R Lab 4

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2025-04-07

6.5.1 Subset Selection Methods

Best Subset Selection

```
library(ISLR2)
```

```
## Warning: package 'ISLR2' was built under R version 4.4.3
```

```
names(Hitters)
```

```
##  [1] "AtBat"     "Hits"      "HmRun"     "Runs"      "RBI"       "Walks"
##  [7] "Years"     "CAtBat"    "CHits"     "CHmRun"    "CRuns"     "CRBI"
## [13] "CWalks"    "League"    "Division"  "PutOuts"   "Assists"   "Errors"
## [19] "Salary"    "NewLeague"
```

```
dim(Hitters) # Dimension of data matrix
```

```
## [1] 322  20
```

```
sum(is.na(Hitters$Salary)) # Sums up NA values for salary of hitters
```

```
## [1] 59
```

```
Hitters <- na.omit(Hitters) # Omitting observations where any variable is NA
sum(is.na(Hitters))
```

```
## [1] 0
```

We can use the **leaps** library.

```
# install.packages("leaps")
library(leaps)
```

```
## Warning: package 'leaps' was built under R version 4.4.3
```

```
# regsubsets() automatically does best subset selection using RSS
regfit.full <- regsubsets(Salary ~ ., data=Hitters)
summary(regfit.full)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##          AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 ) " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 ) "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 ) " "   "*"  " "   " "  " " "*"   " "   "*"    "*"   "*"    " "   " "
## 8  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   " "    " "   "*"    "*"   " "
##          CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 ) " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 ) "*"    " "     "*"       "*"     " "     " "    " "
```

Asterisks indicate that the variable is included in the corresponding model. The **nvmax** parameter in the function lets us predetermine the max amount of variables.

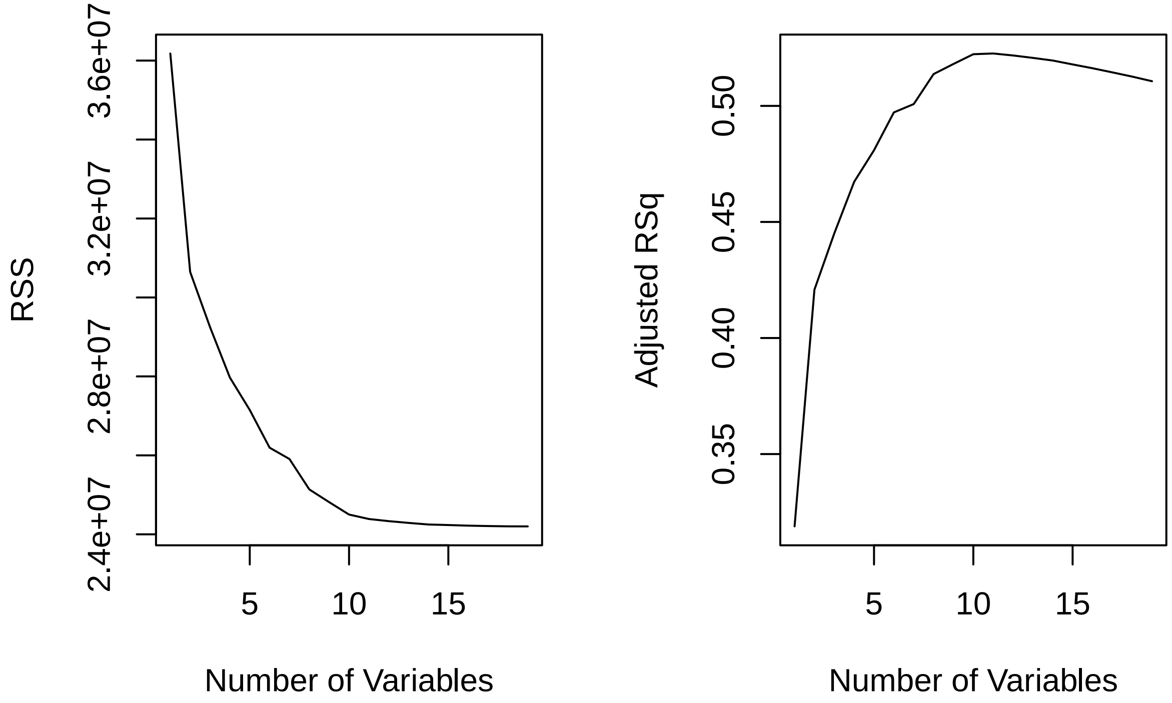
```
regfit.full <-regsubsets(Salary ~ ., data = Hitters, nvmax = 19) # 19 vars
reg.summary <- summary(regfit.full)

# Columns of summary object
names(reg.summary)
```

```
## [1] "which"  "rsq"    "rss"    "adjr2"  "cp"     "bic"    "outmat" "obj"
```

```
# R^2
reg.summary$rsq
```

```
##  [1] 0.3214501 0.4252237 0.4514294 0.4754067 0.4908036 0.5087146 0.5141227
##  [8] 0.5285569 0.5346124 0.5404950 0.5426153 0.5436302 0.5444570 0.5452164
## [15] 0.5454692 0.5457656 0.5459518 0.5460945 0.5461159
```

```
# Plotting the subset RSS  and adjusted R^2
par(mfrow = c(1, 2))
plot(reg.summary$rss, xlab = "Number of Variables",
 ylab = "RSS", type = "l")
plot(reg.summary$adjr2, xlab = "Number of Variables",
 ylab = "Adjusted RSq", type = "l")
```



