A Very Interesting Title

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Abstract

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1 Introduction

An agent starts at $(0, 0)$ with an object and is moving in a straight line towards $(1, 0)$ to deliver it. Because the start may fail, we deploy a second agent (called the *finisher*), starting at position (x, y) that can pick up the object and finish the delivery to $(1, 0)$. Suppose the starter fails at $(0, 0)$ with probability p and at $(1, 0)$ with probability $1 - p$. We only consider the object to be delivered when the finisher and the object are co-located at $(1, 0)$. Both agents always move at a maximum speed of 1. The finisher can start, stop, and change direction instantaneously.

Let's start by considering the case where $(x - 1)^2 + y^2 \leq 1$. In this case, the finisher can always reach the object before it reaches $(1, 0)$. Observe the point $(m, 0)$ where $m = \frac{x^2+y^2}{2x}$ is the point where, supposing the starter does not fail, the two agents can meet at the same time (time $t = m$). Let's consider three candidate algorithms:

1. A_0: The finisher moves directly to $(0, 0)$, then to $(1, 0)$, guaranteeing a delivery time of $1 + \sqrt{x^2 + y^2}$.
2. A_1: The finisher moves directly to $(1, 0)$. Then, with probability p the delivery time is $2+\sqrt{(x - 1)^2 + y^2}$, and with probability $1 - p$ the delivery time is 1.
3. A_m: The finisher moves to $(m, 0)$, then to $(0, 0)$, then to $(1, 0)$. Then with probability p the delivery time is $2 + 1 + \sqrt{(x - m)^2 + y^2}$, and with probability $1 - p$ the delivery time is 1.

Let D_0, D_1, and D_m be the expected delivery times for algorithms A_0, A_1, and A_m, respectively. Then $D_0 = 1 + \sqrt{x^2 + y^2}$, $D_1 = p(2 + \sqrt{(x - 1)^2 + y^2}) + (1 - p)$, and $D_m = p(m + 1 + \sqrt{(x - m)^2 + y^2}) + (1 - p)$.

Let's consider the case where $(x - 1)^2 + y^2 > 1$ (i.e., the finisher cannot reach the object before it reaches $(1, 0)$). In this case, the only two algorithms that make sense are A_0 and A_1. The expected delivery time for A_0, then is better than A_1 if $1 + \sqrt{x^2 + y^2} < p(2 + \sqrt{(x - 1)^2 + y^2}) + (1 - p)$. Solving for y, we find that this is true when

$$y < 2\frac{(1 - p)p(p - x)(1 - p - x)}{(1 - 2p)^2} \tag{1}$$

Theorem 1.1. *The curve defined by Equation 1 has an oblique asymptote at* $y = sgn(x) \cdot 2\sqrt{\frac{(1-p)p}{(1-2p)^2}}x + \frac{1}{2}$.

Proof. JC: This is just a sketch of the proof. Let $f(x) = 2\frac{(1-p)p(p-x)(1-p-x)}{(1-2p)^2}$. Taking the limit of $f'(x)$ as $x \to \infty$, yields $2\sqrt{\frac{(1-p)p}{(1-2p)^2}}$. Thus, the slope of $f(x)$ approaches a constant as $x \to \infty$. Then we know there must exist an oblique asymptote of the form $y = mx + b$ where $m = 2\sqrt{\frac{(1-p)p}{(1-2p)^2}}$. To find b, we must find the value of b such that $\lim_{x\to\infty} f(x) - (mx + b) = 0$. This yields $b = \frac{1}{2}$. Thus, the oblique asymptote is $y = \text{sgn}(x) \cdot 2\sqrt{\frac{(1-p)p}{(1-2p)^2}}x + \frac{1}{2}$. $\square$