# Introduction to Computer Vision

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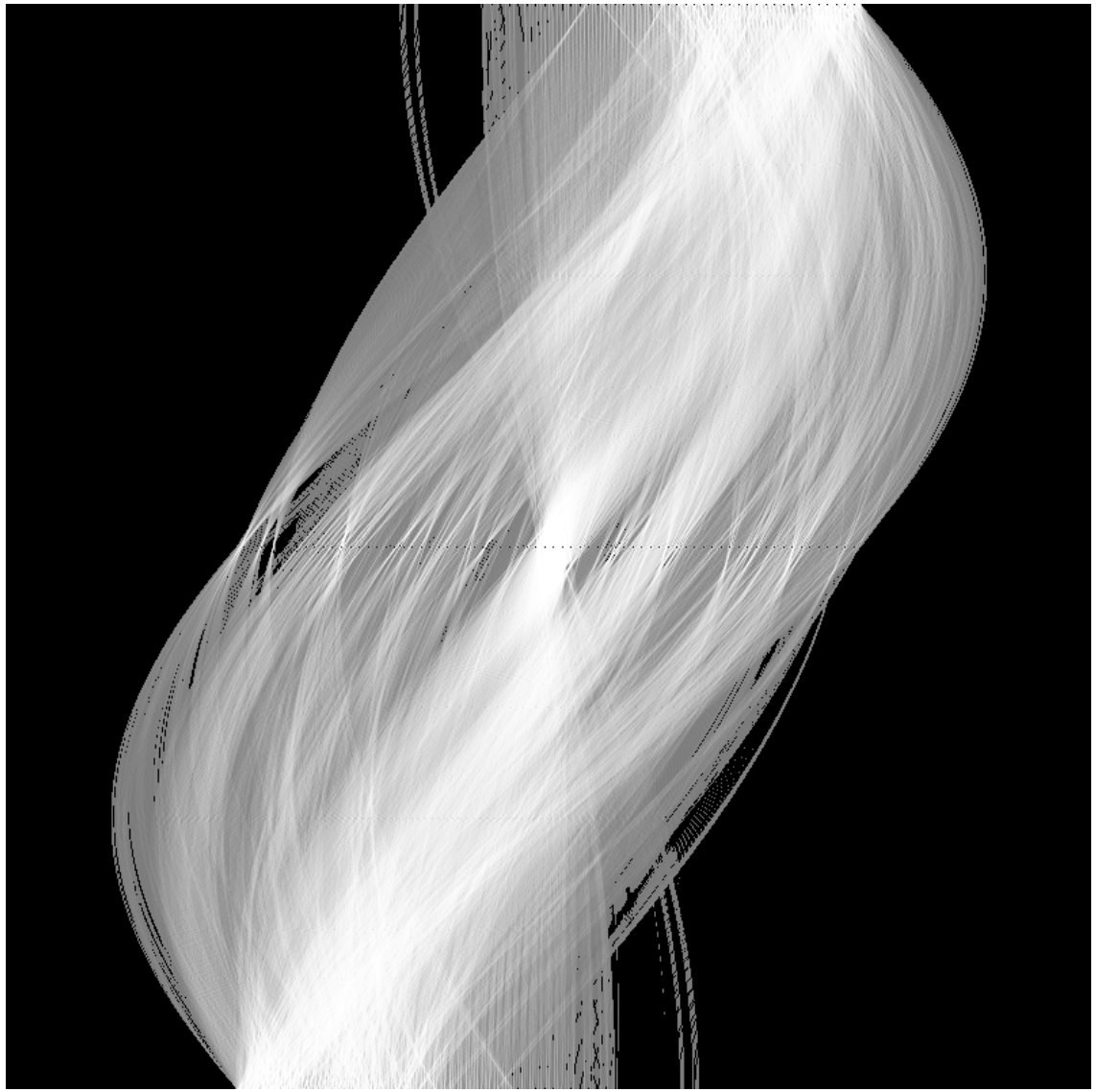
Computer Science Department
Colorado School of Mines

**Lecture 7**

# Learning Outcomes

- Hough Transform
- Line Detection
- Optical Flow



# Hough transform

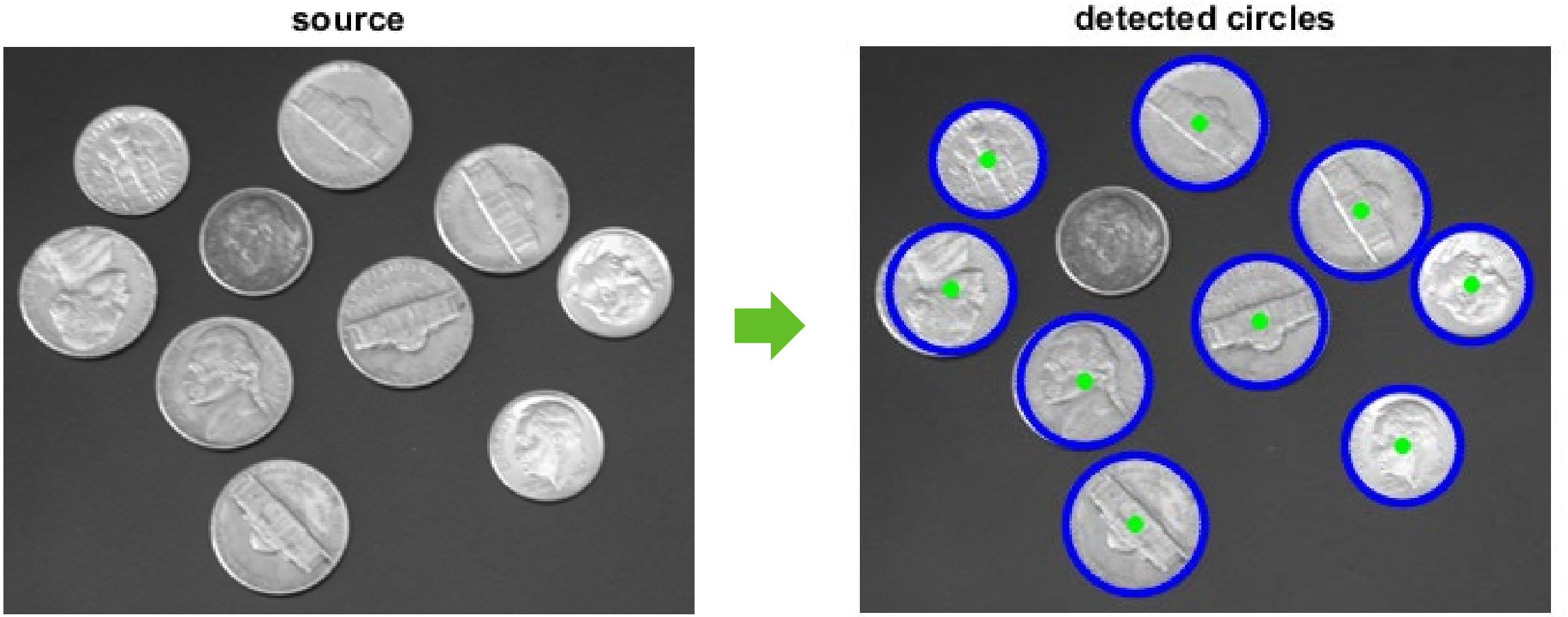
Hough transform is a **generic framework** for detecting a parametric model

Example: Hough transform can detect lines

Why Hough transform? E.g., vs. Canny edge detection?

- Edges don't have to be connected
- Lines can be occluded
- Key idea: edges **vote** for the possible models

# Circles




Example: Hough transform for circle detection
What is its application?

# Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

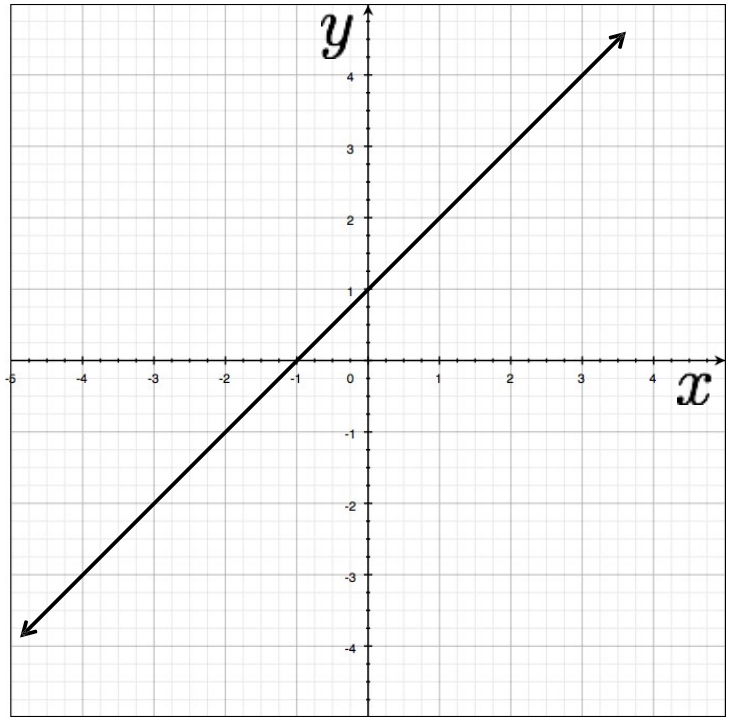
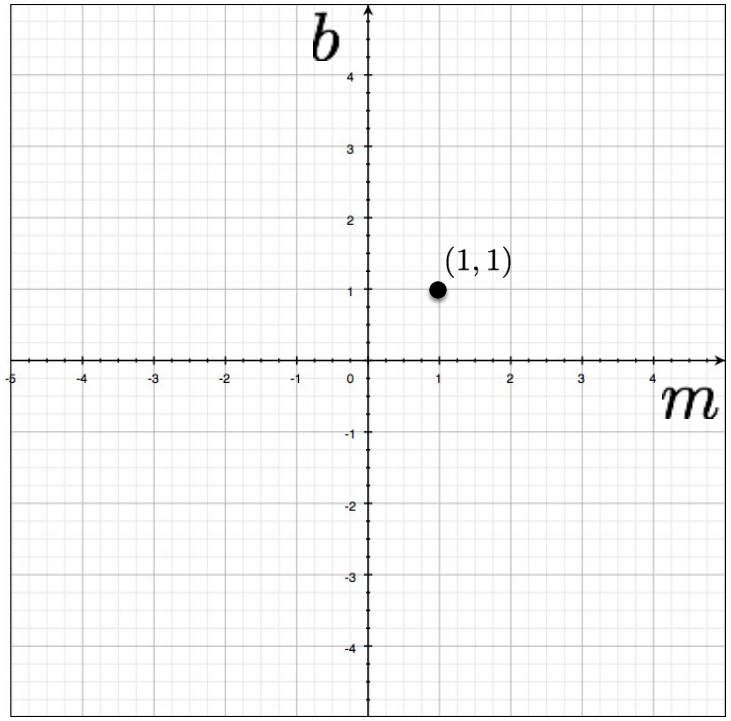



Image space

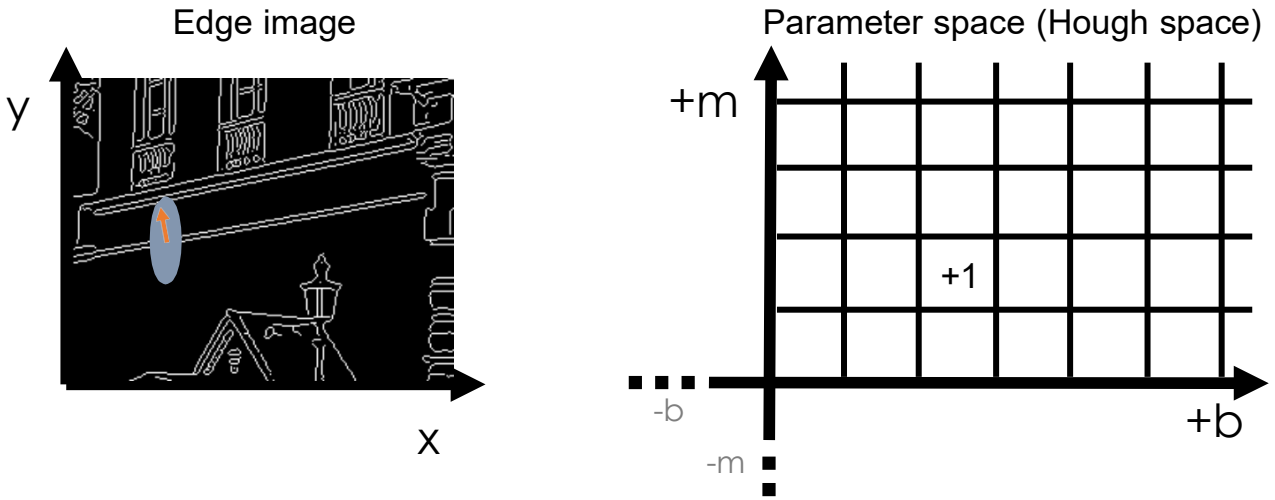
a line becomes a point




Parameter space

# Hough Transform: Outline

- Create a *grid* of candidate m,b parameter values: **accumulator**
  - Why a grid?
  - m,b are continuous; grid discretizes into hypotheses.
- Each edge pixel votes for a set of parameters, which increments those values in grid



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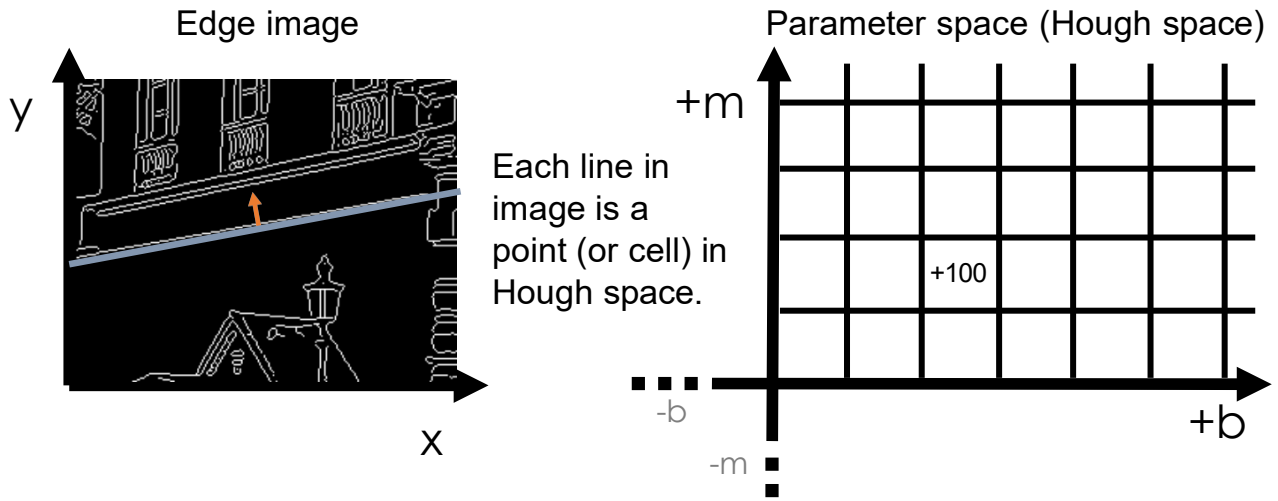

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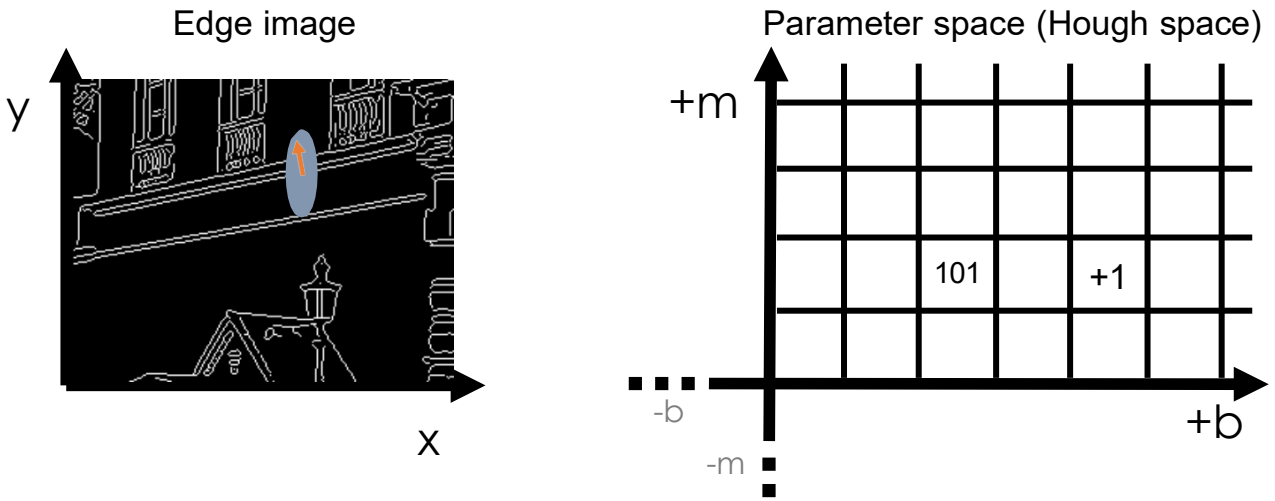