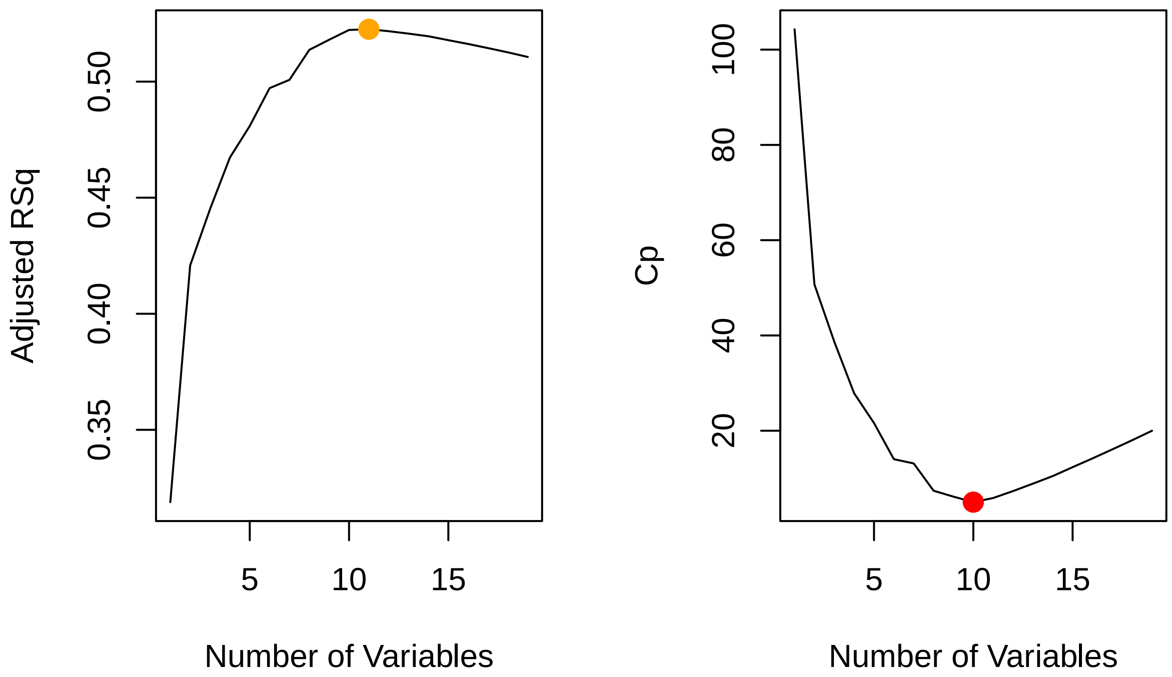
```r
# Finding max adj R^2
which.max(reg.summary$adjr2) # Max # of vars
```

```
## [1] 11
```

```r
plot(reg.summary$adjr2, xlab = "Number of Variables", ylab = "Adjusted RSq", type = "l") # Plotting adj
points(11, reg.summary$adjr2[11], col = "orange", cex = 2, pch = 20) # Plotting point on line

# Analagous procedure for Cp
plot(reg.summary$cp, xlab = "Number of Variables", ylab = "Cp", type = "l")
which.min(reg.summary$cp)
```

```
## [1] 10
```

