MEGN688 Final Project

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Introduction

This project deals with a classic mixed integer linear programming (MILP) problem in the power system optimization space: the unit commitment problem. The unit commitment problem is essentially an extension of the economic dispatch problem (a linear program) with the introduction of binary decision variables. The objective we try to solve is to minimize the short-run costs of meeting electricity demand. Unlike the economic dispatch problem, the unit commitment model considers extra binary decision variables. This decision variables include commitment (decision to commit thermal resource) and start/stop (startup or shutdown decision), which is why it is a mixed-integer problem. In this project, we attempt to solve the unit commitment model using Julia. We will also use efficient formulation techniques to attempt building a more efficient model. The optimization framework used is JuMP and the optimizer used is HiGHS. This project was heavily inspired by the "Power Systems Optimization" course by professor Michael Davidson (UCSD) and professor Jesse Jenkins (Princeton) and is an embellishment of their work. The link to the GitHub course is here.

Unit Commitment Model

The unit commitment model is similar to the economic dispatch model in that it aims to minimize cost while still being able to meet demand and obey certain engineering constraints. As mentioned before, we introduce new binary variables and constraints. This creates difficulties that we do not encounter in the ED model, as binary decision variables increase the computational complexity of solving. The general form of the model is as follows.

SETS

G - the set of all generators
$\mathbf{G_{thermal}} \subset \mathbf{G}$ - the subset of thermal generators for which commitment is necessary
T - the set of all time periods over which we are optimizing commitment and dispatch decisions

PARAMETERS

$\mathbf{Pmin_g}$ - the minimum operating bounds for generator *g*
$\mathbf{Pmax_g}$ - the maximum operating bounds for generator *g*
$\mathbf{D_t}$ - demand (in MW)
$\mathbf{VarCost_g}$ - the variable cost of generator *g*
$\mathbf{StartCost_g}$ - the startup cost of generator *g*
$\mathbf{MinUp_g}$ - the minimum up time of generator *g*
$\mathbf{MinDown_g}$, the minimum down time of generator *g*

VARIABLES

$\mathbf{GEN_{g,t}}$ - generation (in MW) produced by each generator *g*
$\mathbf{START_{g,t}}$ - startup decision (binary) of thermal generator *g* at time *t*
$\mathbf{SHUT_{g,t}}$ - shutdown decision (binary) of thermal generator *g* at time *t*
$\mathbf{COMMIT_{g,t}}$ - commitment status (binary) of generator *g* at time *t*

OBJECTIVE

$$\min cost : \sum_{g \in G, t \in T} VarCost_g \times GEN_{g,t} + \sum_{g \in G_{thermal}, t \in T} StartCost_g \times START_{g,t} \tag{1}$$

CONSTRAINTS

$$\sum_{g} GEN_{g,t} = D_t \qquad \forall \quad t \in T \tag{2}$$

$$GEN_{g,t} \leq Pmax_{g,t} \qquad \forall \quad g \notin G_{thermal}, t \in T \tag{3}$$

$$GEN_{g,t} \geq Pmin_{g,t} \qquad \forall \quad g \notin G_{thermal}, t \in T \tag{4}$$

$$GEN_{g,t} \leq Pmax_{g,t} \times COMMIT_{g,t} \qquad \forall \quad g \in G_{thermal}, t \in T \tag{5}$$

$$GEN_{g,t} \geq Pmin_{g,t} \times COMMIT_{g,t} \qquad \forall \quad g \in G_{thermal}, t \in T \tag{6}$$

$$COMMIT_{g,t} \geq \sum_{t't-MinUp_g}^{t} START_{g,t} \qquad \forall \quad g \in G_{thermal}, t \in T \tag{7}$$

$$1 - COMMIT_{g,t} \geq \sum_{t't-MinDown_g}^{t} SHUT_{g,t} \qquad \forall \quad g \in G_{thermal}, t \in T \tag{8}$$

$$COMMIT_{g,t+1} - COMMIT_{g,t} = START_{g,t+1} - SHUT_{g,t+1} \quad \forall \quad G_{thermal} \in G, t = 1..T-1 \tag{9}$$

This model follows the logic of having to commit thermal generators, whereas economic dispatch models assume that generators are always on. Equation (1) is the objective function, which minimizes the variable cost of generating $GEN_{g,t}$ MW of energy plus the fixed cost of starting generator g. (2) is the demand constraint. It is a hard constraint, which ensures that the minimal cost will be incurred when optimizing at the expense of computational complexity. (3) and (4) ensures that each generator in each time period will operate within their operating bounds. (5) and (6) are the same but are dependent on the commitment decision of each generator in each time period. (7) ensures that once a generator starts, it must remain operational for at least its minimum up time period. (8) is the inverse of (7), in that once a generator shuts off, it must remain off for the minimum down time. (9) enforces the logic that if a generator is committed in the next time period, then a start/stop action must reflect the action of the commitment decision (i.e. if generator g is committed in period $t+1$ when it wasn't in t, then the generator must be started up in $(t+1)$.

