Bayesian Dynamic Linear Models for Strategic Asset Allocation

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Purpose

- To improve the forecasting model of the joint distribution of asset returns, particularly for assets of different classes

How should an investor optimally create a portfolio?

- Establish beliefs about future performance of assets $(\hat{\mu}_t, \hat{\Sigma}_t)$ using all current data:
    - Asset excess returns $\boldsymbol{r}_1, ..., \boldsymbol{r}_{t-1}$, where $\boldsymbol{r}_t = (r_{1t}, ..., r_{mt})'$
    - Predictive covariates $\boldsymbol{x}_1, ..., \boldsymbol{x}_{t-1}$, where $\boldsymbol{x}_t = (x_{1t}, ..., x_{pt})'$
- Choose how much to invest based on these beliefs

Literature on Predicting Risk Premia

$$r_t = \boldsymbol{x}'_{t-1}\boldsymbol{\beta} + \nu_t, \quad var(\nu_t) = \sigma^2$$

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- Predictors
- Model uncertainty
- Time-variation: rolling windows, structural breaks, dynamic linear models, stochastic volatility

Literature on Predicting Multi-Class Risk Premia

- Vector Auto-Regression (VAR)
- Other computationally intensive models needing MCMC
- Features found in univariate literature not all incorporated

How should covariates inform excess returns?

- We look at two methods
  - Assets share a vector of covariates:

$$
\begin{aligned}
r_{jt} &= \boldsymbol{x}'_{t-1}\boldsymbol{\beta}_{jt} + \nu_{jt} \\
\Rightarrow \boldsymbol{r}'_t &= \boldsymbol{x}'_{t-1}B_t + \boldsymbol{\nu}_t
\end{aligned}
$$

  - Include contemporaneous returns of other assets:

$$
\begin{aligned}
r_{jt} &= \boldsymbol{x}'_{j,t-1}\boldsymbol{\beta}_{jt} + \boldsymbol{r}'_{-j,t}\gamma_{jt} + \nu_{jt} \\
\Rightarrow \boldsymbol{r}_t &= \boldsymbol{\mu}_t + \Gamma_t \boldsymbol{r}_t + \boldsymbol{\nu}_t \\
\boldsymbol{r}_t &= (I - \Gamma_t)^{-1}(\boldsymbol{\mu}_t + \boldsymbol{\nu}_t)
\end{aligned}
$$

where $\boldsymbol{\mu}_t = (\boldsymbol{x}'_{1,t-1}\boldsymbol{\beta}_{1t}, ..., \boldsymbol{x}'_{m,t-1}\boldsymbol{\beta}_{mt})'$ and $j = 1, ..., m$

Wishart DLM - West and Harrison (1997)

Model

$$r_{jt} = \boldsymbol{x}'_{t-1}\boldsymbol{\beta}_{jt} + \nu_{jt}, \qquad \boldsymbol{\nu}_t|\Sigma_t \sim N(\mathbf{0}, \Sigma_t)$$
$$B_t = B_{t-1} + \Omega_t, \qquad \Omega_t|\Sigma_t \sim N(\mathbf{0}, W_t, \Sigma_t)$$

Prior

$$(B_0, \Sigma_0|D_0) \sim NW^{-1}_{n_0}(M_0, C_0, S_0)$$

Discount

$$n_t = \delta_v n_{t-1} + 1 \qquad \delta_v \in (0, 1]$$
$$W_t = \left(\frac{1}{\delta_\beta} - 1\right) C_{t-1} \qquad \delta_\beta \in (0, 1]$$

Simultaneous Graphical DLM - Gruber and West (2016)

Model

$$r_{jt} = \boldsymbol{x}'_{j,t-1}\boldsymbol{\beta}_{jt} + \boldsymbol{r}'_{-j,t}\boldsymbol{\gamma}_{jt} + \nu_{jt}, \qquad \nu_{jt} \sim N(\mathbf{0}, \lambda_{jt}^{-1})$$

$$\boldsymbol{\theta}_{jt} = \begin{pmatrix}\boldsymbol{\beta}_{jt} \\ \boldsymbol{\gamma}_{jt}\end{pmatrix} = \begin{pmatrix}\boldsymbol{\beta}_{j,t-1} \\ \boldsymbol{\gamma}_{j,t-1}\end{pmatrix} + \boldsymbol{\omega}_{jt}, \qquad \boldsymbol{\omega}_{jt} \sim N(\mathbf{0}, W_{jt})$$