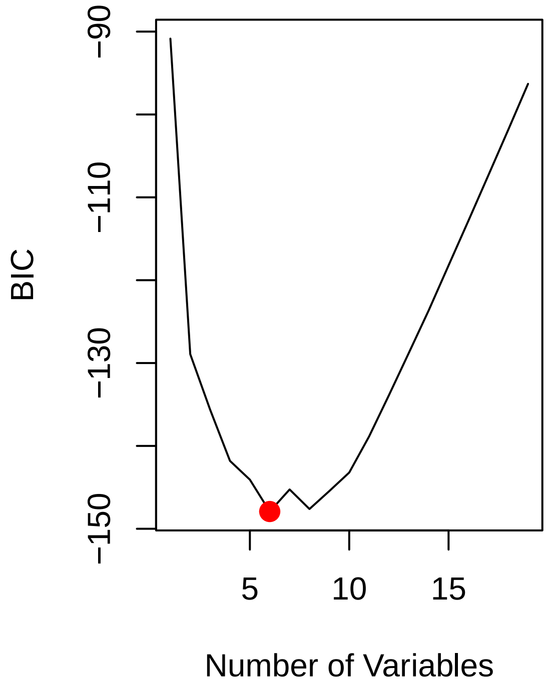
```r
points(10, reg.summary$cp[10], col = "red", cex = 2, pch = 20)
```



```r
# Analagous procedure for BIC
plot(reg.summary$bic, xlab = "Number of Variables", ylab = "BIC", type = "l")
which.min(reg.summary$bic)
```

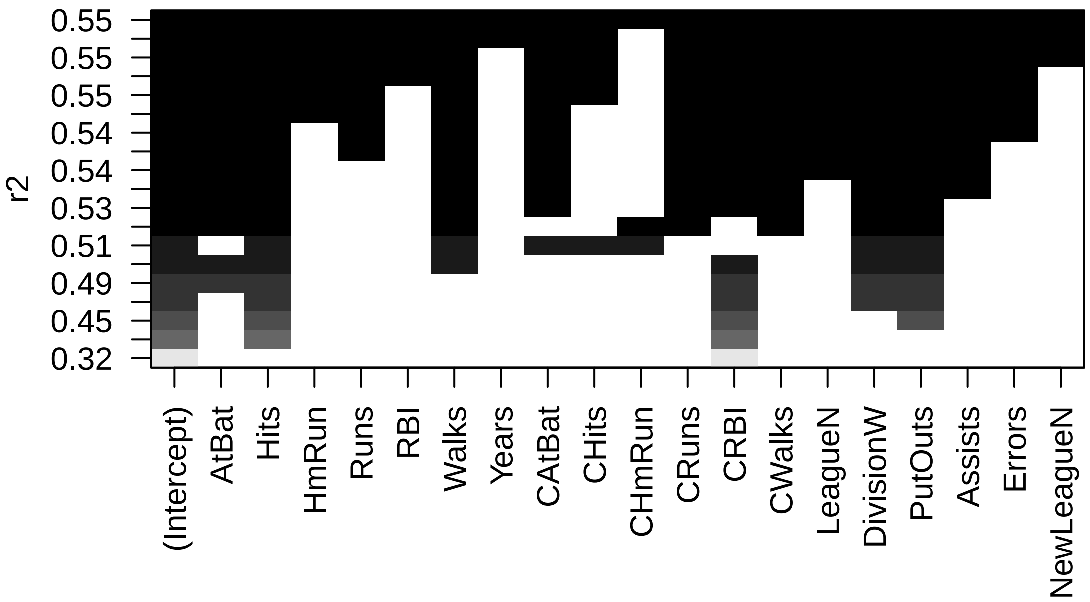
```
## [1] 6
```

```r
points(6, reg.summary$bic[6], col = "red", cex = 2, pch = 20)
```

