Homework 1 - Monte Carlo Methods

Leave your name and uni here

Problem 1

The standard Laplace distribution has density $f(x) = 0.5e^{-|x|}, x \in (-\infty, \infty)$. Please provide an algorithm that uses the inverse transformation method to generate a random sample from this distribution. Use the $U(0, 1)$ random number generator in **R**, *write a* **R**-function to implement the algorithm. Use visualization tools to validate your algorithm (i.e., illustrate whether the random numbers generated from your function truely follows the standard Laplace distribution.)

Answer: your answer starts here...

```r
#Your R codes/functions
```

#Problem 2

Use the inverse transformation method to derive an algorithm for generating a Pareto random number with $U \sim U(0, 1)$, where the Pareto random number has a probability density function

$$f(x; \alpha, \gamma) = \frac{\gamma\alpha^{\gamma}}{x^{\gamma+1}} I\{x \geq \alpha\}$$

with two parameters $\alpha > 0$ and $\gamma > 0$. Use visualization tools to validate your algorithm (i.e., illustrate whether the random numbers generated from your function truely follows the target distribution.)

Answer: your answer starts here...

```r
#Your R codes/functions
```

#Problem 3

Construct an algorithm for using the acceptance/rejection method to generate 100 pseudorandom variable from the pdf

$$f(x) = \frac{2}{\pi\beta^2}\sqrt{\beta^2 - x^2}, \ -\beta \leq x \leq \beta.$$

The simplest choice for $g(x)$ is the $U(-\beta, \beta)$ distribution but other choices are possible as well. Use visualization tools to validate your algorithm (i.e., illustrate whether the random numbers generated from your function truely follows the target distribution.)

Answer: your answer starts here...

```r
#Your R codes/functions
```

#Problem 4

Develop two Monte Carlo methods for the estimation of $\theta = \int_0^1 e^{x^2} dx$ and implement in **R** .

Answer: your answer starts here...

```r
#Your R codes/functions
```

#Problem 5 Show that in estimating $\theta = E\sqrt{1-U^2}$ *it is better to use* U^2 *rather than* U *as the control variate, where* $U \sim U(0,1)$*. To do this, use simulation to approximate the necessary covariances. In addition, implement your algorithms in* **R**.

Answer: your answer starts here...

```r
#Your R codes/functions
```

#Problem 6 Obtain a Monte Carlo estimate of

$$\int_1^\infty \frac{x^2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

by importance sampling and evaluate its variance. Write a **R** *function to implement your procedure.*

Answer: your answer starts here...

```r
#Your R codes/functions
```