Prior

$$(\boldsymbol{\theta}_{jt}, \boldsymbol{\lambda}_{j0} | D_0) \sim NG(\boldsymbol{m}_{j0}, C_{j0}, n_{j0}, s_{j0})$$

Discount

$$n_{jt} = \delta_v n_{j,t-1} + 1 \qquad \delta_v \in (0,1]$$

$$W_{jt} = \left(\frac{1}{\delta} - 1\right)^{***} C_{j,t-1} \qquad \delta_\beta, \delta_\gamma \in (0,1]$$

Specific Research Questions

1. Should we put the extra care into modeling assets' correlation?
2. Is there unincorporated information in X?
   - Does predictability exist/are markets efficient? Should X contain predictors or not?
3. Does it matter how information is incorporated?
   - Should the coefficient parameters B_t and/or volatility Σ_t vary over time, or not $(\delta_\beta = \delta_\gamma = \delta_v = 1)$?

Model Uncertainty

- We don't know the ideal X, δ_β, δ_γ, δ_v *a priori*.
- Solution: average and share strength across models.
  - Predictability: model for each pair of the stock predictors and bond predictors
  - Time-variation: 3 values of each δ are considered, equally spaced in the range $\delta_\beta, \delta_\gamma \in [0.98, 1.0]$, $\delta_v \in [0.95, 1.0]$.
- Weight the model average based on past performance, via score, akin to Bayesian model averaging

Model Evaluation: Forward-looking OOS Measures

- Economic: annualized certainty equivalent returns (CER)
  - Power utility investor, leverage constrained to invest within $[-2, 3]$ in any single asset.
  - $CER_\tau = \left(\left[(1-\gamma) \frac{1}{\tau} \sum_{t=1}^{\tau} U_t \right]^{\frac{12}{1-\gamma}} - 1 \right) * 100$
  - $U(wealth) = \frac{1}{1-\gamma} (wealth)^{1-\gamma}$
- Statistical fit measure: average log score (ALS)
  - $ALS_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} log \left(N[\boldsymbol{r}_t | \hat{\boldsymbol{\mu}}_t, \hat{\Sigma}_t] \right)$

Data Description

- Assets
  - US Treasurey Bonds with 5 and 10 year maturities
  - Large-, Mid-, and Small-cap stock portfolios
  - Risk-free T-Bill rate
- Bond Predictors
  - Cochrane and Piazzesi (2005)'s linear combination of forward rates
  - Fama and Bliss (1987)'s forward spread
  - Ludvigson and Ng (2009)'s macro economic factor
- Stock Predictors, from Welch and Goyal (2008)
  - Dividend Yield
  - Dividend/Payout ratio
  - Stock Variance

Methodology

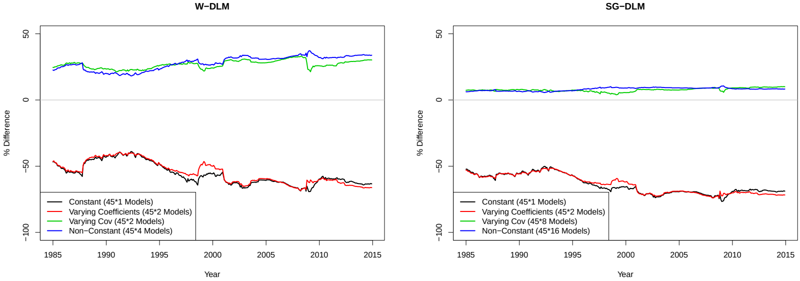
- The dataset contains monthly observations, spanning 1962 - 2014
    - Prior created on 1962-1971
    - Models evaluated on 1985-2014

Models

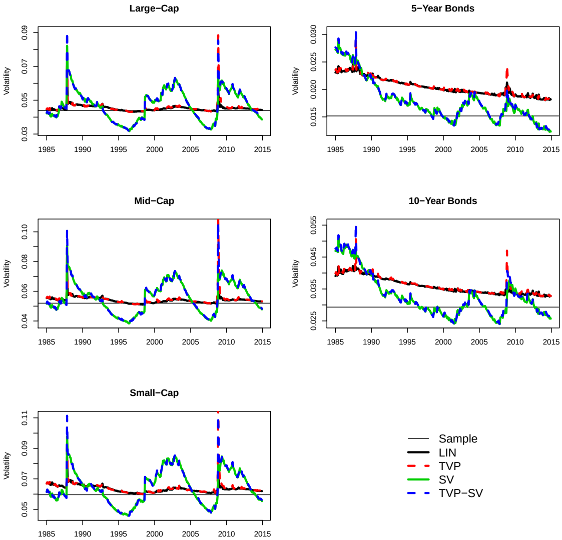
- Different Models
  - SG-DLM
  - W-DLM
- Different Feature Sets