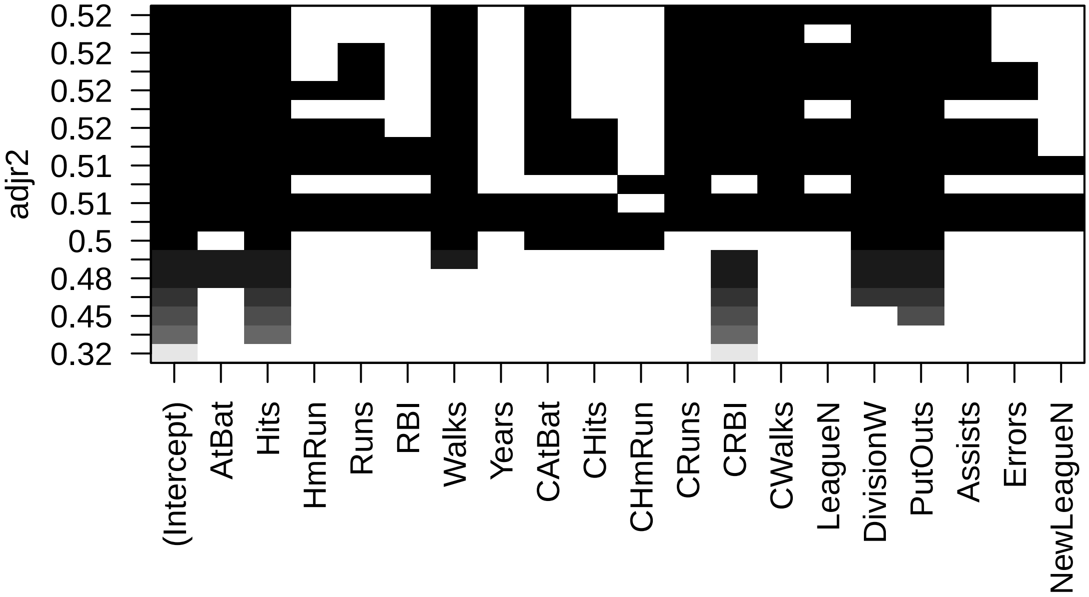


The **regsubsets()** function also has a built in plot command which can display selected variables for a model according to different metrics. The top row contains a black square for a selected variable in the optimal model