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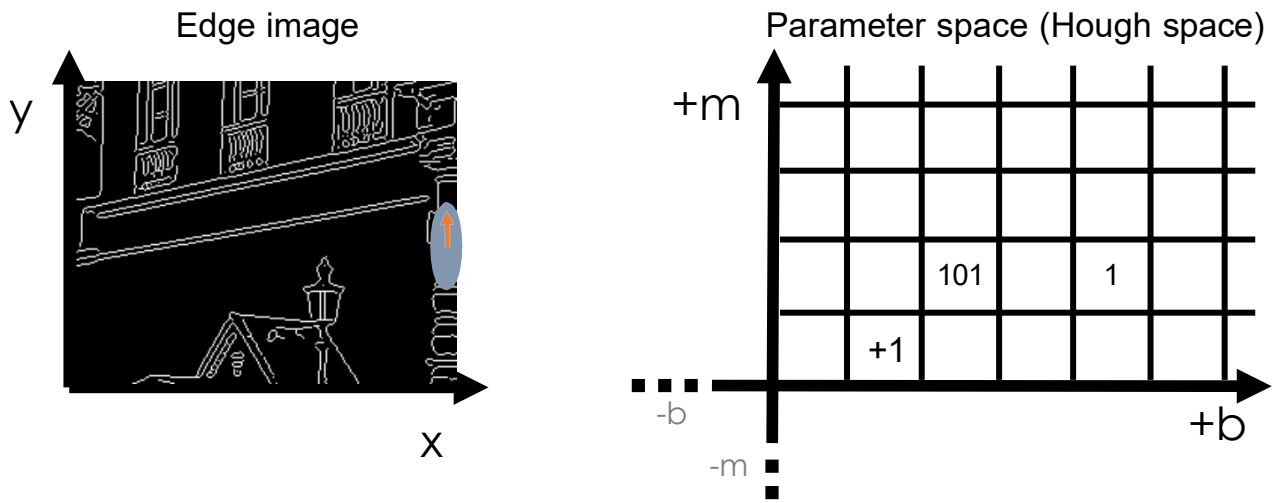

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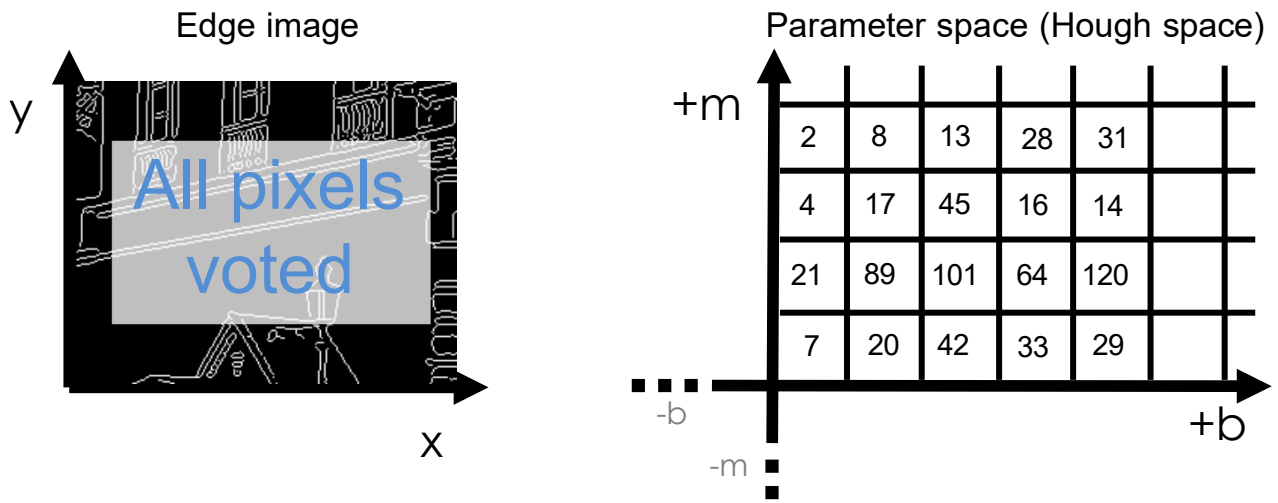

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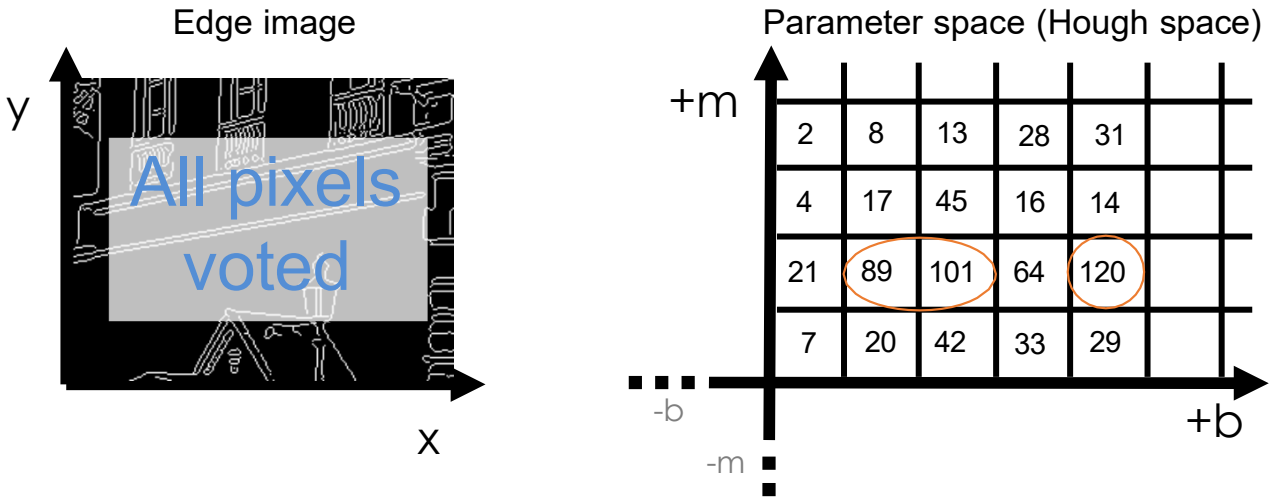
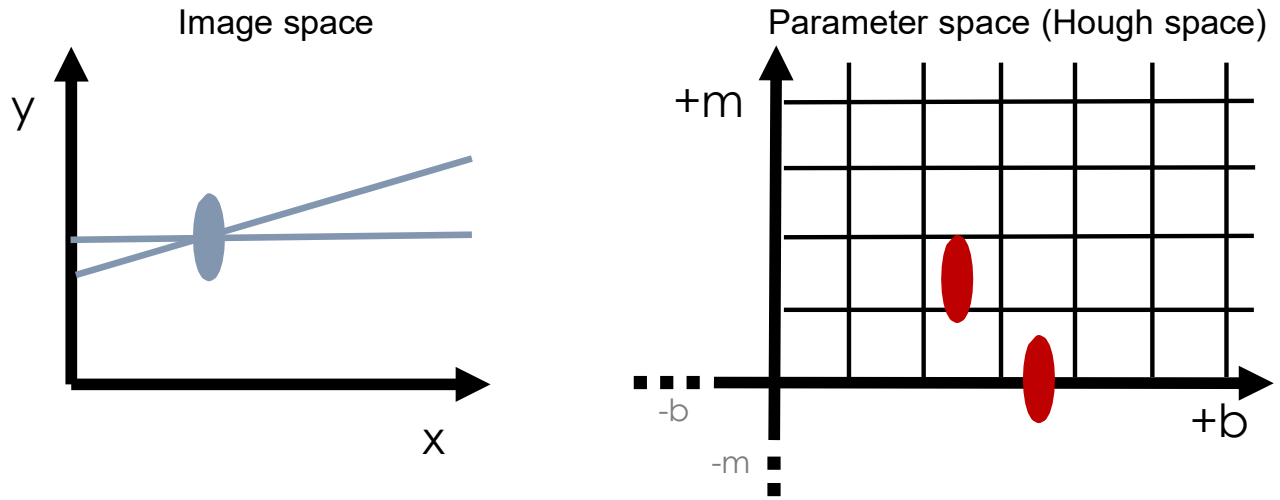
- Each edge pixel votes for a set of parameters, which increments those values in grid
- Find maxima – our line candidates.



# Hough Transform: Step back

- Hough space represents all possible lines.
- With gradient information constraint:
  - Edge is single point in Hough space.
- Without gradient orientation information?



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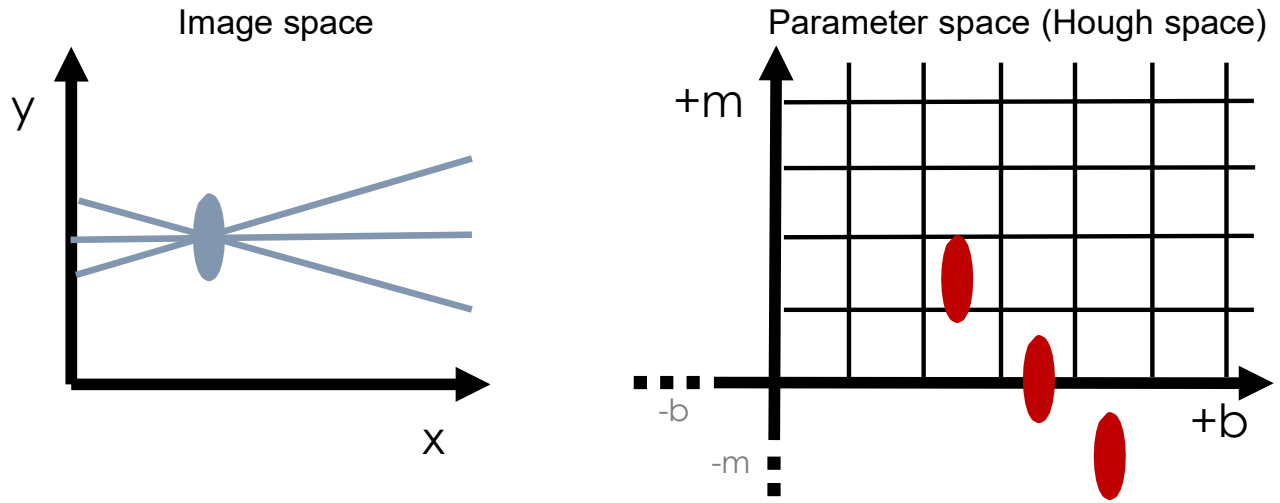

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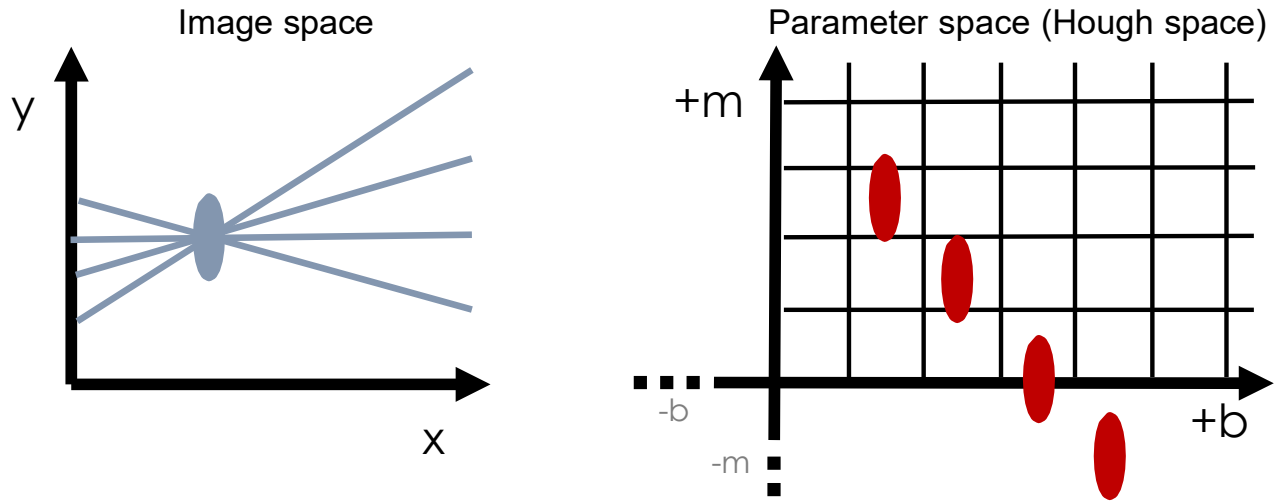

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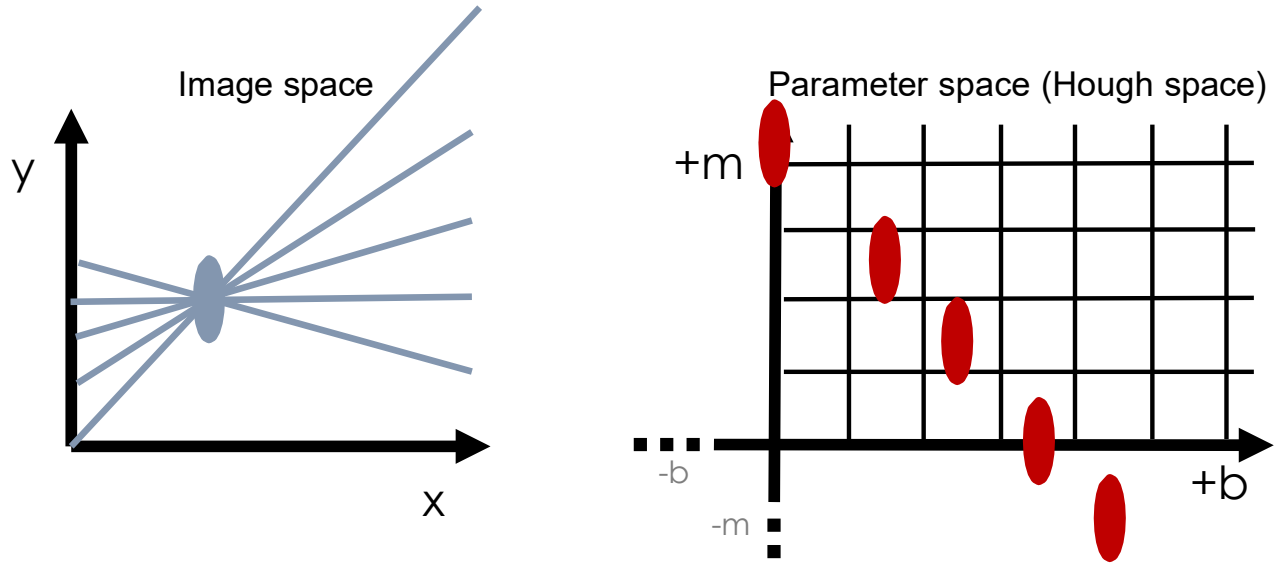
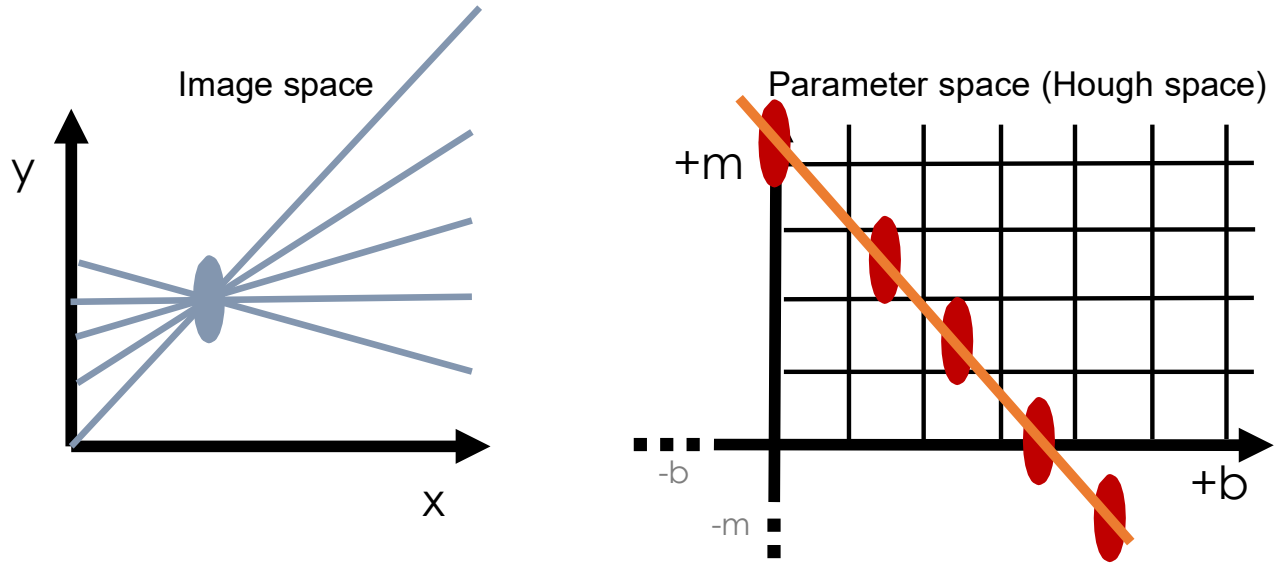


