3010 Submodule Applied Crypto

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NTU, SCSE, Week 10A

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

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Overview Week 10 Lecture-A

- Intro Crypto
- Caesar Cipher
- Simple Substitution
- Vignere Cipher
- One-Time Pad
- Randomness

What is Cryptography - Informal

- The Science and Math of scrambling data (using a specific method with a 'key') into meaningless gibberish to render data incomprehensible to eavesdropper.
- In use since ancient times (>2000 yrs)

Cryptography can be found EVERYWHERE!

- Cellphone calls
- Microsoft office password protection
- E-commerce with Amazon etc
- ATM card
- Smart Cards
- Whatsapp messages sent over the air
- Surfing https websites
- Your digital signatures

At 11:55am a girl dropped a note to a boy

- She asked him to find out the meaning of the note by 12 noon & go to LT 13 to meet her if he manages to understand the message.
- So boy has 5 minutes to figure this note:

This is the Note –What does it mean?

L OLNH X

Steps to Decrypt

- L & X are single letters.
- Assuming text in English and all letters are just letters
- Only possibilities of L & X
  - A, I, O, U

Steps to Decrypt

L OLNH X

I I U

Steps to Decrypt - looks like shift 3 letters

L OLNH X

I I U

Steps to Decrypt - looks like shift 3 letters-YES

L OLNH X

I LIKE U

Steps to Decrypt - looks like shift 3 letters-YES

L OLNH X

I LIKE U

-Caesar Cipher

Caesar Cipher

- Used 2000 years ago!
- Extremely easy to break, just shift 3 letters to left to break it.
- How to make cipher harder?
- Use all possible shifts, 25 of them (26 alphabets)!
- Easy for computers – instant break!
- It's the simplest mono-alphabetic encryption
  - Means each letter means uniquely some other letter (1-1 match)

Caesar Cipher – Main Weakness

- Only 25 possible keys-- trivial for computers to try all keys
- We Say Key space

- Too small!

How Big Should Key Space be

- Depends who you are guarding against.
- A 3 GHz Pc can roughly crack a key space of 2^34 in 1 day
- I am jumping the gun here, but I will state now:
- Present day Minimum acceptable security – 2^128!
- This is an extraordinary huge number

Ancient Famous Story

- A king wants to reward a wise man for his contribution to his kingdom
- King asked him what he wants as a reward
- Tricky question for the wise man
- Ask too much, his head might be gone
- Too little, he would have wasted an opportunity to get rich
- He told the king:

Ancient Famous Story

- Find a field and draw 8x8 squares (like a chessboard)
- Put 1 grain of rice in 1^{st} square, 2 grains in 2^{nd}, 4 grains in 3^{rd},…
- My reward: I want all the grains added up to the 64^{th} square
- King thought it's a reasonable request. He agreed in front of court.
- Ordered the servants to do what wise man had asked.
- After a while the servants reported to the king.
- WE don't have enough grain in the whole kingdom for this wise man!
- What happened? (tutorial)

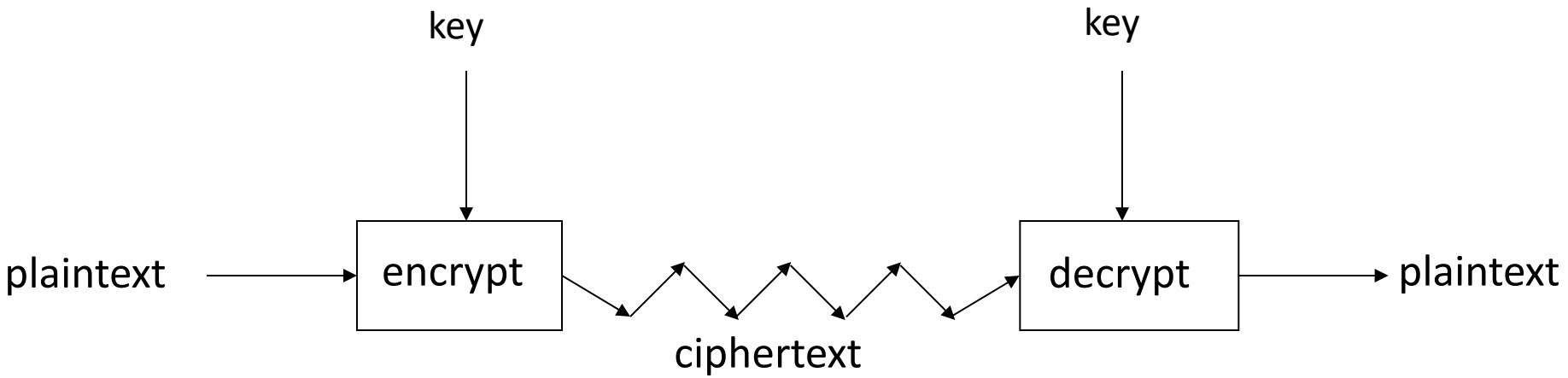
Crypto Terminology

- **Cryptology** — The art and science of making and breaking “secret codes”
- **Cryptography** — making “secret codes”
- **Cryptanalysis** — breaking “secret codes”
- **Crypto** — all of the above (and more)