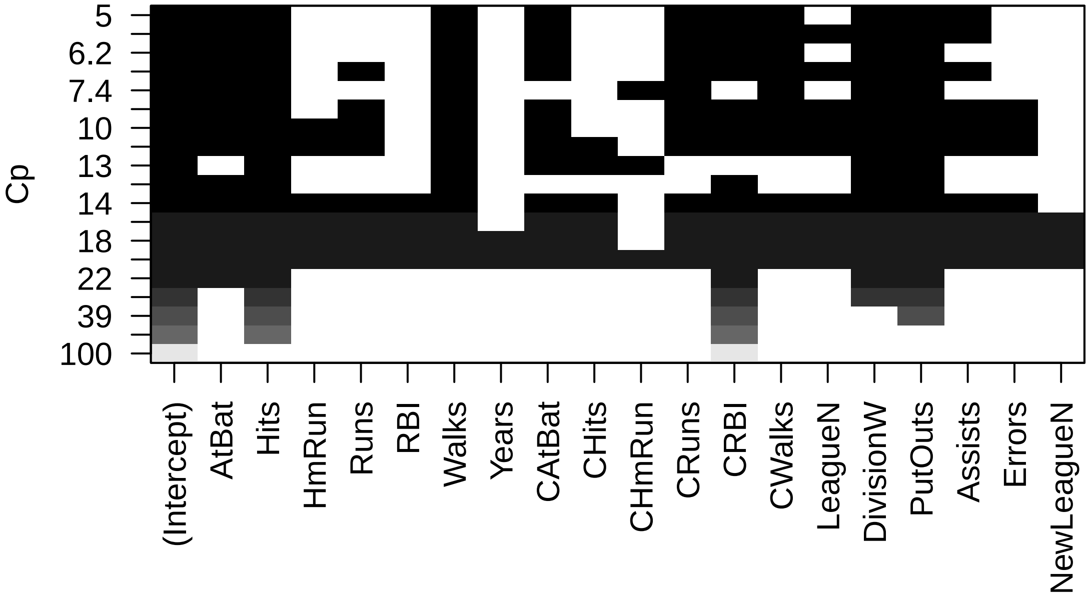
```r
plot(regfit.full, scale = "r2") # R^2
```



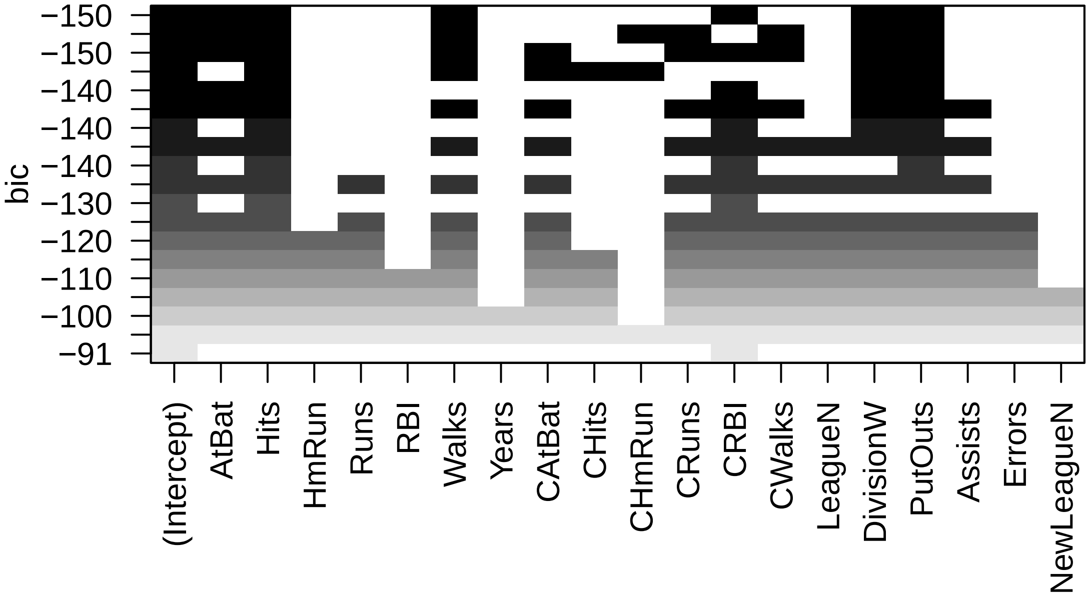
```
plot(regfit.full, scale = "adjr2") # Adj R^2
```



```
plot(regfit.full, scale = "Cp") # C_p
```



```
plot(regfit.full, scale = "bic") # BIC
```



```r
# Coefficients of 6th model
coef(regfit.full, 6)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI    DivisionW 
##   91.5117981   -1.8685892    7.6043976    3.6976468    0.6430169 -122.9515338 
##      PutOuts 
##    0.2643076
```

