

Use of Bayesian Estimates to determine the Volatility Parameter Input in the Black-Scholes and Binomial Option Pricing Models

Shu Wing Ho^a, Alan Lee^b, Alastair Marsden^c

a The University of Auckland, Department of Statistics, Auckland, New Zealand
Email: shuwing.ho@gmail.com

b The University of Auckland, Department of Statistics, Auckland, New Zealand
Email: aj.lee@auckland.ac.nz

^cCorresponding author: The University of Auckland, Department of Accounting and Finance,
Auckland, New Zealand
Email: a.marsden@auckland.ac.nz, ph: (64) (9) 3737-599 Ext: 88564

ABSTRACT

The valuation of options and many other derivative instruments requires an estimation of ex-ante or forward looking volatility. This paper adopts a Bayesian approach to estimate stock price volatility. We find evidence that overall Bayesian volatility estimates more closely approximate the implied volatility of stocks derived from traded call and put options prices compared to historical volatility estimates sourced from IVolatility.com (“IVolatility”). Our evidence suggests use of the Bayesian approach to estimate volatility can provide a more accurate measure of ex-ante stock price volatility and will be useful in the pricing of derivative securities where the implied stock price volatility cannot be observed.

KEY WORDS: Option pricing, volatility estimate, Bayesian statistics

JEL Classification: C11, G13

1. INTRODUCTION

Options pricing and management of derivative positions are an important area of finance. The derivatives market is now also very large. According to recent estimates by BIS Quarterly Review (2011) as at June 2011, the notational amount of outstanding option OTC derivative contracts was 78.8 trillion US dollars. As such, options and other derivative instruments are traded extensively on many global exchanges by hedgers, speculators and arbitrageurs. The derivative markets play a key role in the transfer of risk between these different parties.

The development of the Black-Scholes option pricing model and its equivalent binomial option pricing model represented a major breakthrough in the pricing of corporate stock options (Black and Scholes, 1973; Merton, 1973; and Cox, Ross and Rubenstein, 1979). Since the development of the Black-Scholes and binomial models there have been numerous refinements to the models to account for stock dividends (Merton, 1973), American style options (Barone-Adesi and Whaley, 1986, 1987), foreign exchange options (Garman and Kohlhagen, 1983) and other more exotic options (Hull, 2011). The Black-Scholes option pricing model assumes prices follow a geometric Brownian motion and volatility is constant. A number of authors have also attempted to modify the Black-Scholes and binomial option pricing models to allow for stochastic and time varying volatility (Cox and Ross, 1976). Other adjustments to the Black-Scholes model include models where jumps are superimposed on a continuous return change (e.g. Merton, 1976).

A problem with any option pricing model is the estimation of the parameter inputs and in particular the ex-ante volatility. In this paper we examine if a Bayesian estimate of volatility can provide a better estimate of ex-ante stock price volatility compared to a simple historical volatility estimate, as an input into the Black-Scholes and binomial option pricing models. Specifically, we compare the difference between both Bayesian and historical volatility estimates to the underlying implied stock price volatility. Our implied volatility estimates are supplied by IVolatility and calculated from publicly traded call and put option prices using the binomial option pricing model.

Despite numerous variants to the Black-Scholes model and the binomial options pricing model, these models still remain extensively used in practice. For instance,

International Financial Reporting Standard No 2 notes that valuers may use the Black-Scholes/ binomial options pricing model, or variants thereof, to price employee stock options that must now be expensed in accordance with international financial reporting rules. Similarly, since December 2004, Statement of Accounting Concepts 123R issued by the Financial Accounting Standards Board, requires firms to recognise the value of employee stock options in their financial statements. In a study of the comparative inputs to estimate the fair value of employee stock options, Choudhary (2011) reported that 96% of her sample of US firms used the Black-Scholes model, 4% used the binomial options pricing model and 0.3% used an unspecified model. Similarly Balsam et al. (2007) reported that 86% of firms surveyed in their study used the Black-Scholes model. In practice, valuers and option traders that use the Black-Scholes or binomial option pricing models often adjust the standard models to account for a volatility smile in the pricing of stock options with different exercise prices (Hull, 2011).¹

In summary, despite its well-documented shortcomings, the use of the Black-Scholes and the simple binomial option pricing models to price options remains widespread. Therefore, as noted by Darsinos and Satchell (2007), it is of interest to investigate how the Black-Scholes and binomial models might be improved, while still retaining their essential simplicity. An obvious possibility for improvement is to refine the estimate of volatility, within the Black-Scholes assumption of a geometric Brownian motion for the underlying asset, or equivalently of a random walk for the log price. Although this assumption is not precisely satisfied, as already noted, the Black-Scholes and binomial model formula is regarded to be sufficiently robust to departures from this assumption to still be extensively used, despite the plethora of more sophisticated models that have been proposed.

In this paper, we provide a relatively simple method to estimate volatility using a Bayesian approach that can be applied in practice as an alternative to using historical volatility as a proxy for ex-ante volatility. A more reliable estimate of ex-ante volatility compared to historical stock price volatility may be useful to price options on stocks that have no existing options traded and where an implied volatility estimate cannot be observed.

¹ See Jackwerth and Rubinstein (1996) and Hull (2011) for evidence on volatility smiles for equity stock options.

As in Karolyi (1993) and Darsinos and Satchell (2007), we find that use of the Bayes estimate represents an improvement over the historical volatility. A feature of these two papers is the use of an inverse gamma prior for the volatility parameter. However, as we demonstrate in Section 3, the actual distribution of 30-day volatilities for the sample of stocks that we examine is much better modelled by a gamma distribution (or mixture of gammas) rather than an inverse gamma distribution, and use of this prior leads to a better estimate.