How to Speak Crypto

- A *cipher* or *cryptosystem* is used to *encrypt* the *plaintext*
- The result of encryption is *ciphertext*
- We *decrypt* ciphertext to recover plaintext
- A *key* is used to configure a cryptosystem
- A *symmetric key* cryptosystem uses the same key to encrypt as to decrypt
- A *public key* cryptosystem uses a *public key* to encrypt and a *private key* to decrypt -2nd half course

Crypto as Black Box



A generic view of symmetric key crypto

Caesar Cipher-Simple Substitution

- Plaintext: fourscoreandsevenyearsago
- Key:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Ciphertext:
  IRXUVFRUHDQGVHYHQBHDUVDJR
- Shift by 3 is “Caesar’s cipher”

Caesar Cipher Decryption

- Suppose we know a Caesar's cipher is being used:

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Given ciphertext:
  VSRQJHEREVTXDUHSDQWV
- Plaintext: spongebobsquarepants

Easy to harden Caesar Cipher

- Instead of shifting all letters by 3 letters, we can scramble the 26 letters A-Z into other random permutations of A-Z.
- How many possible permutations of A-Z?
- 26! (26x25x24x….x3x2x1)
- Then encrypt accordingly based on your scrambled key.
- Decrypt using same scrambled key!

Simple Substitution: General Case

- In general, simple substitution key can be any **permutation** of letters
  - Not necessarily a shift of the alphabet
- For example

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Plaintext																										
Ciphertext	J	I	C	A	X	S	E	Y	V	D	K	W	B	Q	T	Z	R	H	F	M	P	N	U	L	G	O

- Then $26! > 2^{88}$ possible keys (tutorial)-beyond BF

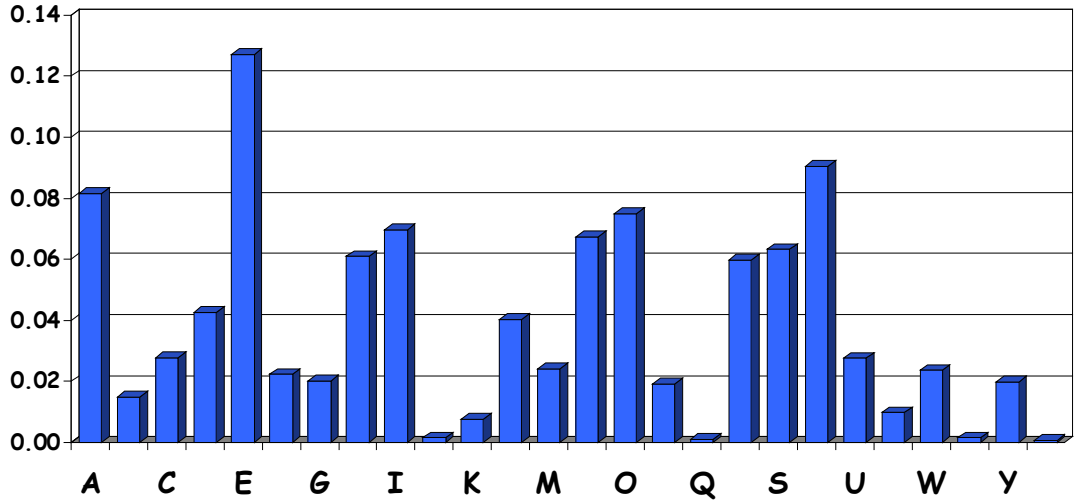
Cryptanalysis II: Be Clever

- We know that a simple substitution is used
- But not necessarily a shift by n
- Find the key given the ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQWAXBVCXQW
AXFQJVWLEQNTOZQGGQLFXQWAKVWLXQWAEBIPBFXFQVXGTVJVWLBTPQWAEBFP
BFHCVLXBQUFEVWLXGDPEQVPQGVPPBFTIXPFHXZHVFAGFOTHFEFBQUFTDHZBQPO
THXTYFTODXQHFTDPTOGHFQPBQWAQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFW
PFHPBFIPBQWKFABVYYDZBOTHPBQPQJTQOTOGHFQAPBFEQJHDXXQVAVXEBQPEFZ
BVFOJIWFFACFCCFHQWAUVWFLQHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFHF
QAITIXPFHXAFQHEFZQWGFLVWPTOFFA