Solve Results

The data used for this experiment was pulled from the GitHub course that this project was based off of. It is data gathered from WECC for the San Diego Gas and Electric (SDGE) region. The data contains a time-series of loads from the region of 1-hour increments for the year of 2020. Additionally, it contains information about generators in the region, information about their variability, and data about fuels and their respective costs. By using the data, we are able to deduce the optimal cost and commitment status of each generator in each time period. In this analysis, we will observe the different solutions across a time horizon of 1-7 days and how they vary in time, cost, and amount of iterations the model takes to solve. It is realistic to only use the commitment model for a short-run time horizon, which is usually not longer than a week.

When solving with the original model with a 1% MIP gap, the solve time increases in a linear fashion over time. From day 1-7, the solve times were [.34,1.39,0.22,4.25,5.82,6.31,7.38] seconds, respectively. The solver (HiGHS in this case) was able to arrive at an optimal solution in a reasonable time. Below, we will observe 2 instances of solving a day's unit commitment. One will be on April 11 2020, and the other will be on April 14 2020, with the former date being a Saturday, and the latter being a Tuesday. The purpose is to demonstrate the difference of the unit commitment model output between a week day and a weekend day, namely the difference in generation and curtailment. For this case, we are interested in the cumulative generation of variable resources (onshore wind, solar photovoltaic, and hydroelectricity) in consideration of a high capacity solar case, where the capacity of solar photovoltaic output is fixed to 3500 MW, and of the difference in curtailment. These will be examined over a period of 24 hours.

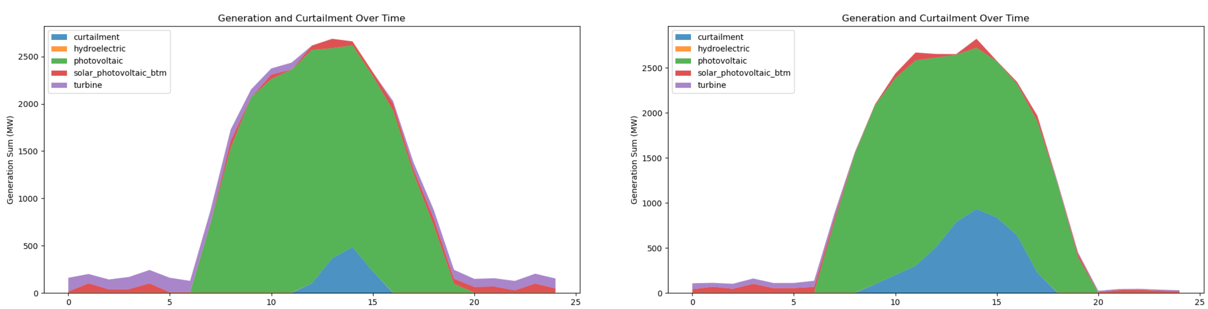

Figure 1: Saturday (left) and Tuesday (right) Generation and Curtailment

Figure 1 depicts a graph of the cumulative generation of variable resources and curtailment. As can be seen, the generation of each variable resource is similar for each hour, assuming because weather conditions were similar on each day. However, we can note that the curtailment on Tuesday was significantly larger than it was on Saturday, indicating that less of the energy produced by these renewable resources were being used. Speculatively, an intuitive answer for this would be due to economic reasons. During the weekend, there is much less industrial activity as opposed to week days, as many people do not work on weekends. A higher demand for electricity would most likely entail increase in prices for dispatching electricity, thus causing more variable energy to be curtailed, as it is often more expensive to dispatch. This can also be inferred from the optimal cost solutions, which were \$736083.88 on Saturday on and \$860327.20 on Tuesday. This shows that the lower loads for Saturday pushed down the cost of dispatching in comparison to Tuesday.