Name	Single Asset Formula	
Prevailing Mean	$r_{jt} = \beta_{j0} + \nu_{jt},$	$\nu_t \sim N(0, \sigma^2)$
LIN	$r_{jt} = \beta_{j0} + \beta_{j1} x_{t-1} + \nu_{jt},$	$\nu_t \sim N(0, \sigma^2)$
TVP	$r_{jt} = \beta_{j0t} + \beta_{j1t} x_{t-1} + \nu_{jt},$	$\nu_t \sim N(0, \sigma^2)$
SV	$r_{jt} = \beta_{j0} + \beta_{j1} x_{t-1} + \nu_{jt},$	$\nu_t \sim N(0, \sigma_t^2)$
TVP-SV	$r_{jt} = \beta_{j0t} + \beta_{j1t} x_{t-1} + \nu_{jt},$	$\nu_t \sim N(0, \sigma_t^2)$

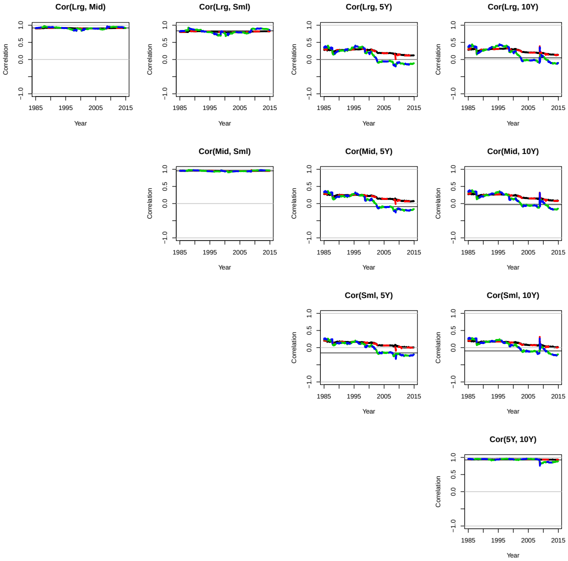
Time series of score-based weights by feature set



Time series of predicted volatilities for SG-DLM models

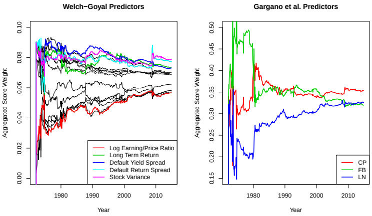


Time series of predicted correlations for SG-DLM models

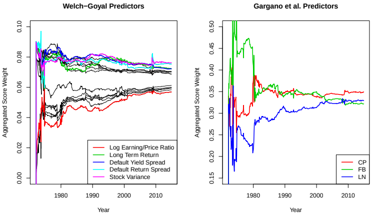


Time-series of score-based weights by predictor

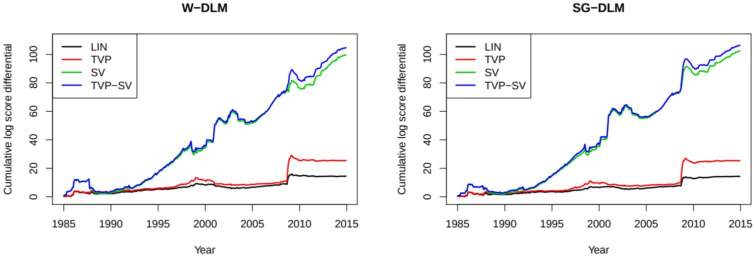
Panel A: W-DLM



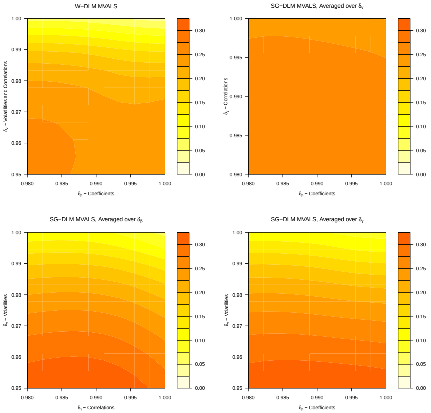
Panel B: SG-DLM



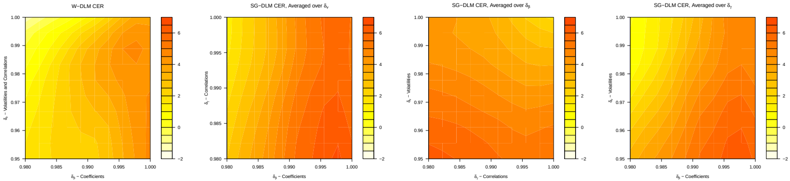
Cumulative sum of the multivariate log score differentials for W-DLM and SG-DLM models