Forward and Backward Stepwise Selection

We can also use **regsubsets()** to perform stepwise selection using the 'method' parameter. Note that compared to the full best subset selection, the best models for 1-6 variables are all the same, so we will look at a 7 variable model.

```r
# Forward
regfit.fwd <-regsubsets(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
# Backward
regfit.bwd <-regsubsets(Salary ~ ., data = Hitters, nvmax = 19, method = "backward")
summary(regfit.bwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "backward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: backward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    "*"   " "
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " "
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " "
## 4  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " "
## 5  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " "
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " "
## 7  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
coef(regfit.full, 7)
```

```
##  (Intercept)         Hits        Walks       CAtBat        CHits       CHmRun
##   79.4509472    1.2833513    3.2274264   -0.3752350    1.4957073    1.4420538
##    DivisionW      PutOuts
## -129.9866432    0.2366813
```

```
coef(regfit.fwd, 7)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI       CWalks
##  109.7873062   -1.9588851    7.4498772    4.9131401    0.8537622   -0.3053070
##    DivisionW      PutOuts
## -127.1223928    0.2533404
```

```
coef(regfit.bwd, 7)
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns       CWalks
##  105.6487488   -1.9762838    6.7574914    6.0558691    1.1293095   -0.7163346
##    DivisionW      PutOuts
## -116.1692169    0.3028847
```

Choosing Among Models Using the Validation-Set Approach and Cross-Validation

We start with the validation set approach.

```
set.seed(1)
# Create random vector of elements of TRUE or FALSe if the corresponding observation is in the training
train <-sample(c(TRUE, FALSE), nrow(Hitters), replace = TRUE)
test <- (!train)

# Apply regsubsets() to training data
regfit.best <-regsubsets(Salary ~ ., data = Hitters[train, ], nvmax = 19)

# Make test data matrix for model
test.mat <- model.matrix(Salary ~ ., data = Hitters[test, ])

val.errors <-rep(NA, 19)
# Using test matrix to compute test MSE
for (i in 1:19) {
 coefi <-coef(regfit.best, id = i)
 pred <- test.mat[, names(coefi)] %*% coefi
 val.errors[i] <-mean((Hitters$Salary[test]- pred)^2)
}

val.errors
```

```
##  [1] 164377.3 144405.5 152175.7 145198.4 137902.1 139175.7 126849.0 136191.4
##  [9] 132889.6 135434.9 136963.3 140694.9 140690.9 141951.2 141508.2 142164.4
## [17] 141767.4 142339.6 142238.2
```

```
which.min(val.errors) # Min errors for 7 vars
```

```
## [1] 7
```

```
coef(regfit.best, 7) # Coefs for 7 var model
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns       CWalks
##   67.1085369   -2.1462987    7.0149547    8.0716640    1.2425113   -0.8337844
##    DivisionW      PutOuts
## -118.4364998    0.2526925
```

```
# This function just mimics the code above
predict.regsubsets <- function(object, newdata, id, ...) {
 form <-as.formula(object$call[[2]])
 mat <-model.matrix(form, newdata)
 coefi <-coef(object, id = id)
 xvars <-names(coefi)
 mat[, xvars] %*% coefi
}

## Now we apply the regsubsets() to the full data set ##
regfit.best <-regsubsets(Salary ~ ., data = Hitters, nvmax = 19)
# NOTE: the subset of variables is different than the set of variables from doing this on the training
coef(regfit.best, 7)
```