# Hough Transform: Step back

- Hough space represents all possible lines.
- With gradient information constraint:
  - Edge is single point in Hough space.
- Without gradient orientation information?
  - Point is line in Hough space



# Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$y - mx = b$$

parameters

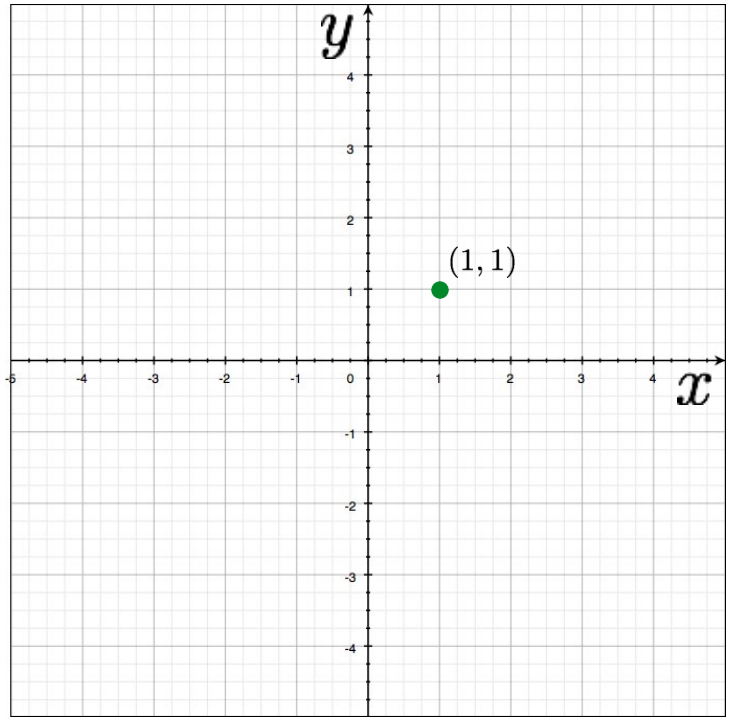
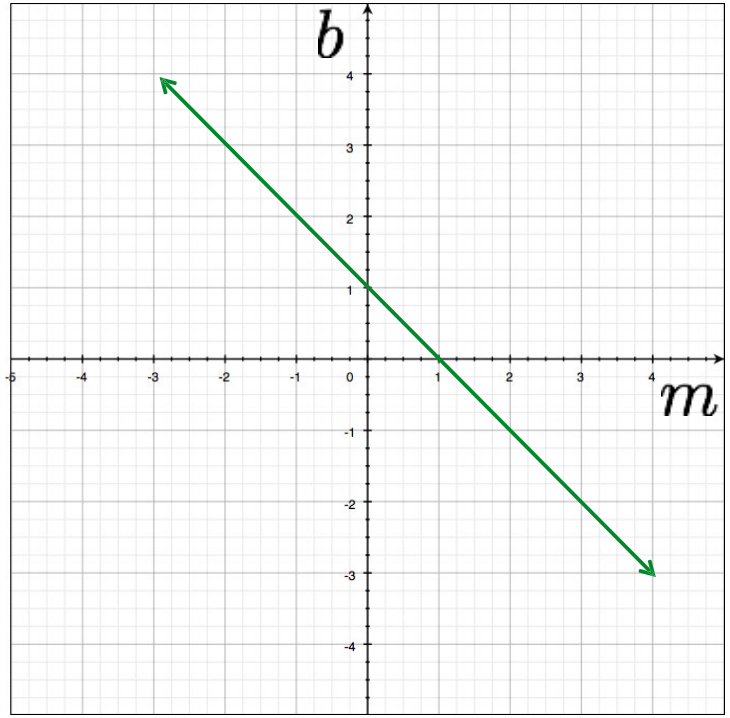



Image space

a point becomes a line

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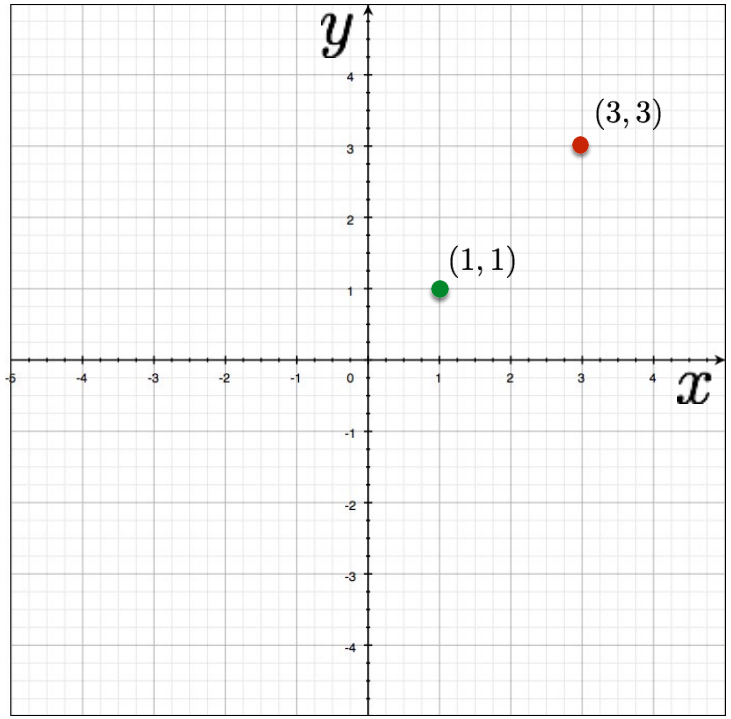
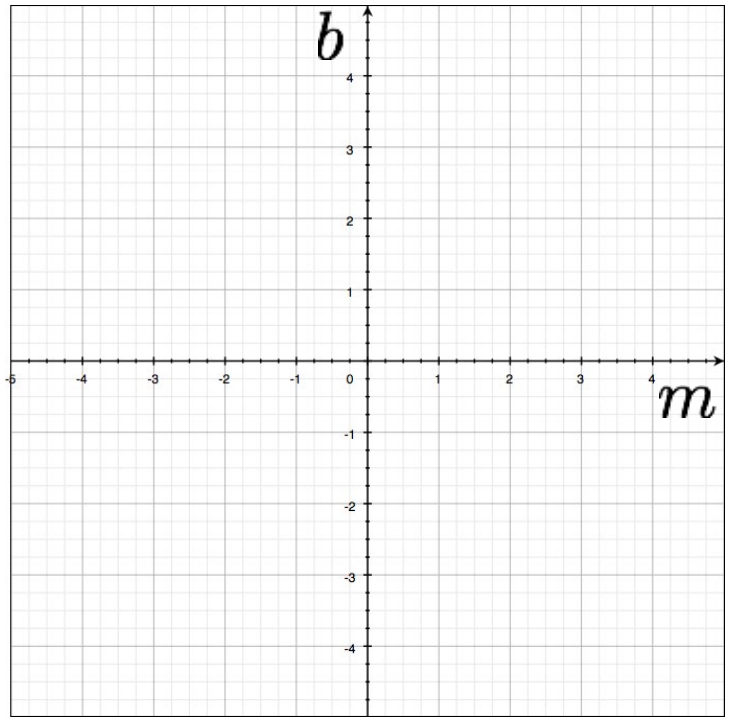



Image space

two points become ?

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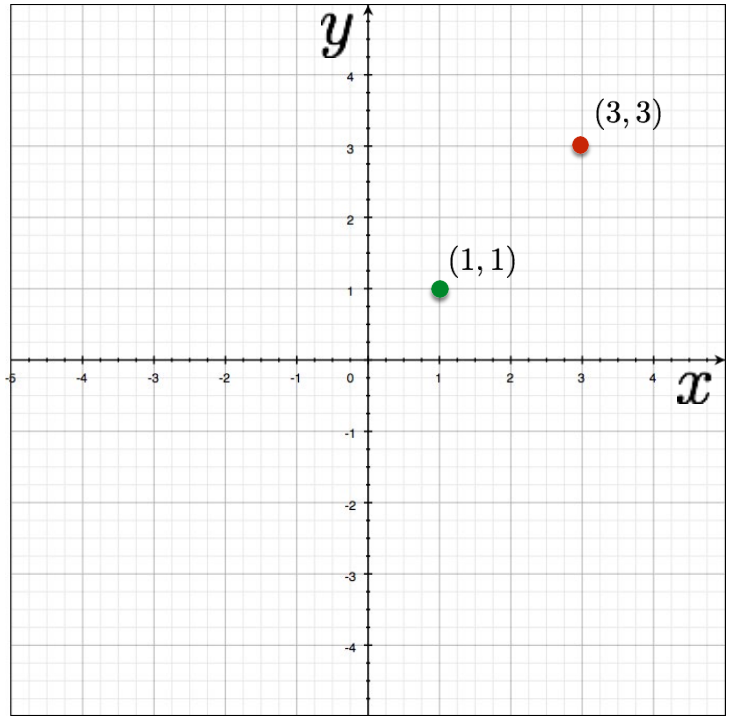


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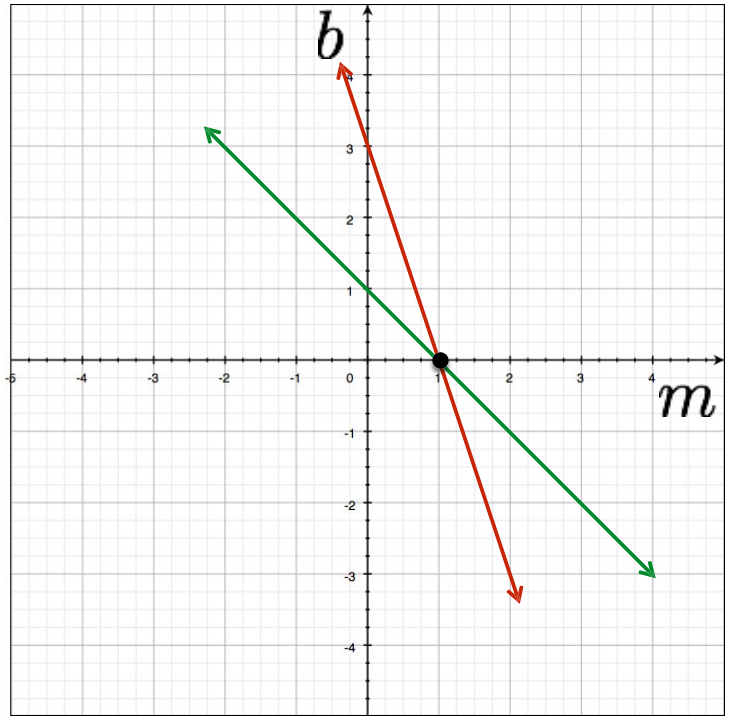


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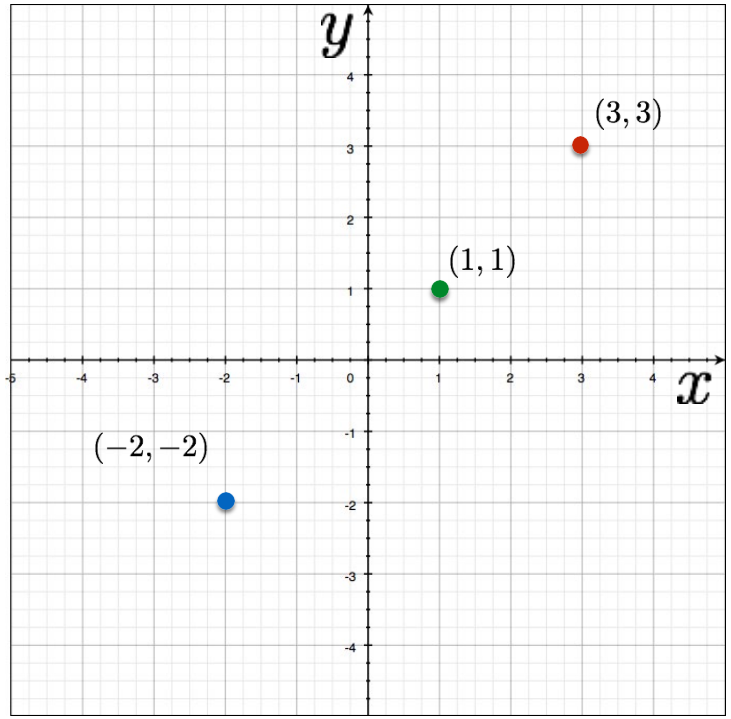
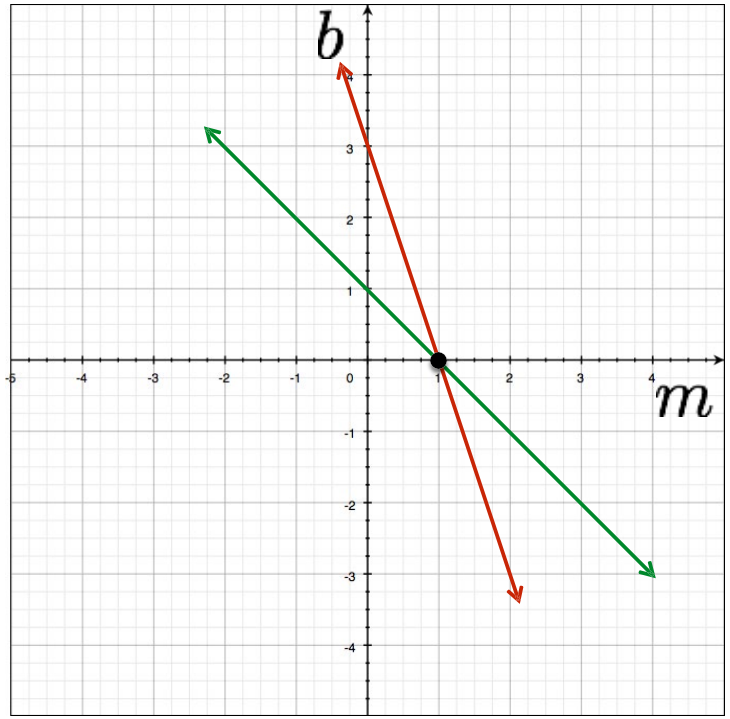



Image space

three points become ?