Our finding that Bayesian volatility estimates outperform historical volatility estimates in the determination of an implied volatility estimate will therefore be of interest to both practitioners and academics.

The remainder of this paper is structured as follows. Section 2 reviews the prior literature. Section 3 describes our data. Section 4 develops the mathematical framework to determine our Bayesian volatility estimates. Section 5 compares the differences in errors between the historical and our Bayesian option volatility estimates compared to the implied option volatility sourced from IVolatility. Section 6 concludes.

2. PRIOR RESEARCH

A Bayesian framework has previously been employed in finance literature by Vasicek (1973) to estimate beta in the capital asset pricing model and hence determine a company's cost of equity capital. Elton et al. (1978), Eubank and Zumwalt (1979), Blume (1975) and Klemkosky and Martin (1975) empirically show that a Bayesian estimate of beta in the capital asset pricing model provided a better estimate of beta than the traditional estimate of beta.

Several authors have also applied Bayesian methods to the estimation of stock price volatility in connection with pricing options, both under the assumption of geometric Brownian motion and under more complicated models for time-varying volatility. For example, Bauwens and Lubrano (2002) assume that the underlying asset follows a GARCH process and apply Bayesian methods to the calculation of option prices. Martin, Forbes and Martin (2005) and Flynn, Grose, Martin and Martin (2005) also assume a GARCH model for the underlying asset, and derive the form of the risk-neutral probability density $q(\theta)$. They

use observed option prices to estimate the posterior distribution of the parameters θ . Forbes, Martin and Wright (2007) develop a joint model for the asset price and the option price, again using observed option prices. Jacquier and Jarrow (2000) use regression models linking the option price to the asset price. In work more in the spirit of ours, Karolyi (1993) and Darsinos and Satchell (2007) assume a random walk model for the asset price and present a Bayesian analysis, assuming a normal model for the log-returns and an inverse gamma prior on the volatility.

A number of studies have examined if implied or historical volatility estimates provide a better forecast estimate of future realised volatility. Early studies on Chicago Board Options Exchange data for stocks by Latane and Rendleman (1976), Chiras and Manaster (1978) and Beckers (1981) concluded that implied volatility estimates provided a superior estimate of future realised volatility compared to the use of a simple historical volatility measure. However, contrary conclusions were reached by Canina and Figlewski (1993) who reported no strong correlation between implied and actual future volatility.

The mixed conclusions of these early studies were critiqued by Jorion (1995), who noted these earlier results may reflect either flawed test procedures or could be the result of inefficient option markets. Using options on futures markets, Jorion (1995) showed that the implied volatility estimate was efficient but a biased forecast of the volatility achieved in the future. In a further study, Christensen and Prabhala (1998) found that implied volatility forecasts were better than historical volatility forecasts in predicting actual future volatility using options on the S&P 100 index option, over the period 1983 to 1995 with tests based on non-overlapping series. This was to partly address concerns in prior studies that over-lapping data sets suffer from serial correlation. More recent evidence also shows that implied volatility estimates provide better estimates than historical volatility of the actual realised volatility in the future (Shu and Zhang, 2003; Szakmary et al., 2003). A study, by Li and Yang (2009), using Australian stock index data provides further evidence that both call and put implied volatilities derived from the Black-Scholes model were superior to the historical volatility in forecasting future realized volatility. Li and Yang (2009) found that the implied call volatility is close to an unbiased forecast of future volatility.

Lastly, a survey article of volatility forecasting research by Poon and Granger (2005) reports that option implied volatility models provide more accurate forecasts than time series models. Amongst the time series models, Poon and Granger (2005) suggest a possible order ranking with, first the historical volatility estimates, followed by more complex models that incorporate generalised autoregressive conditional heteroscedasticity and stochastic volatility.

In this paper, we use the implied volatility drawn from the Black-Scholes / binominal option pricing model formula in conjunction with the observed option price as the proxy for the market estimate of the ex-ante stock volatility. Our approach is to judge the performance of historical and Bayesian volatility estimates by its closeness to the implied volatility.

3. DATA

To test if Bayesian estimates of volatility provide an improved estimate of implied volatility compared to historical stock price volatility, we first collect data on daily adjusted stock prices for 8,461 US traded stocks drawn from CRSP² between June 1, 2007 and Dec 31, 2009. This enables us to compute the prior distribution of the abnormal return of all stocks.

From this sample of 7,084 stocks we deleted stocks that did not trade each day over the prior 30 day trading period for each of 19 “end dates” in the period between August 16, 2007 and November 17, 2009. We also deleted a small number of stocks that represent outliers (see Section 4 below).³

From the remaining samples of between 5,462 and 6,031 stocks (see Table 1) we then randomly selected 500 stocks. In this random sample of 500 stocks, between 243 and 275 stocks had available data on the implied volatility on both call and put traded options sourced from IVolatility in all the periods between August 16, 2007 and November 17, 2009. IVolatility is a data service provider that specializes in equity options in the US and is widely used in the industry.⁴ IVolatility calculates the implied volatility using the option prices for

² CRSP stands for Center for Research in Security Prices and is a provider for historical stock data.

³ Poon and Granger (2005) note that outliers can have a big impact on volatility estimation and suggest outliers might be separately examined with the use of a crisis model or using extreme value theories.

⁴ IVolatility and OptionMetrics are the two most widely used databases for options as pointed out in le Roux (2005).

the four nearest “by strike” or exercise price to the stock price. These prices are converted to an implied volatility measure using a binomial option pricing model and then averaged using a proprietary weighting technique that takes into account the delta and vega of each option.

