20201210-p8133_probset13_jsg2145

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Problem 13

Part a

$X_1 \sim N(\mu t_1,\ 1) \perp X_2 \sim N(\mu t_2,\ 1)$

Z $= \omega_1 X_1 + \omega_2 X_2$

$\omega_1^2 + \omega_2^2 = 1$

w, t > 0

$Pr(|Z| > z_{0.025})\ N(\mu(\omega_1 t_1 + \omega_2 t_2),\ 2) > z_{0.025}$

```
errI = 1 - pnorm(0.9875, sd = sqrt(2))
```

The type I error rate is 0.2425.

Part b

The power is the probability that the null will be rejected given that the null is false.

Z $\sim N(\mu(\omega_1 t_1 + \omega_2 t_2,\ 2))$

$\frac{Z-\mu(\omega_1 t_1 + \omega_2 t_2)}{\sqrt{2}} \sim N(0,1)$

```
errI2 = (1 - pnorm(0.9875))*sqrt(2)
```

The power is 0.2287 + $\mu(\omega_1 t_1 + \omega_2 t_2)$.

Part c

By intuition, $\omega_1 = \omega_2 = \sqrt{0.5}$

Part d

Given: $\mu = 0$

Z ~ N(0, 2)

X_1 ~ N(0, 1)

$Pr(X_1 > z_{0.005}\ or\ Z > z_{0.02}) \leq 0.025$

$Pr(X_1 > z_{0.005})$

$Pr(Z > z_{0.025})$

The left-hand equation will be smaller because the alpha value is smaller. Therefore, fewer observations will satisfy the inequality in the left-hand equation compared to the right hand equation.

Part e

```
errI3 = (1-pnorm(0.99))*sqrt(2)
```

The value of the power will be $0.2278 + \mu(\omega_1 t_1 + \omega_2 t_2)$.

A smaller value of alpha may affect the power of the test. However, power and type I error measure different things.

One tests a null when the null is true and the power tests the null when the alternative is true.

The test still depends on μ, ω, and t.