Parameter space

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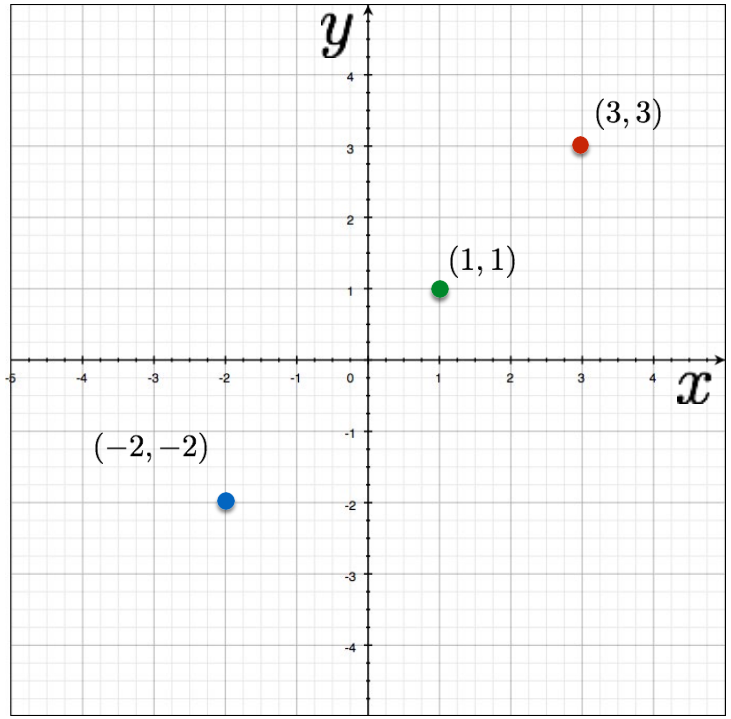


Image space

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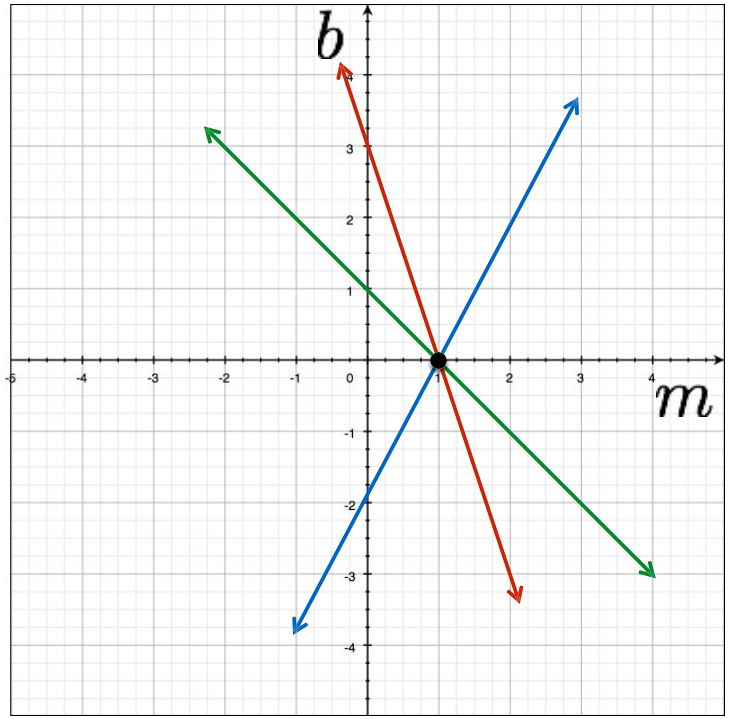


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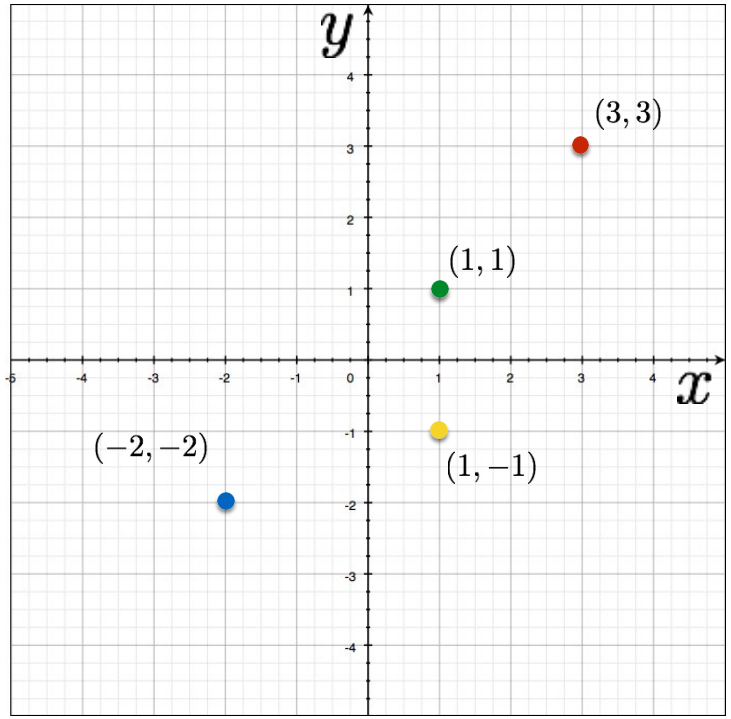
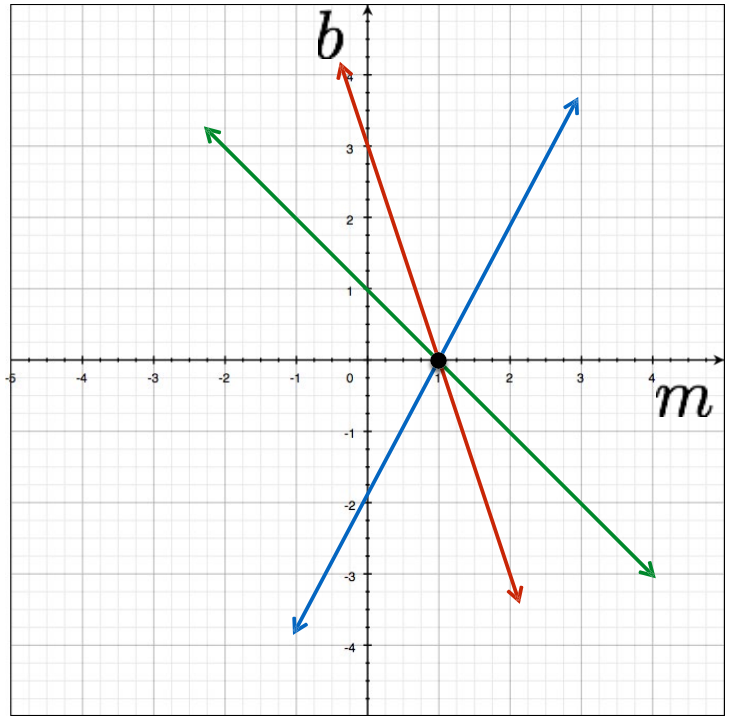



Image space

four points become ?

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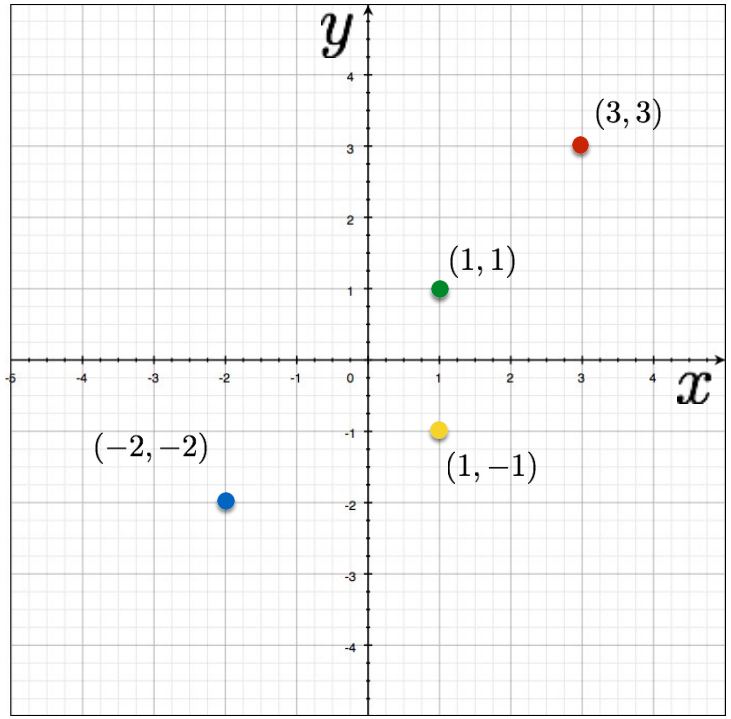


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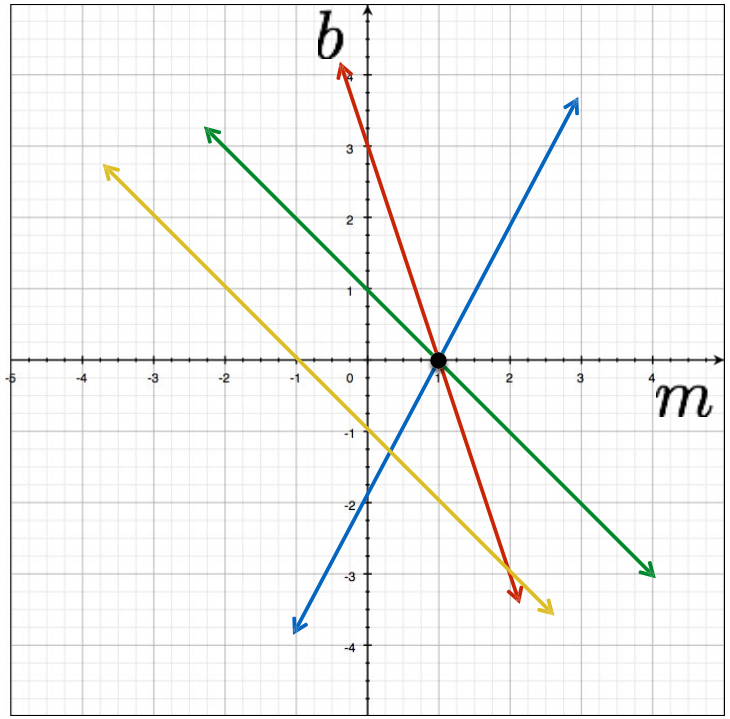
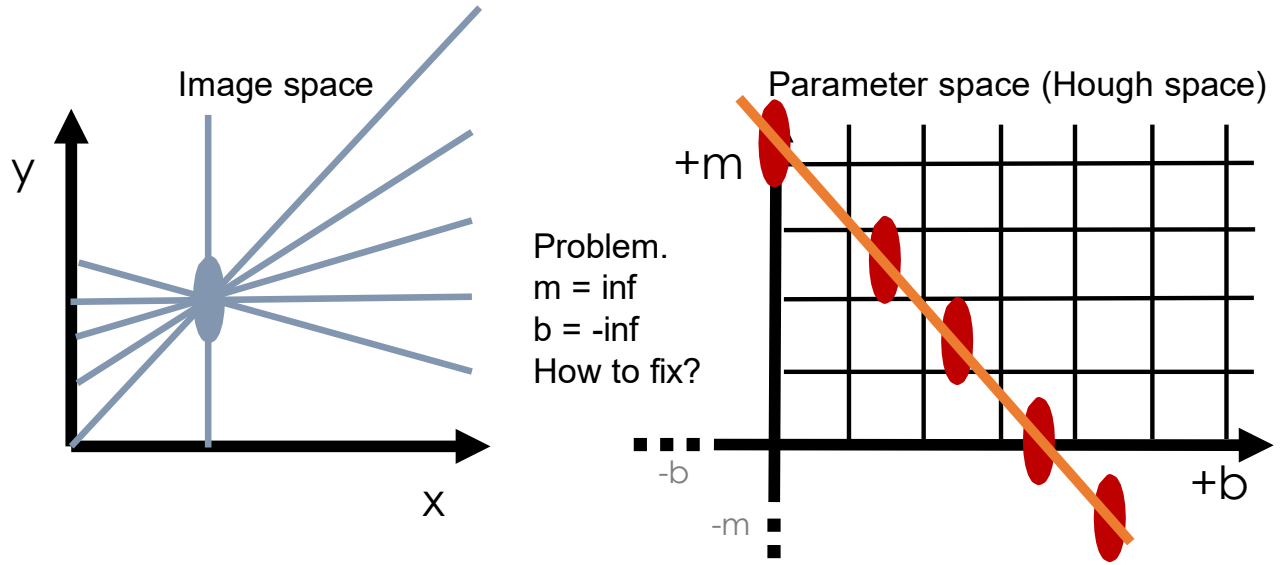



Parameter space

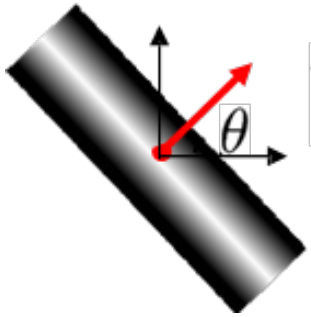
# Hough Transform: Step back

- Hough space represents all possible lines
- With gradient information constraint:
  - Edge is single point in Hough space
- Without gradient orientation information?
  - How big is Hough space?

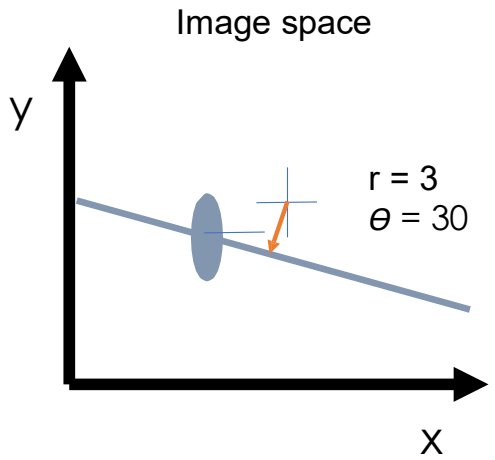


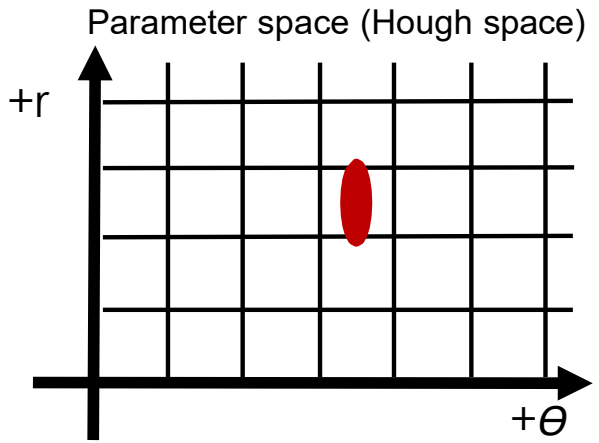
# Hough Transform: Line Normal Form

- Use $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$
  - Space is 0 to 360
- Use r = distance to line from some origin
  - $r_i = x_i \cos\theta_i + y_i \sin\theta_i$
  - Space is $\pm\sqrt{\max\_x^2 + \max\_y^2}$



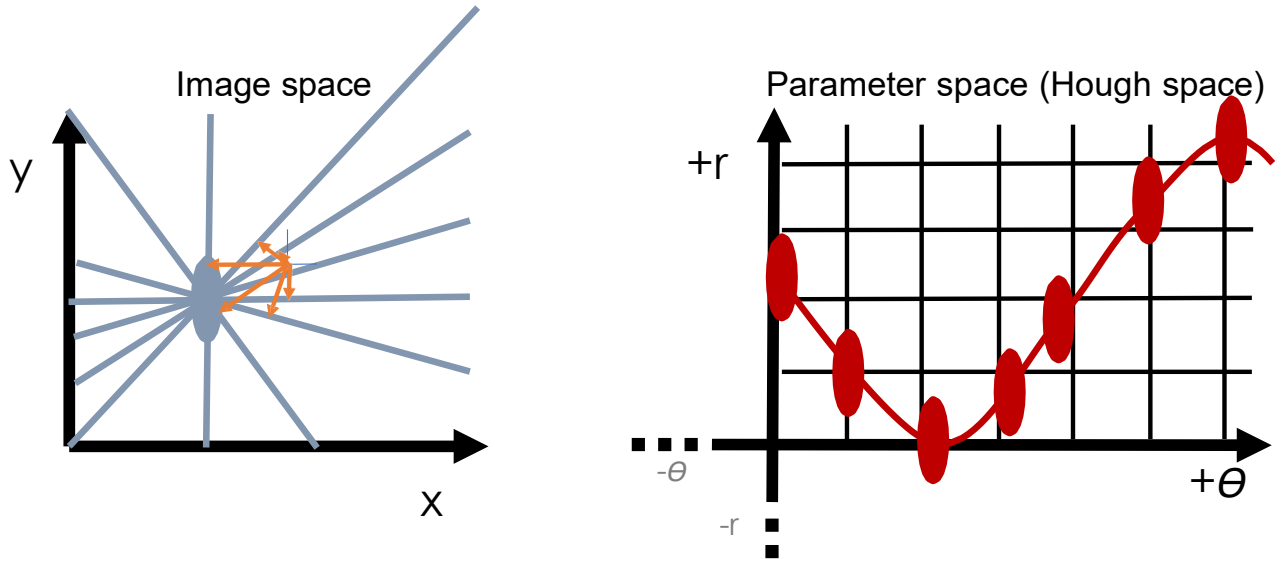

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$



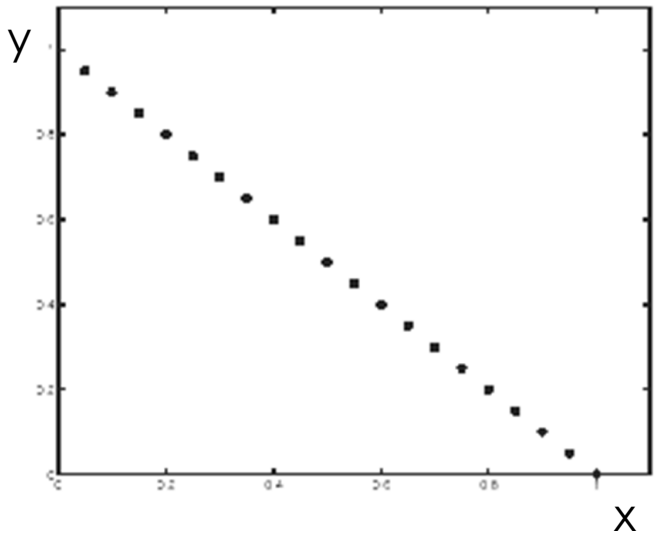
# Hough Transform: Line Normal Form

- In this line form, unoriented edge draws a sinusoid in Hough space:

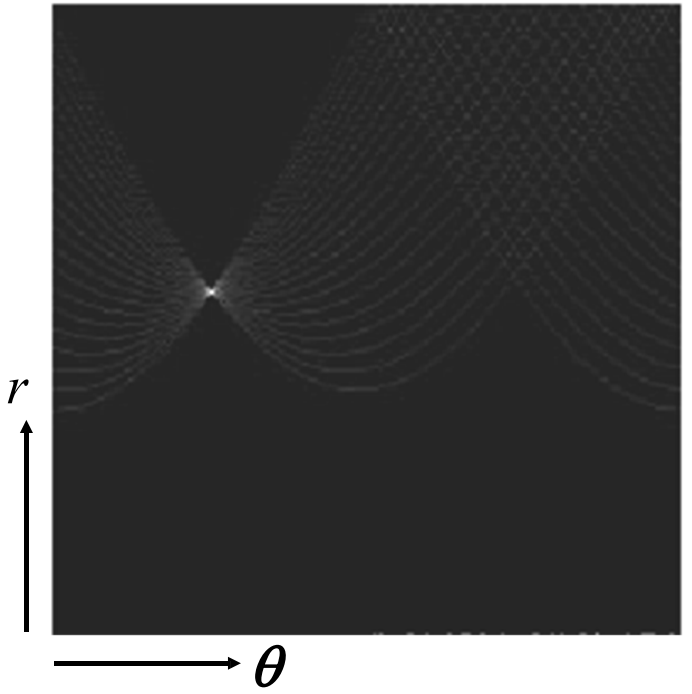


# Hough transform - experiment

Next few images *ignore* edge orientation.
Each point is one sinusoid.