In this study we focus on the 30 day standardized implied volatility provided by IVolatility for at or near-the-money call and put options. Christensen and Prabhala (1998), Flemming (1998) and Li (2002) provide evidence that implied volatility of options that are at-the-money have the best forecasting ability for future volatility even if the Black-Scholes model may not be a valid model to price options.

Table 1 summarises the CRSP and IVolatility sample sizes for each period having an ending date between August 16, 2007 and November 17, 2009.

Insert Table 1 about here.

Table 2 provides descriptive statistics of the historical annualized volatility⁵ and the IVolatility implied volatility for the 30 day trading period as at November 17, 2009. This date is the time period that we chose to determine if a gamma distribution provides a good fit to the conjugate prior of the squared volatility, in applying our Bayesian framework.

For the November 17, 2009 time period the mean (median) annualized 30-day historical volatility of 5,671 stocks in the sample (prior to the deletion of 10 outliers) was 46.0% (39.9%) p.a.. The distribution of the historical volatility estimates is wide, with maximum and minimum values of 272% and 0.3% respectively.

For the sample of 275 stocks where IVolatility data on the implied volatility was obtained, the mean (median) annualized historic volatility was 42.6% (40.5%), compared to the mean (median) volatility estimate of 47.5% (44.2%) for call options and the mean (median) volatility estimate of 47.9% (43.7%) for put options respectively. Also compared to the full sample of 5,671 stocks, the maximum and minimum values are less extreme. The maximum (minimum) historic volatility estimate for this sample of 275 stocks is 126.5%

⁵ See equation 5 for details of this calculation.

(8.5%) and the maximum (minimum) implied IVolatility estimate for call and put options is 131.9% (9.5%) and 129.8% (10.4%) respectively.

Insert Table 2 about here.

4. MATHEMATICAL FRAMEWORK

4.1 A Bayes estimate using a conjugate prior

To construct a Bayes estimate of volatility, we can follow Karolyi (1993) and Darsinos and Satchell (2007) and assume a conjugate prior for the volatility. Assuming the daily abnormal returns, conditional on the volatility are normally distributed with mean zero, the conjugate prior for the squared volatility is an inverse gamma distribution (this is equivalent to the reciprocal of the squared volatility having a gamma distribution).

The gamma density depends on two parameters r (the shape parameter) and λ (the scale parameter), and is given by:

$$f(x) = \frac{1}{\Gamma(r)\lambda^r} x^{r-1} e^{-x/\lambda}, x > 0 \quad (1)$$

The posterior distribution of the volatility is also of the inverse gamma form, and the posterior mean can be written as:

$$\gamma \hat{\sigma}^2 + (1 - \gamma) \times \text{prior mean} \quad (2)$$

where $\gamma = \frac{n}{n + 2r}$ and $\hat{\sigma}^2$ is the usual estimate of squared volatility when the mean is taken

to be zero, namely $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n x_i^2$. We call this estimate the “conjugate Bayes estimate”.

This formula is appealing, as it characterizes the Bayes estimate as the usual estimate pulled towards the prior mean. However, if we examine the actual volatilities of stocks, the inverse gamma does not appear to fit the volatility distribution well. Figure 1 shows a quantile-quantile plot of the fitted gamma distribution. The non-linear appearance of the plot indicates the lack of fit of the inverse gamma.

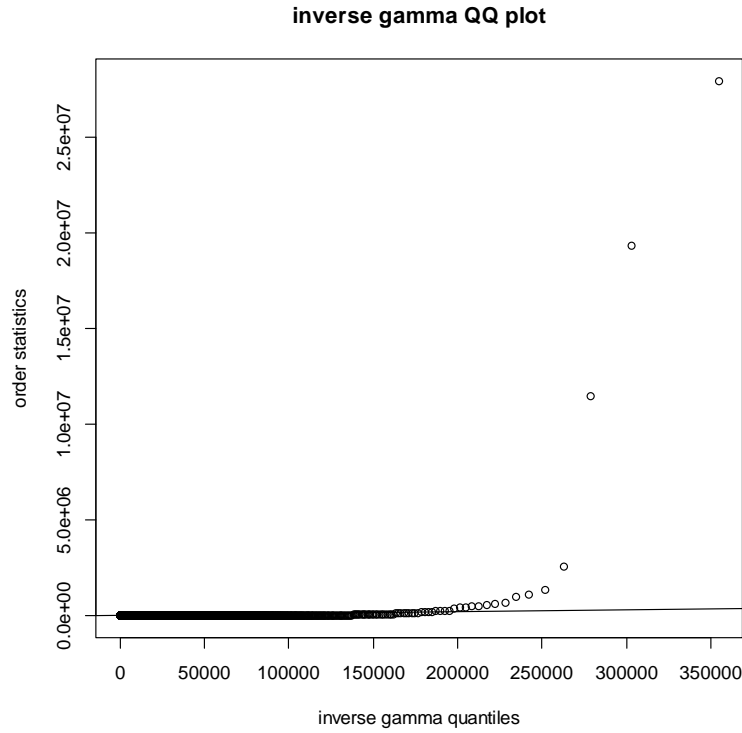


Figure 1 – Inverse gamma Q-Q plot of 30-day historical volatilities 5,671 stocks, period ending 16 Nov 2009.

4.2 An empirical Bayesian approach

To empirically apply a Bayesian approach we first calculated the annualized volatilities of the sample of 5,671 stocks, based on 30 trading days, beginning on November 17, 2009. A normal plot of the data revealed a right skewed distribution, suggesting a gamma distribution (rather than an inverse gamma) might fit this data well.