Cryptanalysis II

- Cannot try all 2^{88} simple substitution keys
- Can we be more clever?
- English letter frequency counts…



Letter Frequencies of English Letter

Letter	Frequency	Letter	Frequency
A	0.0817	N	0.0675
B	0.0150	O	0.0751
C	0.0278	P	0.0193
D	0.0425	Q	0.0010
E	0.1270	R	0.0599
F	0.0223	S	0.0633
G	0.0202	T	0.0906
H	0.0609	U	0.0276
I	0.0697	V	0.0098
J	0.0015	W	0.0236
K	0.0077	X	0.0015
L	0.0403	Y	0.0197
M	0.0241	Z	0.0007

Cryptanalysis: Terminology

- Cryptosystem is **secure** if best known attack is to try all keys (brute force -Exhaustive key search)
- Cryptosystem is termed '**broken'** if ***any*** shortcut attack is known without trying all keys)
- Although substitution = 26! Keys (beyond brute-force range), it succumbs to frequency analysis.
- Main weakness – letters used in English are very unevenly distributed! (eg compare Q with E)
- ***So Don't ever use Substitution ciphers!!!***

Lessons Learned So Far

- Key space must be large
- All letters must be equally likely to happen for ciphers to be maximally secure!
- Average freq for equal appearance is 1/26, which is roughly 4%
- “e” occurs 12% in English text, 3 times higher than average
- “z” appears 0.07%
- So e appears abt 170 times more frequent than z!

Vigenere Cipher

- Took 1500 years to see a meaningful improvement of the Caesar cipher - Vigenère cipher, created in the 16th century

Vigenere Cipher

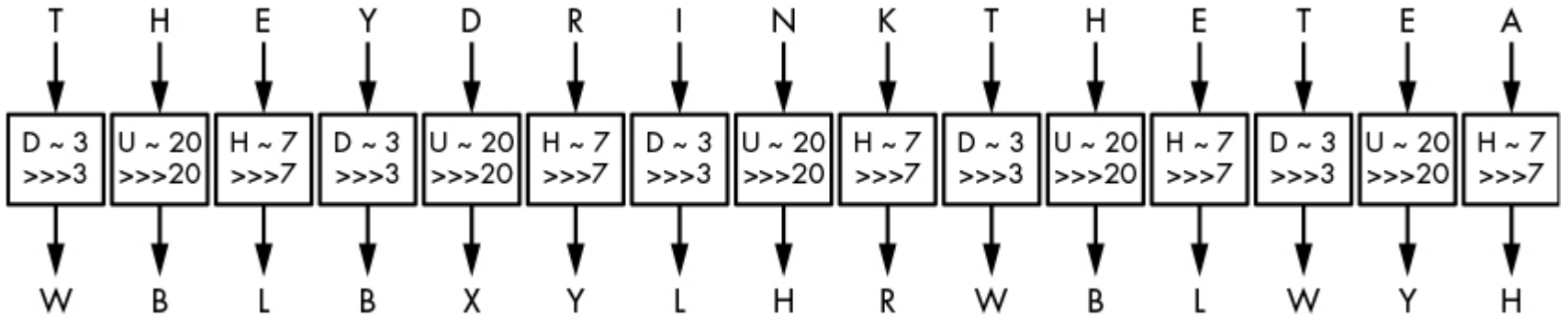
- Vigenère cipher is similar to the Caesar cipher, except that letters aren't shifted by three places but rather by values defined by a *key*, a collection of letters that represent numbers based on their position in the alphabet.
- For example, if the key is DUH, letters in the plaintext are shifted using the values 3, 20, 7
  - because *D* is 3 letters after *A*,
  - *U* is 20 letters after *A*, and
  - *H* is 7 letters after *A*.
- The 3, 20, 7 pattern repeats until you've encrypted the entire plaintext.