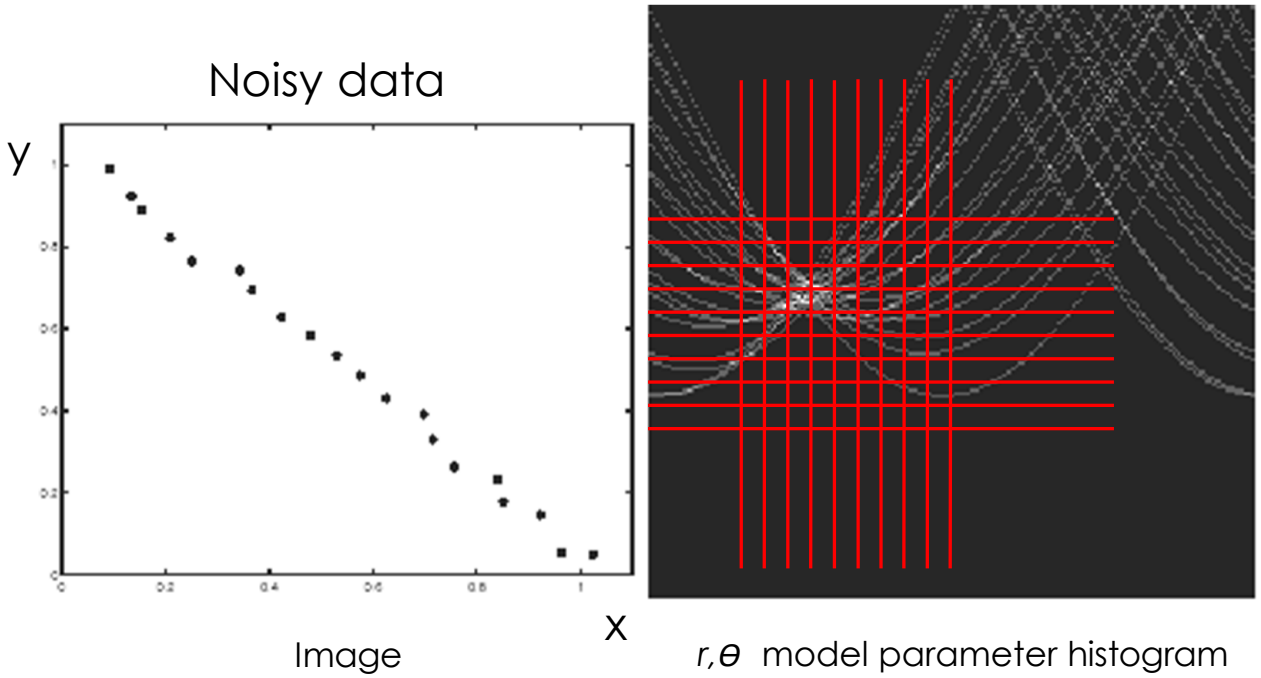



Image




*r*,*θ* model parameter histogram

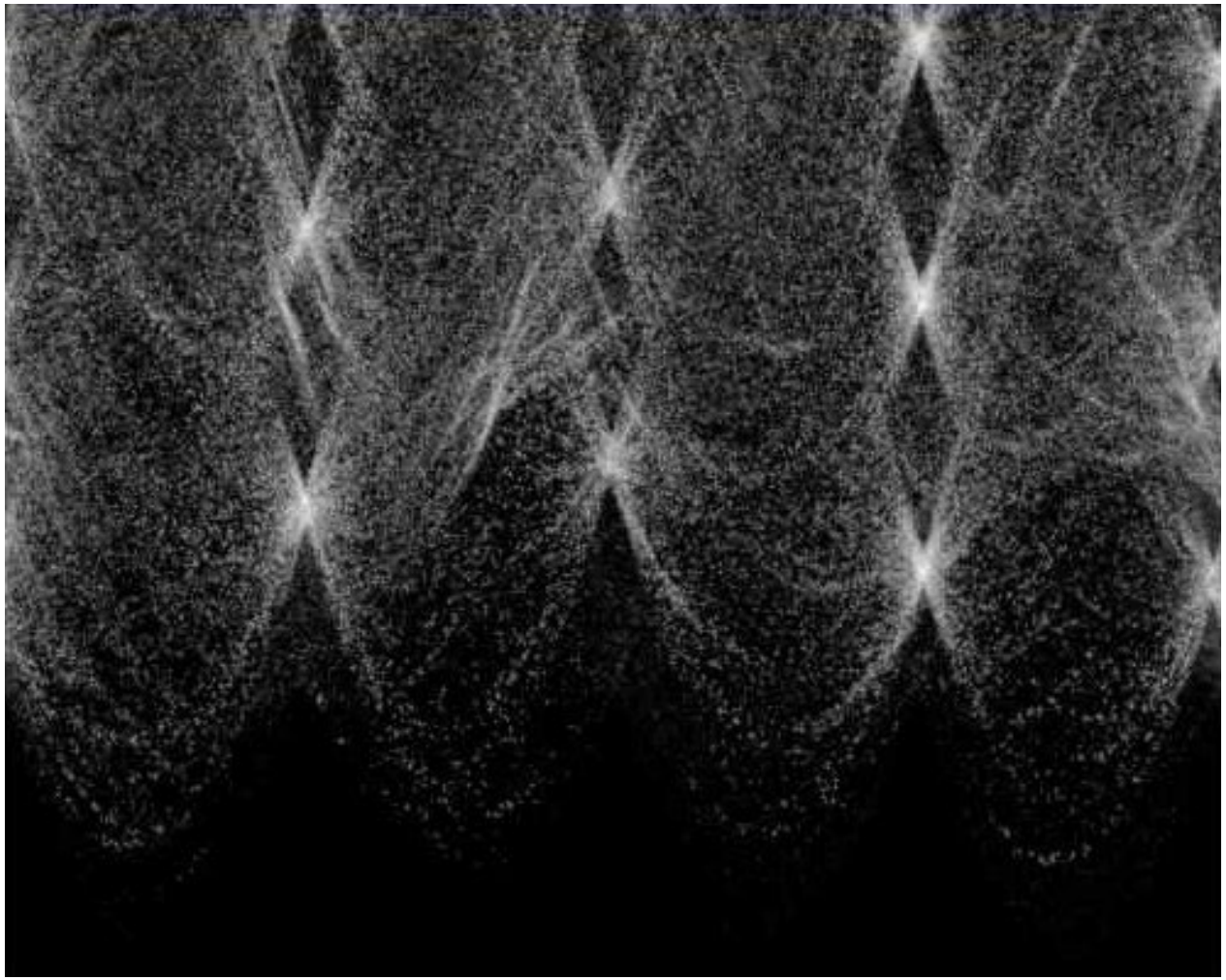
# Hough transform - experiment




Need to adjust grid size or smooth

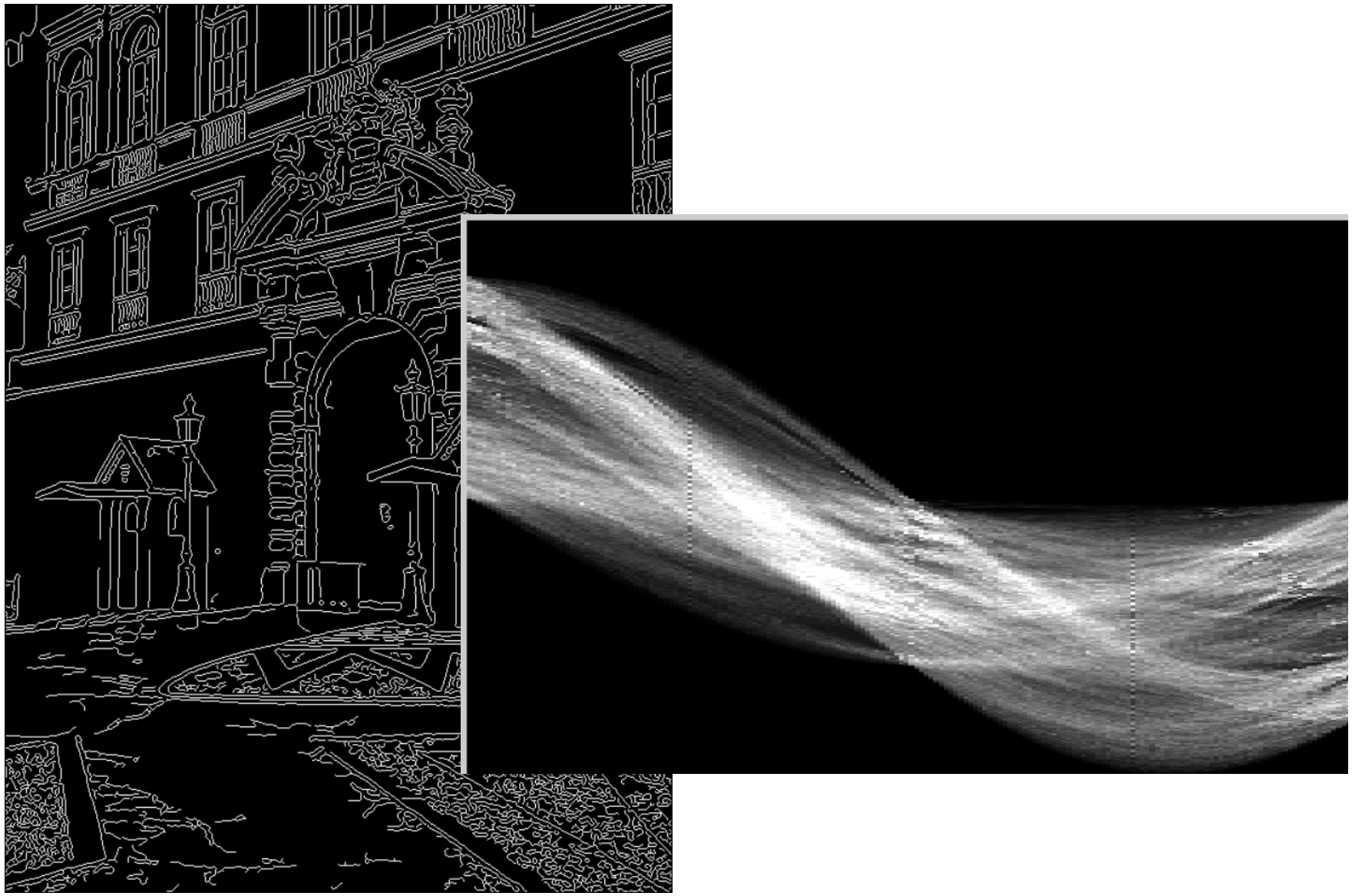
- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

# Hough transform example



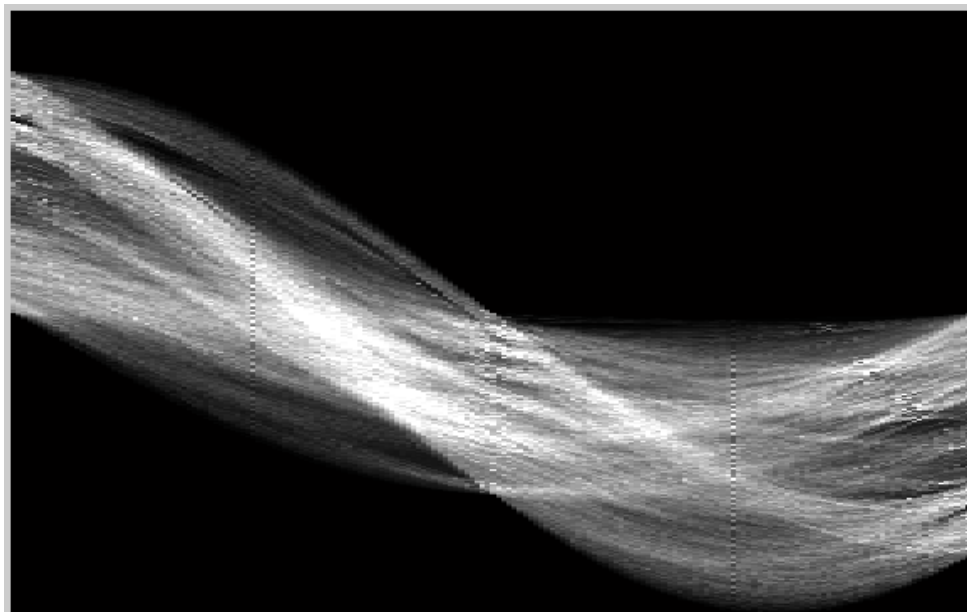
# Image → Canny

# Canny → Hough votes

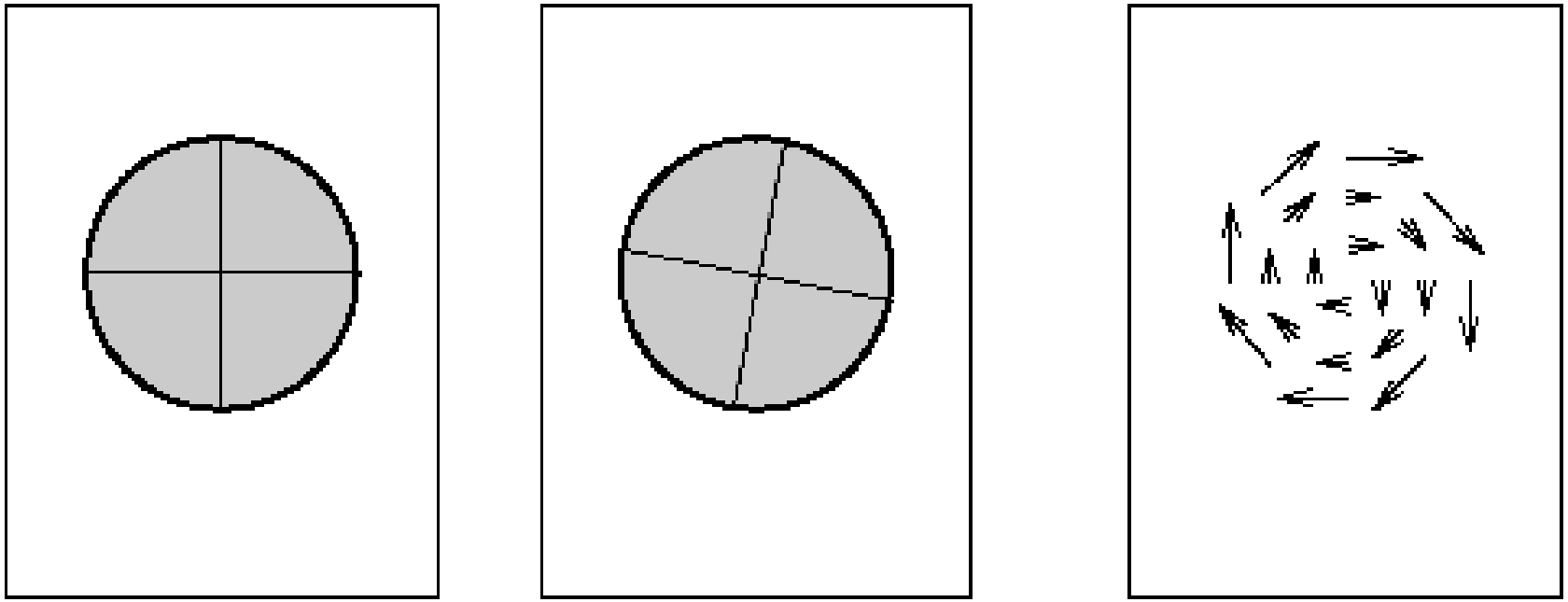


# Hough votes → Edges

Find peaks and post-process.