If the mean and variance of the volatilities are denoted by \bar{v} and $var(v)$ respectively, then method of moments estimates of r and λ are $\hat{r} = \bar{v}^2 / var(v)$ and $\hat{\lambda} = \widehat{var(v)} / \bar{v}$. We applied these formulas after removing the 10 most extreme values, obtaining $\hat{r} = 2.604585$ and $\hat{\lambda} = 0.01113419$. The fit can be improved by a more refined estimation technique for the gamma parameters, namely maximum likelihood. This involves maximizing the log-likelihood:

$$-n[\log \Gamma(r) + r \log \lambda - (r - 1) \sum_{i=1}^n \log v_i + \sum_{i=1}^n v_i / \lambda] \quad (3)$$

as a function of r and λ . The maximizing values (maximum likelihood estimates) are

$\hat{\tau} = 3.051479$ and $\hat{\lambda} = 0.009503573$. The quantile plot with these estimates is shown in Figure 2.

The plot represents a considerable improvement over the inverse-gamma fit, being much closer to the 45° line. A plot showing the fitted gamma density and a kernel density estimate of the volatilities is shown in Figure 3. The figure shows a good degree of fit between the estimates.

To construct a Bayes estimator for the volatility of a particular stock, we can assume a prior distribution for the volatility coinciding with the gamma distribution above, and use the posterior mean as our estimate.

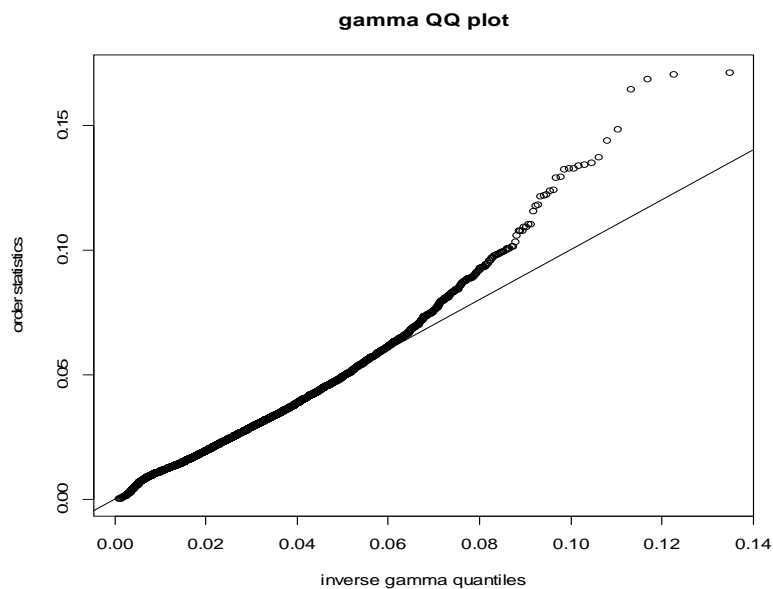


Figure 2 – Gamma Q-Q plot after removal of 10 outliers from stocks in Figure 1, parameters estimated by maximum likelihood.

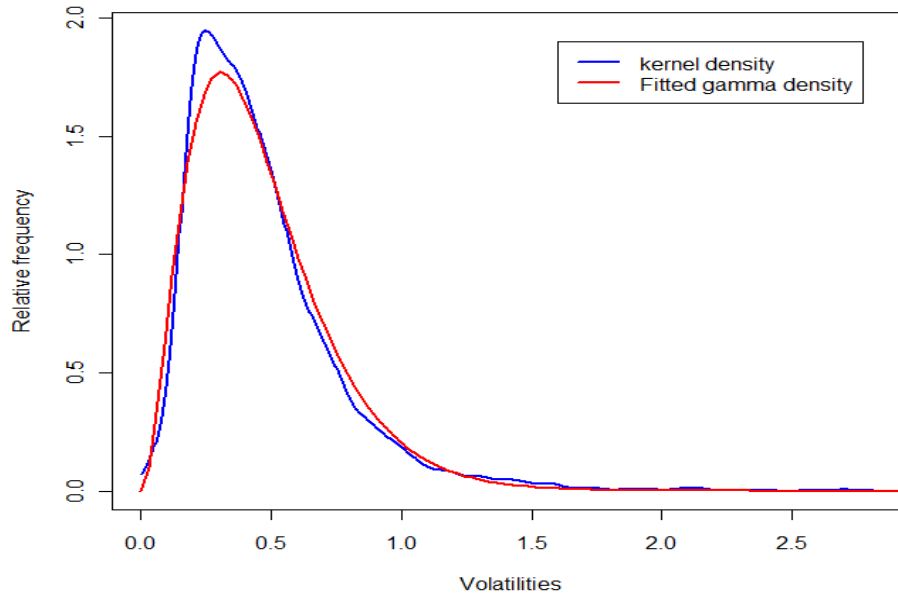


Figure 3. Fitted gamma density and kernel density estimator for volatilities in Figure 1, together with a kernel density estimator.

This results in the estimate

$$V^{(B)} = \frac{\int_0^\infty vg(v,n,r,\lambda,V)dv}{\int_0^\infty g(v,n,r,\lambda,V)dv} \quad (4)$$

where $g(v,n,r,\lambda,V) = v^{r-n-1}\exp(-\frac{v}{\lambda} - \frac{V^2}{2v^2})$, and V is the usual historic annualized volatility, given by:

$$V = \sqrt{\frac{252}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (5)$$

based on n daily returns x_1, \dots, x_n . In our calculations, n was 30, corresponding to 30 day options.