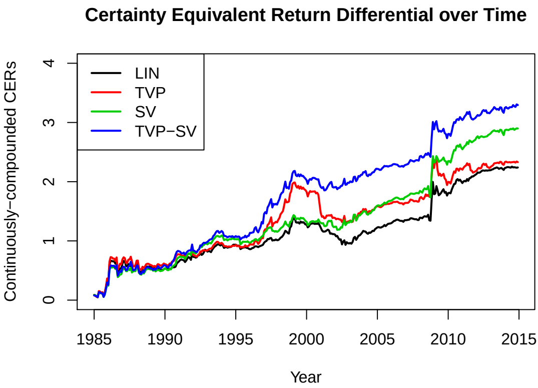
Heat map of multivariate average log scores for different discount factors



Heat map of certainty equivalent returns from discount factor combinations



Continuously compounded CER differentials



Weighted Mean-squared forecast errors

	W-DLM		SG-DLM	
	Equal	Score	Equal	Score
LIN	**0.989**	0.991	**0.989**	0.991
TVP	0.996	0.999	0.996	0.998
SV	**0.989**	0.990	0.989	0.990
TVP-SV	0.996	0.997	0.996	0.997

$$WMSFE_i = \frac{\sum_{\tau=\underline{t}}^{T} \boldsymbol{e}'_{i\tau} \left[\widehat{Cov(\boldsymbol{r}_t)}\right]^{-1} \boldsymbol{e}_{i\tau}}{\sum_{\tau=\underline{t}}^{T} \boldsymbol{e}'_{bcmk,\tau} \left[\widehat{Cov(\boldsymbol{r}_t)}\right]^{-1} \boldsymbol{e}_{bcmk,\tau}}$$

Multivariate Average log score differentials

	W-DLM		SG-DLM	
	Equal	Score	Equal	Score
LIN	0.043	0.040	0.042	0.040
TVP	0.069	0.070	0.069	0.070
SV	0.265	0.277	0.257	0.285
TVP-SV	0.280	**0.291**	0.270	**0.296**

$$MVALS_i = \frac{1}{T - \underline{t} + 1} \sum_{\tau=\underline{t}}^{T} \left(\ln \left(S_{i,\tau} \right) - \ln \left(S_{bcmk,\tau} \right) \right)$$

Annualized certainty equivalent returns

	W-DLM		SG-DLM	
	Equal	Score	Equal	Score
LIN	4.608	3.804	4.526	3.814
TVP	2.220	1.808	2.442	2.100
SV	**5.612**	5.293	5.865	**5.870**
TVP-SV	4.941	4.583	5.459	5.657

$$CER_i = 100 \times \left(\left[\frac{1}{T - \underline{t} + 1} \sum_{t=\underline{t}}^{T} \widehat{W}_{it}^{1-A} \right]^{\frac{12}{1-A}} - 1 \right)$$

Conclusions

1. Should we model correlation? Yes, depending on utility function.
2. Is their information to aid predictability? Yes, if incorporated appropriately.
3. Should time-variation be incorporated? Yes, especially stochastic volatility.

Questions, Comments?

Thank you!

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Myopic, leverage-constrained investor with power utility

- Power utility of wealth (CRRA): $\frac{(w_t)^{1-\gamma}}{1-\gamma}$
- Risk aversion $\gamma = 5$
- Investor is limited/restricted to placing [-2,3] times personal wealth in any given asset
- Following Cambell and Viceira (2002)'s example, we derive the investor's expected utility maximization
- For vector of portfolio weights on risky assets $\boldsymbol{w}_t$,

$$
\begin{aligned}
arg \max_{\boldsymbol{w}_t} \quad & \boldsymbol{w}_t' \left(\hat{\boldsymbol{\mu}}_t + \frac{1}{2} diag(\hat{\Sigma}_t) \right) - \frac{\gamma}{2} \boldsymbol{w_t'} \hat{\Sigma}_t \boldsymbol{w_t} && (1) \\
s.t. \quad & \boldsymbol{w}_t' \mathbf{1} \in [-2, 3] && (2) \\
& \boldsymbol{w}_{jt} \in [-2, 3], \ \forall j && (3)
\end{aligned}
$$