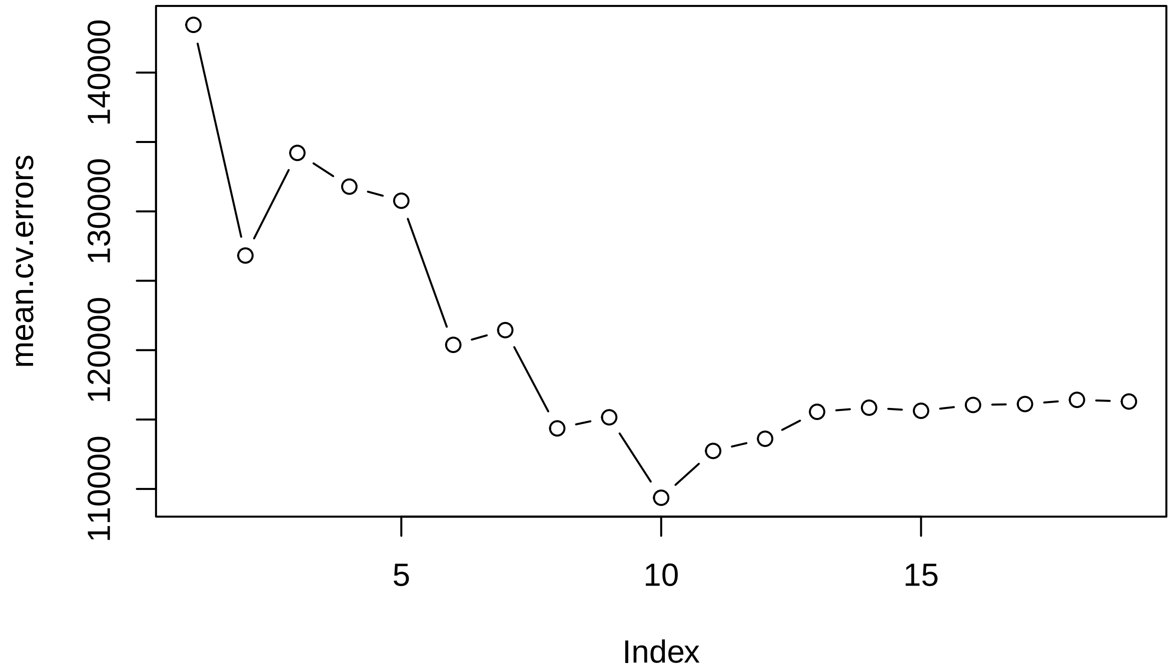
```
##  (Intercept)         Hits        Walks       CAtBat        CHits       CHmRun
##   79.4509472    1.2833513    3.2274264   -0.3752350    1.4957073    1.4420538
##    DivisionW      PutOuts
## -129.9866432    0.2366813
```

Now, we move on to the cross-validation approach.

```
# Set up for k-fold cross validation
k <- 10
n <-nrow(Hitters)
set.seed(1)
folds <-sample(rep(1:k, length = n))
cv.errors <-matrix(NA, k, 19, dimnames = list(NULL, paste(1:19)))

# For loop to perform cross validation
for (j in 1:k) {
 best.fit <-regsubsets(Salary ~. , data = Hitters[folds != j, ], nvmax = 19)
 for (i in 1:19) {
  pred <-predict(best.fit, Hitters[folds == j, ], id = i)
  cv.errors[j, i] <- mean((Hitters$Salary[folds == j]- pred)^2)
 }
}

# Using apply() to average over the columns of the matrix
mean.cv.errors <- apply(cv.errors, 2, mean)
mean.cv.errors # This approach uses the 10 variable model
```

```
##        1        2        3        4        5        6        7        8
## 143439.8 126817.0 134214.2 131782.9 130765.6 120382.9 121443.1 114363.7
##        9       10       11       12       13       14       15       16
## 115163.1 109366.0 112738.5 113616.5 115557.6 115853.3 115630.6 116050.0
##       17       18       19
## 116117.0 116419.3 116299.1
```

```
# Plotting errors. Clear minimizer at 10
par(mfrow=c(1,1))
plot(mean.cv.errors, type = "b")
```



```
# Now doing best subset selection for 10 vars
reg.best <-regsubsets(Salary~., data = Hitters, nvmax = 19)
coef(reg.best, 10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns 
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798    1.4082490 
##         CRBI       CWalks    DivisionW      PutOuts      Assists 
##    0.7743122   -0.8308264 -112.3800575    0.2973726    0.2831680
```