The integrals in the estimate can be evaluated using standard numeric integration. Applying this to our November 17, 2009 data set with 275 implied and historical volatilities, and letting $v_i^{(H)}$, $v_i^{(I)}$ and $v_i^{(B)}$ be respectively the historic volatility, the implied volatility and the Bayes estimate, we get:

$$\frac{1}{275} \sum_{i=1}^{275} (v_i^{(H)} - v_i^{(I)})^2 / v_i^{(I)2} = 0.0417 \quad (6)$$

$$\frac{1}{275} \sum_{i=1}^{275} (v_i^{(B)} - v_i^{(I)})^2 / v_i^{(I)2} = 0.0384 \quad (7)$$

This indicates that the Bayes estimate provides about an 8% improvement.

There is a hint of a mixture in distribution in the volatilities, suggestive of two types of stock with differing volatilities, one more volatile than the other. To further improve the fit of the gamma distribution, we can fit a mixture of two gamma distributions to the volatility data, having density

$$\phi(v) = \pi f(v, r_1, \lambda_1) + (1 - \pi) f(v, r_2, \lambda_2) \quad (8)$$

where f is given by (1), π is the mixing probability and $r_1, r_2, \lambda_1, \lambda_2$ are the shape and scale parameters. The maximum likelihood estimates of these parameters are obtained using the EM algorithm (Dempster, Laird and Rubin, 1977), and a “Mixture Bayesian estimate” may be calculated by the formula:

$$V^{(BI)} = \frac{\int_0^\infty v g(v) dv}{\int_0^\infty g(v) dv} \quad (9)$$

$$\text{where } g(v) = \phi(v) e^{-\frac{v^2}{2v^2}}. \quad (10)$$

The ability of these estimates (the historical volatility, and the three Bayesian estimates) to match the implied volatility is explored in the next section.

For each of the remaining periods (other than November 17, 2009) in the sample period between August 16, 2007 and November 17, 2009, we repeat the same procedure to recalculate the parameters under the gamma distribution to determine the three Bayesian volatility estimates.

4.3 Measurement of estimation error

For each of the three Bayesian estimates (those based on the conjugate prior, the gamma prior and the mixture prior respectively), we calculated a “Bayesian error” using the formula:

$$\text{Bayesian Error} = \left(\frac{\text{Bayesian Volatility estimate} - \text{IVolatility implied volatility}}{\text{IVolatility implied volatility}} \right)^2. \quad (11)$$

These errors were compared to the “historical error”

$$\text{Historical Error} = \left(\frac{\text{Historical Volatility estimate} - \text{IVolatility implied volatility}}{\text{IVolatility implied volatility}} \right)^2, \quad (12)$$

where the Bayesian volatility estimates are calculated in accordance with sections 4.1 and 4.2 above; the historical volatility is calculated in accordance with equation (5) and the IVolatility is the implied volatility supplied by IVolatility.

5. EMPIRICAL RESULTS

The errors calculated above all have extremely right-skew distributions. Accordingly we compare these distributions using medians and the Wilcoxon test, rather than means and the t-test. For each 30-day time period, we made three comparisons by comparing the “historical error” to each of the three “Bayesian errors”. This was done separately for call and put options, although there is not much difference between them. A negative value indicates that the Bayesian error is greater.

Insert Table 3 about here

For all time periods combined, the last lines of both Panels A and B of Table 3 indicate that the Bayesian estimate using the conjugate prior does not perform better than the historical estimate. In Panel A of Table 3 for call options, the difference in medians is small. The proportion of Bayesian errors using the conjugate prior that are less than the historical errors is 45%, so that the Bayesian estimate is significantly stochastically larger than the historical estimate ($p=0.0007$). For put options, Panel B of Table 3, the situation is similar, with no significant difference between the Bayesian approach using the conjugate prior and historical methods.

For the Bayesian estimate using the gamma prior, the situation is reversed. For call options, Panel A of Table 3, the difference in medians is now 0.00217, and the Bayesian estimate is significantly stochastically smaller, with 60% of the Bayesian errors being smaller ($p<0.0001$). The values for the put options, Panel B of Table 3, are 0.00233 and 62% respectively. The results for the mixture prior for both call and put options are very similar to those for the gamma prior.

Table 3, Panels A and B, also show the results for each time period separately. The separate-period analyses further demonstrate the relatively poor performance of the conjugate Bayes estimate. For call options (Table 3, Panel A), the median Bayesian error was less than the historic error in only 7 out of the 19 periods studied. Moreover, if we consider the proportion of stocks for which the Bayesian error was less than the historic error, we find that for only 7 out of 19 periods was this percentage greater than 50%. It seems that use of an inappropriate prior makes the Bayes estimate inferior to the historical volatility.

For call options the position is reversed when we consider the Gamma and mixture priors (Table 3, Panel A). For both of these, the median Bayesian error was less than the historic error in 12 out of the 19 periods studied, with all 12 differences being significant at the 5% level on a Wilcoxon test. For the 7 periods where the historical method was better, 6 were significantly better for the Gamma priors and 5 were significantly better for the mixture priors.

Turning to the proportion of stocks for which the Bayesian error was less than the historic error, we find that for both the gamma and mixture priors, 13 out of the 19 periods studied had the percentage of stocks greater than 50%. Thus, there is a significant benefit in using the Bayesian estimate based on gamma prior over the historic estimate. There does not seem to be much advantage in using the mixture prior, mainly because the mixture is typically dominated by a single component.