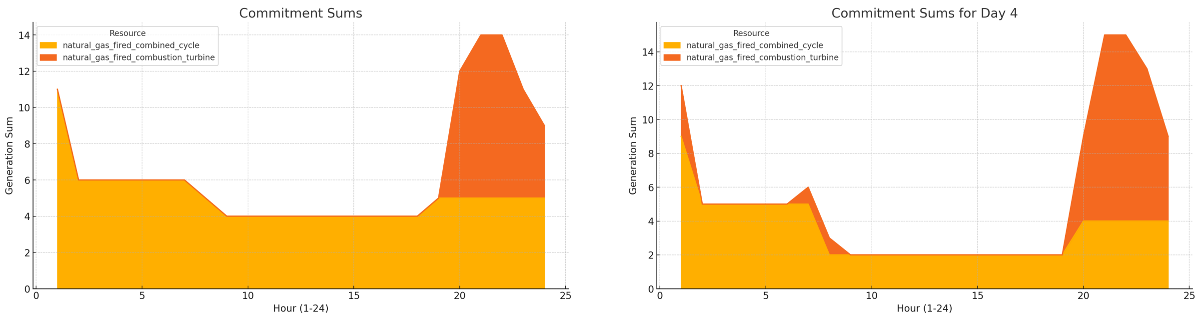

Figure 2: Unit Commitment Statuses on Saturday and Tuesday

Figure 2 depicts the sum of commitment statuses within the 24-hour time period of each day. As we can see, the natural gas combustion turbines were most committed during the night time, starting around 8PM PST. This is intuitive, as people will demand more heat as the sun goes down and the weather starts to chill. Another interesting thing to note is the midday differences between Saturday and Tuesday. Slightly more units were committed for each hour during the day on Saturday. This could perhaps be due to more people being home and demanding thermal energy for cooling.

Efficient Formulations

As an attempt to increase the efficiency of the model, we will use two different methods of efficient formulation. The first method used was variable elimination. To do this, we seek to remove redundant variables that have little importance to the model. In this case, we do that by eliminating generators that have no capacity. We consider a subset of G, G', such that G' contains no variables such that their capacity, κ, is greater than 0. By using this elimination method, we are able to remove redundant variables and tighten the search space of the model.

$$G' \subset G \tag{10}$$
$$G' = \{g \in G : \kappa_g > 0\} \tag{11}$$

The second method used is also manipulating the variables of the model. For this, we will use variable aggregation. In the dataset, there is 15 instances of natural_gas_combustion_turbine. We can aggregate all instances of this variable int one, which will significantly reduce the amount of variables in the set. The downfalls are that this may effect operating bounds such that they no longer obey each individual operating bounds of each generator. Also, this will improve complexity of solving, but at the expense of optimality. Let ng be the aggregation variable of NG combustion turbines, and let $i \in G$ be the subset of variables that are NG combustion turbines and k be the number of combustion turbines. We aggregate relevant parameters as averages of the set of generators:

$$Pmin_{ng} = \frac{1}{n}\sum_{i}^{k} Pmin_i \qquad \text{for } i \in k \tag{12}$$

$$Pmax_{ng} = \frac{1}{n}\sum_{i}^{k} Pmax_i \qquad \text{for } i \in k \tag{13}$$

$$VarCost_{ng} = \frac{1}{n}\sum_{i}^{k} VarCost_i \qquad \text{for } i \in k \tag{14}$$

$$StartCost_{ng} = \frac{1}{n}\sum_{i}^{k} StartCost_i \qquad \text{for } i \in k \tag{15}$$

$$MinDown_{ng} = \frac{1}{n}\sum_{i}^{k} MinDown_i \qquad \text{for } i \in k \tag{16}$$

$$MinUp_{ng} = \frac{1}{n}\sum_{i}^{k} Pmin_i \qquad \text{for } i \in k \tag{17}$$

The above methods were implemented and tested in the optimization model. The efficiency was observed by modeling the solve time, optimal costs, and linear programming iterations to observe efficiency and accuracy over each case of 1 day-7 days. Figures 3-5 depict the evolution of solve time, costs, and LP iterations for each time horizon case.

