PSTAT 170 Assignment 5

2022-11-27

Problem 1

```r
BScall <- function(t=0,T,S,K,r,q=0,sigma,isPut=0) {
# t and T are measured in years; all parameters are annualized
# q is the continuous dividend yield
d1 <- (log(S/K)+(r-q+sigma^2/2)*(T-t))/(sigma*sqrt(T-t))
d2 <- d1-sigma*sqrt(T-t)
binary <- pnorm(-d2)*exp(-r*T)

# Call Delta at t
Delta <- exp(-q*(T-t))*pnorm(d1)
Gamma <- exp(-q*(T-t))*exp(-d1^2/2)/sqrt(2*pi)/S/sigma/sqrt(T-t)
Vega <- S*exp(-q*(T-t))/sqrt(2*pi)*exp(-d1^2/2)*sqrt(T-t)
Theta <- -S*exp(-q*(T-t))*sigma/sqrt(T-t)/2*dnorm(d1) - r*K*exp(-r*(T-t))*pnorm(d2) +
    q*S*exp(-q*(T-t))*pnorm(d1)
Rho <- (T-t)*K*exp(-r*(T-t))*pnorm(d2)


# Black-Scholes formula for Calls
BSprice <- -K*exp(-r*(T-t))*pnorm(d2)+S*Delta

if (isPut==1) {
    Delta <- -exp(-q*(T-t))*pnorm(-d1)
    BSprice <- S*Delta+K*exp(-r*(T-t))*pnorm(-d2)
    Theta <- -S*exp(-q*(T-t))*sigma/sqrt(T-t)/2*dnorm(d1) + r*K*exp(-r*(T-t))*pnorm(-d2) -
    q*S*exp(-q*(T-t))*pnorm(-d1)
    Rho <- -(T-t)*K*exp(-r*(T-t))*pnorm(-d2)
}
Bank <- BSprice-Delta*S

return (list(Delta=Delta,Gamma=Gamma,Theta=Theta,Vega=Vega,Rho=Rho,Price=BSprice,d1=d1,d2=d2,B=Bank))
}
```

Call

```r
# Call
BScall(t=0/365,T=55/365,S=33,K=34,r=0.06,q=0.01,sigma=0.32,isPut=0)$Price
```

```
## [1] 1.304938
```

```
# Delta
BScall(t=0/365,T=55/365,S=33,K=34,r=0.06,q=0.01,sigma=0.32,isPut=0)$Delta
```

```
## [1] 0.4525239
```

```
# Call Next Day
BScall(t=1/365,T=55/365,S=34.5,K=34,r=0.06,q=0.01,sigma=0.32,isPut=0)$Price
```

```
## [1] 2.073432
```

Put

```
# Put
BScall(t=0/365,T=55/365,S=33,K=34,r=0.06,q=0.01,sigma=0.32,isPut=1)$Price
```

```
## [1] 2.048615
```

```
# Delta
BScall(t=0/365,T=55/365,S=33,K=34,r=0.06,q=0.01,sigma=0.32,isPut=1)$Delta
```

```
## [1] -0.5459703
```

```
# Put Next day
BScall(t=1/365,T=55/365,S=34.5,K=34,r=0.06,q=0.01,sigma=0.32,isPut=1)$Price
```

```
## [1] 1.323963
```

Problem 2

```
# 90 day Put
BScall(t=0/365,T=90/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.106753
```

```
BScall(t=0/365,T=90/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4280874
```

```
BScall(t=0/365,T=90/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Gamma
```

```
## [1] 0.06322545
```

```
# 60 day Put
BScall(t=0/365,T=60/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.775211
```

```
BScall(t=0/365,T=60/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4411767
```

```
BScall(t=0/365,T=60/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Gamma
```

```
## [1] 0.07786017
```

```
# P(1/365) for S = 48,50,52
BScall(t=1/365,T=90/365,S=48,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 3.08432
```

```
BScall(t=1/365,T=60/365,S=48,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.805904
```

```
BScall(t=1/365,T=90/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.097047
```

```
BScall(t=1/365,T=60/365,S=50,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.762393
```

```
BScall(t=1/365,T=90/365,S=52,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.362214
```

```
BScall(t=1/365,T=60/365,S=52,K=50,r=0.06,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.029264
```