Vigenere Cipher

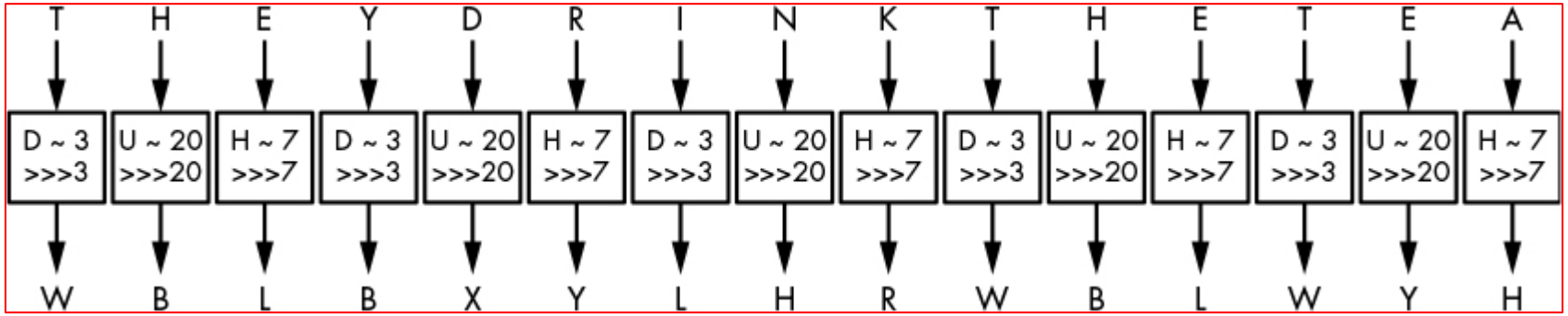
- For example, the word CRYPTO would encrypt to FLFSNV using DUH as the key:
  - *C* is shifted 3 positions to *F*,
  - *R* is shifted 20 positions to *L*, and so on.

- Figure 1-3 illustrates this principle when encrypting the sentence THEY DRINK THE TEA.

Vigenere Cipher



Vigenere Cipher : E maps to both L & Y!



Vigenere Cipher

- In this cipher, same letter can be mapped to different letters
- Also different letters can be mapped into a single letter
- We call this poly-alphabetic substitution
- Much more secure than simple mono-alphabetic sub!

Vigenere Cipher

- Vigenère cipher > Caesar cipher, yet it's still fairly easy to break.
- The first step to breaking it is to figure out the key's length.
- **THEYDRINKTHETEA** encrypts to
- **WBLBXYLHRWBLWYH** , with the key **DUH**.
- Notice in ciphertext WBLBXYLHRWBLWYH, the group of three letters WBL appears twice in the ciphertext at nine-letter intervals.

Vigenere Cipher

- Suggest same 3-letter word was encrypted using the same shift values, producing WBL each time.
- A cryptanalyst can then deduce that the key's length is either **9** or a value that divides nine (that is, **3**).
- Furthermore, they may guess that this repeated 3-letter word is THE and **therefore determine DUH** as a possible encryption key.

Vigenere Cipher -Status

- Long keywords implies stronger Vigenere (Tutorial)
- Short message implies stronger Vigenere (Tutorial)
- Not good enough for modern use!

Classical Ciphers

- The previous class of ciphers are all known as classical ciphers
- There are many more such ciphers, but all insecure due to computers
- Used before WW2, way before invention of computers

- Question:
- Any unbreakable ciphers, even with computers in attacker's hand?
- YES- ONE-TIME PAD!

Perfect Encryption: The One-Time Pad

- Essentially, a classical cipher can't be secure unless it comes with a huge key, but encrypting with a huge key is impractical.
- However, the one-time pad is such a cipher, and it is the most secure cipher.
- In fact, it guarantees *perfect secrecy*:
  - even if an attacker has unlimited computing power, it's impossible to learn anything about the plaintext except for its length.

Perfect Encryption: The One-Time Pad

- The one-time pad takes a plaintext, *P*, and a "random key", *K*, that's the same length as *P* and produces a ciphertext *C*, defined as $C = P \oplus K$,

where *C*, *P*, and *K* are bit strings of the same length and where $\oplus$ is the bitwise exclusive OR operation (XOR), defined as

$0 \oplus 0 = 0$,

$0 \oplus 1 = 1$,

$1 \oplus 0 = 1$,

$1 \oplus 1 = 0$.