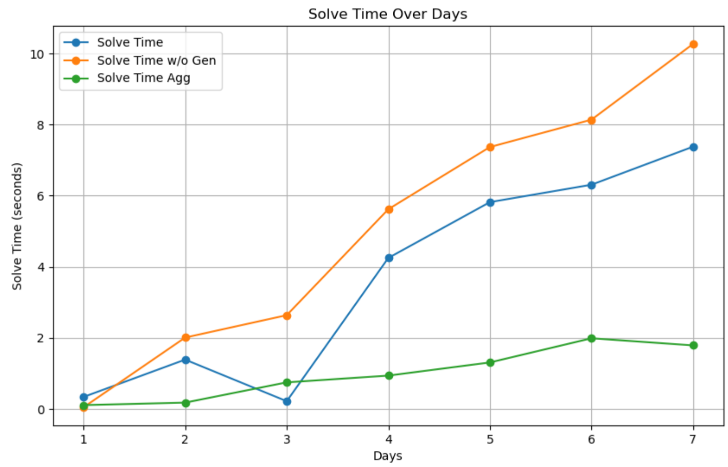

Figure 3: Solve Times

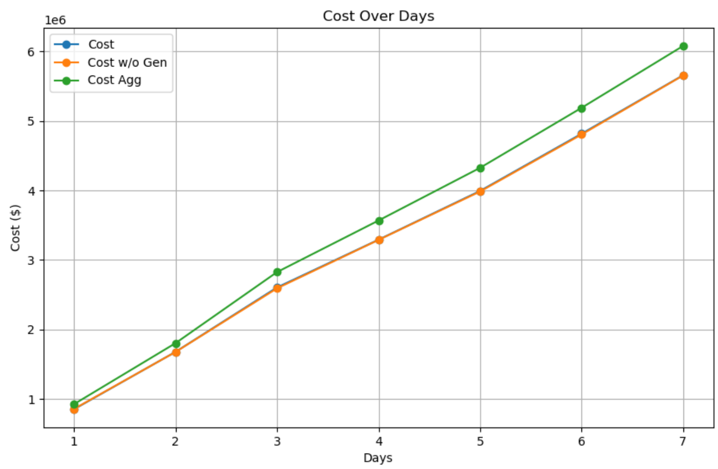

Figure 4: Costs

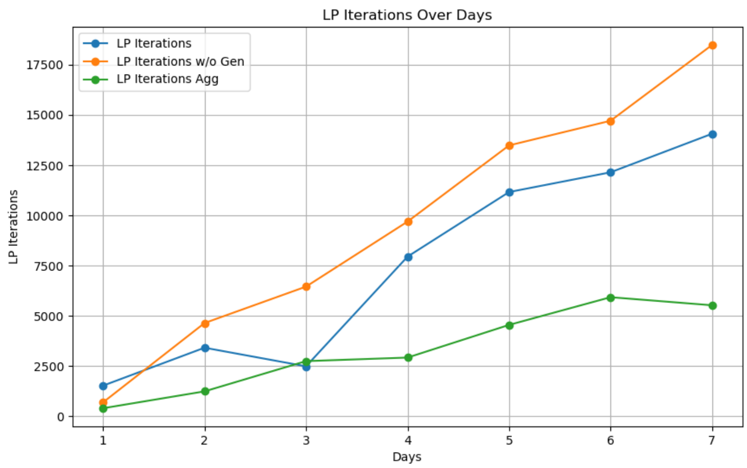

Figure 5: LP Iterations

Each line represents the integer programming model with different formulations; the blue line is the original model, the orange line is the model with eliminated variables, and the green line represents the model with aggregated variables. As we can observe, the model with aggregated variables has much faster solve times and LP iterations than the original model. The trade-off is that the model is outputting sub-optimal cost solutions. The use of this model may only be useful for cases where the normal model would take an unreasonable amount of time to solve (such as a 2-year time horizon). The variable elimination model, oddly enough, performed very poorly in comparison to the other models. While it did improve on optimal cost solutions compared to the original models, the changes were negligible, as they often only saved around $1000. However, the model is more computationally expensive than the original model; the solve times and LP iterations were higher and increasing at a faster rate than the original model. This is unintuitive, as one would expect for solve times to decrease as the search space is tightened.

Conclusion

This study explored the unit commitment problem within the context of power system optimization, employing mixed integer linear programming (MILP) techniques to address the challenges of optimizing power system dispatch. It was heavily inspired by the aforementioned professors and their course; all credit goes to them. The study utilized Julia's JuMP optimization framework and the HiGHS optimizer, and code will be included on my GitHub.

Though the original model provided a robust framework for approaching the unit commitment problem. However, efficient formulation techniques were used to attempt to improve the efficiency of the model. The model using aggregated variables demonstrated significant improvements in solve times and linear programming iterations, but at the expense of achieving sub-optimal cost solutions. This suggests that while aggregation can reduce computational demand, it does so by sacrificing the granularity of data, which could lead to less precise decision-making. Conversely, the variable elimination model did not perform as expected. Although it theoretically should have enhanced the model's efficiency by reducing the search space, it resulted in increased computational costs and only marginal improvements in cost efficiency. This outcome underscores the complexity of the unit commitment problem and suggests that not all reductions in variable space will lead to straightforward gains in performance.

Future work could explore several avenues to enhance the current model. One area could involve a deeper analysis of the trade-offs between model complexity and execution time, particularly in how different solver settings and parameter tunings impact the outcomes. Another promising direction would be the application of novel learning-to-optimize methods that could help system operators optimize and manage with real-time data adjustments.

Ultimately, this project not only showcases the challenges in power system optimization but also highlights the potential of advanced mathematical modeling techniques to inform and improve energy management strategies. The insights of being able to build more robust UC models could be instrumental for system operators, policy makers, and researchers, especially as the world transitions to a new age of renewable energy.

References

[1] Jenkins, J., Davidson, M. (n.d.). *Power Systems Optimization Course.* Accessed May 10, 2024. https://github.com/Power-Systems-Optimization-Course/power-systems-optimization.