

$$T(n) = aT(n/2) + O(n^d)$$

RSA:

$$y = x^e \bmod N$$

$$x = y^d \bmod N$$

$$d = e \bmod (p-1)(q-1)$$

$$(tn \ tn-1 \ \dots)(b^n)(p(n))$$

$$(\dots)(r-b)^{d+1}$$

function multiply(x, y)

Input: Two n -bit integers x and y , where $y \geq 0$

Output: Their product

if $y = 0$: return 0

$z = \text{multiply}(x, \lfloor y/2 \rfloor)$

if y is even:

return $2z$

else:

return $x + 2z$

function modexp(x, y, N)

Input: Two n -bit integers x and N , an integer exponent y

Output: $x^y \bmod N$

if $y = 0$: return 1

$z = \text{modexp}(x, \lfloor y/2 \rfloor, N)$

if y is even:

return $z^2 \bmod N$

else:

return $x \cdot z^2 \bmod N$

function extended-Euclid(a, b)

Input: Two positive integers a and b with $a \geq b \geq 0$

Output: Integers x, y, d such that $d = \gcd(a, b)$ and $ax + by = d$

if $b = 0$: return $(1, 0, a)$

$(x', y', d) = \text{Extended-Euclid}(b, a \bmod b)$

return $(y', x' - \lfloor a/b \rfloor y', d)$

function primality(N)

Input: Positive integer N

Output: yes/no

Pick a positive integer $a < N$ at random

if $a^{N-1} \equiv 1 \pmod{N}$:

return yes

else:

return no

$$f = \Omega(g) \text{ means } g = O(f)$$

$$f = \Theta(g) \text{ means } f = O(g) \text{ and } f = \Omega(g).$$

procedure prim(G, w)

Input: A connected undirected graph $G = (V, E)$ with edge weights w_e

Output: A minimum spanning tree defined by the array prev

for all $u \in V$:

cost(u) = ∞

prev(u) = nil

Pick any initial node u_0

cost(u_0) = 0

$H = \text{makequeue}(V)$ (priority queue, using cost-values as keys)

while H is not empty:

$v = \text{deletemin}(H)$

for each $\{v, z\} \in E$:

if cost(z) > $w(v, z)$:

cost(z) = $w(v, z)$

prev(z) = v

decreasekey(H, z)

procedure kruskal(G, w)

Input: A connected undirected graph $G = (V, E)$ with edge weights w_e

Output: A minimum spanning tree defined by the edges X

for all $u \in V$:

makeset(u)

$X = \{\}$

Sort the edges E by weight

for all edges $\{u, v\} \in E$, in increasing order of weight:

if find(u) \neq find(v):

add edge $\{u, v\}$ to X

union(u, v)

$$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a. \end{cases}$$

procedure dijkstra(G, l, s)

Input: Graph $G = (V, E)$, directed or undirected;

positive edge lengths $\{l_e : e \in E\}$; vertex $s \in V$

Output: For all vertices u reachable from s , dist(u) is set to the distance from s to u .

for all $u \in V$:

dist(u) = ∞

prev(u) = nil

dist(s) = 0

$H = \text{makequeue}(V)$ (using dist-values as keys)

while H is not empty:

$u = \text{deletemin}(H)$

for all edges $(u, v) \in E$:

if dist(v) > dist(u) + $l(u, v)$:

dist(v) = dist(u) + $l(u, v)$

prev(v) = u

decreasekey(H, v)

procedure shortest-paths(G, l, s)

Input: Directed graph $G = (V, E)$;

edge lengths $\{l_e : e \in E\}$ with no negative cycles;

vertex $s \in V$

Output: For all vertices u reachable from s , dist(u) is set to the distance from s to u .

for all $u \in V$:

dist(u) = ∞

prev(u) = nil

dist(s) = 0

repeat $|V| - 1$ times:

for all $e \in E$:

update(e)