# Optical Flow



# Fundamental Equations of Computer Vision

1. Image Filtering $$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

2. Optical Flow $$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$

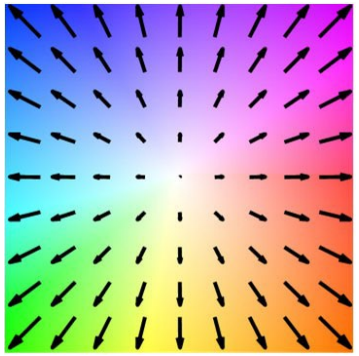
3. Camera Geometry $$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R}\ \mathbf{t}\end{bmatrix}\mathbf{X} \qquad x^T F x' = 0$$

4. Machine Learning $$\arg\min_{\mathbf{S}} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \|\mathbf{x} - \boldsymbol{\mu}_i\|^2. \qquad y = \varphi(\sum_{i=1}^{n} w_i x_i + b) = \varphi(\mathbf{w}^T \mathbf{x} + b)$$

# Optical Flow: Dense Correspondence Over Time

Input [Liu *et al.* CVPR'08]

Optical flow (2D motion vector)



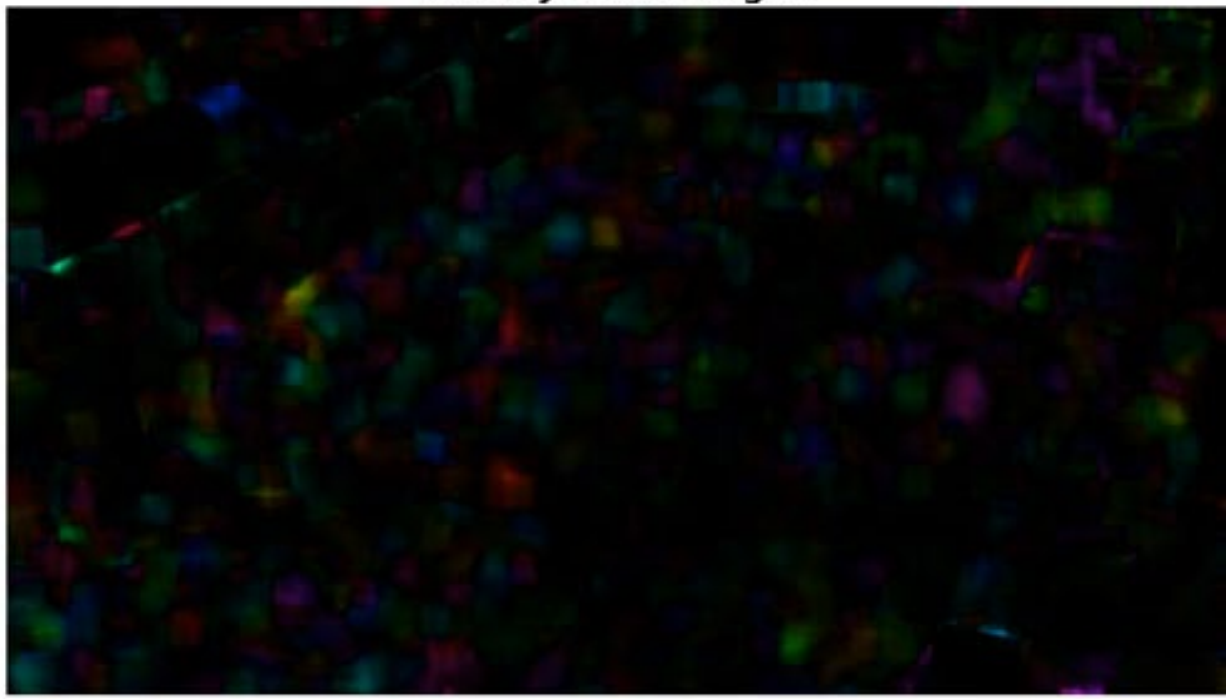
Color key
[Baker *et al.* IJCV'11]

Sandipan Dey (UMBC)

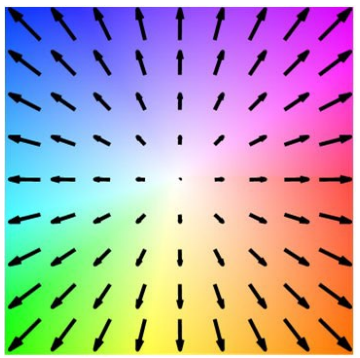
# Optical Flow to find the Walk Direction of people in a Video with OpenCV-python

input

Velocity Vector Angles



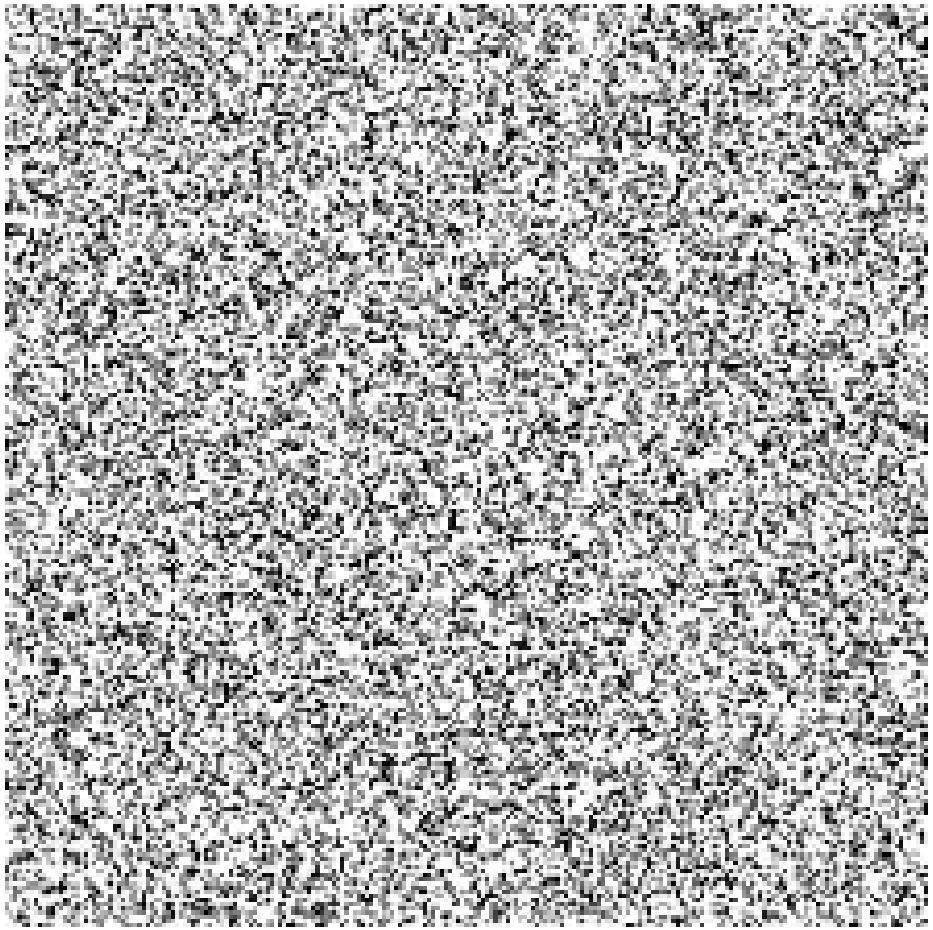
https://www.youtube.com/watch?v=e_TeY6QRp4c



Color key

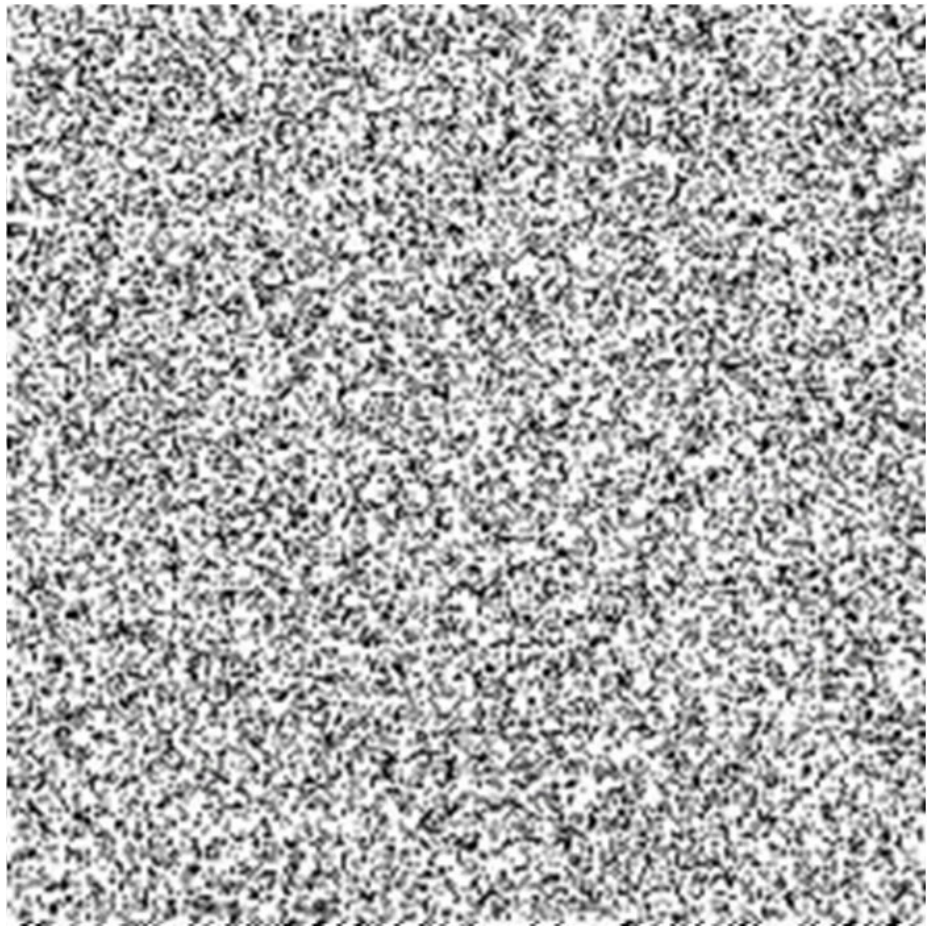
# Correspondence and perceptual organization

Sometimes correspondence/motion is the only cue



# Correspondence and perceptual organization

Sometimes correspondence/motion is the only cue

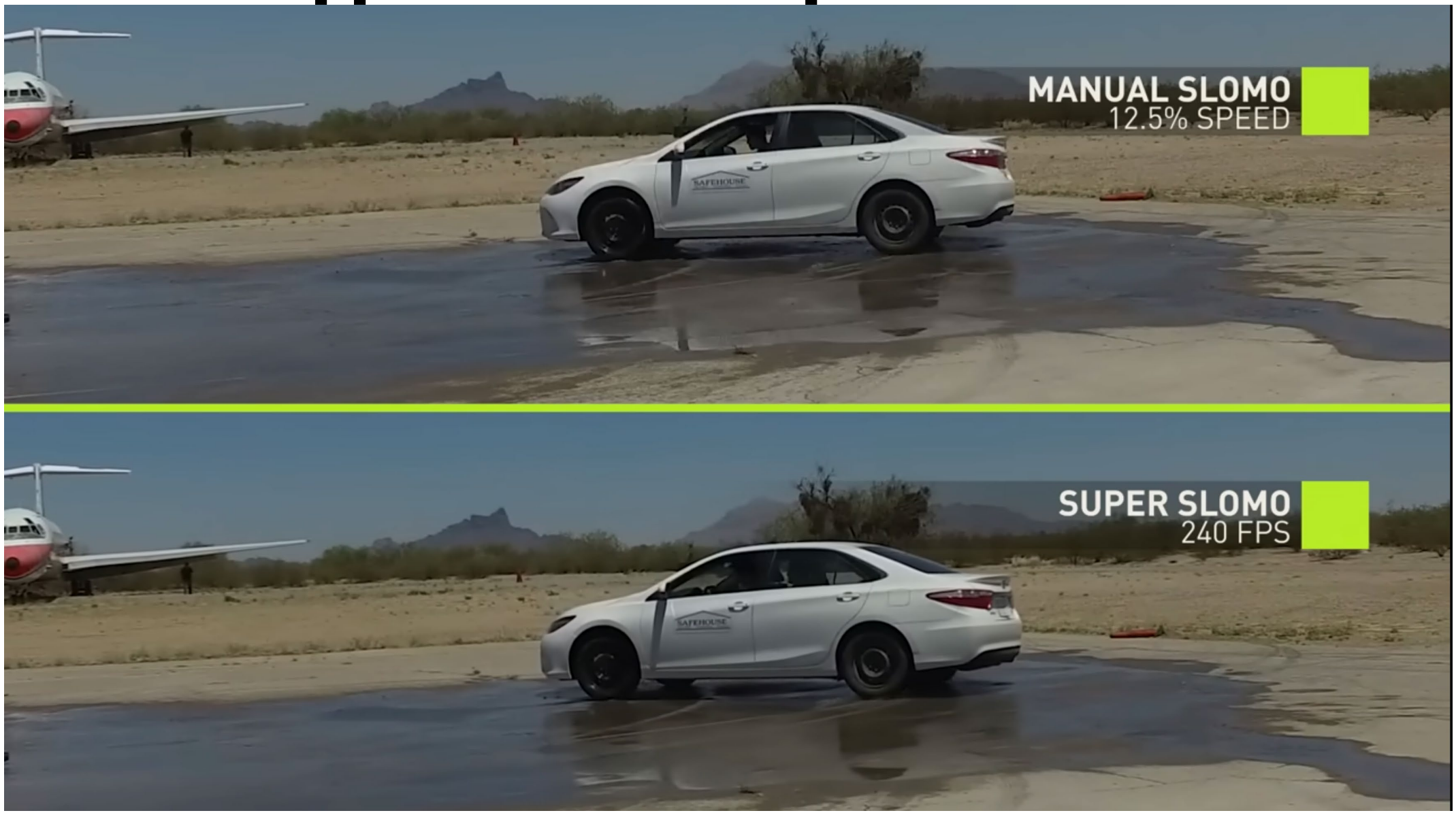


https://www.youtube.com/watch?v=mNrqvcS0oI0

# Applications of optical flow


Video Superresolution
[Liu & Sun CVPR 2011, TPAMI 2014]

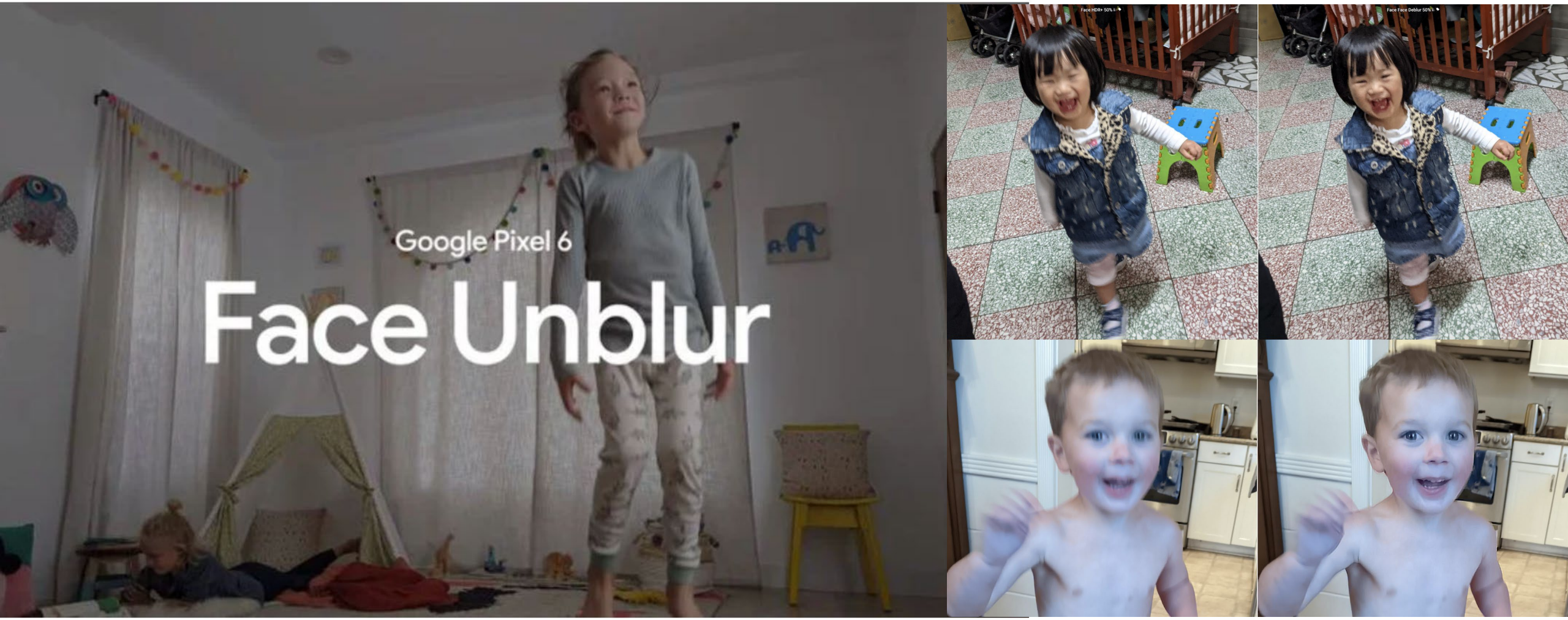
# Applications of optical flow



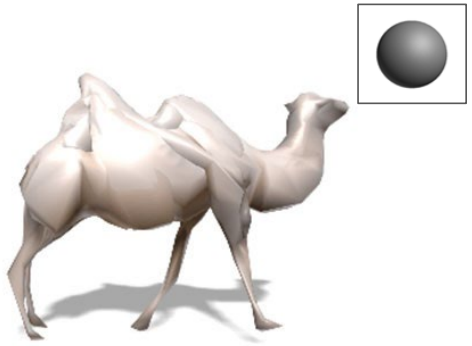

https://youtu.be/MjViy6kyiqs?si=zXenEX6O6VJkEaQF

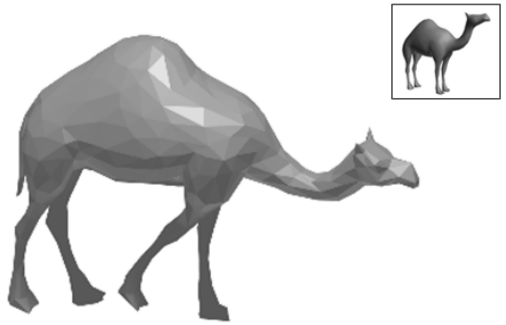
Super SloMo [Jiang, Sun, *et al.* CVPR 2018 Spotlight] Incorporated into **NVIDIA NGX** SDK for the Turing GPU.