For put options (Table 3, Panel B), the situation is broadly similar. For the conjugate prior, 6 out of 19 periods have the median Bayesian error less than the historic, and only in 7 out of the 19 periods was the historical volatility percentage error greater than 50%. For the gamma prior, the results are 14/19 and 13/19 respectively, and for the mixture prior 14/19 and 14/19.

Overall the evidence suggests that the Bayesian volatility estimates based on the gamma and mixture priors provide a more accurate estimate of the implied volatility compared to the historical estimate of stock volatility.

Insert Table 4 about here

In our Bayesian estimates, we have assumed that the 30-day sequences of log-returns are normally distributed. We tested the sequences for normality at each time point, and typically found about 23% failed the Shapiro-Wilk normality test. The detailed results are shown in Table 4. This indicates that overall the estimates may be quite robust with respect to this assumption.

6. CONCLUSION

Options and derivative contracts are extensively traded on many exchanges and play an important role in the transfer of risk between hedgers and speculators. In pricing these instruments a key parameter input is an estimate of ex-ante volatility.

This paper investigates if Bayesian volatility estimates can provide a more accurate and reliable estimate of a stock's implied volatility compared to a historical volatility estimate. The implied volatility of the stock is computed by IVolatility using traded options price, for at or near-the-money call and put options, on a sample of US stocks over the period between August 16, 2007 and November 17, 2009. Prior research suggests that implied volatility estimates from option prices provide a more accurate estimate of actual realised volatility compared to historical estimates.

Overall our results provide evidence that Bayesian volatility estimates based on the Gamma and mixture priors may provide a better estimate of implied option volatility than the historical volatility estimate. A more reliable estimate of ex-ante volatility compared to historical stock price volatility can be useful to price options on stocks that have no existing options traded and where an implied volatility estimate cannot be observed. For example, many US and offshore companies will have employee stock option schemes but no publicly traded options. However, these employee stock options must now be valued and expensed in accordance with US and international accounting standards. One drawback from our Bayesian approach is that the conjugate prior is not used, and therefore the posterior has to be solved numerically. However, since this involves integration with only one variable, it can still be solved easily without resorting to simulations.

Areas for future research include exploring the accuracy of our Bayesian estimates to the implied volatility of options with different terms to maturity other than 30 days and for call and put options that are well out or well in-the-money. Other avenues of research include comparing the relationship between Bayesian and historical volatility estimates, using pricing models other than the Black-Scholes or binomial options pricing model or comparing Bayesian forecasts to forecast volatility estimates based on exogenous variables such as GDP change, interest rates and other macro-economic indicators.

REFERENCES

- Balsam, S., O’Keefe, S. and Weidemer, M., 2007, Frontline reaction to FASB 123-12, *Journal of Accounting*, 203, No.4, 54-56.
- Barone-Adesi, G. and Whaley, R.E., 1986, The valuation of American call options and the expected ex-dividend stock price decline, *Journal of Financial Economics*, 17, 58-91.
- Barone-Adesi, G. and Whaley, R.E., 1987, Efficient analytic approximation of American Option Values, *Journal of Finance*, June, 42, 301-320.
- Bauwens, L. and Lubrano, M., 2002, Bayesian option pricing using asymmetric GARCH models, *Journal of Empirical Finance*, 9, 321-342.
- Beckers, S., 1981, Standard deviation implied in option prices as predictors of future stock price variability, *Journal of Banking and Finance*, 5(3), 363-381.
- BIS Quarterly Review, 2011, Dec, http://www.bis.org/publ/qtrpdf/r_qsl1112.pdf
- Black, F. and Scholes, M., 1973, The valuation of options and corporate liabilities, *Journal of Political Economy*, 81, 637–654.
- Blume, M.E., 1975, Betas and their regression tendencies, *Journal of Finance*, 30, No. 3, 785-795.
- Canina, L. and Figlewski, S., 1993, The informational content of implied volatility, *Review of Financial Studies*, 6, 659-681.
- Chiras, D.P. and Manaster, S., 1978, The information content of prices and a test of market efficiency, *Journal of Financial Economics*, 6, 213-234.
- Choudhary, P., 2011, Evidence on differences between recognition and disclosure: A comparison of inputs to estimate fair value of employee stock options, *Journal of Accounting and Economics*, 51, 77-94.
- Christensen, B.J. and Prabhala, N.R., 1998, The relation between implied realized volatility, *Journal of Financial Economics*, 50, No. 2, November, 125-150.
- Cox, J.C. and Ross, S.A., 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics*, 3, 145-166.
- Cox, J.C., Ross, S.A. and Rubenstein, M., 1979, Option pricing: A simplified approach, *Journal of Financial Economics*, 7, 229-264.
- Darsinos, T. and Satchell, S., 2007, Bayesian analysis of the Black-Scholes option price, in Satchel, S (Ed), *Forecasting Expected Returns in the Financial Markets*, Academic Press, NY, 117-150.

Dempster, A.P., Laird, N.M. and Rubin, D.B., 1977, Maximum Likelihood from Incomplete Data via the EM Algorithm, *Journal of the Royal Statistical Society, Series B*, 39, 1-38.

Elton, E.J., Gruber, M.J. and Ulrich, T.J., 1978, Are Betas Best?, *Journal of Finance*, 33, No. 5 1375-1384.

Eubank, Jr. A. and Zumwalt, J.K., 1979, An Analysis of the Forecast Error Impact of Alternative Beta Adjustment Techniques and Risk Classes, *Journal of Finance*, 34, No. 3, 761-776.

Fleming, J., 1998, The quality of market volatility forecasts implied by S&P 100 Index option prices, *Journal of Empirical Finance*, 5, 317-345.

