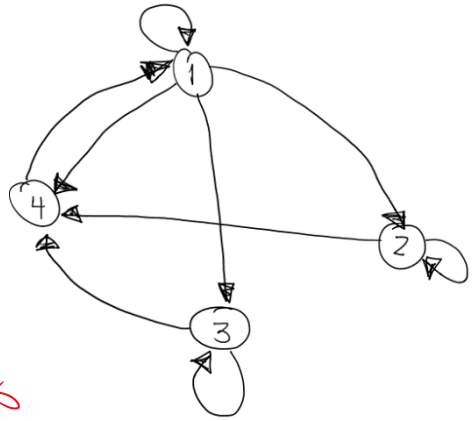


Figure 5

   a. (5 pts) List all TEXTBOOK paths of length 3 starting from vertex 2

2→2→2→2
2→2→2→4
2→2→4→1
2→4→1→1
2→4→1→2
2→4→1→3
2→4→1→4
} paths of L=3

   b. (5 pts) List all ZYBOOK paths of length 3 starting from vertex 2

2→4→1→3

For question 9 let…

$$A = \begin{bmatrix} 3 & 3 & 1 \\ 6 & -2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 2 & -1 \\ 8 & 7 & 3 \\ 4 & -1 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 2 & 2 \\ 6 & 5 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 9 & 2 \\ 5 & 4 \end{bmatrix}$$

9. (20 points) If possible, compute each of the following:

a. (4 pts) B + C

3x3 + 3x3 = 3x3

$$B + C = \begin{bmatrix} 3 & 3 & -1 \\ 5 & 9 & 5 \\ 10 & 4 & 5 \end{bmatrix}$$

b. (6 pts) BA

not possible

c. (8 pts) AD

not possible

This should have been DA – my mistake is your benefit.