Math 5001: Introduction to Numerical Analysis
Homework assignment 2

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Show all relevant work in detail to justify your conclusions. Partial credit depends upon the work you show. For each numerical experiment, all the .m files of your Matlab code should be electronically submitted to hex@mst.edu together with a .txt file which copies all the information in the Matlab command window when you run the code to obtain the numerical results.

Problem #1: Using Newton's method with a special function $f(x)$, derive the iterative scheme

$$x_{k+1} = x_k(2 - Rx_k)$$

to calculate the reciprocal of a number R.

Problem #2: Describe the scheme in detail for Newton's method to compute the value of x that satisfies the equation $\int_0^x e^{t^2}\ dt = 1$.

Problem #3: Program the Newton's method $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ in Matlab. Consider $f(x) = arctan(x)$ which has a unique zero $z = 0$. Choose all the tolerances to be 10^{-10}. Starting with

$$x_0 = 0.5, 1, 1.3, 1.4, 1.35, 1.375, 1.3875, 1.39375, 1.390625, 1.3921875,$$

iterate until one of the tolerance is satisfied or 20 iterations are completed. Output the numerical solution at each iteration step. What behavior do you observe?

Problem #4: Program the secant method $x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$ in Matlab. Consider $f(x) = arctan(x)$ which has a unique zero $z = 0$. Choose all the tolerances to be 10^{-10}. Starting with

$$(x_0, x_1) = (0.5, 1), (1, 1.3), (1.4, 1.5), (10, 11),$$

iterate until one of the tolerance is satisfied or 20 iterations are completed. Output the numerical results at each iteration step.

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