Flynn, D.B., Grose, S.D., Martin, G.M. and Martin, V.L., 2005, Pricing Australian S&P 200 options: A Bayesian approach based on general distributional forms, *Australian and New Zealand Journal of Statistics*, 47, 101-17.

Forbes, C.S., Martin, G.M. and Wright, J., 2007, Inference for a class of stochastic volatility models using option and spot prices: application of a bivariate Kalman filter, *Econometric Reviews*, 26, 387-418.

Garman, M.B. and Kohlhagen, S.W., 1983, Foreign currency option values, *Journal of International Money and Finance*, 2, 231-237.

Hull, J.C., 2011, *Options, Futures and Other Derivatives*, Eight Edition, Pearson Education Limited.

Jackwerth, J.C. and Rubinstein, M., 1996, Recovering probability distributions from option prices, *Journal of Finance*, 51, December, 1611-1631.

Jacquier, E. and Jarrow, R., 2000, Bayesian analysis of contingent claim model error, *Journal of Econometrics*, 94, 145-180.

Jorion, P., 1995, Predicting volatility in the foreign exchange market, *Journal of Finance*, 50, 507-528.

Karolyi, G.A., 1993, A Bayesian approach to modelling stock return volatility for option valuation. *Journal of Financial and Quantitative Analysis*, 28, 579-594.

Klemkosky, R.C., and Martin, J.D., 1975, The adjustment of beta forecasts, *Journal of Finance*, 30, No. 4, 1123-1128.

Latane, H. And Rendleman, R.J., 1976, Standard deviations of stock price ratios implied in option prices, *Journal of Finance*, 31, No. 2, 369-381.

le Roux, M., 2005, A long-term model of the dynamics of the S&P 500 implied volatility surface, *North American Actuarial Journal* 11, 4, 61-75.

Li, K., 2002, Long-memory versus option-implied volatility prediction, *Journal of Derivatives*, Vol. 9, No. 3, Spring, 9-25.

Li, S and Yang, Q., 2009, The relationship between implied and realized volatility: evidence from the Australian stock index option market, *Review of Quantitative Finance and Accounting* 32, 405-419.

Martin, G.M., Forbes, C.S. and Martin, V.L., 2005, Implicit Bayesian inference using option prices, *Journal of Time Series Analysis*, 26, 437-462.

Merton, R.C., 1973, Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4, Spring, 141-183.

Merton, R.C., 1976, Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics*, 3, 125-144.

Poon, S. and Granger, C., 2005, Practical issues in forecasting volatility, *Financial Analysts Journal*, 61, 45-56.

Shu, J., and Zhang, J.E., 2003, The relationship between implied volatility and realised volatility of S&P 500 index, *Technical Article 4, WILMOTT Magazine*, 83-91.

Szakmary, A., Ors, E., Kim, J.K. and Davidson, W.N., 2003, The predictive power of implied volatility: evidence from 35 futures markets, *Journal of Banking and Finance*, 27, 2151-2175.

Vasicek, O.A., 1973, A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas, *Journal of Finance*, 28, No. 5, 1233-1239.

Table 1

End Date	CRSP sample Size (prior to deletion of outliers)	Ivolatility sample size
August 16, 2007	6,031	243
October 1, 2007	5,912	249
November 13, 2007	5,942	255
December 28, 2007	5,961	252
February 13, 2008	6,018	253
March 31, 2008	5,899	250
May 13, 2008	5,816	259
June 26, 2008	5,889	258
August 11, 2008	5,751	263
September 24, 2008	5,739	256
November 6, 2008	5,750	247
December 22, 2008	5,637	248
February 24, 2009	5,462	245
April 8, 2009	5,557	256
May 22, 2009	5,557	266
July 8, 2009	5,573	267
August 20, 2009	5,577	270
October 5, 2009	5,674	272
November 17, 2009	5,671	275

CRSP and IVolatility sample sizes for each 30-day period.

Table 2

Sample	N	Minimum	First quartile	Mean	Median	Third quartile	Maximum
Historical volatility of entire sample on CRSP	5,671	0.0030	0.2614	0.4604	0.3985	0.5756	2.720
Historical volatility of stocks of interest	275	0.0850	0.2907	0.4258	0.4053	0.5205	1.2650
Implied volatility (call)	275	0.0954	0.3353	0.4750	0.4423	0.5756	1.3190
Implied volatility (put)	275	0.1042	0.3437	0.4785	0.4371	0.5744	1.2980

Descriptive statistics of historical volatility and implied volatility for the data from CRSP and IVolatility. The IVolatility data is 30 day implied volatility data collected for call and put options as at November 17, 2009.