# Face Unblur for Pixel 6



# Articulated shape reconstruction from a monocular video



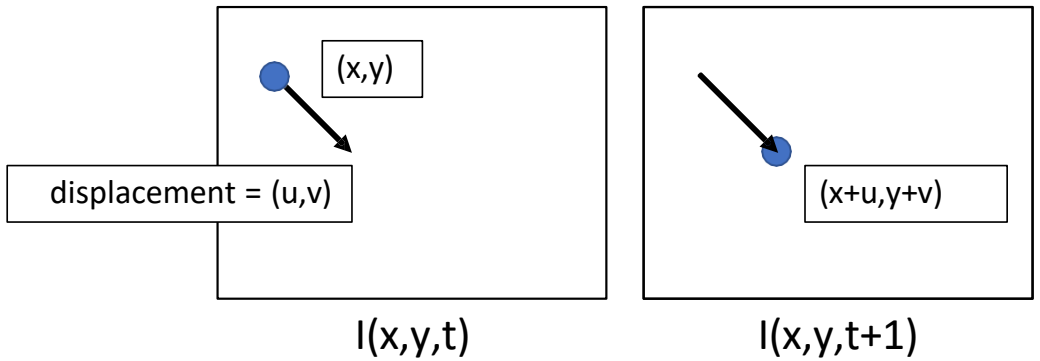


A-CSM: Kulkarni *et al.* CVPR 2020

VIBE: Kocabas *et al.* CVPR 2020

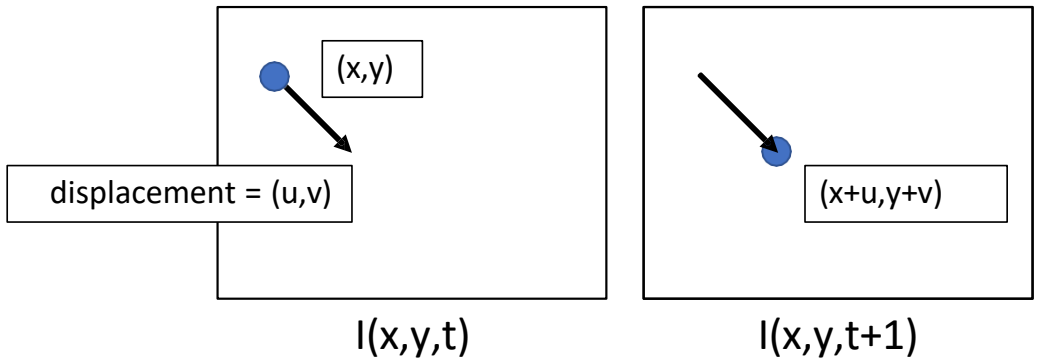
**Challenge**: Solving non-rigid 3D shape from 2D measurements without template or category prior is highly *under-constrained*

# Optical Flow




Brightness constancy: $I(x, y, t) = I(x + u, y + v, t + 1)$

# Optical Flow




Brightness constancy: $I(x, y, t) = I(x + u, y + v, t + 1)$

Recall Taylor Expansion: $I(x + u, y + v, t + 1) = I(x, y, t) + I_x u + I_y v + I_t \ldots$

# Optical Flow Equation

$$
\begin{aligned}
& I(x+u, y+v, t+1) = I(x,y,t) \\
0 &= I(x+u, y+v, t+1) - I(x,y,t) \\
&\approx I(x,y,t) + I_t + I_x u + I_y v - I(x,y,t) \\
&= I_t + I_x u + I_y v \\
&= I_t + \nabla I \cdot [u, v]
\end{aligned}
$$

Taylor Expansion

When is this approximation **bad**?

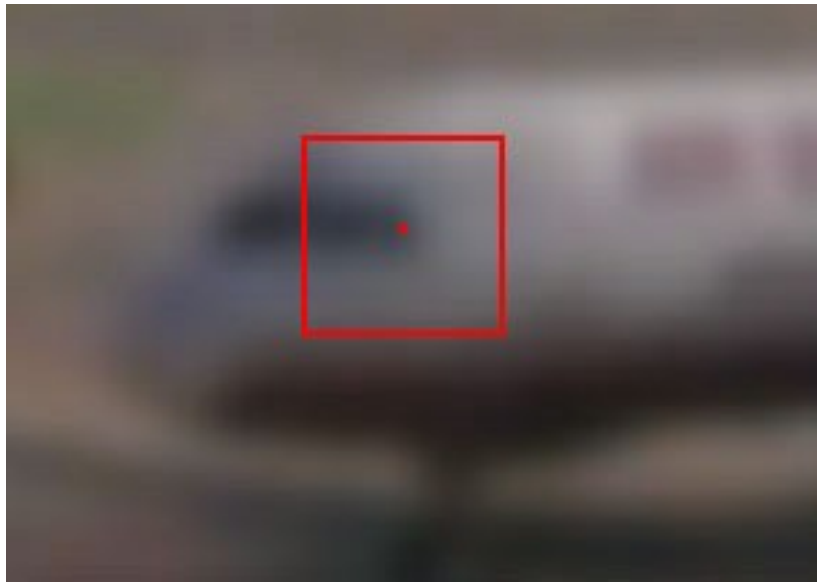
**u or v big**

# Optical Flow Equation

Brightness constancy equation

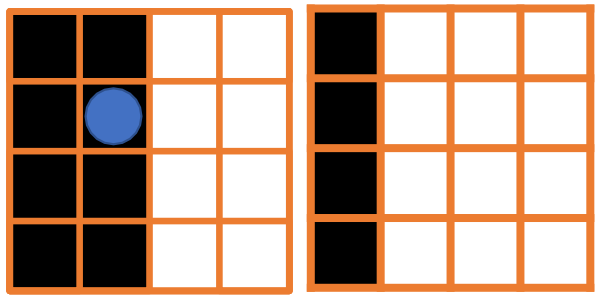
$$I_x u + I_y v + I_t = 0$$

What do static image gradients have to do with motion estimation?



# Brightness Constancy Example

$$I_x u + I_y v + I_t = 0$$



t t+1

It = 1-0 = 1

@  Iy = 0

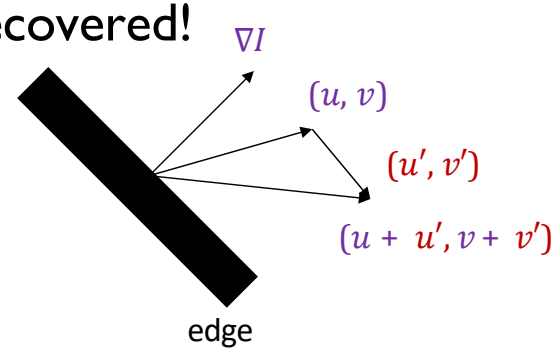
Ix = 1-0 = 1

**What's u? What's v?**

# The Brightness Constancy Constraint

$$I_x u + I_y v + I_t = 0$$

- Given the gradients $I_x$, $I_y$ and $I_t$, can we uniquely recover the motion $(u, v)$?
  - One equation, two unknowns
  - Suppose $(u, v)$ satisfies the constraint: $\nabla I \cdot (u, v) + I_t = 0$
  - Then $\nabla I \cdot (u + u', v + v') + I_t = 0$ for any $(u', v')$ s. t. $\nabla I \cdot (u', v') = 0$
  - Interpretation: the component of the flow perpendicular to the gradient (i.e., parallel to the edge) cannot be recovered!



# Solving Ambiguity – Lucas Kanade

- 2 unknowns [u,v], 1 eqn per pixel How do we get more equations?
- Assume **spatial coherence**: pixel's neighbors have move together / have same [u,v]
- 5x5 window gives 25 new equations

$$I_t + I_x u + I_y v = 0$$

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{25}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

# Solving for u,v

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ \vdots \\ I_t(p_{25}) \end{bmatrix} \qquad \underset{25x2}{\boldsymbol{A}} \; \underset{2x1}{\boldsymbol{d}} = \underset{25x1}{\boldsymbol{b}}$$

**What's the solution?**

$$(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})\boldsymbol{d} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} \quad \rightarrow \quad \boldsymbol{d} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b}$$

Intuitively, need to solve (sum over pixels in window)

$$\underbrace{\begin{bmatrix} \sum \boldsymbol{I_x I_x} & \sum \boldsymbol{I_x I_y} \\ \sum \boldsymbol{I_x I_y} & \sum \boldsymbol{I_y I_y} \end{bmatrix}}_{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum \boldsymbol{I_x I_t} \\ \sum \boldsymbol{I_y I_t} \end{bmatrix}}_{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b}}$$