Problem 3

```
# Greeks
BScall(t=0/365,T=1,S=50,K=40,r=0.05,q=0.03,sigma=0.3,isPut=1)
```

```
## $Delta
## [1] -0.16343
##
## $Gamma
## [1] 0.016273
##
## $Theta
## [1] -1.591732
##
## $Vega
## [1] 12.20475
##
## $Rho
## [1] -9.682504
##
## $Price
## [1] 1.511002
##
## $d1
## [1] 0.9604785
##
## $d2
## [1] 0.6604785
##
## $B
## [1] 9.682504
```

```r
BScall(t=0/365,T=1,S=50,K=45,r=0.05,q=0.03,sigma=0.3,isPut=1)
```

```
## $Delta
## [1] -0.2766373
##
## $Gamma
## [1] 0.02196674
##
## $Theta
## [1] -2.042092
##
## $Vega
## [1] 16.47505
##
## $Rho
## [1] -16.88243
##
## $Price
## [1] 3.050568
##
## $d1
## [1] 0.5678684
##
## $d2
## [1] 0.2678684
##
## $B
```

```
## [1] 16.88243
```

```
BScall(t=0/365,T=1,S=50,K=40,r=0.05,q=0.03,sigma=0.302,isPut=1)
```

```
## $Delta
## [1] -0.1644983
##
## $Gamma
## [1] 0.01623303
##
## $Theta
## [1] -1.609375
##
## $Vega
## [1] 12.25593
##
## $Rho
## [1] -9.760379
##
## $Price
## [1] 1.535463
##
## $d1
## [1] 0.9561111
##
## $d2
## [1] 0.6541111
##
## $B
## [1] 9.760379
```

```
BScall(t=0/365,T=1,S=50,K=45,r=0.05,q=0.03,sigma=0.302,isPut=1)
```

```
## $Delta
## [1] -0.2772199
##
## $Gamma
## [1] 0.02184314
##
## $Theta
## [1] -2.05883
##
## $Vega
## [1] 16.49157
##
## $Rho
## [1] -16.94453
##
## $Price
## [1] 3.083534
##
## $d1
## [1] 0.566101
```

```
##
## $d2
## [1] 0.264101
##
## $B
## [1] 16.94453
```

Problem 4

```
# Determining d1 and d2
BScall(t=0/365,T=1,S=100,K=103,r=0.06,q=0.05,sigma=0.15,isPut=0)
```

```
## $Delta
## [1] 0.454605
##
## $Gamma
## [1] 0.02526026
##
## $Theta
## [1] -3.005242
##
## $Vega
## [1] 37.89039
##
## $Rho
## [1] 40.60813
##
## $Price
## [1] 4.852373
##
## $d1
## [1] -0.05539201
##
## $d2
## [1] -0.205392
##
## $B
## [1] -40.60813
```

```
# 200 simuations
r <-0.06; delt <- 0.05; S0 <- 100; sigm <- 0.15; T <- 1
S_T <- S0*exp( (r-delt-0.5*sigm^2)*T + sigm*sqrt(T)*rnorm(200))
exp(-r*T)*mean( pmax((S_T-100)*(S_T>103)), 0)
```

```
## [1] 5.000418
```

```
# 10000 simulations
r <-0.06; delt <- 0.05; S0 <- 100; sigm <- 0.15; T <- 1
S_T <- S0*exp( (r-delt-0.5*sigm^2)*T + sigm*sqrt(T)*rnorm(10000))
exp(-r*T)*mean( pmax((S_T-100)*(S_T>103)), 0)
```

```
## [1] 5.931481
```

Problem 5

```
# Scenario 1
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.610472
```

```
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4684985
```

```
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.278965
```

```
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4727118
```

```
BScall(t=2/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.878374
```

```
BScall(t=2/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4777135
```

```
BScall(t=3/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.344376
```

```
BScall(t=3/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.484237
```

```
# Scenario 2
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.610472
```

```
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4684985
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.278965
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4727118
```

```r
BScall(t=2/52,T=4/52,S=99,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.396858
```

```r
BScall(t=2/52,T=4/52,S=99,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.5592604
```

```r
BScall(t=3/52,T=4/52,S=98,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.539742
```

```r
BScall(t=3/52,T=4/52,S=98,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.7065083
```

```r
# Scenario 3
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.610472
```

```r
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4684985
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.278965
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4727118
```

```r
BScall(t=2/52,T=4/52,S=99,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.396858
```

```r
BScall(t=2/52,T=4/52,S=99,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.5592604
```

```r
BScall(t=3/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 1.344376
```

```r
BScall(t=3/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.484237
```

```r
# Scenario 4
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.610472
```

```r
BScall(t=0/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4684985
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 2.278965
```

```r
BScall(t=1/52,T=4/52,S=100,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.4727118
```

```r
BScall(t=2/52,T=4/52,S=96,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 4.421933
```

```r
BScall(t=2/52,T=4/52,S=96,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.7813367
```

```r
BScall(t=3/52,T=4/52,S=97,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Price
```

```
## [1] 3.294133
```

```r
BScall(t=3/52,T=4/52,S=97,K=100,r=0.04,q=0,sigma=0.25,isPut=1)$Delta
```

```
## [1] -0.7992809
```