Table 3

Panel A	Call Options								
	Conjugate			Gamma			Mixture		
End date	Difference of medians	Wilcoxon P-value	Historical percent greater	Difference of medians	Wilcoxon P-value	Historical percent greater	Difference of medians	Wilcoxon P-value	Historical percent greater
August 16, 2007	-0.00123	0.8988	46	0.00382	0.0000	63	0.00299	0.0000	62
October 1, 2007	-0.00032	0.0018	37	0.00450	0.0000	71	0.00460	0.0000	71
November 13, 2007	-0.00023	0.0264	38	-0.00001	0.0000	73	0.00053	0.0000	72
December 28, 2007	-0.00089	0.0855	50	0.00006	0.0000	64	0.00153	0.0000	66
February 13, 2008	0.00269	0.0000	61	-0.00454	0.0755	46	-0.00395	0.2807	47
March 31, 2008	-0.00048	0.0522	40	0.00336	0.0000	64	0.00391	0.0000	68
May 13, 2008	0.00132	0.8680	45	0.00489	0.0000	63	0.00540	0.0000	64
June 26, 2008	-0.00309	0.0000	12	0.01130	0.0000	88	0.00970	0.0000	83
August 11, 2008	0.00247	0.0000	64	-0.00240	0.0015	41	-0.00119	0.0200	43
September 24, 2008	-0.00184	0.0286	51	0.00042	0.0000	62	-0.00113	0.6299	53
November 6, 2008	0.00765	0.0000	84	-0.02174	0.0000	32	-0.02939	0.0000	16
December 22, 2008	0.00857	0.0000	93	-0.03283	0.0000	21	-0.03563	0.0000	9
February 24, 2009	-0.00037	0.0636	41	0.00239	0.0000	66	0.00100	0.0000	60
April 8, 2009	0.00051	0.0000	73	-0.00715	0.0000	29	-0.00754	0.0000	26
May 22, 2009	0.00165	0.0000	66	-0.00285	0.0004	39	-0.00260	0.0022	41
July 8, 2009	-0.00330	0.0000	12	0.00907	0.0000	88	0.00988	0.0000	88
August 20, 2009	-0.00194	0.0000	28	0.00520	0.0000	73	0.00512	0.0000	72
October 5, 2009	-0.00372	0.0000	9	0.01221	0.0000	94	0.01560	0.0000	95
November 17, 2009	-0.00021	0.0000	31	0.00095	0.0000	65	0.00109	0.0000	66
All periods	0.00020	0.0007	45	0.00217	0.0000	60	0.00193	0.0000	58

For each end date, the table shows (a) the difference between the median historical error and the median Bayes error, with a negative indicating the Bayes error is greater, (b) the p-value of the Wilcoxon one-sample test comparing the historical and Bayes errors, and (c) the percentage (taken over all stocks) of times the historical error exceeds the Bayes error. Data is for calls only.

Table 3 - continued

Panel B	Put options								
	Conjugate			Gamma			Mixture		
End date	Difference of medians	Wilcoxon P-value	Historical percent greater	Difference of medians	Wilcoxon P-value	Historical percent greater	Difference of medians	Wilcoxon P-value	Historical percent greater
August 16, 2007	-0.00102	0.6806	44	0.00136	0.0000	63	0.0002	0.0000	61
October 1, 2007	-0.00211	0.0000	32	0.00455	0.0000	79	0.0020	0.0000	77
November 13, 2007	-0.00207	0.0048	36	0.00466	0.0000	73	0.0050	0.0000	72
December 28, 2007	0.00041	0.0416	49	0.00078	0.0000	66	0.0013	0.0000	67
February 13, 2008	0.00097	0.0000	59	-0.00424	0.7031	49	-0.0046	0.7632	51
March 31, 2008	-0.00134	0.0341	40	0.00131	0.0000	66	0.0013	0.0000	68
May 13, 2008	-0.00072	0.4641	43	0.00408	0.0000	66	0.0039	0.0000	68
June 26, 2008	-0.00272	0.0000	12	0.00666	0.0000	87	0.0070	0.0000	83
August 11, 2008	0.00118	0.0000	65	0.00053	0.0344	44	0.0006	0.2023	46
September 24, 2008	-0.00002	0.0398	51	0.00382	0.0000	62	0.0023	0.4593	54
November 6, 2008	0.00382	0.0000	84	-0.02300	0.0000	30	-0.0264	0.0000	15
December 22, 2008	0.00801	0.0000	93	-0.03216	0.0000	22	-0.0385	0.0000	10
February 24, 2009	-0.00086	0.0098	39	0.00513	0.0000	69	0.0038	0.0000	62
April 8, 2009	0.00152	0.0000	68	-0.00580	0.0000	32	-0.0059	0.0000	28
May 22, 2009	-0.00040	0.0000	67	-0.00656	0.0002	38	-0.0067	0.0013	40
July 8, 2009	-0.00336	0.0000	12	0.00873	0.0000	87	0.0096	0.0000	88
August 20, 2009	-0.00217	0.0000	25	0.00437	0.0000	76	0.0037	0.0000	75
October 5, 2009	-0.00418	0.0000	9	0.01328	0.0000	94	0.0160	0.0000	95
November 17, 2009	-0.00048	0.0000	27	0.00453	0.0000	69	0.0048	0.0000	71
All periods	0.00000	0.14635	46	0.00233	0.0000	62	0.00227	0.0000	60

For each end date, the table shows (a) the difference between the median historical error and the median Bayes error, with a negative indicating the Bayes error is greater, (b) the p-value of the Wilcoxon one-sample test comparing the historical and Bayes errors, and (c) the percentage (taken over all stocks) of times the historical error exceeds the Bayes error. Data is for puts only.

Table 4

End date	Count	N	Percent
August 16, 2007	86	243	35
October 1, 2007	39	249	16
November 13, 2007	80	255	31
December 28, 2007	42	252	17
February 13, 2008	62	253	25
March 31, 2008	51	250	20
May 13, 2008	84	259	32
June 26, 2008	57	258	22
August 11, 2008	71	263	27
September 24, 2008	72	256	28
November 6, 2008	54	247	22
December 22, 2008	25	248	10
February 24, 2009	38	245	16
April 8, 2009	27	256	11
May 22, 2009	68	266	26
July 8, 2009	47	267	18
August 20, 2009	81	270	30
October 5, 2009	60	272	22
November 17, 2009	71	275	26
All periods	1,115	4,884	23

Results of Shapiro-Wilk Normality tests for the 19 30-day time periods. Count: number of stocks failing the test. N: Number of stocks having 30 days of data in the time period. Percent: percent failing test.