# Challenges for traditional methods

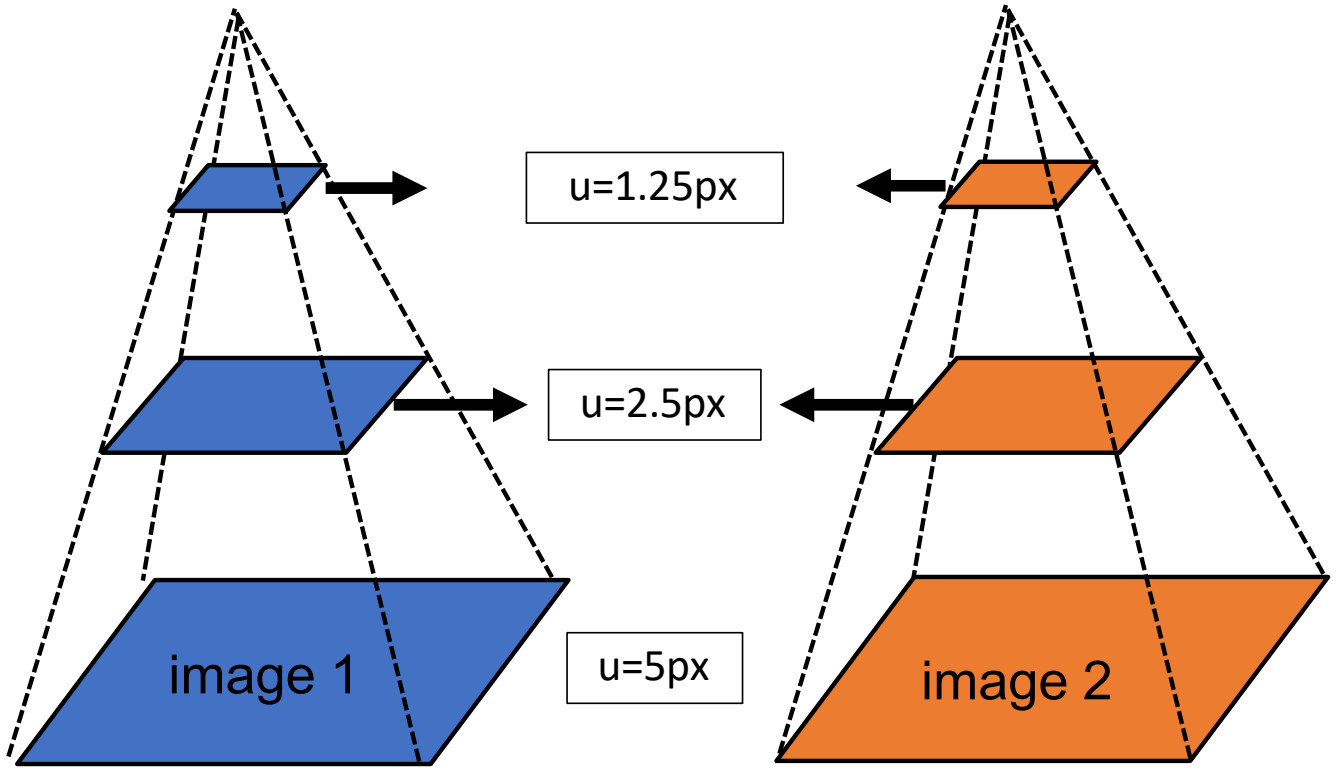
Large motion, motion blur

Textureless regions

Occlusions

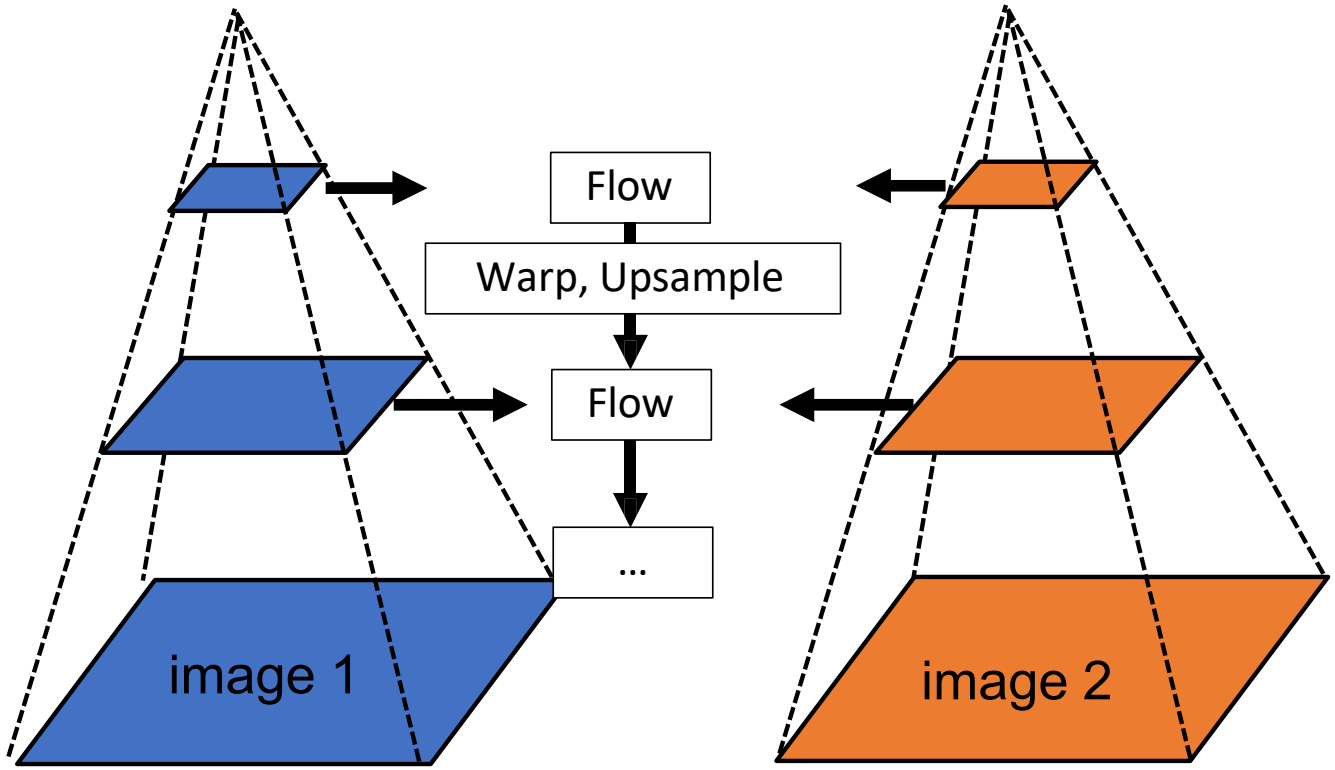
Lighting changes, noise …

# Coarse-to-fine optical flow estimation

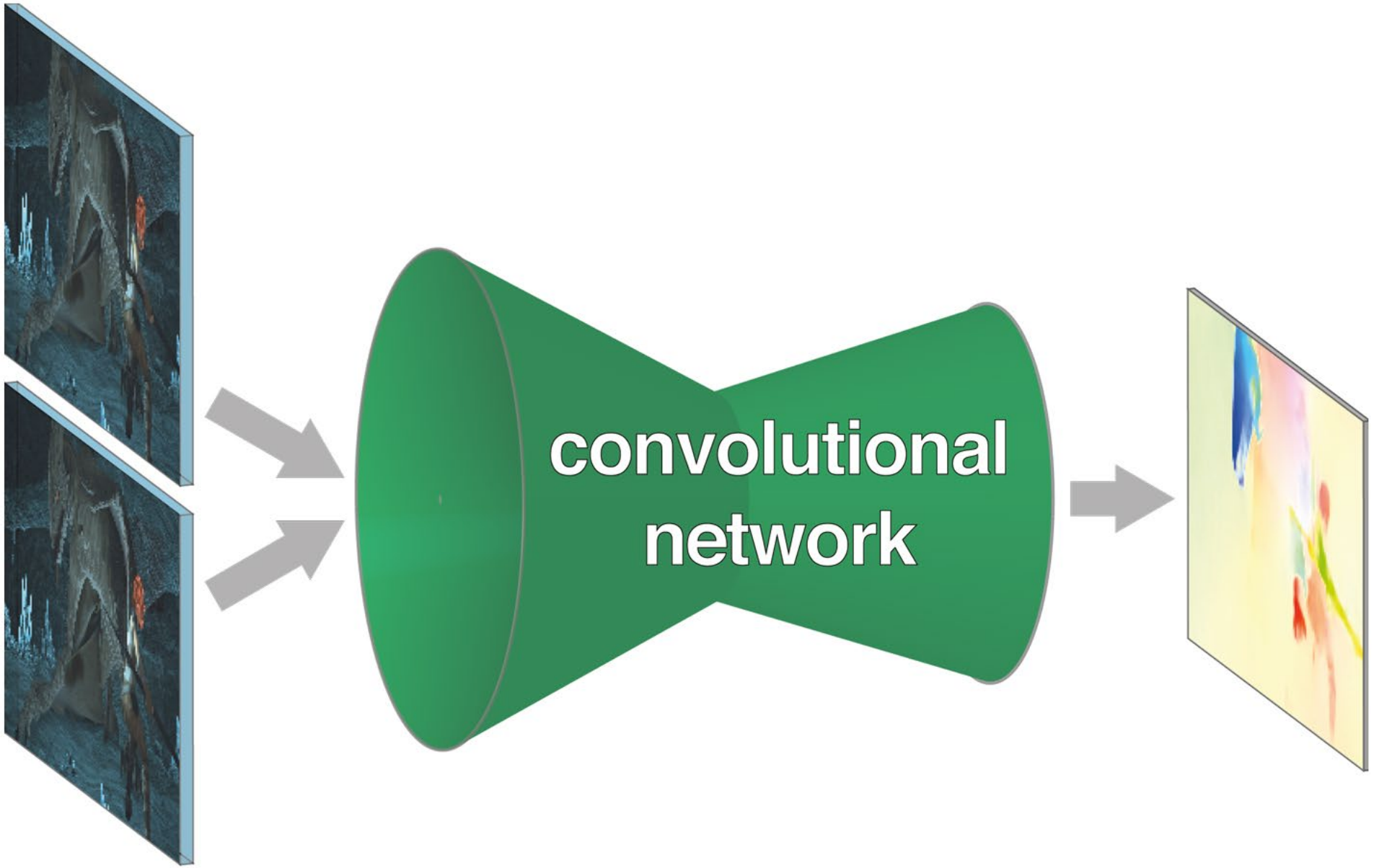



Typically called Gaussian Pyramid

# Coarse-to-fine optical flow estimation



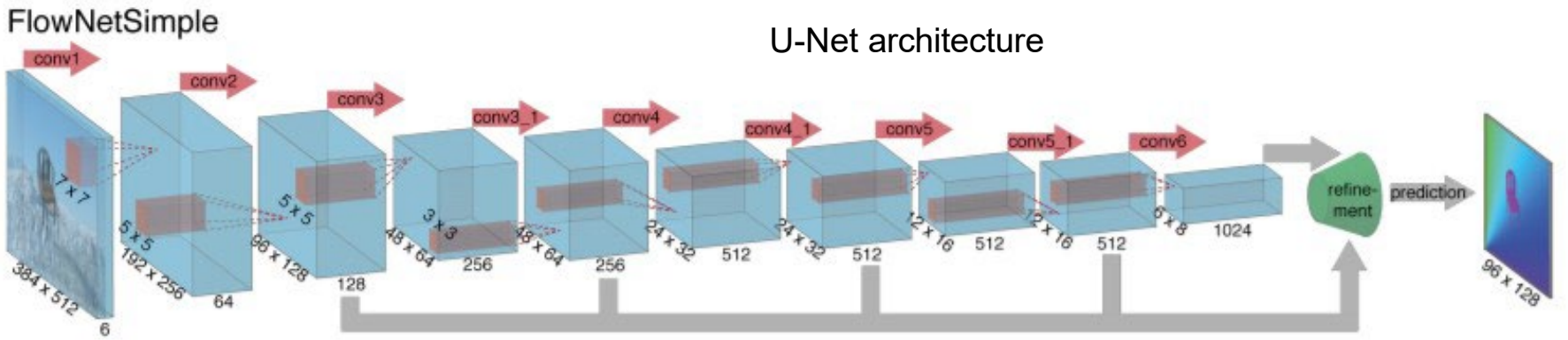
# Typical DL Pipeline for Optical Flow

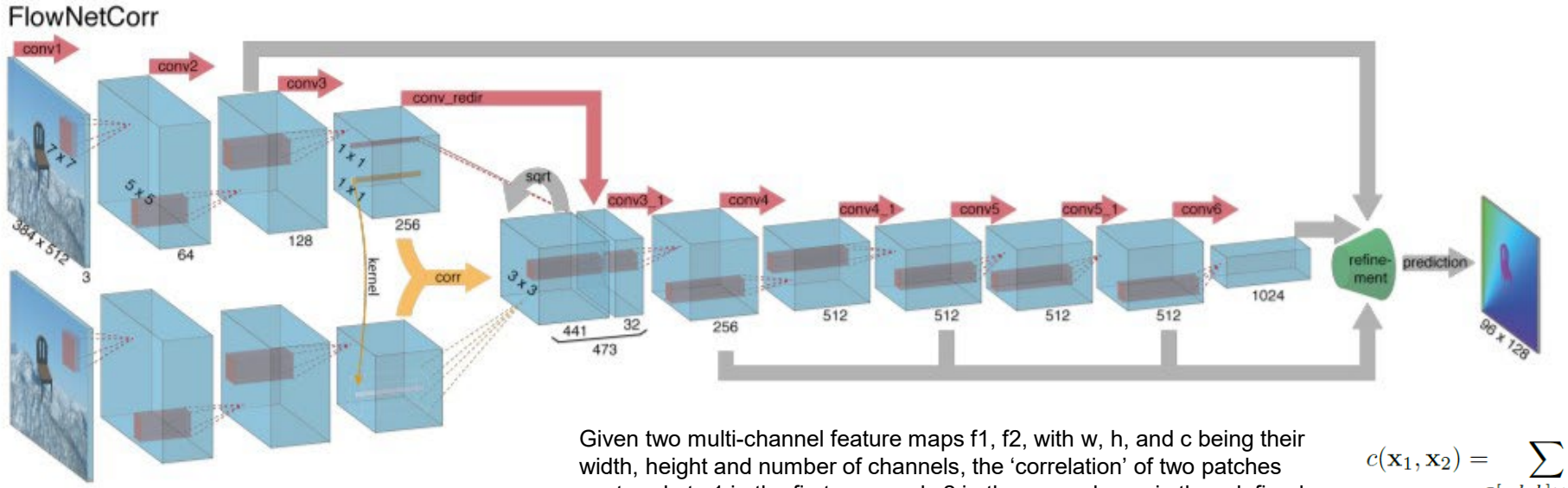


# FlowNetS: mapping from images to flow

[Dosovitskiy *et al.* ICCV'15]

U-Net architecture

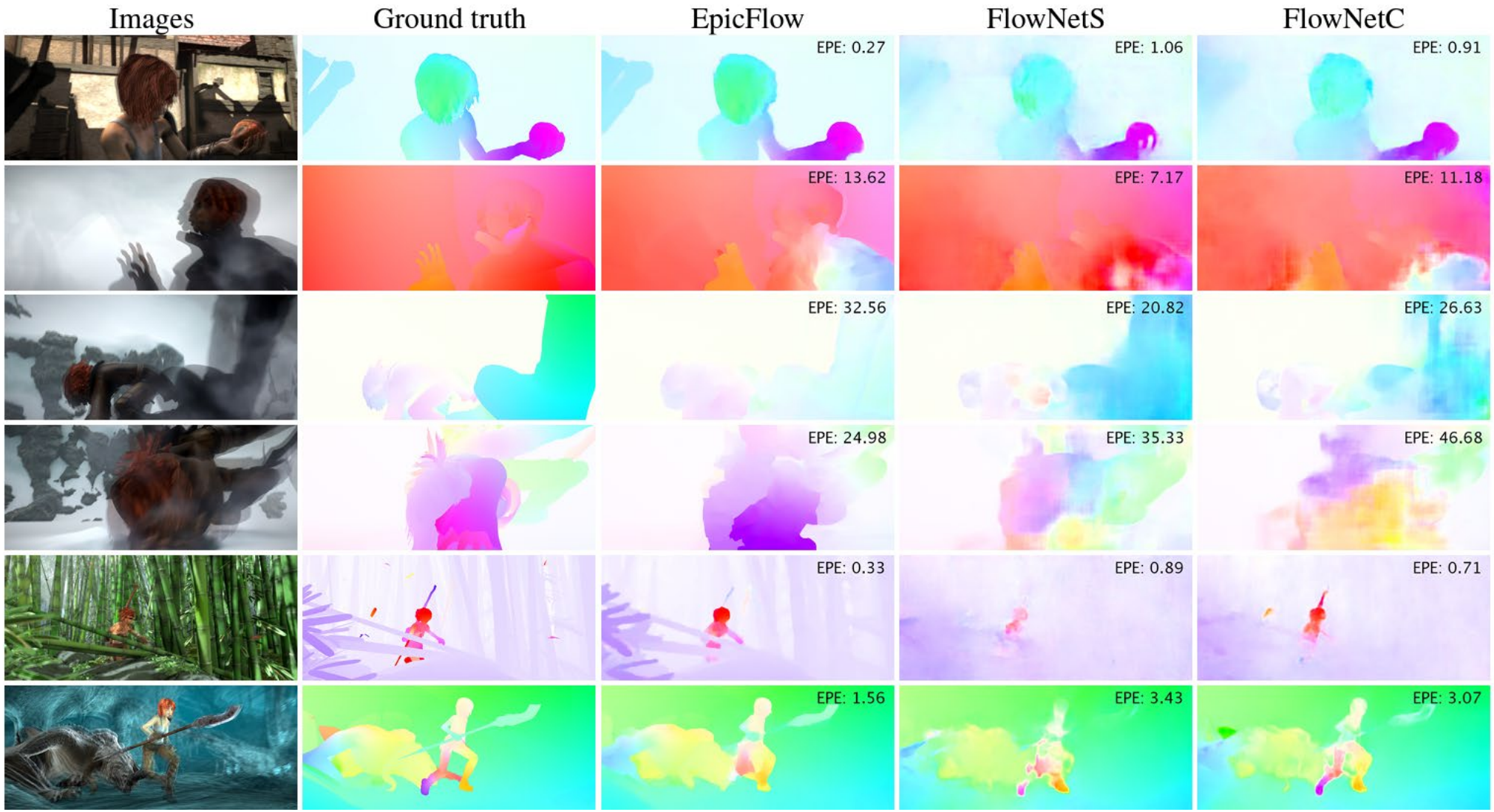



Given two multi-channel feature maps f1, f2, with w, h, and c being their width, height and number of channels, the 'correlation' of two patches centered at x1 in the first map and x2 in the second map is then defined as:

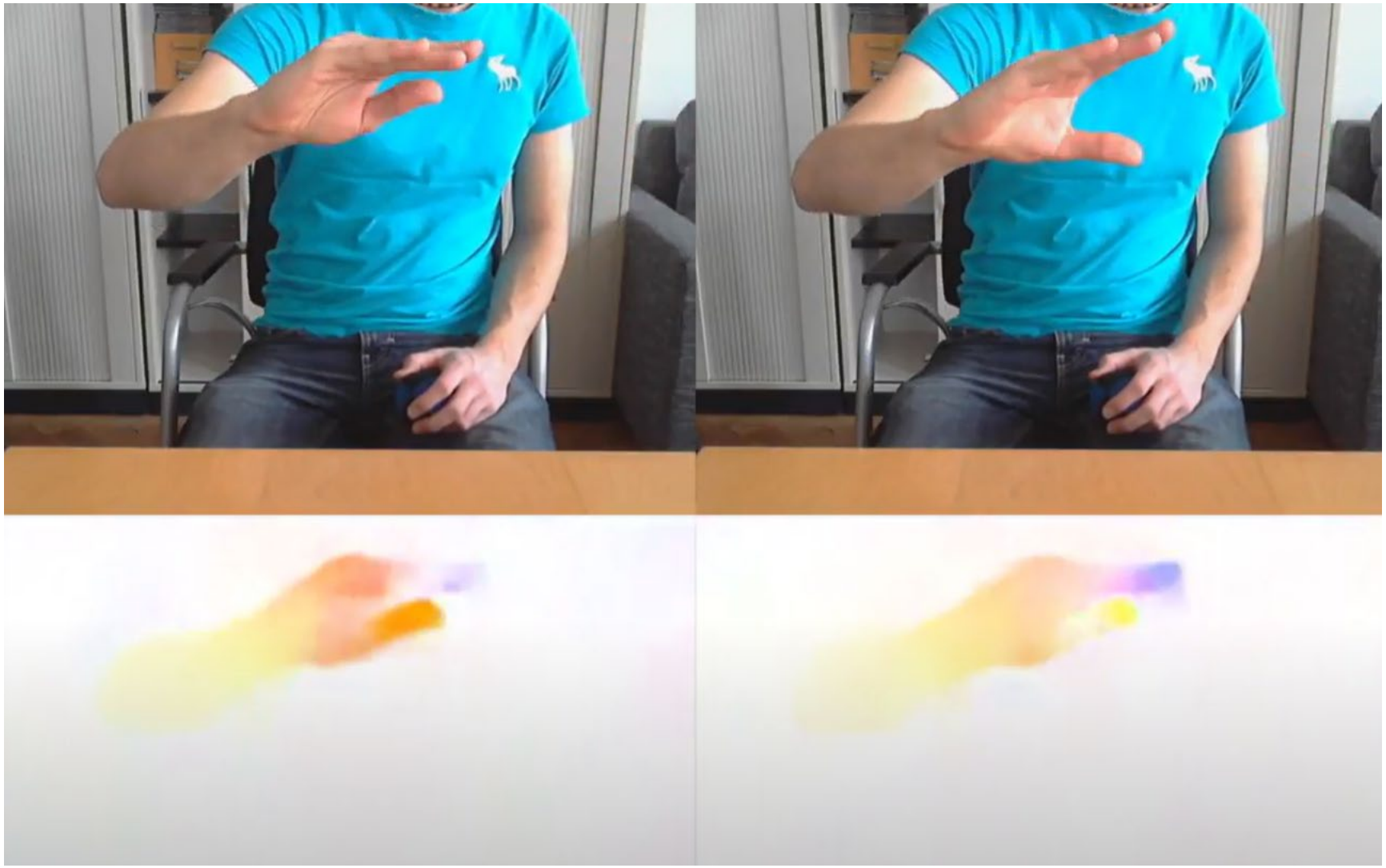
$$c(\mathbf{x}_1, \mathbf{x}_2) = \sum_{\mathbf{o} \in [-k,k] \times [-k,k]} \langle \mathbf{f}_1(\mathbf{x}_1 + \mathbf{o}), \mathbf{f}_2(\mathbf{x}_2 + \mathbf{o}) \rangle$$

# FlowNetS: mapping from images to flow




**EPE**: endpoint error

# FlowNetS: mapping from images to flow



https://youtu.be/k_wkDLJ8lJE?si=wJj1RePbw-NA75lU