Easy Encryption: The One-Time Pad

- Since $C = P \oplus K$, *we have*
- $C \oplus K = P \oplus K \oplus K$, *yielding*
- $C \oplus K = P \oplus 0 = P$, *the plaintext!*
- *So for both encrypt & decrypt, just XOR!*
- *XOR is superfast – instantaneous encryption & decryption!*
- *So course should be over.*
- *Everyone just use one-time pad, end of story!*
- *Will answer this question shortly.*

Easy Encryption: The One-Time Pad -Letters

- Can be used for encrypting just letters too
- We need long random pads, as long as the message
- Suppose msg is YES & random pad generated is CAB
- For encryption
  - C – shift 3 to right
  - A – shift 1 to right
  - B- shift 2 to right
- **YES + CAB** (Y+C = B , becos Y shift 3 right –sequence Y,Z,A,B)
- **BFU (E shift 1 right, S shift 2 right)**

Easy Decryption: The One-Time Pad -Letters

- If user receive cipher BFU, and we know one time pad is CAB, we apply “inverse of add – that is minus
- So since encrypt means shift right, Decrypt means shift left
- For decryption
  - C – shift 3 to left
  - A – shift 1 to left
  - B- shift 2 to left
- **BFU-CAB** (B-C=Y , becos B shift 3 left –sequence B,A,Z,Y)
- **YES (F shift 1 left, U shift 2 right)**
- Useful to list ABCDEFGHIJKLMNOPQRSTUVWXYZABCDEFGHIJKLMN…

OTP In Computation (letters)-encryption($)

- Earlier by shifting argument, we have Y + C = B
- If letters to add are large, not easy to see shifted letter by listing
- Use this trick: A=1, B=2,…,Y=25,Z=26
- Y + C (numerical) (mod 26) , α(mod26) means remainder of α when divided by 26.
- = 25 + 3 (mod 26)
- = 28 (mod 26)
- = 2
- = B , same answer as before

OTP In Computation (letters)-decrytion

- Earlier by shifting argument, we have B - C = Y
- If letters to subtract are large, not easy to see shifted letter by listing
- Use this trick: A=1, B=2,…, Y=25, Z=26
- B - C (numerical) (mod 26)
- = 2 - 3 (mod 26)
- = -1 (mod 26)
- = -1 + 26 (mod 26) = 25
- = Y, same answer as before.

One-Time Pad Summary

- **Provably** secure
  - Ciphertext gives **no** useful info about plaintext
  - All plaintexts are ***equally likely, so for a 3 letter cipher, we will not be able to tell if plaintext is YES or NOT !***
- BUT, only when it is used correctly
  - Pad must be random – (why? – Tutorial)
  - Pad used only once (why? -Tutorial)
  - Keep track of bits used (why? –Tutorial)
  - Pad is known only to sender and receiver
- Note: pad (key) is same size as message
- Got to generate a long random pad to your buddy so that you do not need to frequently meet up.

OTP instantaneous & unbreakable –End of Story?

- We have noted OTP operations are instantaneous using computers
- And its unbreakable.
- So should be end of story for crypto.
- Why not?

Challenges of Using OTP

- How to generate truly random LONG one-time pad (OTP)?
- How to store OTP securely?
- How to encrypt and decrypt securely
- Both parties have to keep in synchronization portions of pad that has already been used, so that both can keep on talking
- How to agree on new OTP if old OTP is used up or compromised?

Informal Notions of Random used in crypto

- Suppose X_i is bit *i* of OTP.
- Random OTP (bits) informally means

1. P(X_i = 0) = P(X_i = 1) = 0.5, both equally likely
2. Successive bits are indep of each other i.e. P(X_{i+1}/precedingX_i s) = P(X_{i+1})

- One way out: Think of unbiased coin and repeatedly toss it & record heads or tails
- In practice: how to manufacture unbiased coin?
- Truly random source – from eg radioactive decay… slow to generate

Randomness

- Randomness is found everywhere in cryptography:
  - in the generation of secret keys,
  - in encryption schemes, and
  - even in the attacks on cryptosystems.

- Without randomness, cryptography would be impossible because all operations would become predictable, and therefore insecure.

Randomness

- This section introduces you to the concept of randomness in the context of cryptography and its applications.
- We discuss pseudorandom number generators and how operating systems can produce reliable randomness, and we conclude with real examples showing how flawed randomness can impact security.

Randomness –how they look like

Is the 8-bit string 11010110 more random than 00000000? (tutorial)

Randomness

- This example illustrates two types of errors people often make when identifying randomness:
- **Mistaking non-randomness for randomness** Thinking that an object was randomly generated simply because it *looks* random.
- **Mistaking randomness for non-randomness** Thinking that patterns appearing by chance are there for a reason other than chance.
- The distinction between random-looking and actually random is crucial. Indeed, in crypto, non-randomness is often synonymous with insecurity.

Informal Notions of Random used in crypto

- Please note crypto notion of randomness needed is much more stringent than randomness needed in simulations (monte-carlo) used in video games and computing probabilities of complicated events
- It is also more stringent than random numbers generated from pseudo-random number generators
- This type of generation won't produce truly robust random numbers needed for crypto.
- Later in course I will suggest some famously good CSPRNG, crypto-secure pseudo random number generators